Mean Field Methods for Nuclear Structure

Part 1: Hartree-Fock and Hartree-Fock-Bogoliubov for Ground States

Part 2: RPA and QRPA for Excitations

Outline of part 1

- - Introduction
- - Energy density functional
- - Hartree-Fock (HF) and Hartree-Fock-Bogoliubov (HFB)
- - Quasiparticle continuum: exact and approximate treatments
- - Illustrative examples
- - Summary

Microscopic approaches to many-body, finite nuclear systems

- Theoretical models based on effective interactions between nucleons:
- - Nuclear shell model
- - Mean field approaches (and beyond)
- - molecular dynamics

- * going away from stability regions, we need a theoretical framework which can be predictive and able to handle new situations (continuum, pairing correlations in continuum).
- * the Hartree-Fock-Bogoliubov + Quasiparticle Random Phase Approximation can be used from unstable nuclei to neutron star crust.

Hartree-Fock and HF-Bogoliubov

$$\left|HF\right\rangle \doteq a_{i1}^{+} a_{i2}^{+} \ldots\ldots a_{iA}^{+} \left|0\right\rangle$$

$$\left|HFB\right\rangle = \prod_{i} a_i^{\dagger} \prod_{p} \left(u_p + v_p a_p^{\dagger} a_{p'}^{\dagger}\right) \left|0\right\rangle$$

$$\beta_k^{\dagger} = \sum_i \left(u_{ik} a_i^{\dagger} + v_{ik} a_i\right) \qquad \beta_k \left|HFB\right\rangle = 0$$

Energy Density Functional in Hartree-Fock

$$H = \sum_i^A -\frac{\hbar^2}{2m}\nabla_i^2 + \sum_{i<j}^A V_{ij}$$

$$E \equiv \int \mathcal{E} d^3r$$

$$= \sum_{\alpha\beta} \langle \varphi_\alpha | -\frac{\hbar^2}{2m}\nabla^2 | \varphi_\beta \rangle \rho_{\alpha\beta} + \frac{1}{2}\sum_{\alpha\beta\gamma\delta} \langle \varphi_\alpha \varphi_\beta | V_{12}(1 - P_{12}) | \varphi_\gamma \varphi_\delta \rangle \rho_{\alpha\gamma}\rho_{\delta\beta}$$

Effective Interactions

particle-hole channel:
Skyrme interaction

$$
\begin{aligned}
V_{12} &= t_0(1+x_0P^\sigma)\delta(\mathbf{r}) \\
&+ \frac{1}{2}t_1(1+x_1P^\sigma)(\mathbf{P}'^2\delta(\mathbf{r})+\delta(\mathbf{r})\mathbf{P}^2) \\
&+ t_2(1+x_2P^\sigma)(\mathbf{P}'\cdot\delta(\mathbf{r})\mathbf{P}) \\
&+ iW_0\sigma\cdot(\mathbf{P}'\times\delta(\mathbf{r})\mathbf{P}) \\
&+ \frac{1}{6}t_3(1+x_3P^\sigma)\rho^\alpha(\mathbf{R})\delta(\mathbf{r}) \ .
\end{aligned}
$$

particle-particle channel:
zero-range

$$V_{pp} = V_0\left[1-\eta\left(\frac{\rho}{\rho_0}\right)^\alpha\right]\delta(\mathbf{r}_1-\mathbf{r}_2)$$

Densities

- Normal density, or density matrix

$$\rho_{ij} = \langle \Phi | a_j^\dagger a_i | \Phi \rangle \qquad \rho^\dagger = \rho$$

- Abnormal density, or pairing tensor

$$\kappa_{ij} = \langle \Phi | a_j a_i | \Phi \rangle \qquad \kappa^T = -\kappa$$

One-body densities in Hartree-Fock

$$\rho_q(\mathbf{r}) = \sum_{i,s} |\varphi_i^q(\mathbf{r},s)|^2 n_i^q \ , \qquad \rho = \rho_n + \rho_p \ ,$$

$$\tau_q(\mathbf{r}) = \sum_{i,s} |\nabla\varphi_i^q(\mathbf{r},s)|^2 n_i^q \ , \qquad \tau = \tau_n + \tau_p \ ,$$

$$\begin{aligned} \mathbf{J}_q(\mathbf{r}) &= \sum_{i,s,s'} \varphi_i^{q*}(\mathbf{r},s')\nabla\varphi_i^q(\mathbf{r},s) \times (s'|\sigma|s) n_i^q \ , \\ \mathbf{J} &= \mathbf{J}_n + \mathbf{J}_p \ . \end{aligned}$$

Generalisation to Hartree-Fock-Bogoliubov

$$\Phi_i(nljm, \mathbf{r}, \sigma) = R_i(nlj, r)\frac{1}{r}\mathcal{Y}_{lj}^{m}(\hat{\mathbf{r}}, \sigma) \ , \ i = 1, 2 \ ,$$

where:

$$\mathcal{Y}_{lj}^{m}(\hat{\mathbf{r}}, \sigma) \equiv \sum_{m_l \, m_\sigma} Y_{lm_l}(\theta, \phi)\chi_{1/2}(m_\sigma)(lm_l\frac{1}{2}m_\sigma|jm) \ .$$

In what follows we use for the upper and lower component of the radial wave functions the standard notation $U_{nlj}(r)$ and $V_{nlj}(r)$.

HFB densities in spherical case

- Nuclear density

$$\rho(r) = \frac{1}{4\pi}\sum_i (2j_i+1) V_i^*(r) V_i(r)$$

- Abnormal (or pairing) density

$$\kappa(r) = \frac{1}{4\pi}\sum_i (2j_i+1) U_i^*(r) V_i(r)$$

- Kinetic energy density

$$\tau(r) = \frac{1}{4\pi}\sum_i (2j_i+1)[(\frac{dV_i}{dr} - \frac{V_i}{r})^2 + \frac{l_i(l_i+1)}{r^2} V_i^2]$$

- Spin density

$$J(r) = \frac{1}{4\pi}\sum_i (2j_i+1)[j_i(j_i+1) - l_i(l_i+1) - \frac{3}{4}]\, V_i^2$$

The Hartree-Fock-Bogoliubov Equations

$$\begin{pmatrix} U_\alpha(\mathbf{r}) \\ V_\alpha(\mathbf{r}) \end{pmatrix} = \frac{1}{r} \begin{pmatrix} u_\alpha(r) \\ v_\alpha(r) \end{pmatrix} \mathbf{Y}_\alpha(\hat{r}, \sigma)$$

$$\begin{pmatrix} h(r) - \lambda & \Delta(r) \\ \Delta(r) & -h(r) + \lambda \end{pmatrix} \begin{pmatrix} U_i(r) \\ V_i(r) \end{pmatrix} = E_i \begin{pmatrix} U_i(r) \\ V_i(r) \end{pmatrix},$$

Hartree-Fock field and pairing field

$$h(r) = h(\rho, \tau, \mathbf{J})$$

$$\Delta(r) = V_0[1 - \eta(\frac{\rho}{\rho_0})^\alpha]\kappa(r).$$

Finite-Temperature HFB

$$\begin{pmatrix} h_T(r)-\lambda & \Delta_T(r) \\ \Delta_T(r) & -h_T(r)+\lambda \end{pmatrix} \begin{pmatrix} U_i(r) \\ V_i(r) \end{pmatrix} = E_i \begin{pmatrix} U_i(r) \\ V_i(r) \end{pmatrix}$$

$$\rho_T(r) = \frac{1}{4\pi}\sum (2j_i+1)[V_i^*(r)V_i(r)(1-f_i)+U_i^*(r)U_i(r)f_i]$$

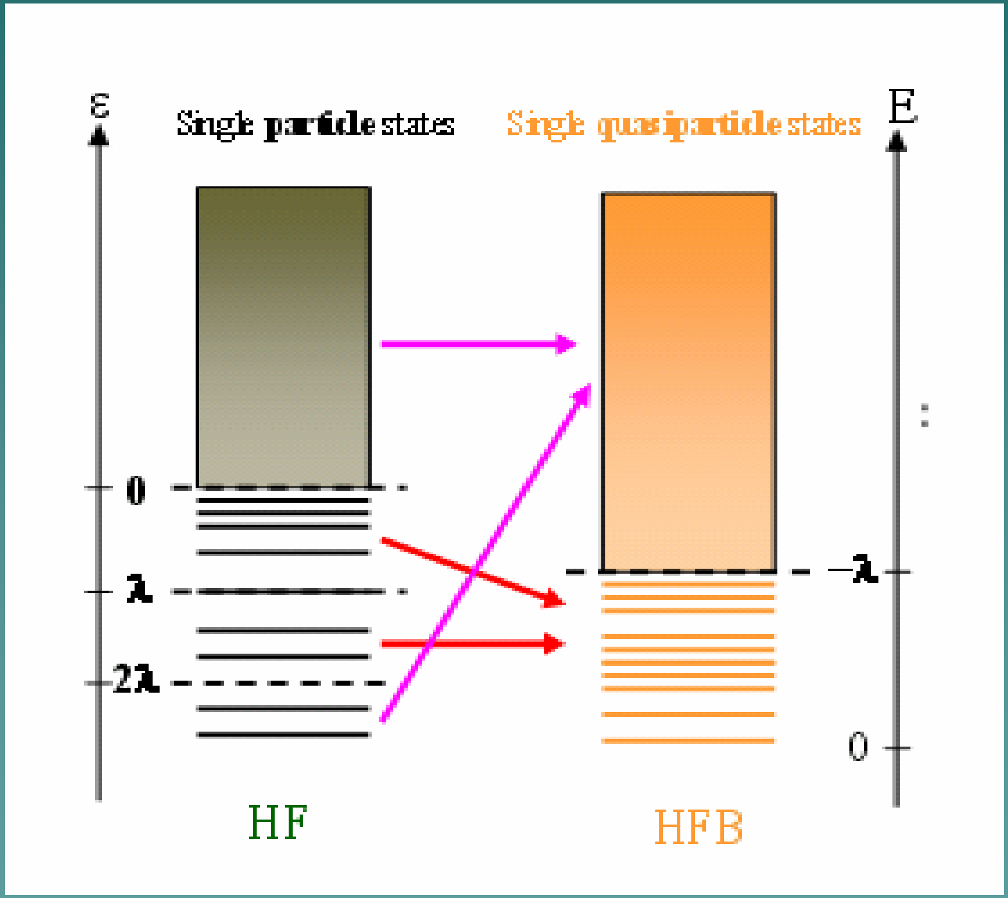
$$\Delta_T(r) = V_{pair}\,\kappa_T(r)$$

$$\kappa_T = \frac{1}{4\pi}\sum (2j_i+1)U_i^*(r)V_i(r)(1-2f_i)$$

where : $f_i = (1+e^{E_i/kT})^{-1}$

N.S, Phys.Rev.**C70** (2004) 025801

Quasiparticle continuum



Treatment of quasiparticle continuum (1)

The asymptotic behaviour of the HFB wave function is determined by the physical condition that at large distances the nuclear mean field $\Gamma(r)$ and the pairing field $\Delta(r)$ vanish.

In the asymptotic region the equations for U_{nlj} and V_{nlj} are decoupled and one can easily see how the physical solutions must behave at infinity.

For a bound system ($\lambda < 0$) there are two well separated regions in the quasiparticle spectrum:

- Between 0 and $-\lambda$ the quasiparticle spectrum is discrete and both upper and lower components of the radial HFB wave function decay exponentially at infinity:

$$\begin{aligned} U_{lj}(E,r) &= A h_l^{(+)}(\alpha_1 r) \ , \\ V_{lj}(E,r) &= B h_l^{(+)}(\beta_1 r) \ , \end{aligned}$$

where $\alpha_1^2 = \frac{2m}{\hbar^2}(\lambda + E)$, $\beta_1^2 = \frac{2m}{\hbar^2}(\lambda - E)$. These solutions correspond to the bound quasiparticle spectrum.

