Astrophysical Constraints on the Nuclear Matter Equation of State

Helmholtz International Summerschool

Nuclear Theory and Astrophysical Applications

Dubna, 07. - 17.08 2007

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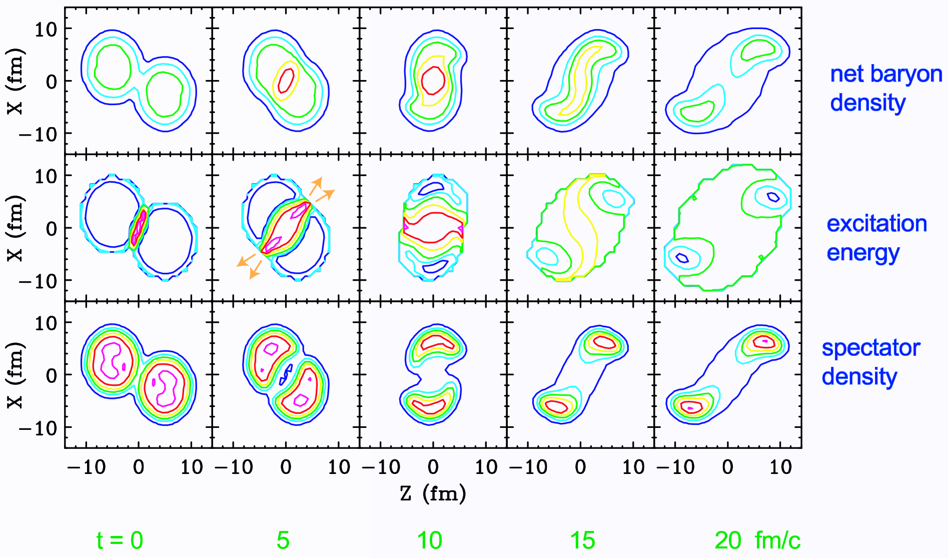
Part 1: Neutron Stars - Masses and Radii

Part 2: Consistence with modern Equations of State (Thursday)

Exploring the Limits - Introduction

On Earth

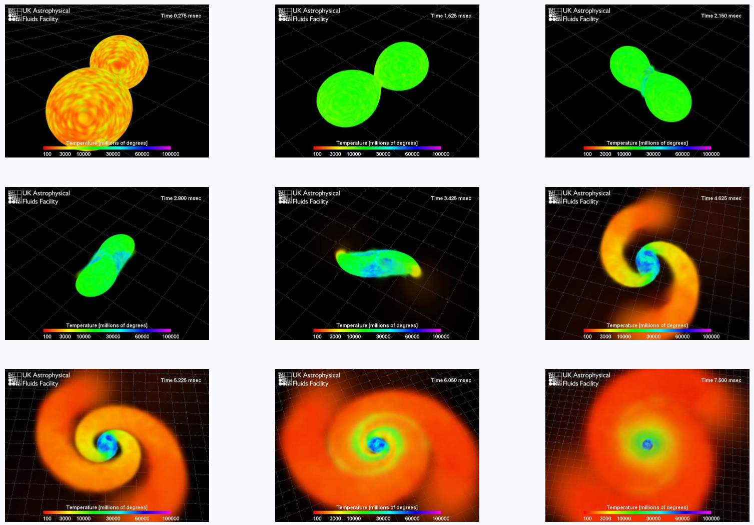


L. Shi et al., PRC **64** (2001) 034601

Colliding ^{124}Sn-Ions

- 10 fm, $2.5 \times 10^2 \mathrm{m_n}$
- up to $4\mathrm{n}_0$
- n, p^+, e^-, μ^-

In Universe



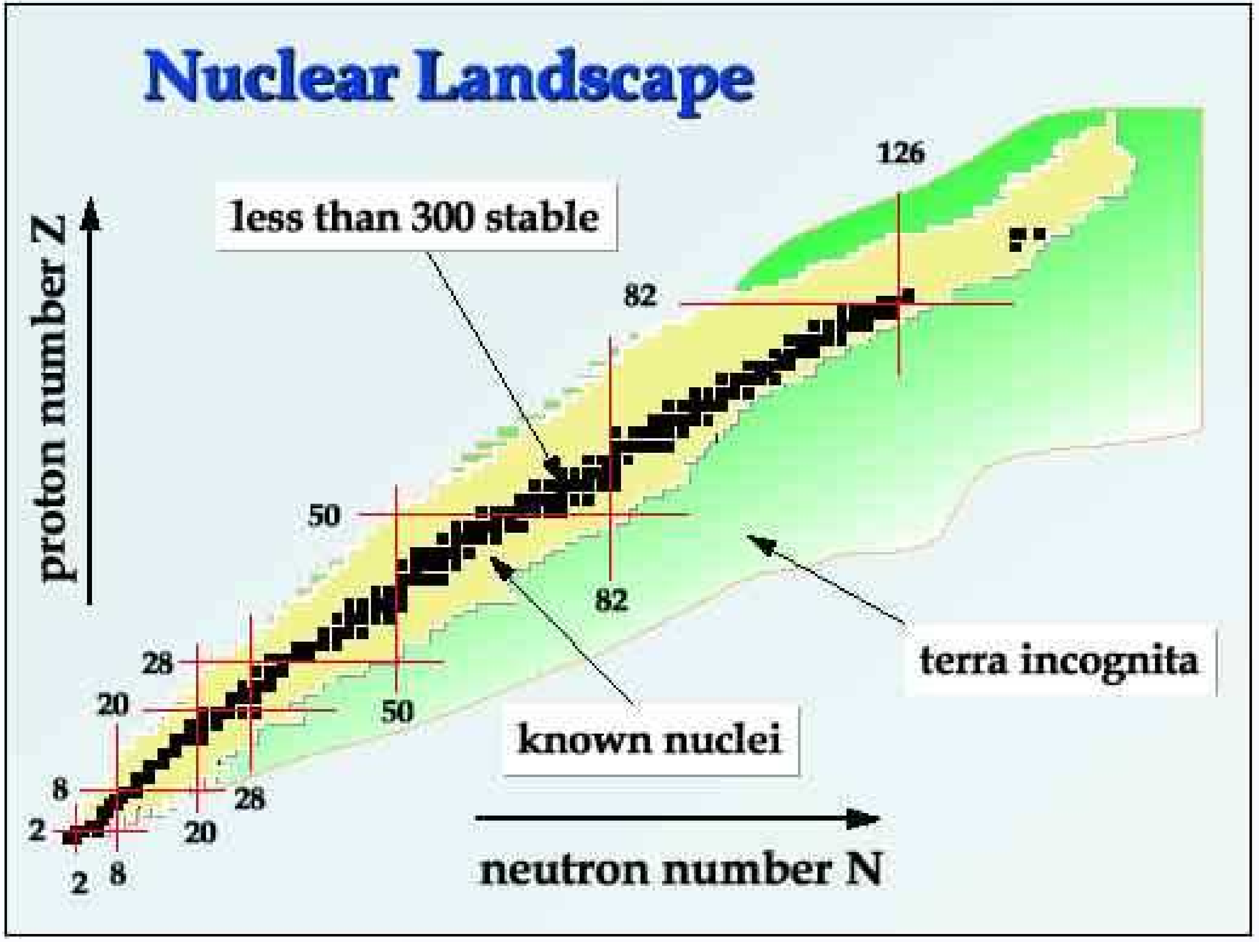
S. Rosswog and R. West,
http://www.ukaff.ac.uk/movies/nsmerger/

Merging Neutron Stars

- 10 km = 10^{19} fm, $10^{57}\mathrm{m_n}$
- up to $9\mathrm{n}_0$
- n, p^+, e^-, μ^-

Same constituents, similar densities, 'just' different scales!

Nuclear Theory : Understanding the Nuclear Landscape



Origin of elements (nucleosynthesis)?

How to describe masses, energy levels, shapes, driplines?

How general is our understanding?

