Hydrodynamic modeling of deconfinement phase transition in heavy-ion collisions at FAIR-NICA energies

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FIAS Frankfurt Institute for Advanced Studies

HIC for FAIR Helmholtz International Center

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Contents

Most famous hydro models

(1+1) – dimensional models:

Landau, 1953 – full stopping of produced matter in Lorentz-contracted volume;
Bjorken, 1983 – partial transparency of colliding nuclei + delayed formation of fluid

(2+1) – dimensional models (transverse hydro + Bjorken longitudinal expansion)**:**

Kolb, Sollfrank & Heinz, 1999; Teaney, Lauret & Shuryak, 2001

(3+1) – dimensional models (starting with cold nuclei)**:**

Harlow, Amsden & Nix, 1976; Stöcker, Maruhn & Greiner, 1979;
Rischke et al., 1995; Hama et al., 2005

Multi-fluid models (partial transparency of nuclei)**:**

Amsden et al., 1978; Clare & Strottman, 1986; Mishustin, Russkikh & Satarov, 1988;
Brachmann et al., 2000; Ivanov, Russkikh & Toneev, 2006

Hydro-cascade models (hydrodynamics + kinetic descripton of freeze-out stage) **:**

Bass et al., 2000; Teaney et al., 2001; Petersen, Bleicher et al., 2008

Viscous hydro models: Romatschke, 2007; Song & Heinz, 2008

Equations of ideal hydrodynamics

Local conservation of 4-momentum and baryon charge

$$\partial_\nu T^{\mu\nu} = 0$$

$$\partial_\mu (n u^\mu) = 0$$

$$\mu, \nu = 0, 1, 2, 3$$

Energy-momentum tensor

$$T^{\mu\nu} = (\varepsilon + P) u^\mu u^\nu - P g^{\mu\nu}$$

Net baryon density

$$n = n_B - n_{\overline{B}}$$

Collective 4-velocity

$$u^\mu = (\gamma, \gamma \vec{v})^\mu \qquad \gamma = \left(1 - \vec{v}^{\,2}\right)^{-1/2}$$

Necessary information

- equation of state (EoS)
- initial conditions

(3+1) – dimensional hydrodynamic modeling of nuclear collisions

$$\partial N/\partial t + \nabla(\vec{v}N) = 0$$

$$\partial \vec{M}/\partial t + \nabla(\vec{v}\vec{M}) = -\nabla P$$

$$\partial E/\partial t + \nabla(\vec{v}E) = -\nabla(\vec{v}P)$$

$\vec{v}, N, \vec{M}, E$ ➡ collective velocity, baryon density, momentum and energy densities in lab frame

$$N = \gamma n$$

$$E = \gamma^2(\varepsilon + P) - P$$

$$\vec{M} = \gamma^2(\varepsilon + P)\vec{v}$$

n, ε, P ➡ baryon density, energy density, pressure in local rest frame

$P = P(n, \varepsilon)$ ➡ equation of state (EoS)