Treatment of quasiparticle continuum (2)

- For $E > -\lambda$ the spectrum is continuous and the solutions are:

$$U_{lj}(E,r) = C[cos(\delta_{lj})j_l(\alpha_1 r) - sin(\delta_{lj})n_l(\alpha_1 r)] ,$$

$$V_{lj}(E,r) = D_1 h_l^{(+)}(\beta_1 r) ,$$

Treatment of quasiparticle continuum (3)

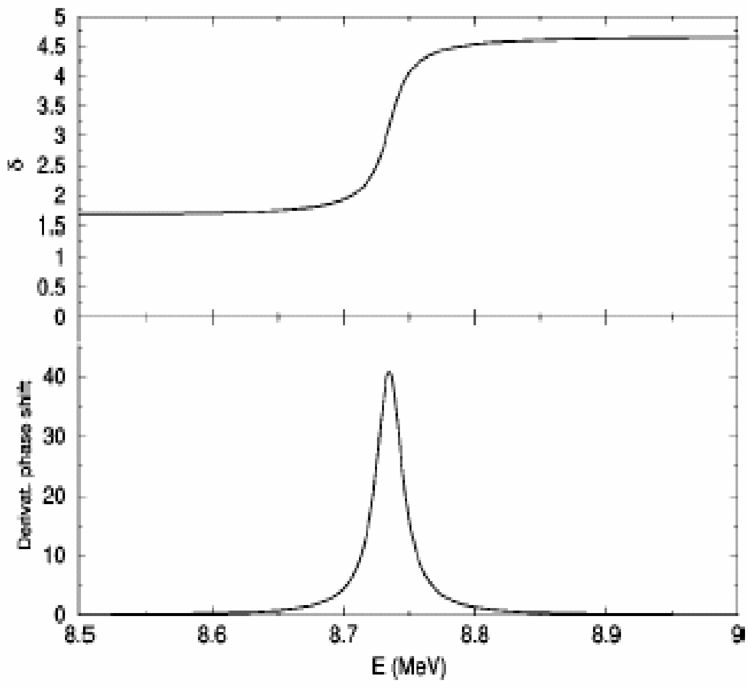


FIG. 1. Phase shift (top) and its derivative (bottom) in the $p_{1/2}$ channel for a square well model.

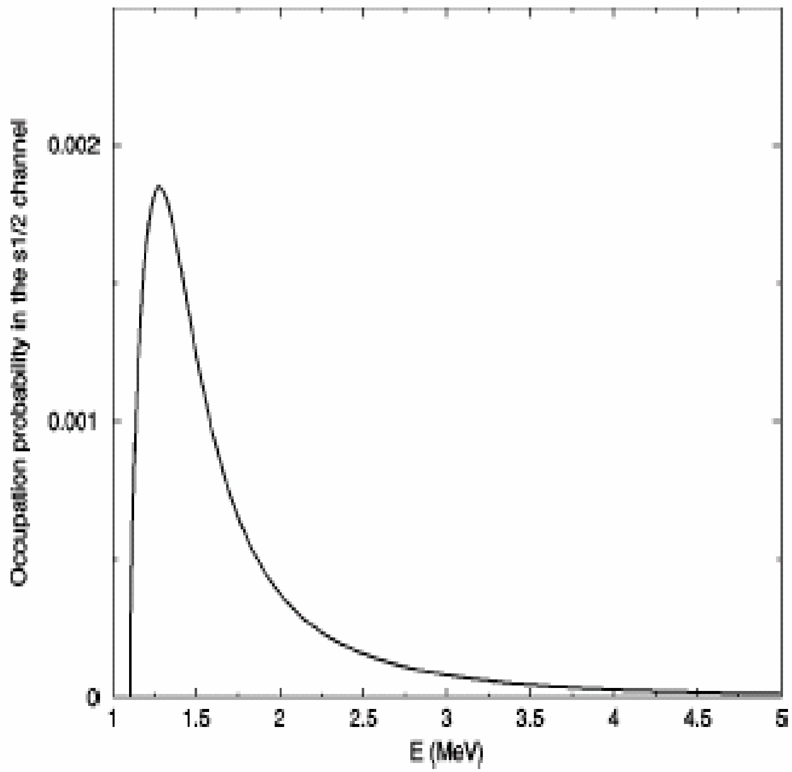


FIG. 2. Occupation probability profile in the $s_{1/2}$ channel for ^{84}Ni.

Densities with continuum

$$\rho(\mathbf{r}) = \sum_{0\le E_\alpha \le -\lambda} |V_\alpha(\mathbf{r})|^2 + \int_{-\lambda}^{E_{cutoff}} dE_\alpha |V_{E_\alpha}(\mathbf{r})|^2 ,$$

$$\kappa(\mathbf{r}) = \sum_{0\le E_\alpha \le -\lambda} U_\alpha(\mathbf{r}) V_\alpha^*(\mathbf{r}) + \int_{-\lambda}^{E_{cutoff}} dE_\alpha U_{E_\alpha}(\mathbf{r}) V_{E_\alpha}^*(\mathbf{r}) .$$

Discretization by box boundary condition

- Alternatively, one can enclose the system in a box of radius R.
- The quasiparticle spectrum is calculated with the boundary condition that the wave function vanishes at r=R.
- One thus obtains a discrete set of states forming a complete basis in the box.

illustration: Ni isotopes

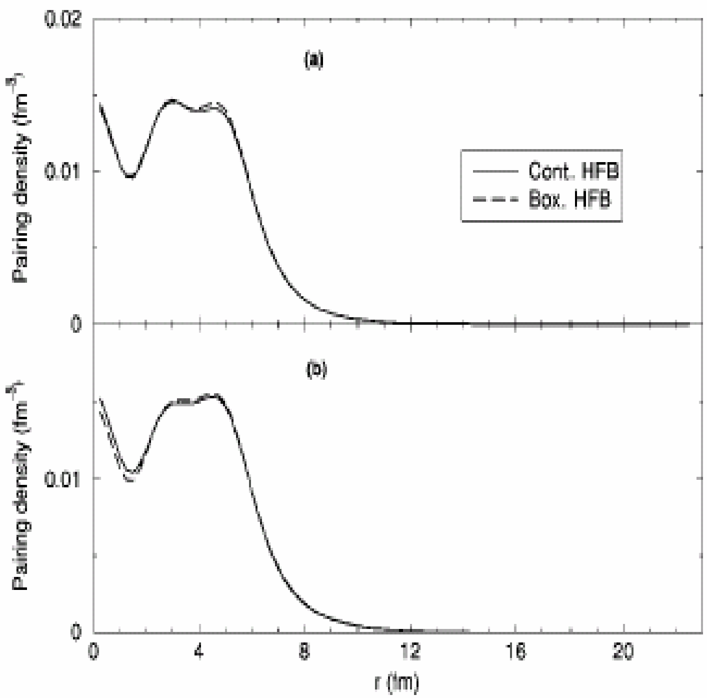


FIG. 4. Neutron pairing densities in HFB calculations in ^{84}Ni (a) and ^{86}Ni (b).

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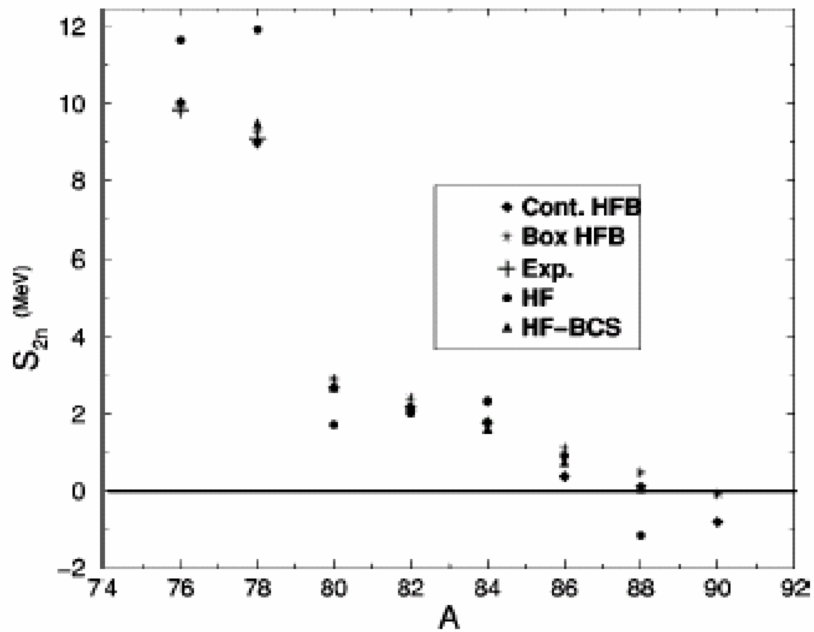
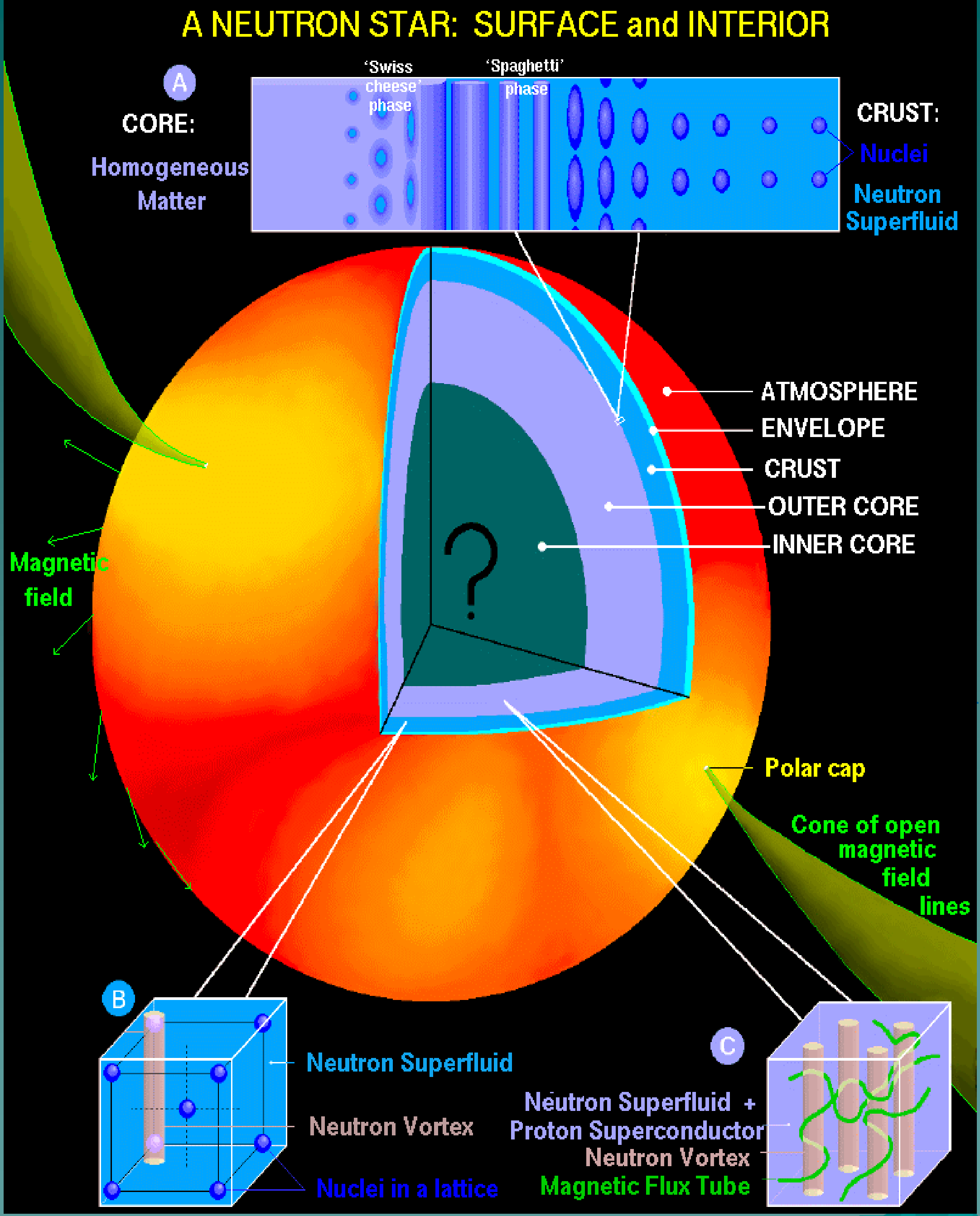
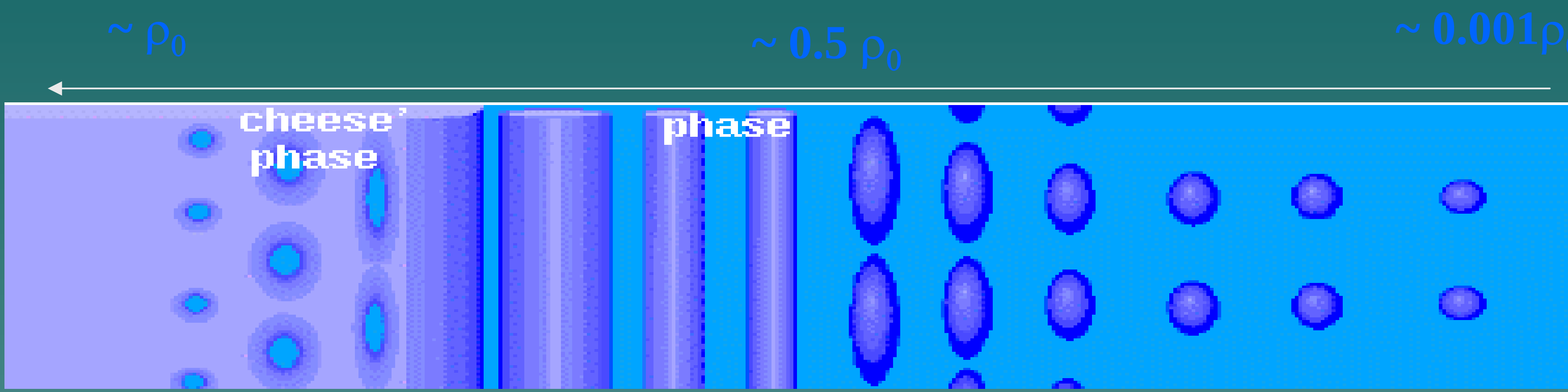
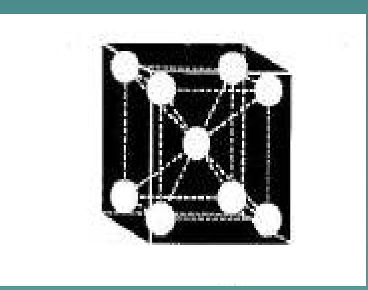


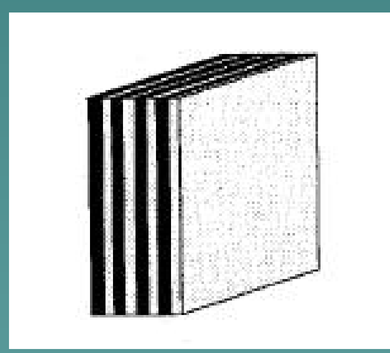
FIG. 5. Two-neutron separation energies in HFB, HF-BCS, and HF approximations. For ^{76}Ni and ^{78}Ni the corresponding values extrapolated from experimental data [18] are also shown.