[1] http://www.phy.anl.gov/ria/index.html

The Bethe-Weizsäcker Mass Formula

Semiempirical mass formula by Weizsäcker (1935)

also by Bethe and Bacher (1936)

$E(N,Z) =$	$a_{vol} A + a_{sf} A^{2/3}$	bulk and surface
	$+ \left(a_{sym} A + a_{ss} A^{2/3}\right) \delta^2$	asymmetric contributions
	$+ \frac{3e^2}{5r_0} Z^2 A^{-1/3}$	Coulomb interaction
	$+ \Delta_n + \Delta_p$	pairing

$\delta = \frac{N-Z}{N+Z} = 1 - 2\frac{Z}{A} = 1 - 2x_p$

$\Delta_{n,p} = \pm\Delta$ for $\frac{\text{even}}{\text{odd}}$ N,Z

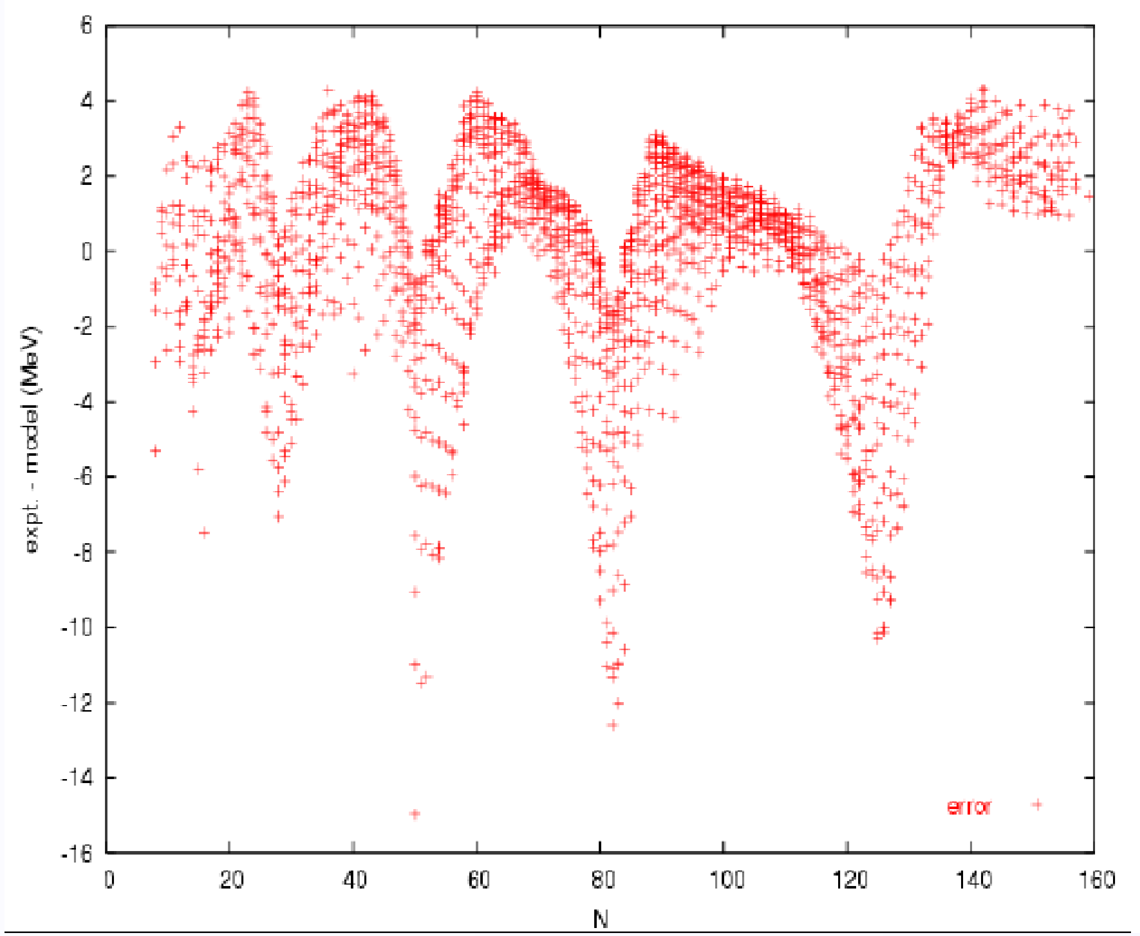
Remark: a_{ss} introduced by Myers and Swiatecki (1966)

The Bethe-Weizsäcker Mass Formula

For N,Z greater than 8 *2149* atomic masses have been measured (until 2003)

Fitting the six parameters of the Weizsäcker Mass Formula yields:

$$
\begin{aligned}
a_{vol} &= -15.70 \text{ MeV} \\
a_{sf} &= 17.66 \text{ MeV} \\
a_{sym} &= 26.31 \text{ MeV} \\
a_{ss} &= -17.00 \text{ MeV} \\
r_0 &= 1.22 \text{ fm} \\
\Delta &= -1.25 \text{ MeV}
\end{aligned}
$$



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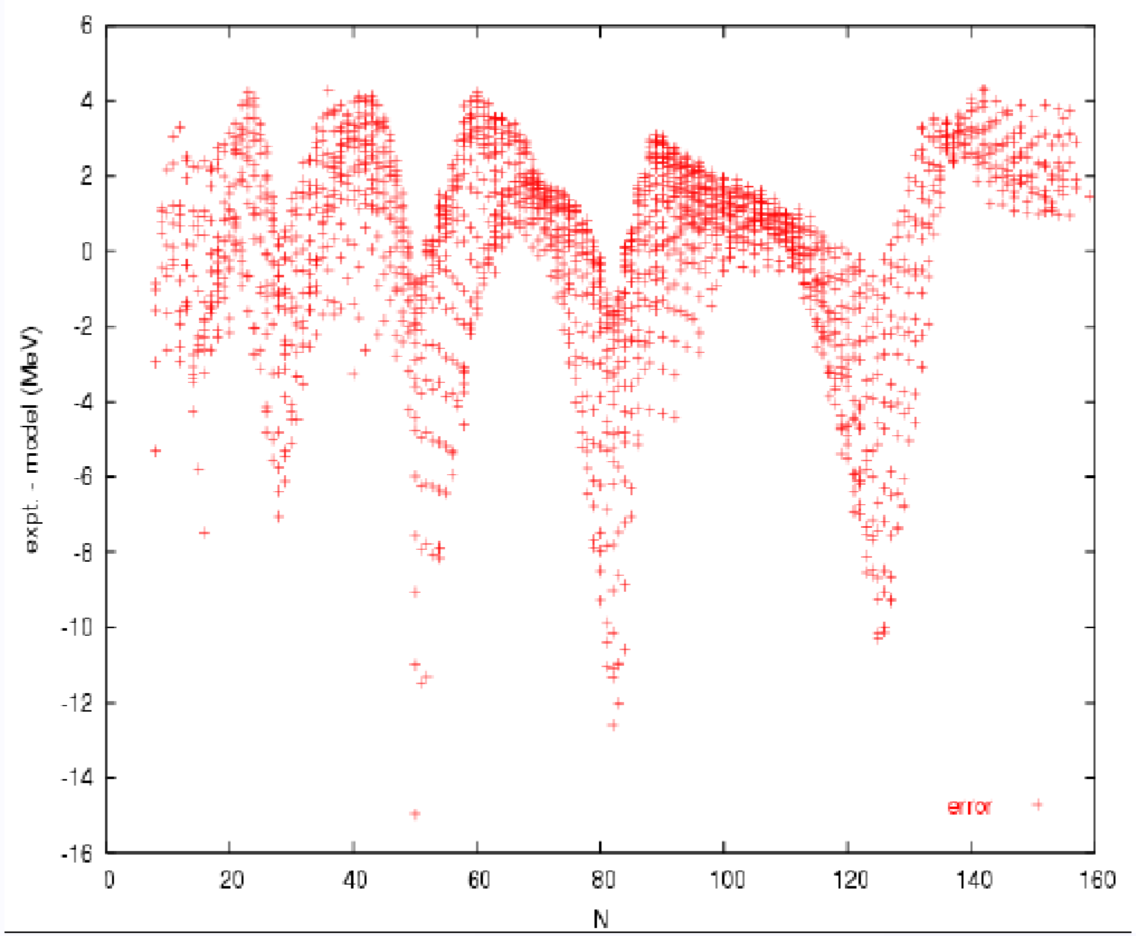

What about large A? (something like A = 10^{57})

The Bethe-Weizsäcker Mass Formula

Large A Limit

Large nuclei, $A \to \infty$

Energy per nucleon:

$$\frac{E}{A} = a_{vol} + a_{sf} A^{-1/3} + \left(a_{sym} + a_{ss} A^{-1/3}\right) \delta^2 + \frac{3e^2}{5r_0}(1-\delta)^2 A^{2/3} + (\Delta_n + \Delta_p) A^{-1}$$

Coulomb term diverges, unless $\delta = 1$ (pure neutron matter!)