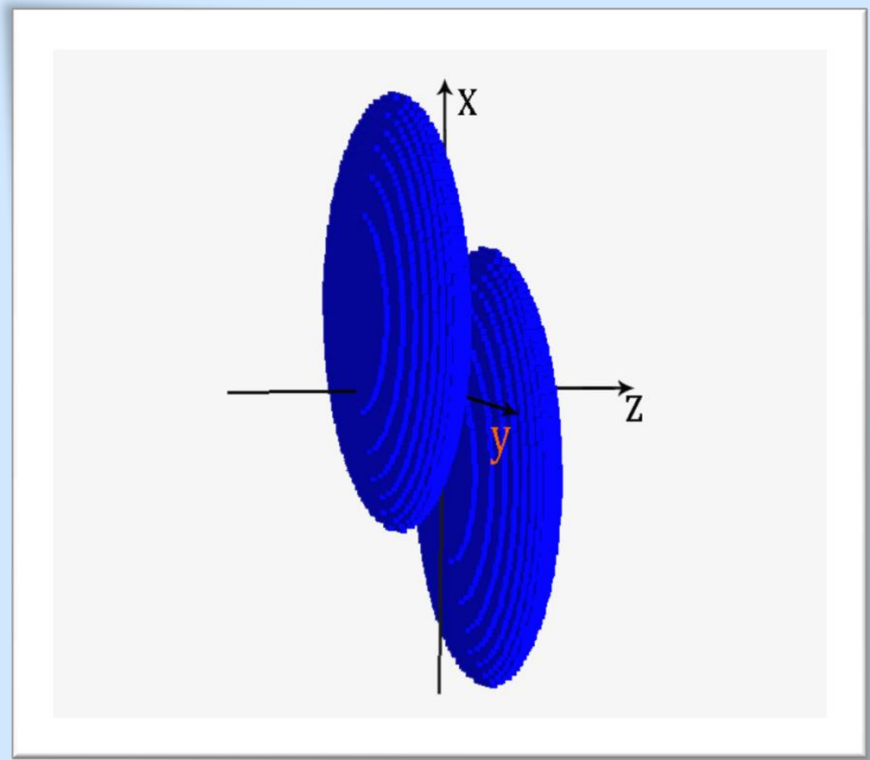
We choose cell size dx=0.1 fm, time step dt=0.01 fm/c

➡ numerically solved by using flux-corrected algorithm SHASTA

Boris & Book (1973), Rischke et al (1995)

Geometry of collision

we start from cold Lorentz-contracted nuclei



Z – beam (longitudinal) axis

X – impact parameter (b) axis

X0Z – reaction plane

X0Y – transverse plane

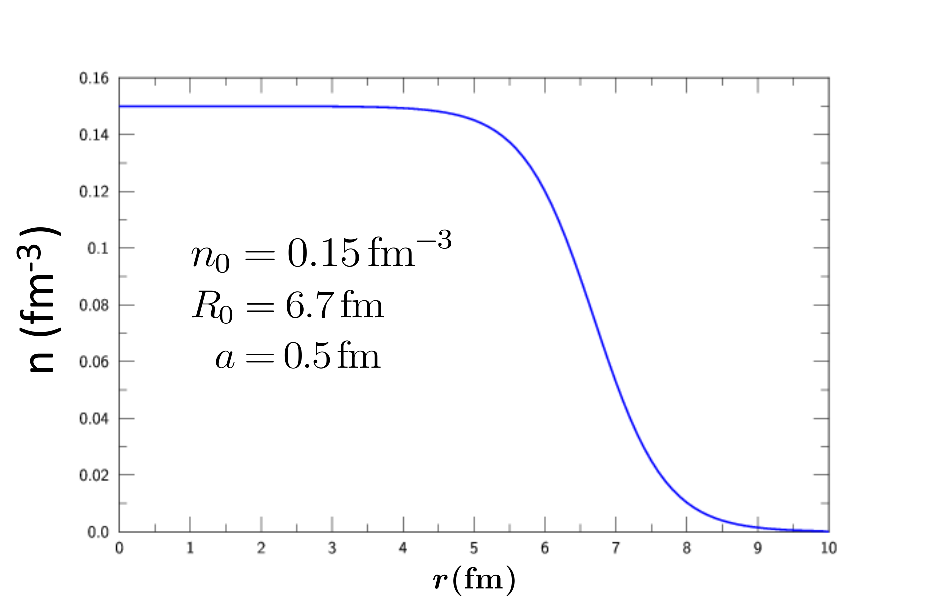
Lorentz contraction along Z - axis:

$$\gamma_0 = \sqrt{1 + E_{\rm lab}/2m_N}$$

Initial conditions

Woods-Saxon distribution for baryon density (in the rest frame of initial nucleus)