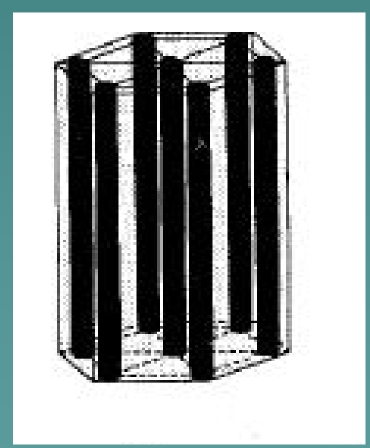

A NEUTRON STAR: SURFACE and INTERIOR
A
CORE:
Homogeneous Matter
'Swiss cheese' phase
'Spaghetti' phase
CRUST:
Nuclei
Neutron Superfluid
ATMOSPHERE
ENVELOPE
CRUST
OUTER CORE
INNER CORE
?
Magnetic field
Polar cap
Cone of open magnetic field lines
B
Neutron Superfluid
Neutron Vortex
Nuclei in a lattice
C
Neutron Superfluid +
Proton Superconductor
Neutron Vortex
Magnetic Flux Tube

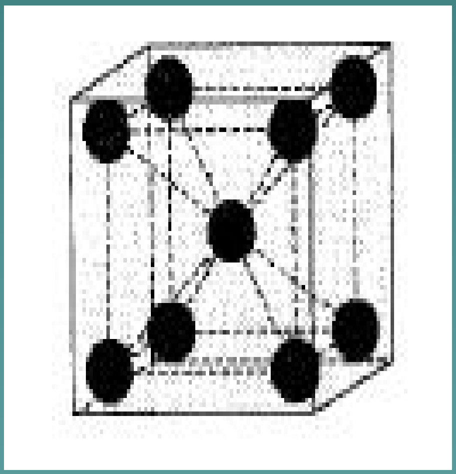
Inner Crust Matter





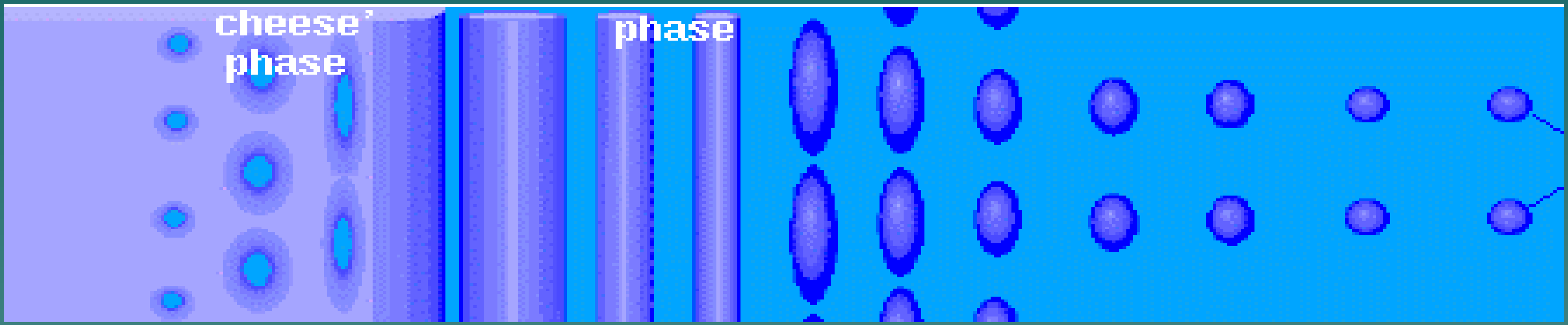


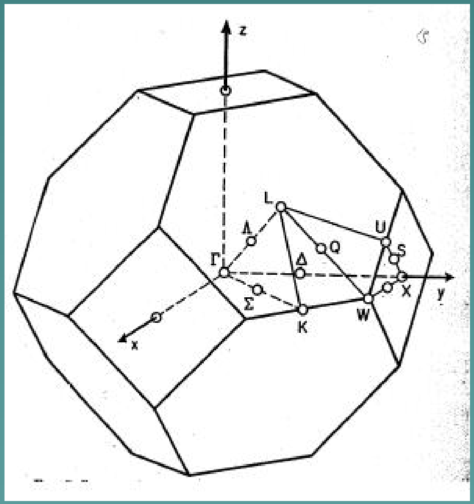


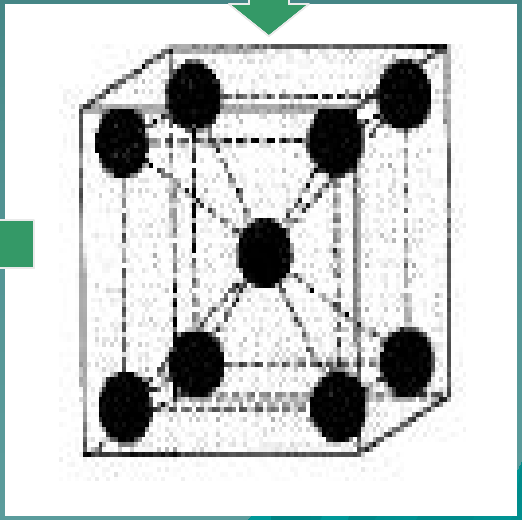


Crystal lattice structures

Elementary cells





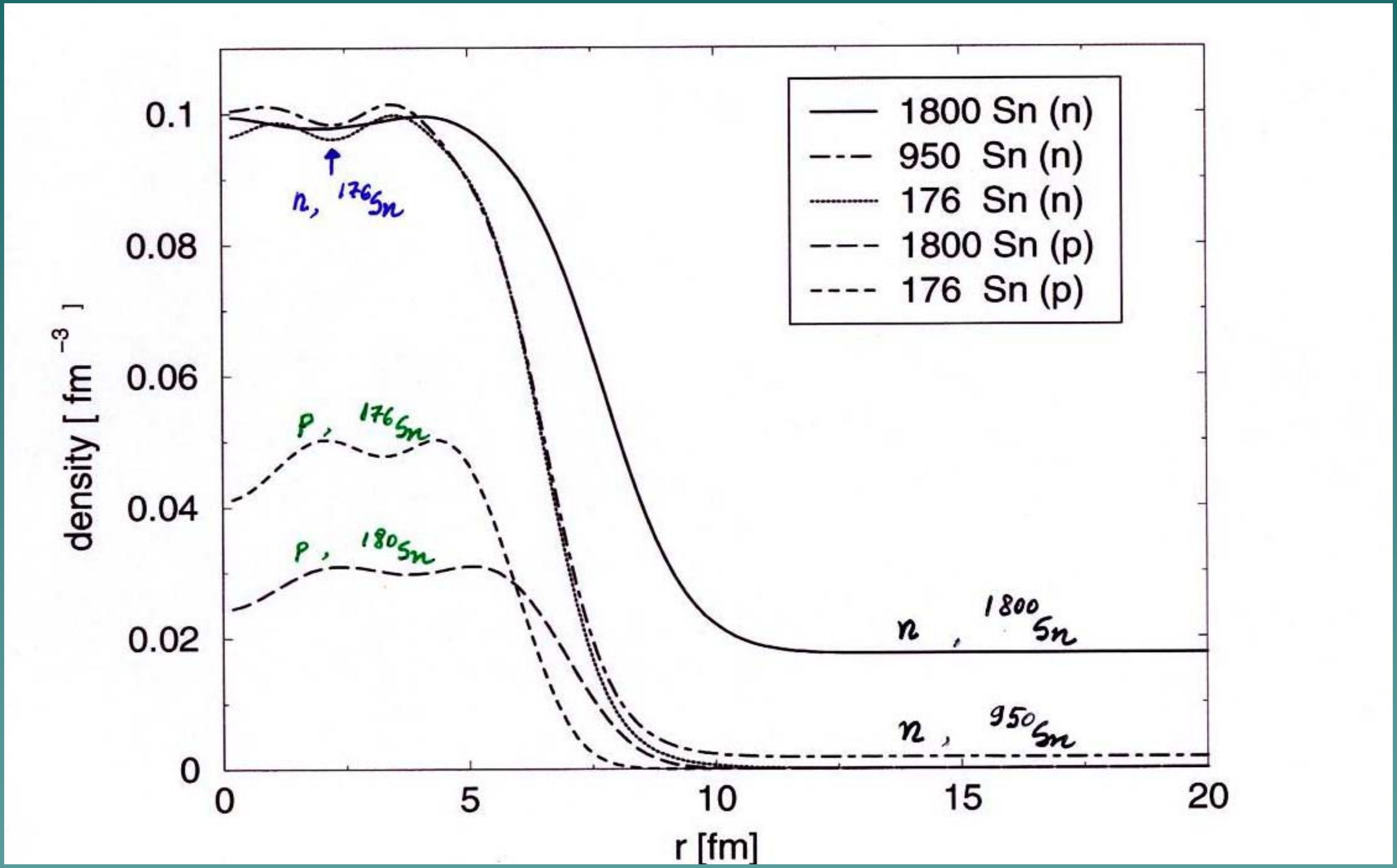


Wigner-Seitz cell

Elementary cell

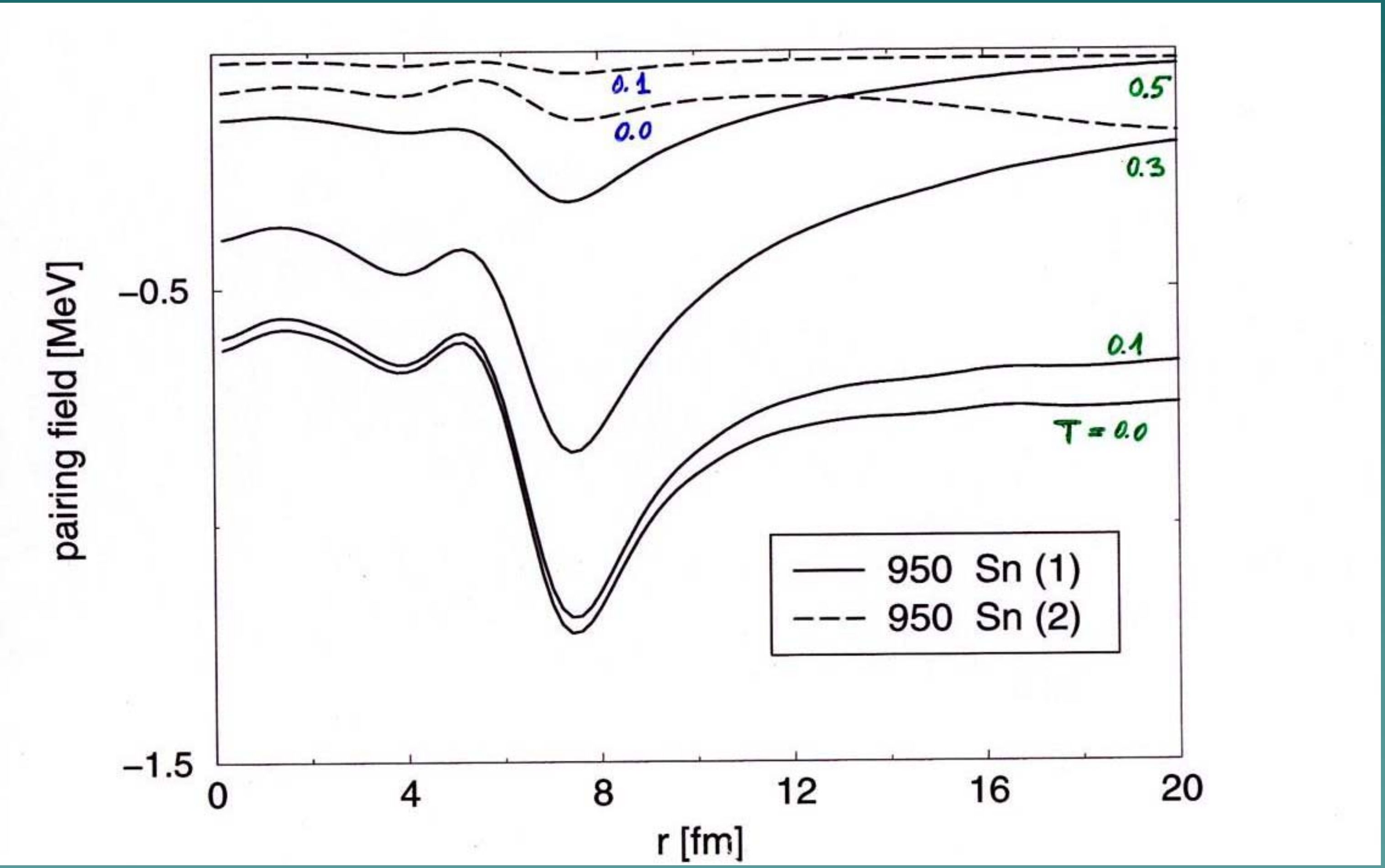
Lattice

Density in the Wigner-Seitz Cells



N.Sandulescu, Nguyen Van Giai,R.J.Liotta, Phys.Rev.C69(2004)045802

Pairing Field in the Wigner-Seitz Cells



*N.S*andulescu, Phys.Rev.**C70** (2004) 025801

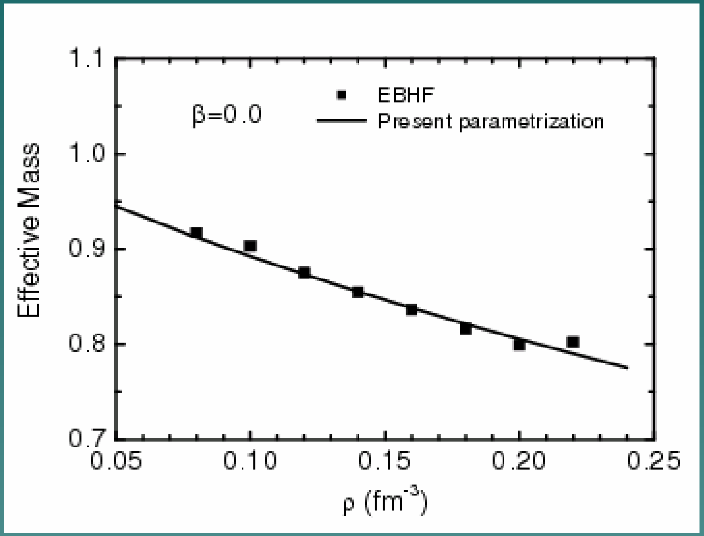
Can one relate Skyrme force to a nuclear matter microscopic theory?

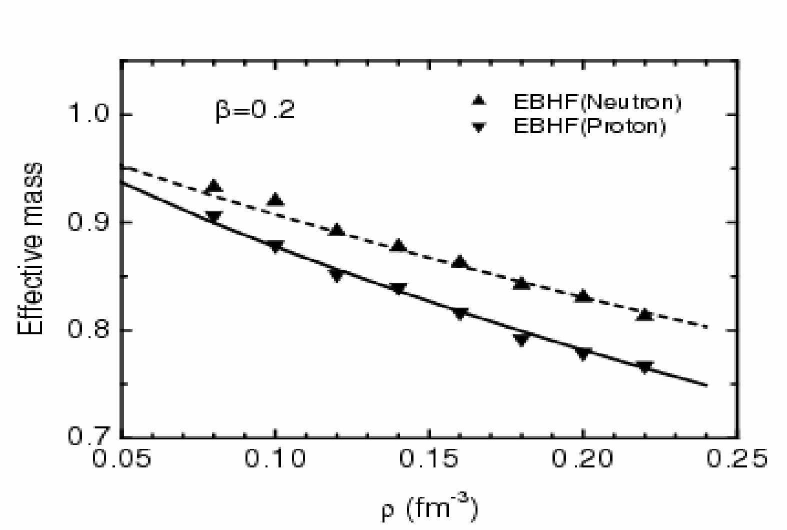
- Density Matrix Expansion (DME) method of Negele-Vautherin (1972)