$\Rightarrow \frac{E}{A}(Z = 0, N \to \infty) = a_{vol} + a_{sym} = +10.6$ MeV

Large neutron systems are unbound...

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Unbound? What about Neutron Stars???

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Large neutron systems are unbound...

Unbound? What about Neutron Stars??? $\to$ bound by gravity

$E_G = -\frac{3GM^2}{5R} = -\frac{3GM^2}{5r_0} A^{5/3}$ (compare to Coulomb!)

The Bethe-Weizsäcker Mass Formula

All ingredients for our first Neutron Star

$\frac{E}{A} = a_{vol} + a_{sym} - \frac{3GM^2}{5r_0} A^{2/3}$ (still assume that $\delta = 1$)

System is bound for $\frac{E}{A} < 0$. This holds for

$A > \left[\frac{5r_0}{3GM^2} (a_{vol} + a_{sym})\right]^{3/2} \approx 0.79 \times 10^{56}$ (very small NS of $\approx 0.1 M_\odot$)

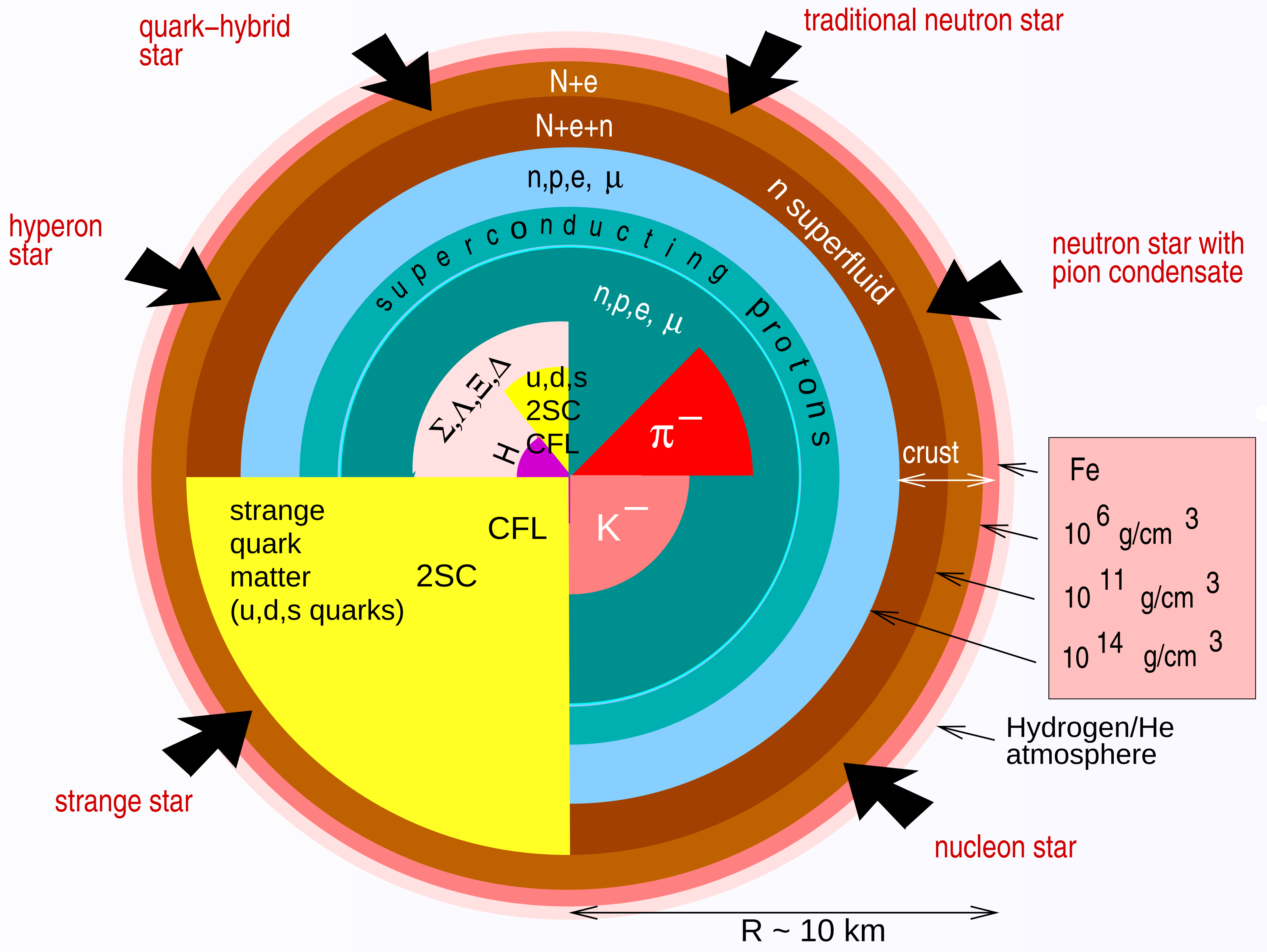
Many things need to be (and have been) done better:

- general relativity
- proton fraction x_p in realistic models is small (<20%) but finite
- densities beyond saturation $n_0 = 0.16$ fm^{-3}
- more exotic states than NM are possible (Kaons, Quark Matter)

Neutron stars are laboratories for the equation of state over a wide range of densities and asymmetries.

There is no equivalent on earth!

Neutron Stars - Different Scenarios



(F. Weber, arXiv:astro-ph/0705.2708v2)

Tolman Oppenheimer Volkov Equations

Space, time and matter are related via Einsteins Field Equations

$$G_{\mu\nu} = -8\pi G T_{\mu\nu}$$

Einstein Tensor $G_{\mu\nu}$ — defined by metric

Energy Momentum Tensor $T_{\mu\nu}$ — defined by equation of state

Approximations

non rotating, spheric symmetry

$$ds^2 = g_{\mu\nu} dx^\mu dx^\mu$$

$$\rightarrow g_{00}(r)dt^2 + g_{11}(r)dr^2 + g_{22}(r)d\theta^2 + g_{33}(r,\theta)d\phi^2$$