$$n(0,\vec{r}) = \frac{n_0}{1+\exp\left(\frac{|\vec{r}|-R_0}{a}\right)}$$



$$v_z = \pm\sqrt{E_{\mathrm{lab}}/(2m_N + E_{\mathrm{lab}})}$$

$$v_x = v_y = 0$$

$$N = \gamma_0 n(0,\vec{r})$$

$$M_z = \gamma_0^2 \varepsilon v_z$$

$$M_x = M_y = 0$$

$$\varepsilon = (m_N + W_0)\, n(0,\vec{r})$$

$$E = \gamma_0^2 \varepsilon$$

-16 MeV

we choose the initial cm distance between centers = $2\,(R_0 + 7a)/\gamma_0$

Equation of state ➡ $\boldsymbol{P = P(n, \varepsilon)}$

Satarov, Dmitriev, and Mishustin, Phys. Atom. Nucl. 72 (2009) 1390

We use two equations of state (EoS):

1) EoS of hadron resonce gas (**EoS-HG**) – purely hadronic system

➡ no exluded volume corrections

Skyrme-like mean field (to stabilize cold nuclei)

2) EoS with deconfinement phase transition (**EoS-PT**) – three phases:

hadronic phase (HP) ➡

hadron resonance gas with excluded volume corrections $v = v_i = 1\,\mathrm{fm}^3$

+ Skyrme-like mean field $U(n) = -\alpha n + \beta n^{7/6}$

same excluded volume for all hadrons

quark-gluon phase (QGP) ➡

bag model with perturbative corrections $B = 344\,\mathrm{MeV/fm}^3, m_s = 150\,\mathrm{MeV}$

mixed phase (MP) ➡

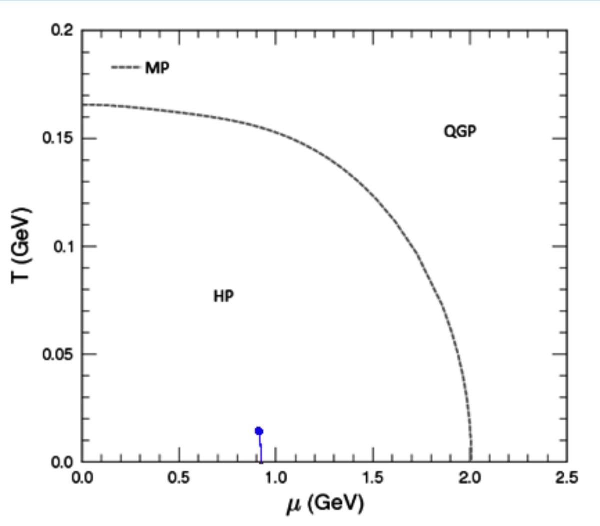
Gibbs condition of equilibrium $P_H(\mu, \mu_S, T) = P_Q(\mu, \mu_S, T)$