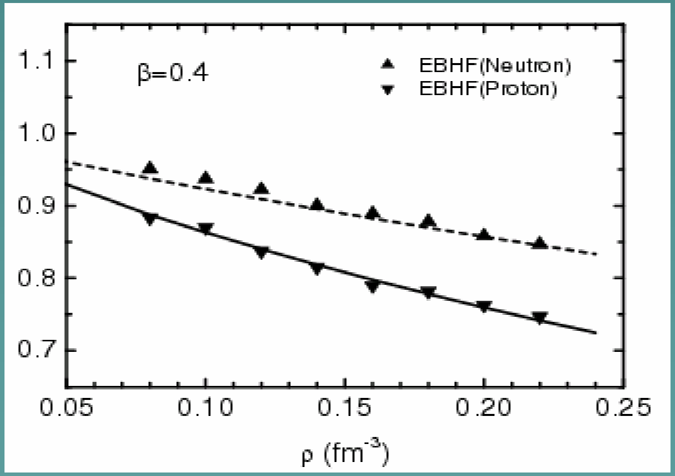
- From Brueckner Theory to Skyrme Energy Functional, L.G. Cao, U. Lombardo, C.W. Shen, NVG (2005):

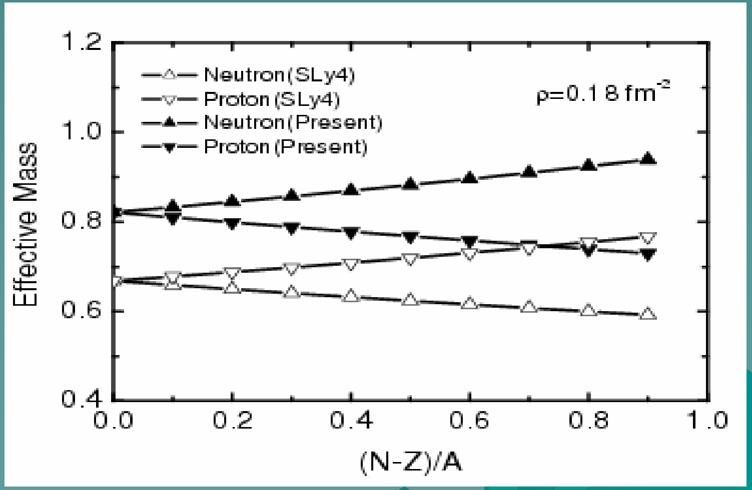
fit of EBHF results in nuclear matter to determine the force parameters.

Effective Masses

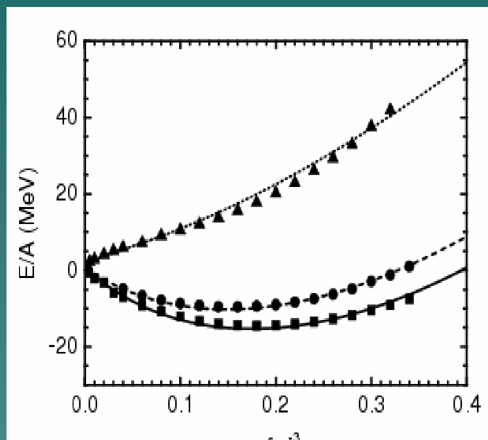








Skyrme parameters from EBHF



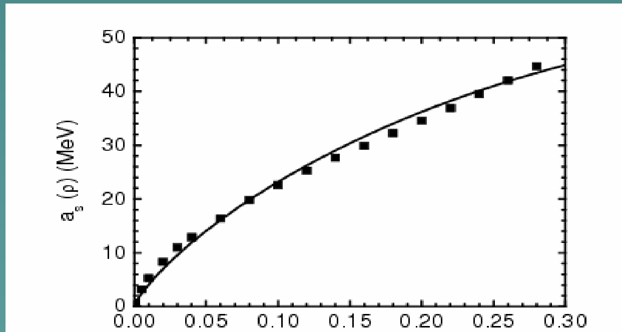


FIG. 5: The symmetry energy from BHF calculations (squares) and the present fit (solid curve).

TABLE I: The Skyrme parameter set and the corresponding bulk properties of infinite nuclear matter.

	LNS
$t_0(MeV fm^3)$	-2484.97
$t_1(MeV fm^5)$	266.735
$t_2(MeV fm^5)$	-337.135
$t_3(MeV fm^{3+3\sigma})$	14588.2
x_0	0.06277
x_1	0.65845
x_2	-0.95382
x_3	-0.03413
σ	0.16667
$W_0(MeV fm^5)$	96.00
$\rho_0(fm^{-3})$	0.1746
$E/A(MeV)$	-15.32
$K_\infty(MeV)$	210.85
$\frac{m^*}{m}(isoscalar)$	0.825
$\frac{m^*}{m}(isovector)$	0.727
$a_s(MeV)$	33.4

Finite Nuclei

- Density profiles

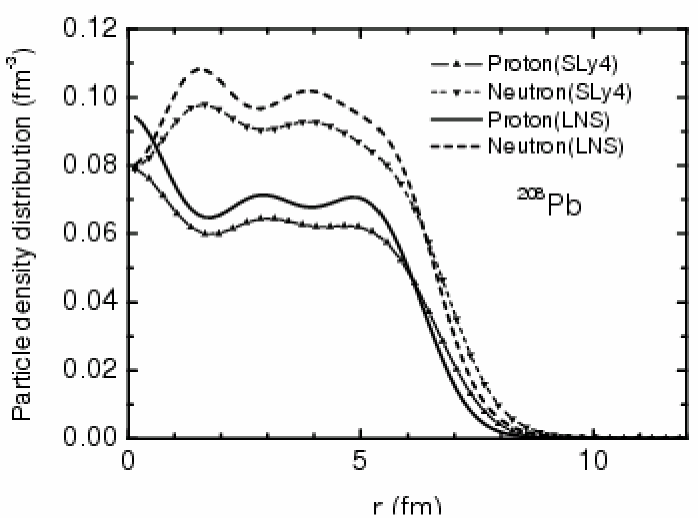
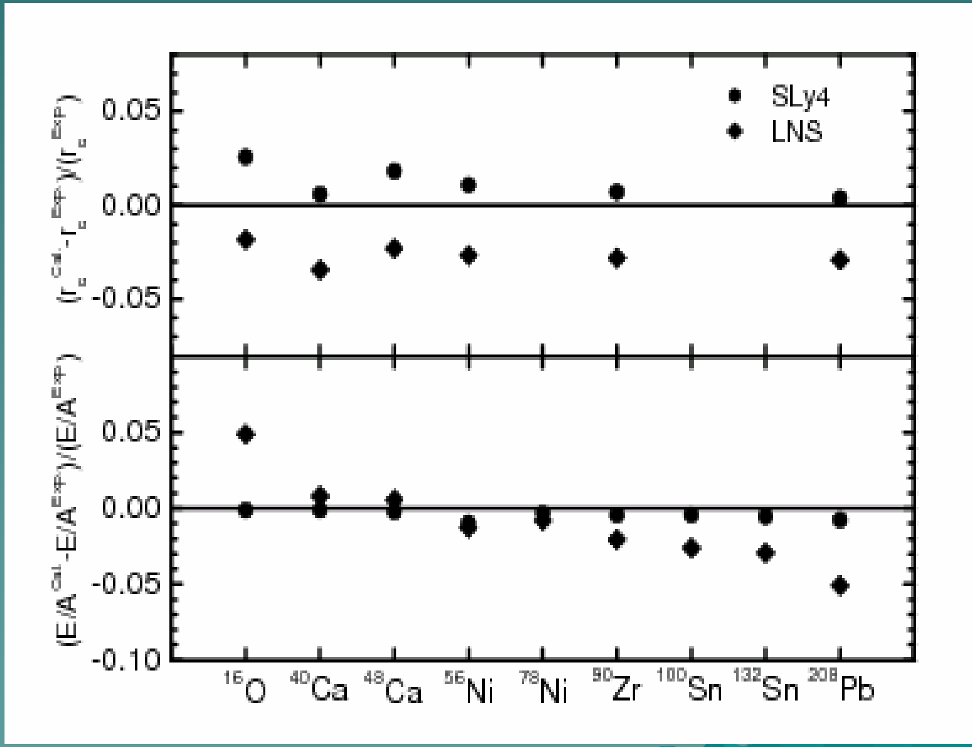


FIG. 7: Neutron and proton densities in ^{208}Pb, calculated with LNS and SLy4 parametrizations.