hydrostatic equilibrium

$$-pg^{\mu\nu} + (p+\varepsilon)u^\mu u^\nu$$

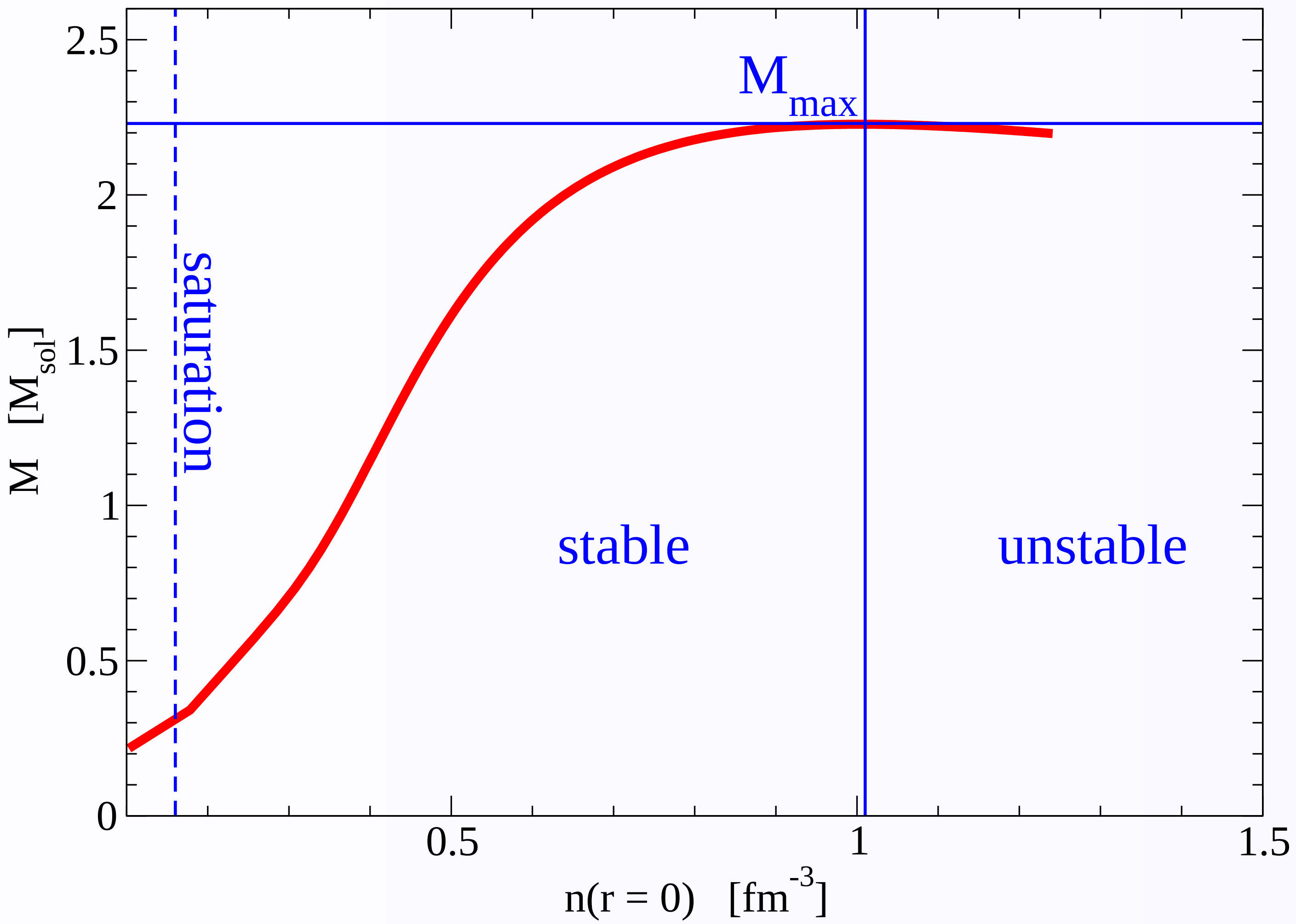
Tolman-Oppenheimer-Volkov (TOV) Equations (1939)

$$\frac{dp(r)}{dr} = -\frac{Gm(r)\varepsilon(r)}{r^2}\left(1+\frac{p(r)}{\varepsilon(r)}\right)\left(1+\frac{4\pi r^3 p(r)}{m(r)}\right)\left(1-\frac{2Gm(r)}{r}\right)^{-1}$$

$$m(r) = 4\pi\int_0^r dr'\, r'^2 \varepsilon(r')$$

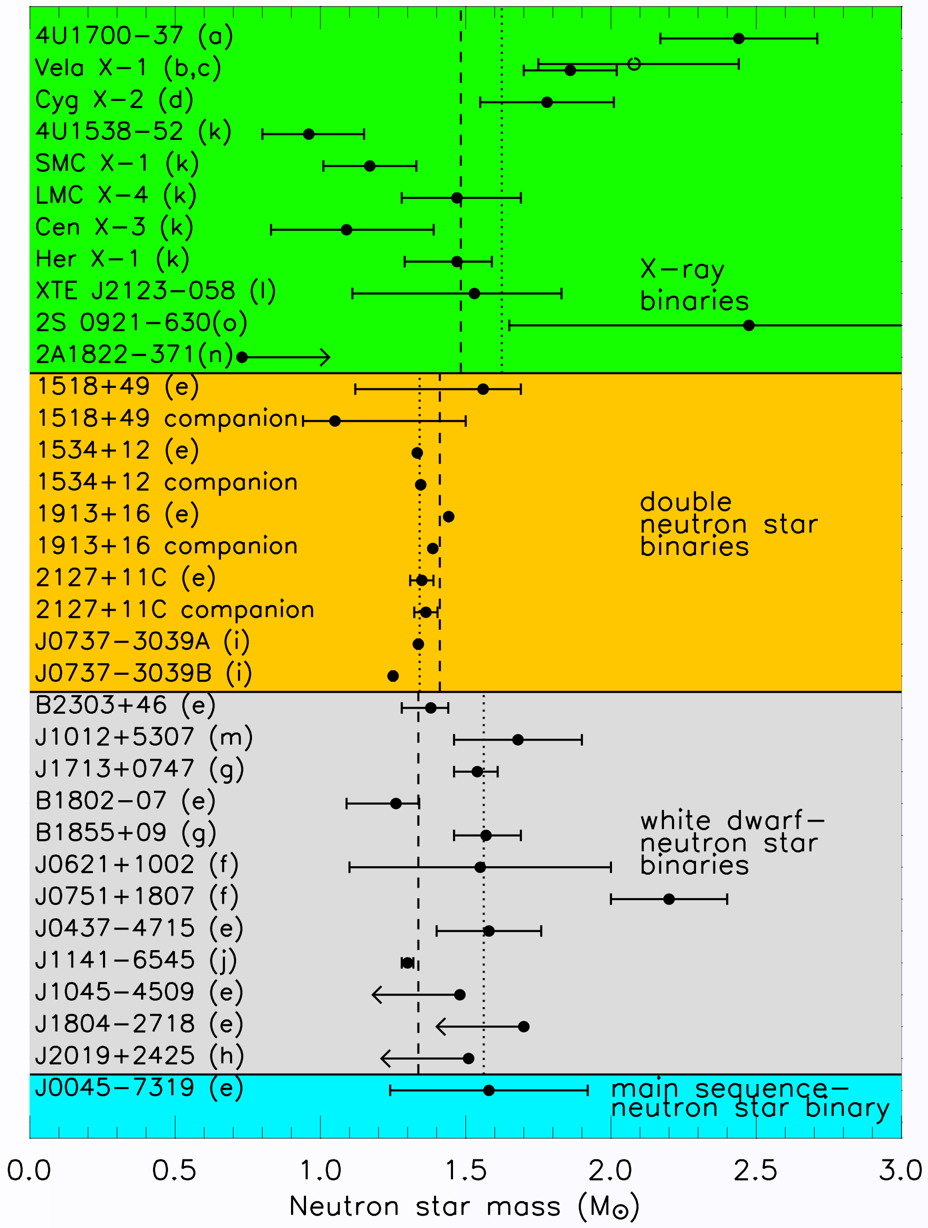
Neutron Star Masses

Solving the TOV-equations gives masses and radii as function of central density.



Question: Is the maximum neutron star mass sensitive to the EoS?

Neutron Star Masses



Neutron Star Masses have been measured for binary objects only

$\longrightarrow$ Keplerian Orbits

too few observables

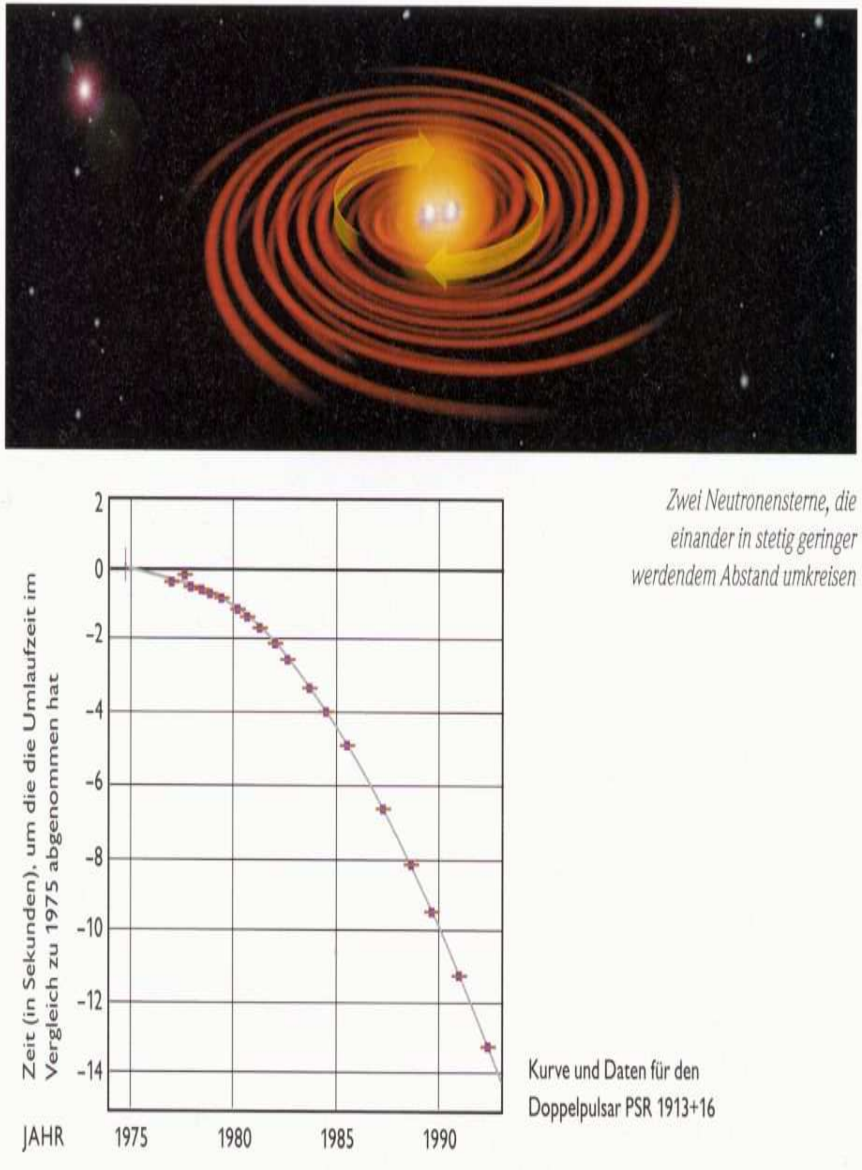
$\longrightarrow$ Relativistic Corrections

evolution of observables

Fig: J. M. Lattimer and M. Prakash, PRL **94**, 111101 (2005)

The Hulse-Taylor pulsar (PSR B1913+16)

Discovered in 1974 (ApJ Lett, January 15, 1975), Nobel Prize in 1993



spin period

$$P_s = 59 \text{ ms}$$

orbital period

$$P = 7.75 \text{ h}$$

measured decrease

$$dP/dt = -(2.4086 \pm 0.0052) \times 10^{-12}$$

predicted by GR

$$dP/dt = -(2.40247 \pm 0.00002) \times 10^{-12}$$

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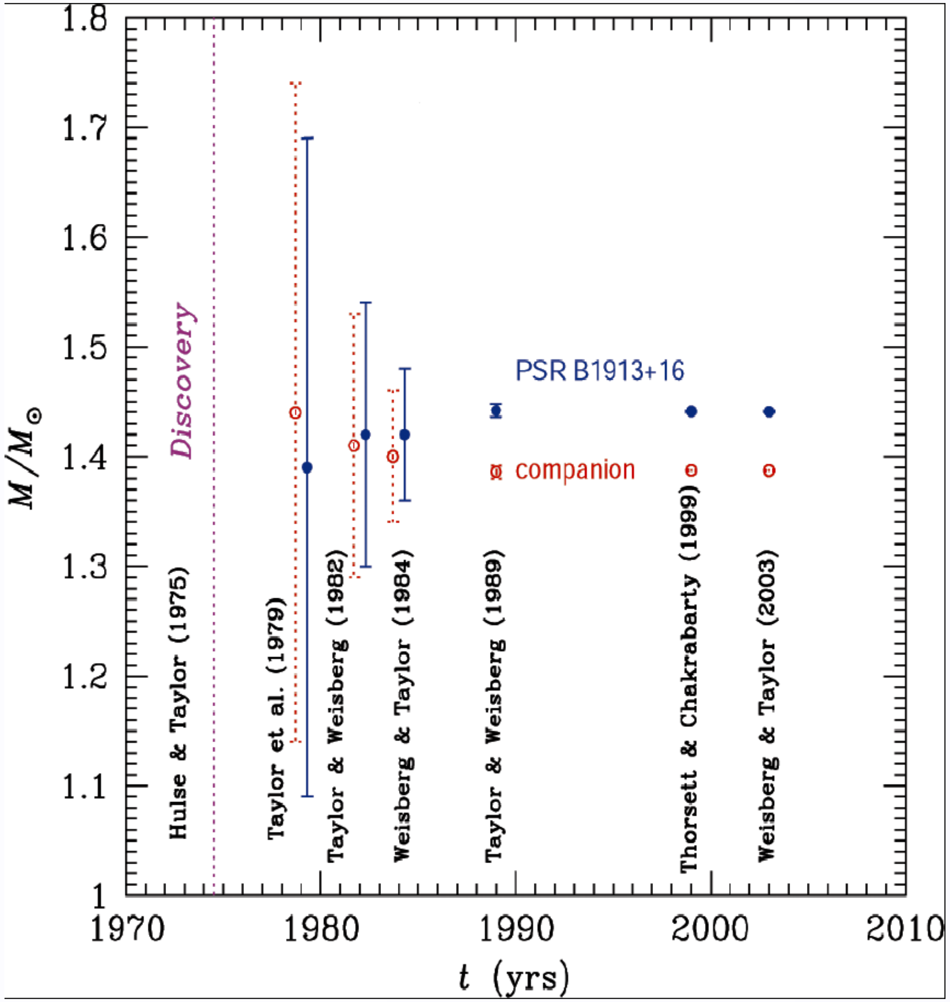


$$M_1 = (1.4408 \pm 0.0006) M_\odot$$
$$M_2 = (1.3873 \pm 0.0006) M_\odot$$

Accuracy increased
and still does.
Good perspectives for
other objects,
just by waiting.

Neutron Star Radii

It is obviously difficult to measure the radius of an object with $R \approx 10 \ldots 15$ km at distances of kpc.

Again: Neutron Stars are relativistic objects.

redshift $z = \left(1 - \frac{2GM}{R}\right)^{-1/2} - 1$

$\rightarrow$ gravitational light bending (van Kerkwijk et al. (1995))

Her X-1: $z = 0.247 - 0.268$, $M = (1.29 - 1.59) M_\odot \Rightarrow 10.1 < R/km < 13.1$

$\rightarrow$ redshifted resonance scattering lines (Cottam et al.(2002), Özel (2006))