- Energies and Radii



SUMMARY

- A self-consistent theory of nuclear ground states.
- Pairing and continuum effects are treated.
- Applications to the description of unstable nuclei.
- Applications to the physics of the inner crust of neutron stars.

Mean Field Methods for Nuclear Structure

Part 1: Hartree-Fock and Hartree-Fock-Bogoliubov for Ground States

Part 2: RPA and QRPA for Excitations

Outline of Part 2:

1. Linear response theory: a brief reminder
2. Non-relativistic RPA (Skyrme)
3. Extension to QRPA
4. Illustrative cases
5. Summary

Linear Response Theory

- In the presence of a time-dependent external field, the response of the system reveals the characteristics of the eigenmodes.
- In the limit of a weak perturbing field, the linear response is simply related to the exact two-body Green's function.
- The RPA provides an approximation scheme to calculate the two-body Green's function. .

- Adding a time-dependent external field:

$$H = H_0 + Af(t)$$

$$i\hbar\frac{\partial}{\partial t}|\Phi_n\rangle_t = H|\Phi_n\rangle_t$$

$$i\hbar\frac{\partial}{\partial t}|\tilde{\Phi}_n\rangle = \tilde{A}f(t)|\tilde{\Phi}_n\rangle$$

First order response as a function of time

$$q(t) \equiv \langle \tilde{\Phi}_0 | \tilde{Q} | \tilde{\Phi}_0 \rangle_t - \langle \tilde{0} | \tilde{Q} | \tilde{0} \rangle$$

$$q(t) = -\frac{1}{\hbar} \int_{-\infty}^{+\infty} R(t-t') f(t') dt'$$

$$R(t-t') = \begin{cases} 0 & \text{if } \ t \leq t', \\ -i \langle \tilde{0} | [\tilde{A}(t'), \tilde{Q}(t)] | \tilde{0} \rangle & \text{if } \ t \geq t' \end{cases}$$

Two-body Green's Function and density-density correlation function [1]

$$G(\mathbf{r},\mathbf{r}';t-t') \equiv -i\langle\tilde{\Phi}_0|T(\psi^\dagger(x)\psi(x)\psi^\dagger(x')\psi(x'))|\tilde{\Phi}_0\rangle$$

$$Re\ G^R(\mathbf{r},\mathbf{r}';\omega) = Re\ G(\mathbf{r},\mathbf{r}';\omega),$$

$$Im\ G^R(\mathbf{r},\mathbf{r}';\omega) = sgn(\omega) Im\ G(\mathbf{r},\mathbf{r}';\omega)$$

$$R(t-t') = \int d^3r\, d^3r' A(\mathbf{r}) Q(\mathbf{r}') G^R(\mathbf{r},\mathbf{r}';t-t')$$

$$\delta\rho(\mathbf{r}) = -\int d^3r' G^R(\mathbf{r},\mathbf{r}';t-t') A(\mathbf{r}')$$

Linear response function and Strength distribution

$$R(\omega) = \int d^3r d^3r' A(\mathbf{r}) Q(\mathbf{r}')$$
$$\times \sum_n \langle 0|\psi^\dagger(\mathbf{r})\psi(\mathbf{r})|n\rangle\langle n|\psi^\dagger(\mathbf{r}')\psi(\mathbf{r}')|0\rangle\left[\frac{1}{\omega_{n0}-\omega-i\eta}+\frac{1}{\omega_{n0}+\omega+i\eta}\right]$$

$$S(\omega) \equiv \sum_n |\langle 0|Q|n\rangle|^2 \delta(\omega-\omega_{n0})$$
$$= \frac{1}{\pi} Im\ R(\omega)$$

Main results:

- The knowledge of the retarded Green's function gives access to:
- Excitation energies of eigenmodes (the poles)
- Transition probabilities (residues of the response function)
- Transition densities (or form factors), transition currents, etc… of each excited state .

TDHF and RPA (1)

1

$$H_{HF} = K + U(\rho_0), \qquad \rho_0(\mathbf{r}) = \sum_{i \le k_F} \varphi_i^*(\mathbf{r})\varphi_i(\mathbf{r})$$

Add a small, time-dependent perturbation $A(e^{-i\omega t} + e^{i\omega t})$

$$\phi_i = \varphi_i + \lambda_i(\mathbf{r})e^{i\omega t} + \chi_i(\mathbf{r})e^{-i\omega t}$$

$$i\frac{\partial}{\partial t}\phi_i = (H(t) - \epsilon_i)\phi_i$$

$$H(t) = H_{HF} + [(A(\mathbf{r}) + \frac{\partial U}{\partial \rho}\delta\rho)e^{-i\omega t} + h.c.]$$

TDHF and RPA (2)

4

$$
\begin{aligned}
\chi_i &= \frac{1}{\omega - H_{HF} + \epsilon_i + i\eta}(A + \frac{\partial U}{\partial \rho}\delta\rho)\varphi_i \\
\lambda_i &= -\frac{1}{\omega + H_{HF} - \epsilon_i + i\eta}(A + \frac{\partial U}{\partial \rho}\delta\rho^*)\varphi_i \\
\delta\rho(\mathbf{r}) &= -\int d^3r' G^{(0)}(\mathbf{r},\mathbf{r}';\omega)[A(\mathbf{r}') + \frac{\partial U}{\partial \rho}\delta\rho(\mathbf{r}')]
\end{aligned}
$$

$$
\begin{aligned}
G^{(0)}(\mathbf{r},\mathbf{r}';\omega) &\equiv \sum_i \varphi_i^*(\mathbf{r})\langle\mathbf{r}|\frac{1}{H_{HF} - \epsilon_i - \omega - i\eta} + \frac{1}{H_{HF} - \epsilon_i + \omega - i\eta}|\mathbf{r}'\rangle\varphi_i(\mathbf{r}') \\
&= \sum_{i<F,m>F} \varphi_i^*(\mathbf{r})\varphi_m(\mathbf{r})[\frac{1}{\epsilon_m - \epsilon_i - \omega - i\eta} + \frac{1}{\epsilon_m - \epsilon_i + \omega - i\eta}]\varphi_m^*(\mathbf{r}')\varphi_i(\mathbf{r}')
\end{aligned}
$$

And by comparing with p.6

$$
\begin{aligned}
G(\mathbf{r},\mathbf{r}';\omega) &= G^{(0)}(\mathbf{r},\mathbf{r}';\omega) + \int d^3r'' G^{(0)}(\mathbf{r},\mathbf{r}'';\omega)\frac{\partial U(\mathbf{r}'')}{\partial \rho}G(\mathbf{r}'',\mathbf{r}';\omega) \\
&= G^{(0)}(\mathbf{r},\mathbf{r}';\omega) + \int d^3r'' G^{(0)}(\mathbf{r},\mathbf{r}'';\omega)\frac{\partial U(\mathbf{r}'')}{\partial \rho}G^{(0)}(\mathbf{r}'',\mathbf{r}';\omega) + \dots
\end{aligned}
$$

Residual p-h interaction

$$H = \sum_{i}^{A} K_i + \sum_{i<j}^{A} v(i,j)$$

$$\begin{aligned} E &\equiv \langle 0|H|0\rangle \\ &= \sum_{\alpha\gamma} \langle\alpha| -\frac{\hbar^2\Delta}{2m}|\gamma\rangle\rho_{\alpha\gamma} + \frac{1}{2}\sum_{\alpha\beta\gamma\delta}\langle\alpha\beta|v(1,2)(1-P_{12})|\gamma\delta\rangle\rho_{\alpha\gamma}\rho_{\beta\delta} \end{aligned}$$

$$\begin{aligned} \langle\alpha|K+U|\gamma\rangle &= \frac{\partial E}{\partial\rho_{\alpha\gamma}} \\ &= \langle\alpha| -\frac{\hbar^2\Delta}{2m}|\gamma\rangle + \sum_{\beta\delta}\langle\alpha\beta|v(1,2)(1-P_{12})|\gamma\delta\rangle\rho_{\beta\delta} \\ &\quad + \frac{1}{2}\sum_{\alpha'\gamma'\beta\delta}\langle\alpha'\beta|\frac{\partial v}{\partial\rho_{\alpha\gamma}}(1-P_{12})|\gamma'\delta\rangle\rho_{\alpha'\gamma'}\rho_{\beta\delta} \end{aligned}$$

$$\frac{\partial U_{\alpha\gamma}}{\partial\rho_{\beta\delta}} = \langle\alpha\beta|v(1,2)(1-P_{12})|\gamma\delta\rangle + \ldots$$

Analytic summation of single-particle continuum [3]

$$\langle \mathbf{r}| \frac{1}{z+i\eta - H_{HF}} |\mathbf{r}'\rangle = \frac{1}{rr'} \sum_{ljm} g_{lj}(r,r';z) \mathcal{Y}^{m\dagger}_{lj}(\hat{r}) \mathcal{Y}^{m}_{lj}(\hat{r}')$$

$$H^{lj}_{HF} = -\frac{d}{dr}(\frac{\hbar^2}{2m^*})\frac{d}{dr} + \frac{\hbar^2}{2m^*}\frac{l(l+1)}{r^2} + U_{lj}(r)$$

$$g_{lj}(r,r';z) = \sum_n \langle r|n\rangle \frac{1}{z+i\eta-\epsilon_n}\langle n|r'\rangle$$

$$g_{lj}(r,r';z) = \frac{2m^*}{\hbar^2}\frac{1}{W(w,u)} u(r_<)w(r_>)$$

1) u, w are regular and irregular solutions satisfying appropriate asymptotic conditions

2) This analytic summation is not possible if potential U is non-local .

Approximate treatments of continuum

- Calculate positive-energy s.p. states with scattering asymptotic conditions, and sum over an energy grid along the positive axis, up to some cut-off
- Sum over discrete states of positive energy calculated with a box boundary condition .

Finite temperature

2

$$\theta(\epsilon_F - \epsilon_i) \;\rightarrow\; f_i(T) = \frac{1}{1 + exp[(\epsilon_i - \mu)/T]}$$

$$\sum_{i<F} \rightarrow \sum_{all\ i} f_i(T)$$

Applications: evolution of escape widths and Landau damping of IVGDR with temperature .

RPA on a p-h basis

$$H = H_{HF} + V_{res}$$

$$V_{res} \neq V \qquad \text{if density} - \text{dependent } V$$

$$d_\alpha^+ |\tilde{0}\rangle = |d_\alpha\rangle, \text{energy } \omega_\alpha$$

$$d_\alpha |\tilde{0}\rangle = 0$$

$$d_\alpha^+ = \sum_{mi} X_{mi}^{(\alpha)} a_m^+ a_i - Y_{mi}^{(\alpha)} a_i^+ a_m$$

$$\begin{vmatrix} A & B \\ -B & -A \end{vmatrix} \begin{vmatrix} X^{(\alpha)} \\ Y^{(\alpha)} \end{vmatrix} = \omega_\alpha \begin{vmatrix} X^{(\alpha)} \\ Y^{(\alpha)} \end{vmatrix}$$

A and B matrices

$$
\begin{aligned}
A_{mi,nj} &= \langle\tilde{0}|[a_i^+ a_m, H, a_n^+ a_j]|\tilde{0}\rangle \\
&\simeq \langle HF|[a_i^+ a_m, H, a_n^+ a_j]|HF\rangle \\
&= (\epsilon_m - \epsilon_i)\delta_{mn}\delta_{ij} + \langle mj|V_{res}|ni\rangle
\end{aligned}
$$

$$
\begin{aligned}
B_{mi,nj} &= -\langle\tilde{0}|[a_m^+ a_i, H, a_n^+ a_j]|\tilde{0}\rangle \\
&\simeq -\langle HF|[a_m^+ a_i, H, a_n^+ a_j]|HF\rangle \\
&= \langle mn|V_{res}|ji\rangle
\end{aligned}
$$

$$
[P, Q, R] = \frac{1}{2}([P, [Q, R]] + [[P, Q], R])
$$

Restoration of symmetries

- Many symmetries are broken by the HF mean-field approximation: translational invariance, isospin symmetry, particle number in the case of HFB, etc…
- If RPA is performed consistently, each broken symmetry gives an RPA (or QRPA) state at zero energy (the spurious state)
- The spurious state is thus automatically decoupled from the physical RPA excitations
- This is not the case in phenomenological RPA .

Sum rules

1

$$m_k = \sum_n |\langle n|Q|0\rangle|^2 \omega_{n0}^k$$
$$= \int S(\omega)\omega^k d\omega$$

- For odd k, RPA sum rules can be calculated from HF, without performing a detailed RPA calculation.
- k=1: Thouless theorem
- k=-1: Constrained HF
- k=3: Scaling of HF

Phenomenological RPA

- The HF mean field is replaced by a parametrized mean field (harmonic oscillator, Woods-Saxon potential, …)
- The residual p-h interaction is adjusted (Landau-Migdal form, meson exchange, …)
- Useful in many situations (e.g., double-beta decay)
- Difficulty to relate properties of excitations to bulk properties (K, symmetry energy, effective mass, …) .

Finite rank form of Skyrme HF-RPA (V.Voronov, Ch. Stoyanov et al.)

Separable interaction

$$V_{mi,nj} \simeq \chi D_{mi} D_{nj}$$

$$A_{mi,nj} = \varepsilon_{mi}\delta_{mn}\delta_{ij} + \chi D_{mi} D_{nj} \quad \sim\sim\sim\sim \quad \varepsilon_{mi} = \varepsilon_m - \varepsilon_i$$

$$B_{mi,nj} = \chi D_{mi} D_{nj}$$

$$\sum_{mi} D_{mi}^2 \left(\frac{1}{\varepsilon_{mi} - \omega} + \frac{1}{\varepsilon_{mi} + \omega} \right) = -\frac{1}{\chi}$$

Finite rank separable form

$$V_{mi,nj} \simeq \chi \sum_k D_{mi}^{(k)} D_{nj}^{(k)}$$

QRPA (1)

- The scheme which relates RPA to linearized TDHF can be repeated to derive QRPA from linearized Time-Dependent Hartree-Fock-Bogoliubov (cf. E. Khan et al., Phys. Rev. C 66, 024309 (2002))
- Fully consistent QRPA calculations, except for 2-body spin-orbit, can be performed (M. Yamagami, NVG, Phys. Rev. C 69, 034301 (2004)) .

QRPA (2)

- If Vpp is zero-range, one needs a cut-off in qp space, or a renormalisation procedure a la Bulgac. Then, one cannot sum up analytically the qp continuum up to infinity
- If Vpp is finite range (like Gogny force) one cannot solve the Bethe-Salpeter equation in coordinate space
- It is possible to sum over an energy grid along the positive axis (Khan - Sandulescu et al., 2002) .

The QRPA Green's Function

The QRPA equation is derived as the small amplitude limit of the time-dependent HFB equation[7]. The linear response equation is

$$\begin{aligned}\delta\rho_\alpha(\boldsymbol{r}) &= \int d\boldsymbol{r}' \int d\boldsymbol{r}'' \sum_{\beta,\gamma} G_0^{\alpha,\beta}(\boldsymbol{r},\boldsymbol{r}') V_{\beta,\gamma}(\boldsymbol{r}',\boldsymbol{r}'')\delta\rho_\gamma(\boldsymbol{r}'') \\ &+ \int d\boldsymbol{r}' \sum_{\beta} G_0^{\alpha,\beta}(\boldsymbol{r},\boldsymbol{r}') F_\beta(\boldsymbol{r}'), \qquad (5)\end{aligned}$$

where $\delta\rho_\alpha$ is the transition density, $G_0^{\alpha,\beta}$ is the unperturbed Green's function and F_α is the external field. The index α runs $\alpha = ph, \Delta, \partial^\pm, l_+^{(\pm)}, l_-^{(\pm)}, pp, hh$. By definition, the QRPA Green's function G relates the perturbing external field to the transition density,

$$\delta\rho_\alpha(\boldsymbol{r}) = \int d\boldsymbol{r}' \sum_{\beta} G^{\alpha,\beta}(\boldsymbol{r},\boldsymbol{r}') F_\beta(\boldsymbol{r}'). \qquad (6)$$

Combining with Eq.(5), we obtain the generalized Bethe-Salpeter type equation,

$$\begin{aligned}G^{\alpha,\beta}(\boldsymbol{r},\boldsymbol{r}') &= G_0^{\alpha,\beta}(\boldsymbol{r},\boldsymbol{r}') \\ &+ \int d\boldsymbol{r}'' \int d\boldsymbol{r}''' \sum_{\gamma,\delta} G_0^{\alpha,\gamma}(\boldsymbol{r},\boldsymbol{r}'') V_{\gamma,\delta}(\boldsymbol{r}'',\boldsymbol{r}''') G^{\delta,\beta}(\boldsymbol{r}''',\boldsymbol{r}'). \qquad (7)\end{aligned}$$

External field and Strength distribution

In the case of transition from the ground state to excited states within the same nucleus, only the (ph, ph) component of G is acting. Because we consider only spin independent response, the strength function is given by

$$S(\omega) = -\frac{1}{\pi} Im \int d\boldsymbol{r} \int d\boldsymbol{r}' F^*_{ph}(\boldsymbol{r}) G^{ph,ph}(\boldsymbol{r}, \boldsymbol{r}') F_{ph}(\boldsymbol{r}'). \tag{8}$$

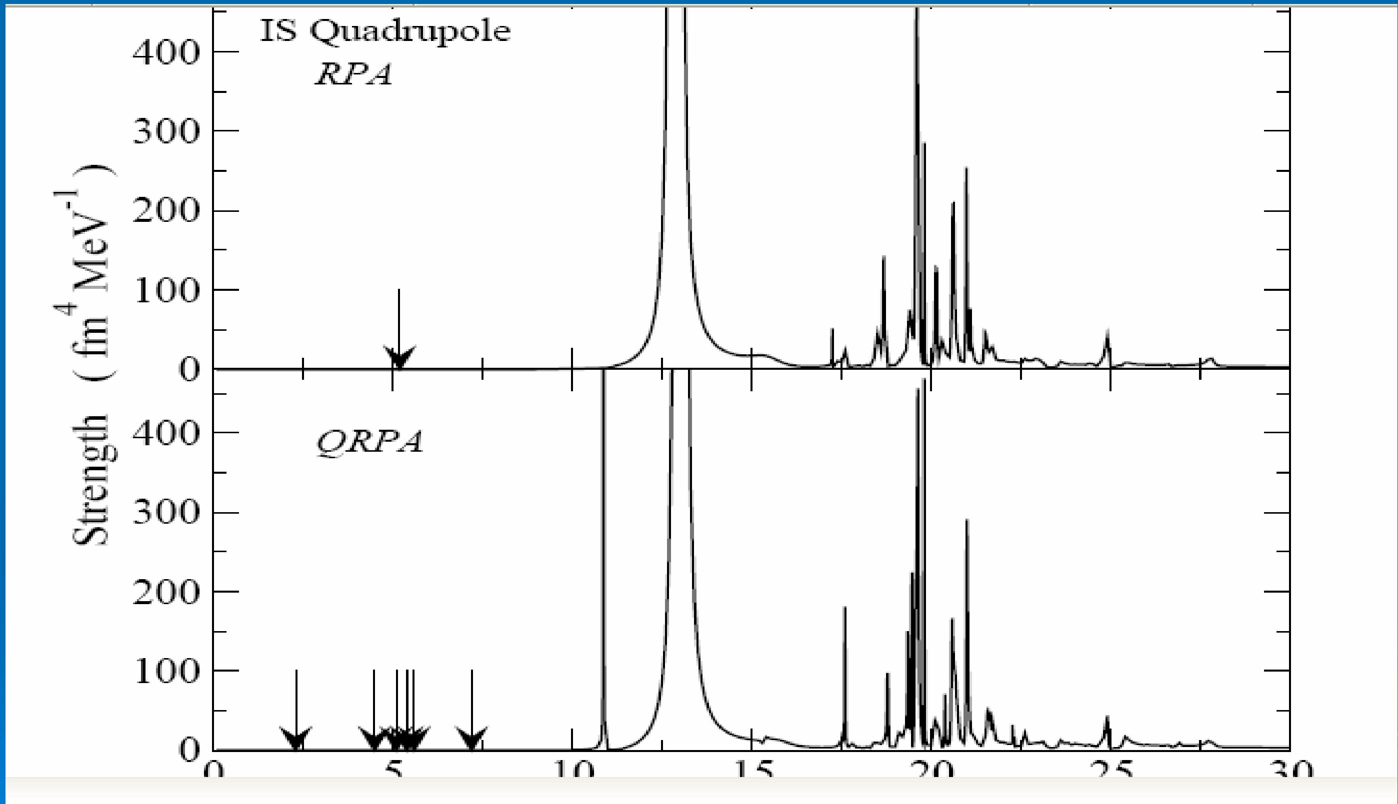
The external field for isoscalar quadrupole response is

$$F^{ISQ}_{ph,\mu} = \sum_i r_i^2 Y_{2\mu}(\hat{\boldsymbol{r}}_i) \tag{9}$$

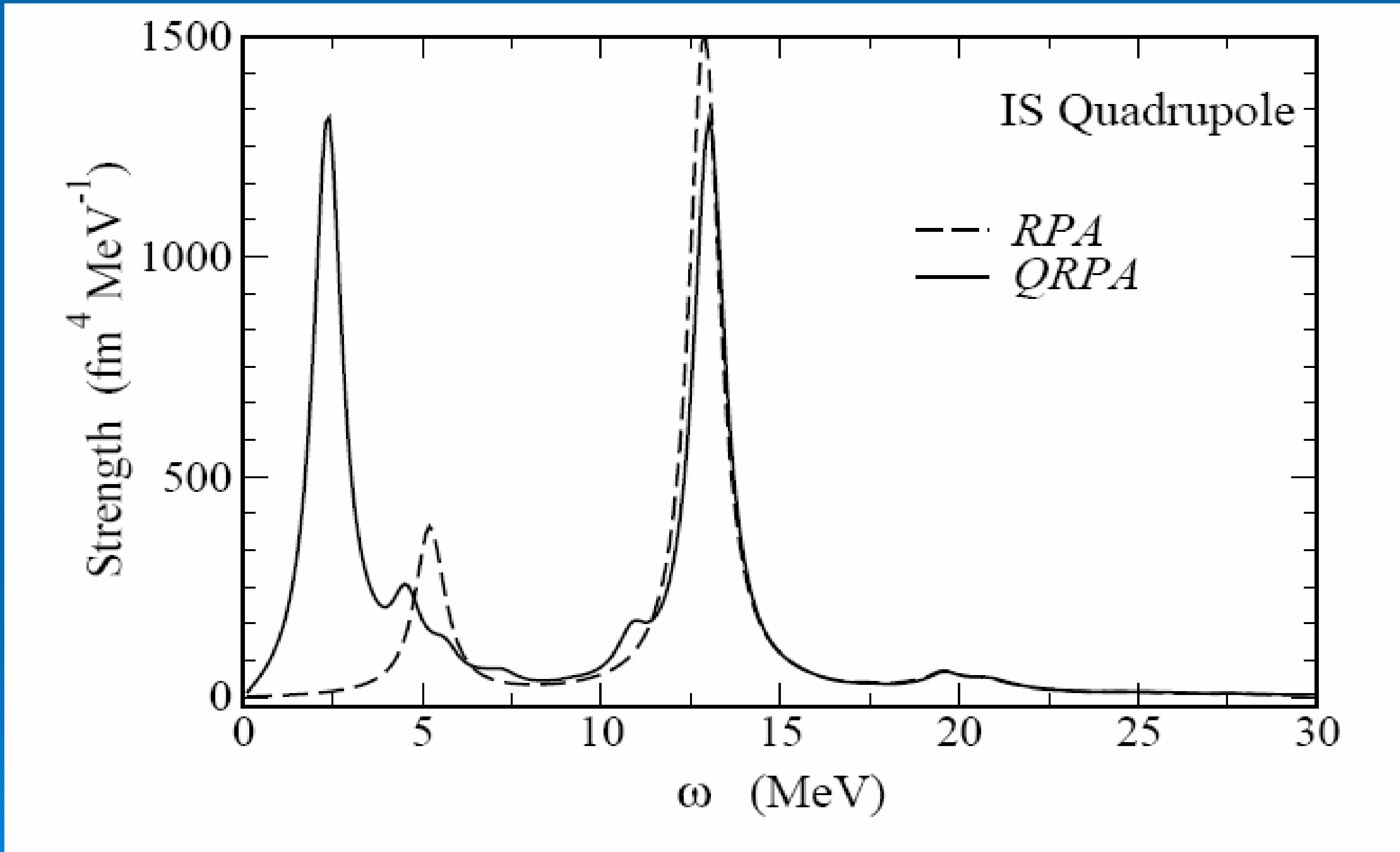
and for isovector dipole response,

$$F^{IVD}_{ph,\mu} = e\frac{Z}{A} \sum_{i_n=1}^{N} r_{i_n} Y_{1\mu}\left(\hat{\boldsymbol{r}}_{i_n}\right) - e\frac{N}{A} \sum_{i_p=1}^{Z} r_{i_p} Y_{1\mu}\left(\hat{\boldsymbol{r}}_{i_p}\right). \tag{10}$$