EXO 0748: Fe XXV and XXVI observed once $\Rightarrow z \approx 0.345$

Measurement of $R_\infty = R(1+z)$ would fix both M and R

However, usually measured quantities are related to R/D

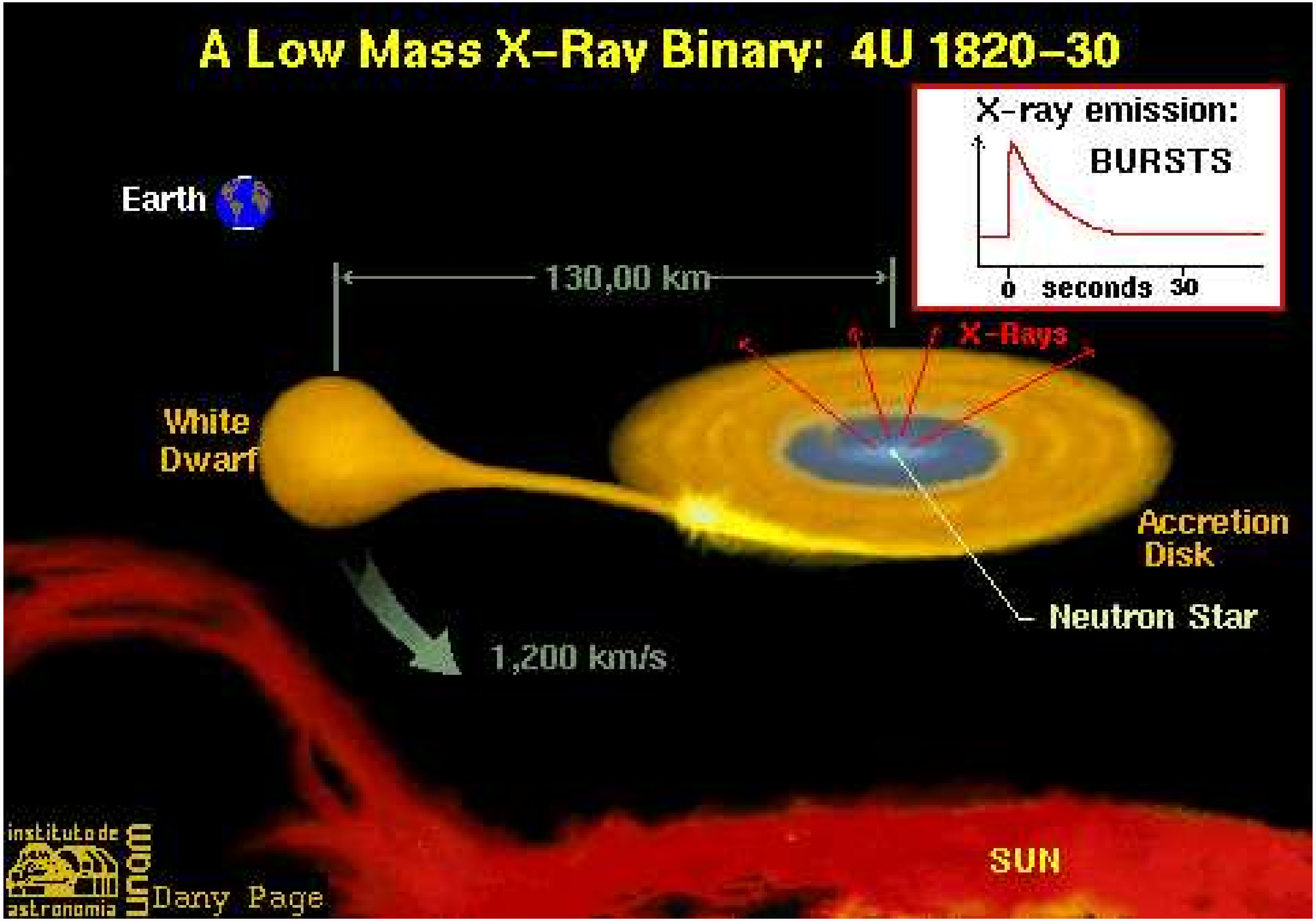
3 Unknowns$\rightarrow$3 Observables

$$F_{edd} = \frac{GM}{D^2 \kappa_{es}} \left(1 - \frac{2GM}{R}\right)^{1/2}$$ maximum flux during bursts

$$F_{cool}/\sigma T_c^4 = f_\infty^2 \frac{R^2}{D^2} \left(1 - \frac{2GM}{R}\right)^{-1}$$ cooling flux (between bursts)

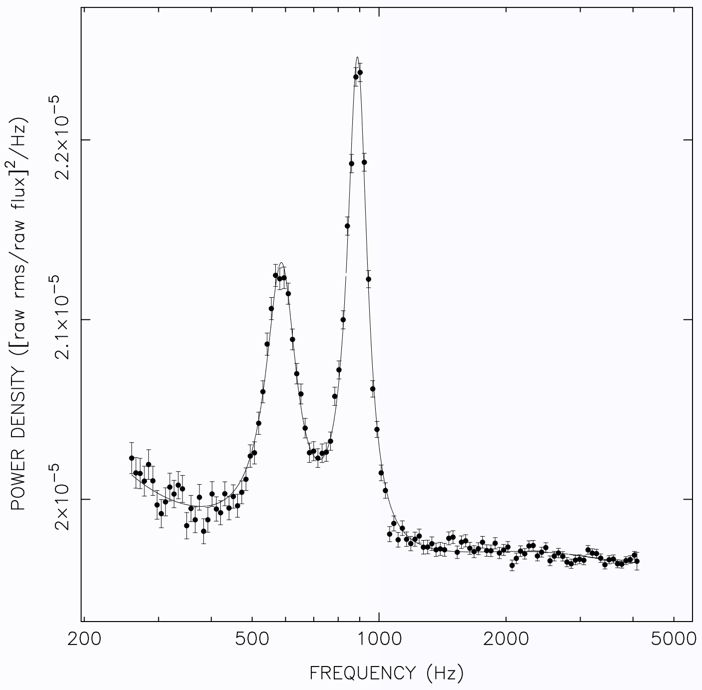
$$z = \left(1 - \frac{2GM}{R}\right)^{-1/2} - 1$$

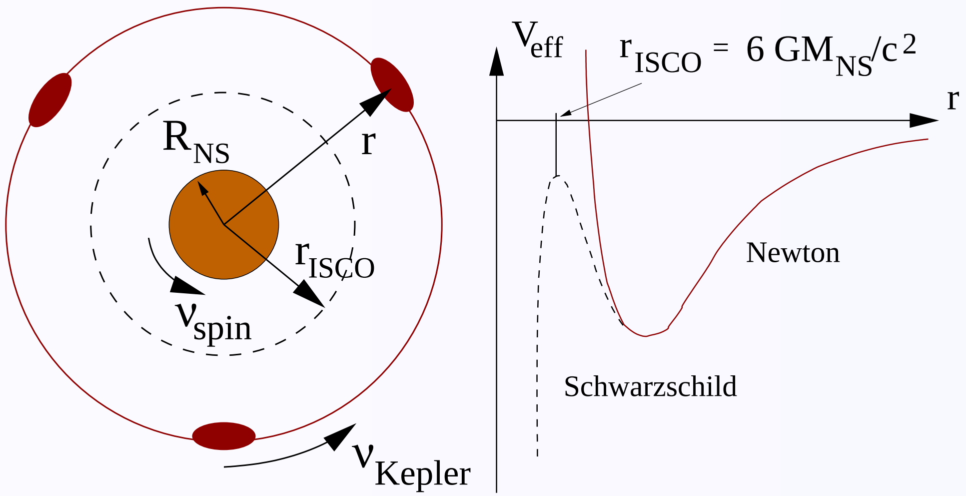
Low Mass X-Ray Binaries (LMXBs)



Mass-Radius Constraints from QPO's

Quasi Periodic Brightness Oscillations





$\nu_{max} \approx \nu_{orbit} < \nu_{ISCO}$

Keplerian Orbit r_K

$$R < r_k = (GM/4\pi^2\nu^2_{max})^{1/3} \rightarrow R_{max}(M)$$

$$M < 2.2M_\odot(1000Hz/\nu_{max})(1+0.75j) \rightarrow M_{max}$$

if(!) $\nu_{max} \approx \nu_{ISCO}$

$$M \approx 2.2M_\odot(1000Hz/\nu_{max})(1+0.75j)$$

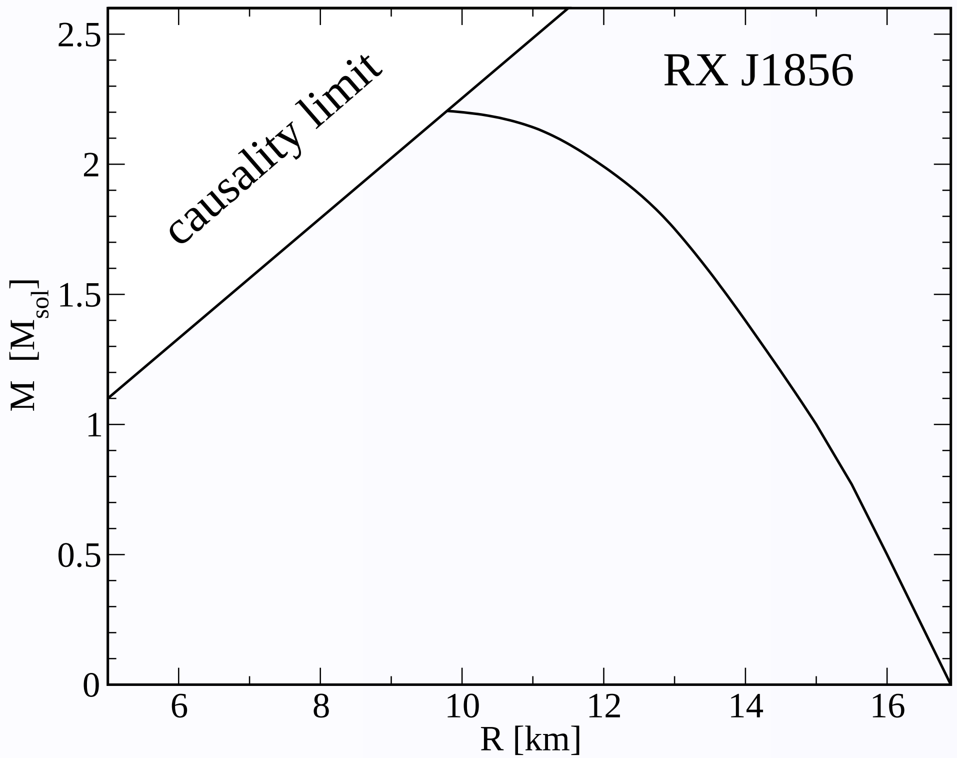
M. van der Klies, ARA&A **38**, 717 (2000)

M-R Constraint from Radio Quiet Isolated NS RXJ1856

RXJ1856 black body spectrum: $T_\infty = 57$ eV

measurement of distance: 60 pc (2002) $\rightarrow$ 117 pc (2004)

$\rightarrow$ photospheric radius: $R_\infty = R(1 - R/R_S)^{-1/2}$ $\quad R_S = 2GM$



J. Trümper et al., Nucl. Phys. Proc. Suppl. 132, 560 (2004)

D. Barret, J.-F. Olive, M.C. Miller, Mon. Not. Roy. Astron. Soc. 361, 855 (2005)

Gravitational Mass $\leftrightarrow$ Baryon Number J0737-3039

Double Pulsar System J0737-3039

Pulsar A $P^{(A)}$ = 22.7 ms, $M^{(A)} \approx 1.338 M_\odot$

Pulsar B $P^{(B)}$ = 2.77 s, $M^{(B)} = 1.249 \pm 0.001 M_\odot$ (record!)

Progenitor ONeMg white dwarf, driven hydrodyn. unstable by

e^- captures on Mg & Ne; no mass-loss during collapse

Observational constraint for $M(M_N)$ from PSR J0737-3039:

- observed NSs gravitational mass (remnant star) $M^{(B)} = (1.248 - 1.250) M_\odot$
- critical baryon mass for ONeMg white dwarf $M_N^{(B)} = (1.366 - 1.375) M_\odot$

Theory: $M(M_N)$ characteristic for remnants EoS

$$M = 4\pi \int_0^R \mathrm{d}r\, r^2 \varepsilon(r) \;;$$

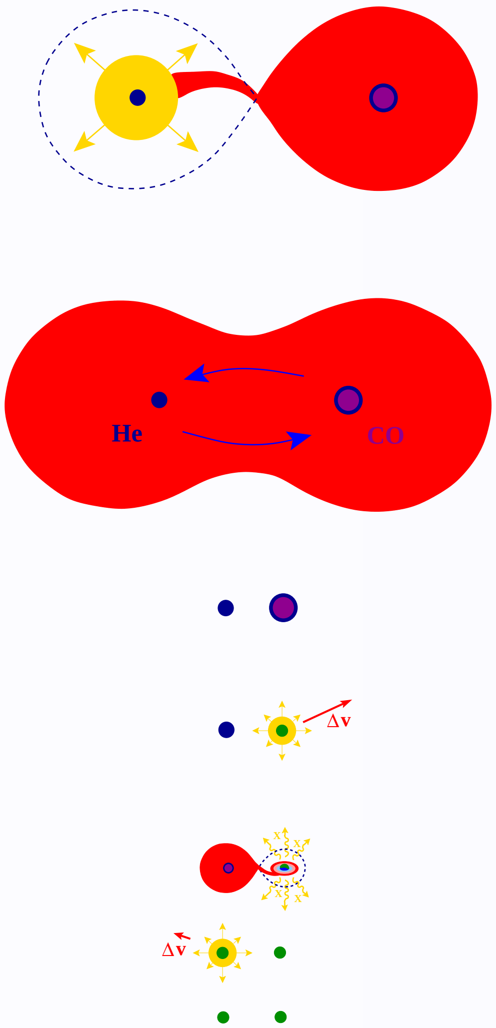
$$M_N = u N_B = 4\pi u \int_0^R \mathrm{d}r \frac{r^2 n(r)}{\sqrt{1 - 2GM(r)/r}}$$

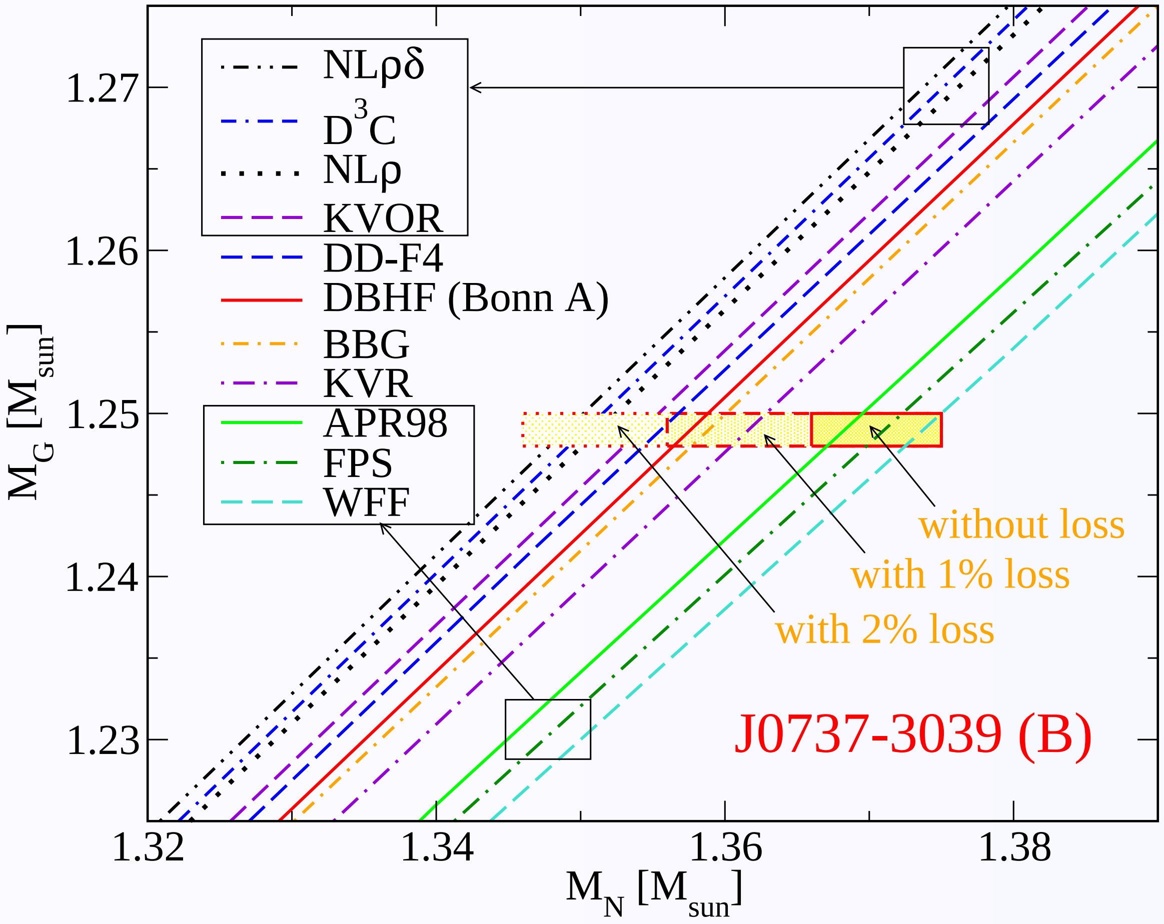
(conversion of baryon number to mass by $u = 931.5$ MeV)