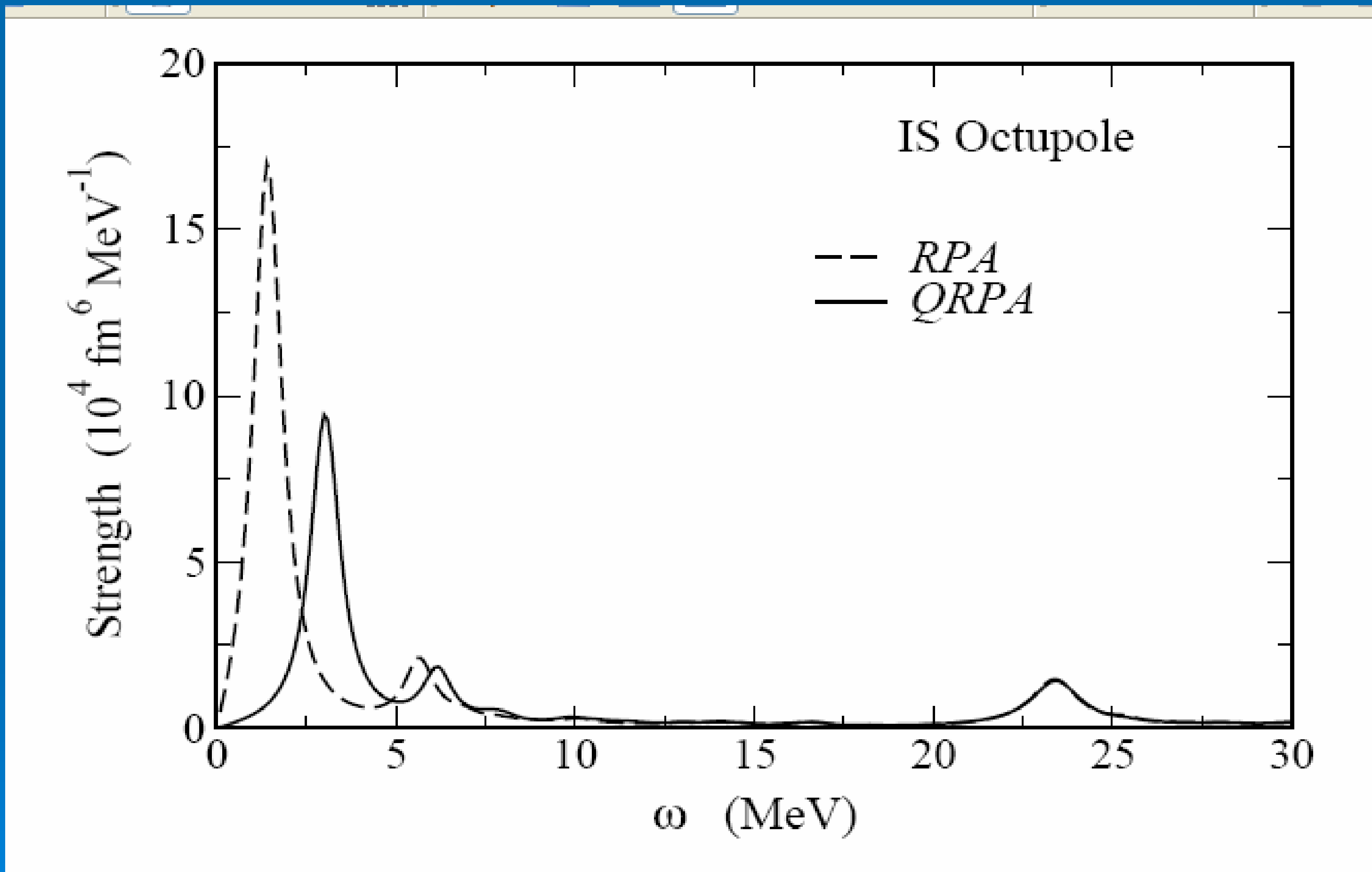
2+ states in 120Sn



2+ states in 120Sn, with smearing

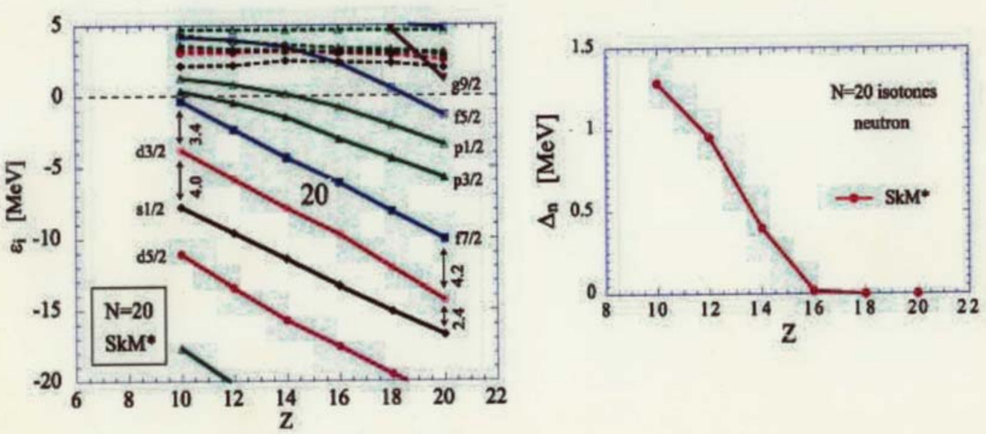


3- states in 120Sn, with smearing



Evolution of pairing in N=20 isotones

Neutron N=20 shell gap in N=20 isotones

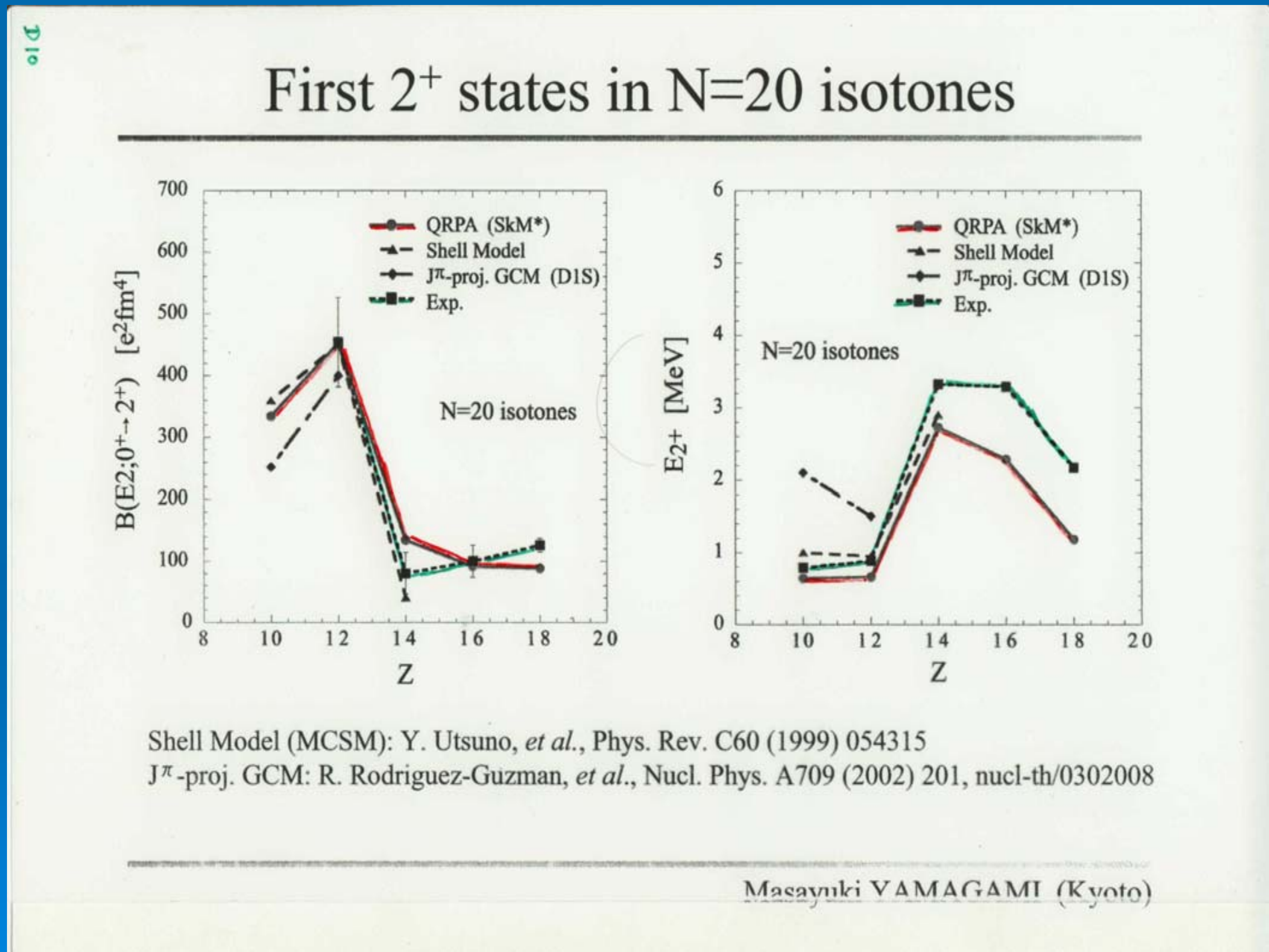


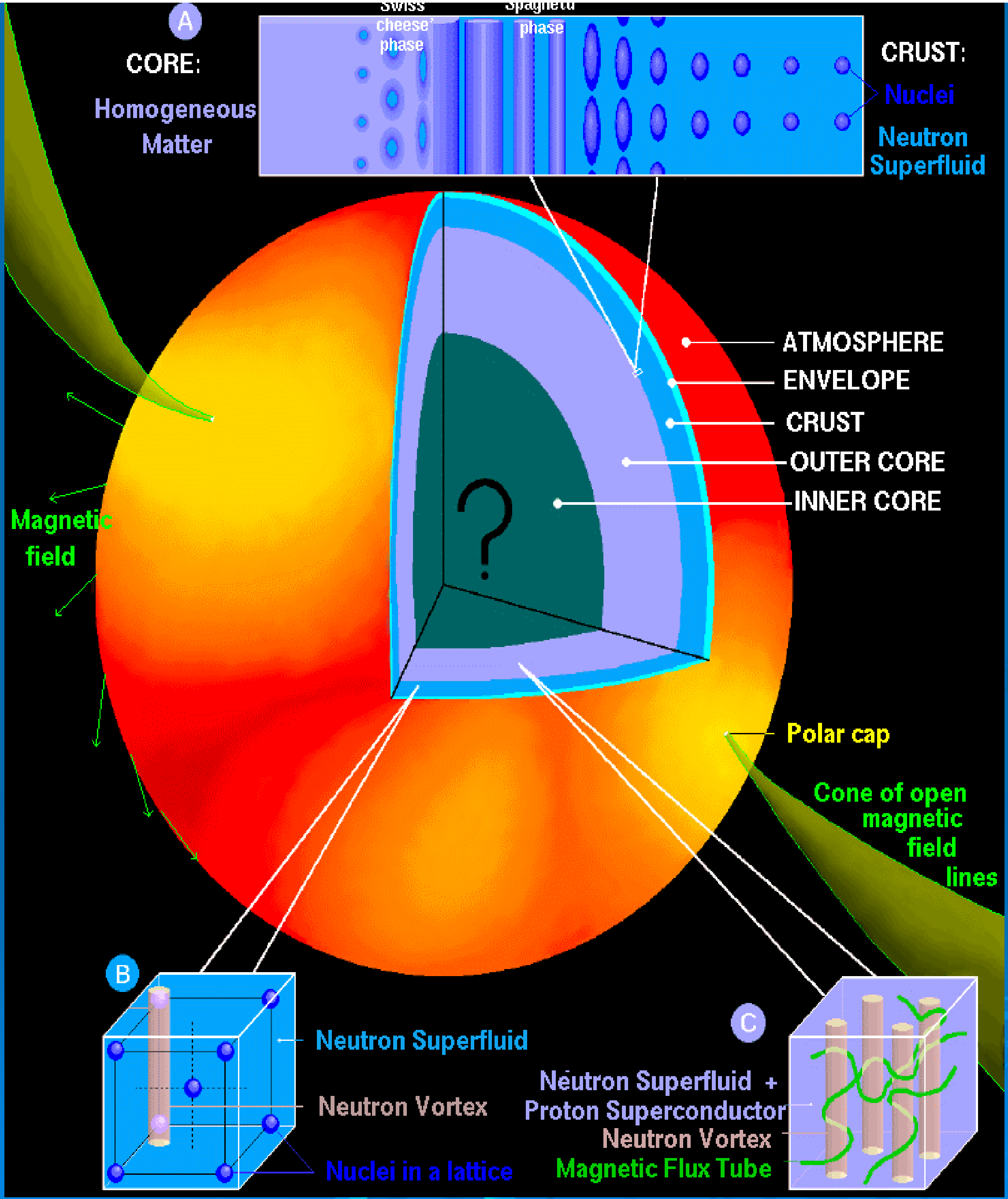
Key point

Different *l*-dependence of single-particle orbits as $\varepsilon_i \Rightarrow 0$