P. Podsiadlowski et al., Mon. Not. Roy. Astron. Soc. **361**, 1243 (2005)

Gravitational vs. Baryon Mass

Double core scenario:



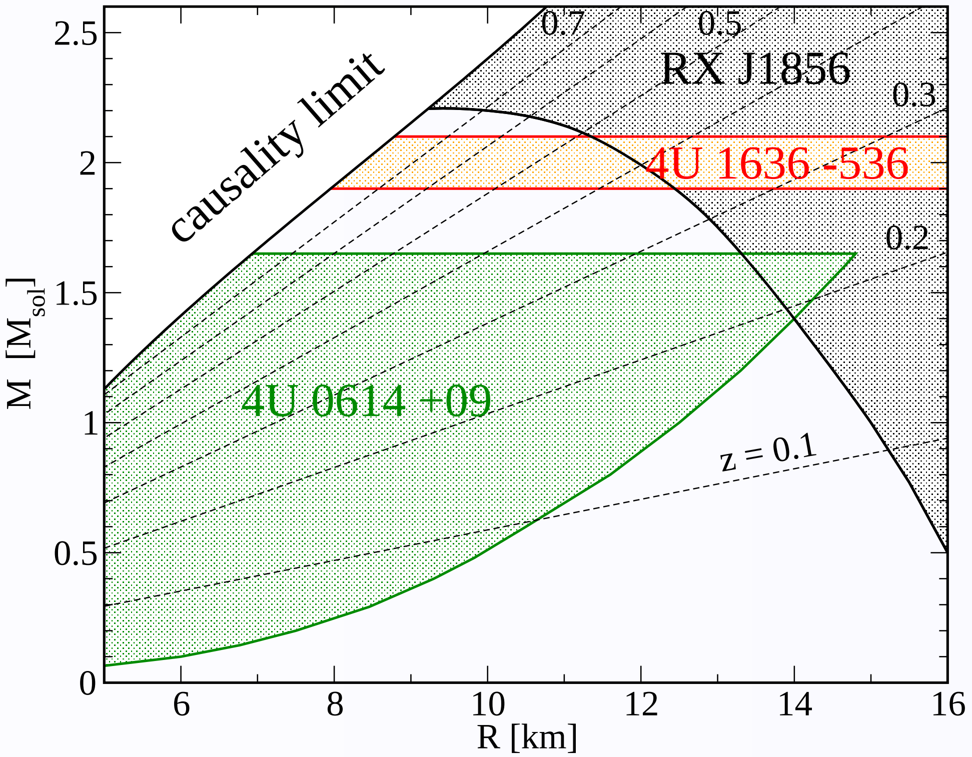


Podsiadlowski et al., MNRAS 361 (2005) 1243

Dewi et al., MNRAS (2006)

David Blaschke, T. K., F. Weber, CBM Theory Book (2007)

Summary on M-R-Constraints



Mass Radius Constraints

QPO : M-R upper limits

ISCO : max. mass constraint

redshift z: compactness

RXJ1856: M-R lower limits

each region...

→ represents a different object

→ should be touched at least once

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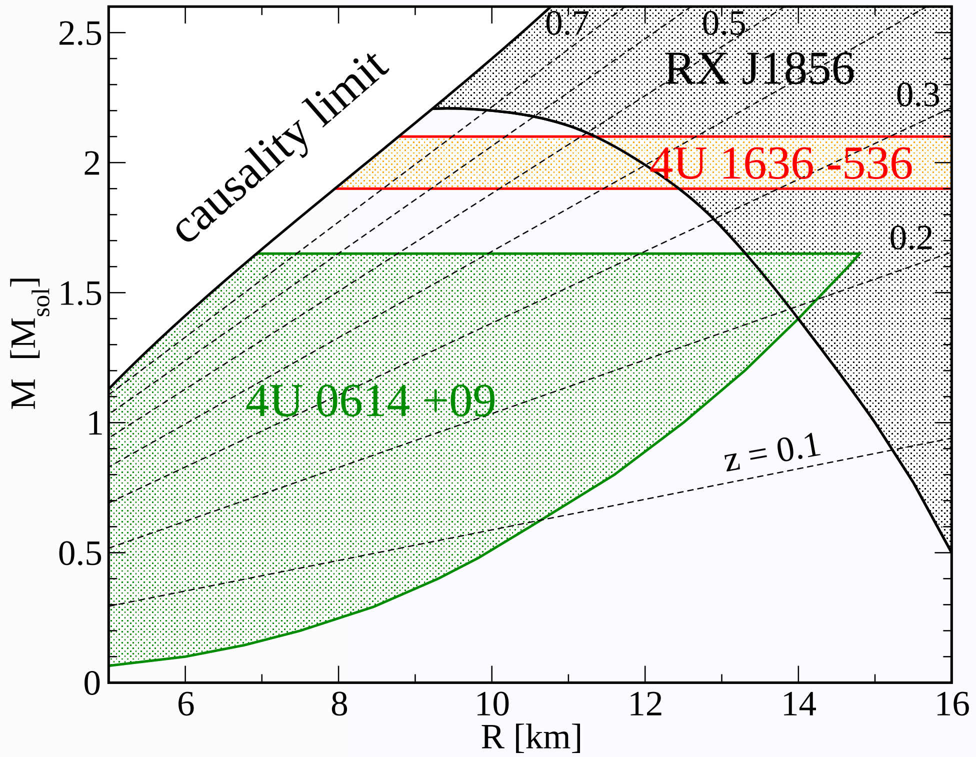

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Consequences from these (and other) astrophysical constraints on modern EosS.

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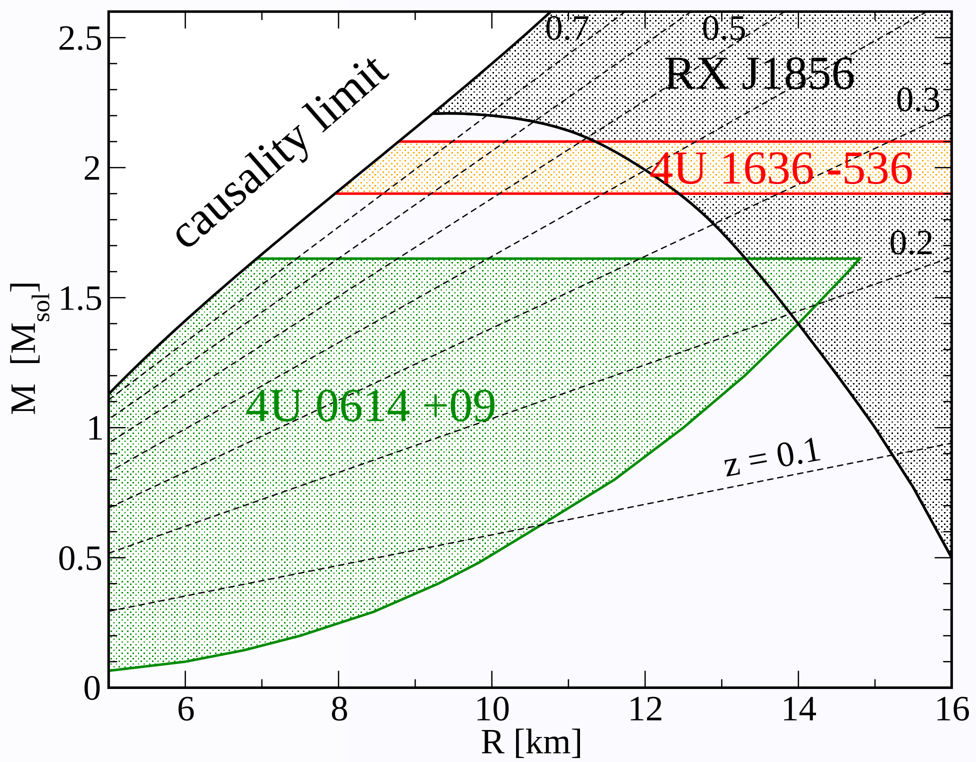

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