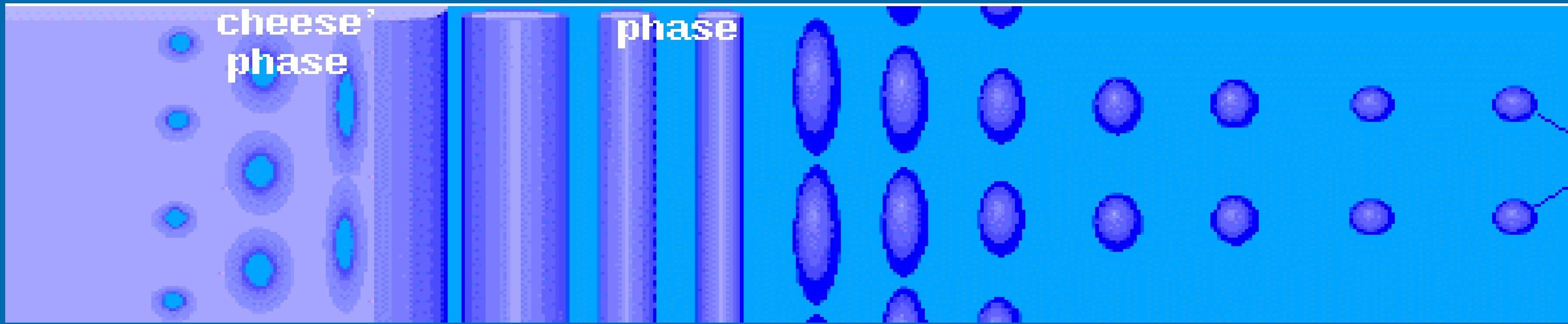
Masayuki YAMAGAMI (Kyoto)

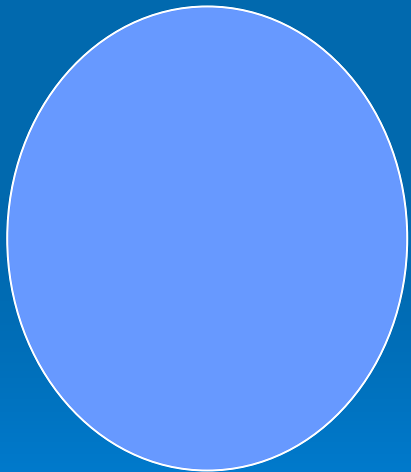
Effect of pairing on 2+ states

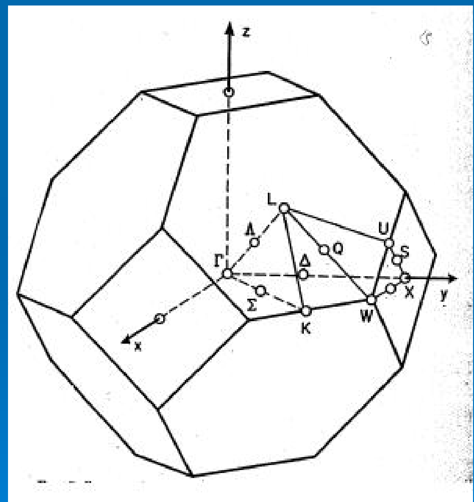


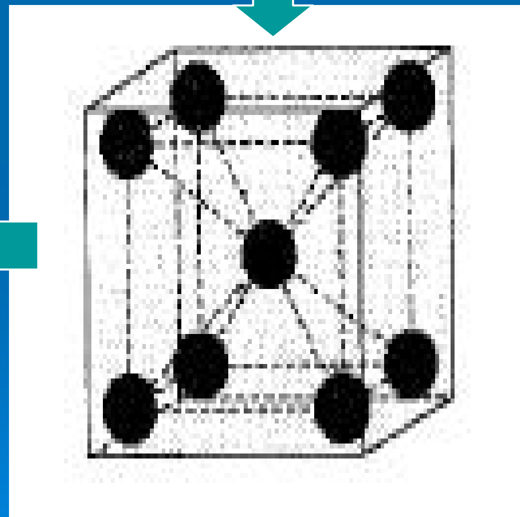

A
CORE:
Homogeneous
Matter
Swiss
cheese
phase
Spaghetti
phase
CRUST:
Nuclei
Neutron
Superfluid
ATMOSPHERE
ENVELOPE
CRUST
OUTER CORE
INNER CORE
?
Magnetic
field
Polar cap
Cone of open
magnetic
field
lines
B
Neutron Superfluid
Neutron Vortex
Nuclei in a lattice
C
Neutron Superfluid +
Proton Superconductor
Neutron Vortex
Magnetic Flux Tube

Wigner-Seitz cells







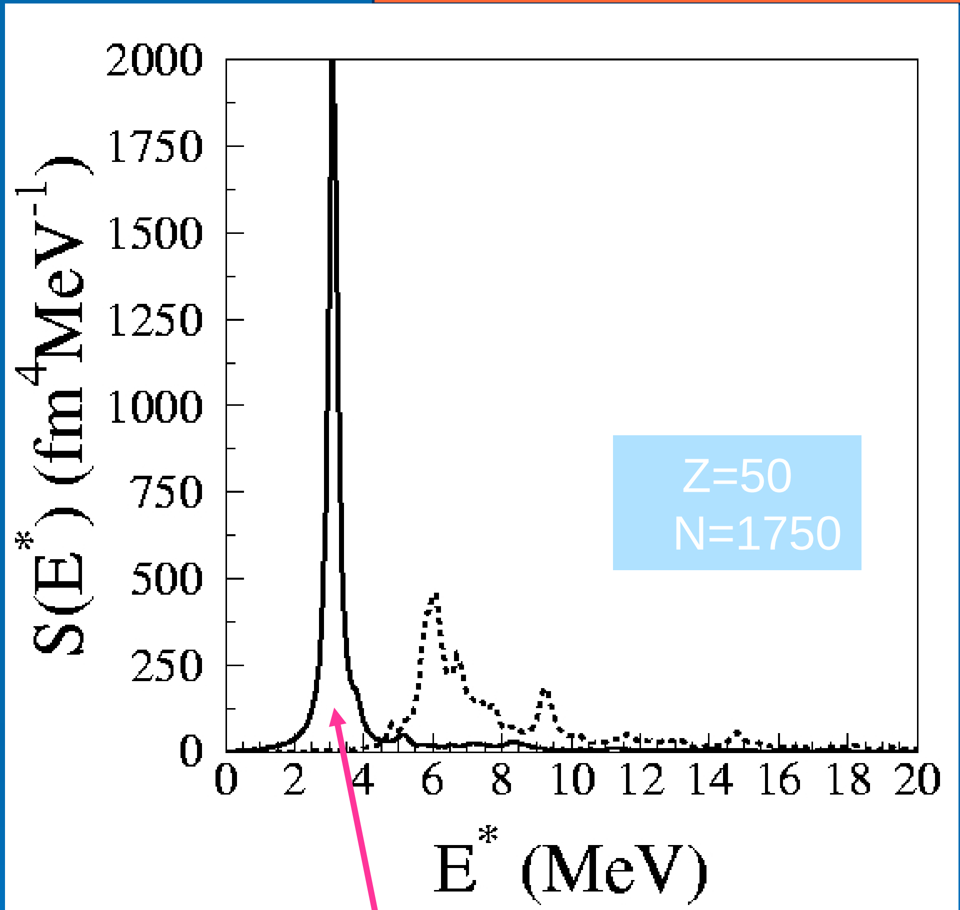


Wigner-Seitz cell

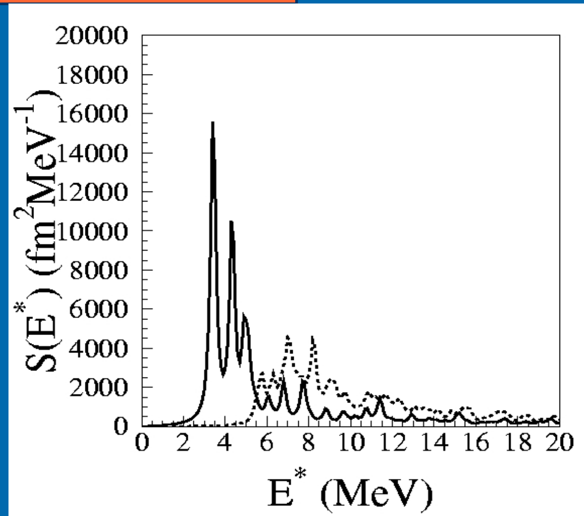
Elementary cell

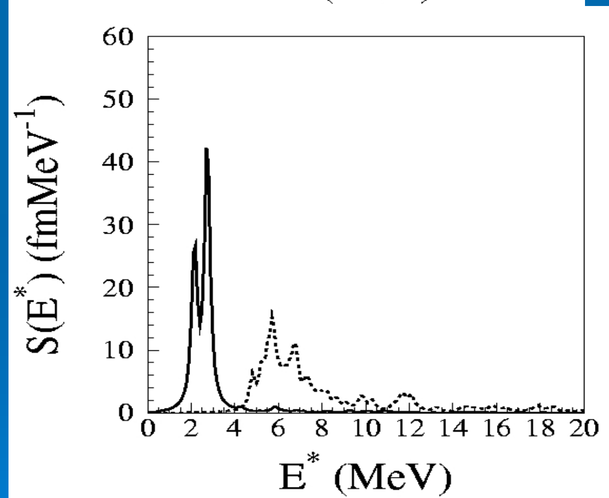
Lattice

Supergiant resonances



71% EWSR





Effect on specific heat ?

E.Khan, N.Sandulescu, Nguyen Van Giai, Phys.Rev.C 71,042801 (R) (2005)

Concluding Remarks

- The RPA and QRPA approach in coordinate space has its own advantage.
- But: very complicated to include 2-body spin-orbit!
- Necessary to work in configuration space (matrix form) for full self-consistency.
- To be explored: deformed RPA and QRPA with Skyrme forces.

Pairing window method

$$D_{\alpha\beta}(i) = \phi_\alpha(r_i)\phi_\beta(r_i)\langle j_\alpha l_\alpha || Y_L || j_\beta l_\beta\rangle \frac{u_\alpha v_\beta + (-)^L v_\alpha u_\beta}{\sqrt{2L+1}}(1+\delta_{\alpha,\beta})^{-1/2}, \quad (15)$$

where v_α^2 is the BCS occupation probability and $u_\alpha^2 = 1 - v_\alpha^2$. E_α is the quasi-particle

as the RPA response function. The free response function in the BCS approximation (14) thus becomes

$$\Pi_0(i,j;\omega) = -\sum_{\alpha\leq\beta} D_{\alpha\beta}(i)D_{\alpha\beta}(j)\left(\frac{1}{E_\alpha+E_\beta-\omega-i\eta}+\frac{1}{E_\alpha+E_\beta+\omega-i\eta}\right)$$
$$-\sum_\alpha \phi_\alpha(r_i)\phi_\alpha(r_j)v_\alpha^2\sum_{j_k l_k}\langle j_\alpha l_\alpha||Y_L||j_k l_k\rangle^2\frac{1}{2L+1}$$
$$\times\left\{\left\langle r_i\left|\frac{1}{E_\alpha+\hat{h}-\lambda-\omega-i\eta}+\frac{1}{E_\alpha+\hat{h}-\lambda+\omega-i\eta}\right|r_j\right\rangle\right.$$
$$\left.-\sum_\beta \delta_{j_k,j_\beta}\delta_{l_k,l_\beta}\phi_\beta(r_i)\phi_\beta(r_j)\left(\frac{1}{E_\alpha+\epsilon_\beta-\lambda-\omega-i\eta}+\frac{1}{E_\alpha+\epsilon_\beta-\lambda+\omega-i\eta}\right)\right\}, \quad (16)$$

where the summations of α and β are restricted to the states within the pairing active space. The last term in Eq. (16) is a correction for a double-counting of excitations within the pairing active space, which stems from the substitution of the completeness relation

K. Hagino, H. Sagawa, Nucl. Phys. A 695, 82 (2001) .