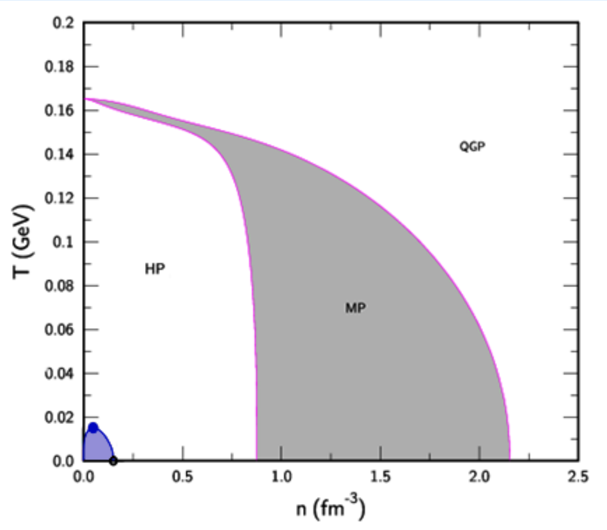
Phase diagram

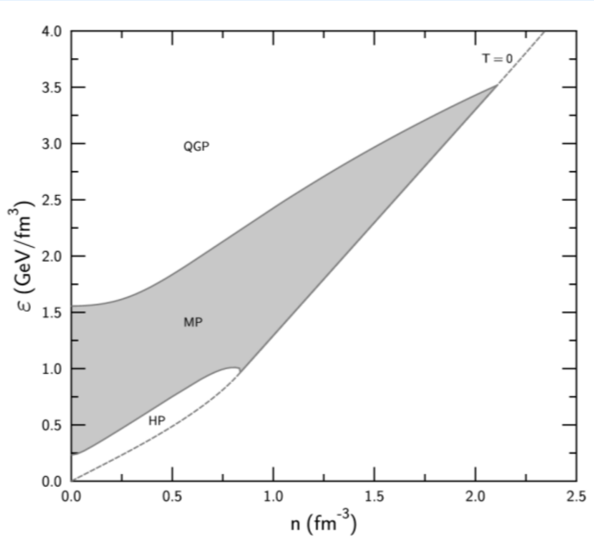
Gibbs condition for mixed phase $P_H(\mu, \mu_S, T) = P_Q(\mu, \mu_S, T)$

critical "line" $T = T_c(\mu, \mu_S)$ in $\mu - T$ plane

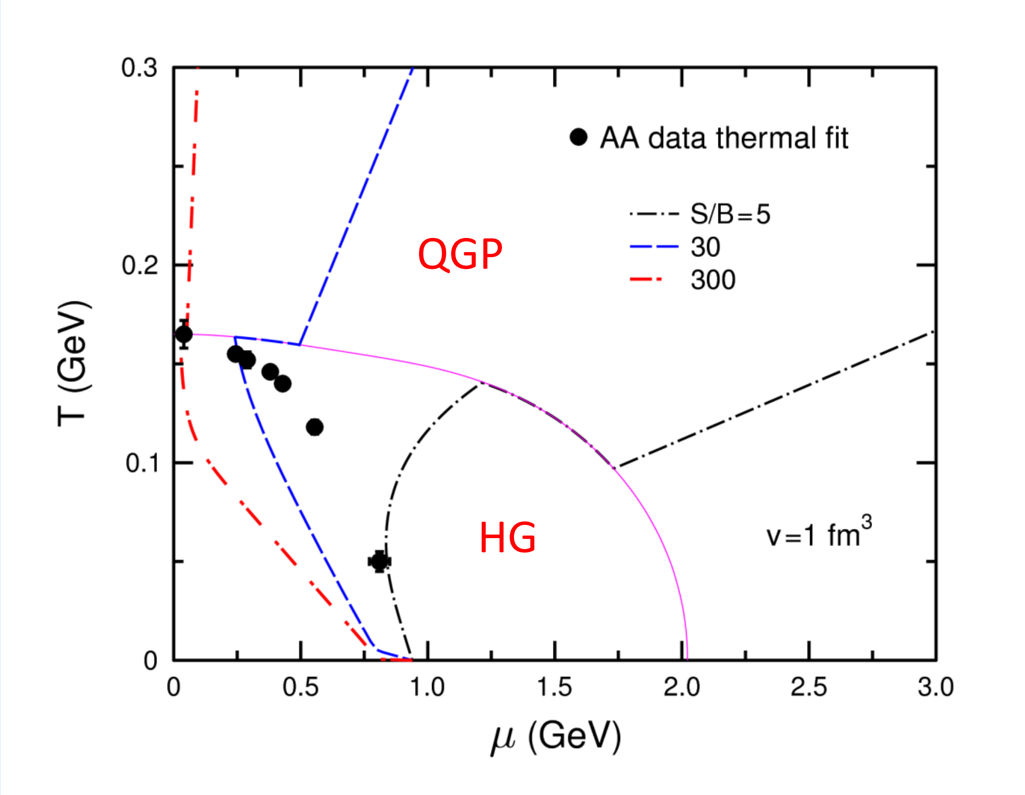
(μ_S from strangeness neutrality)





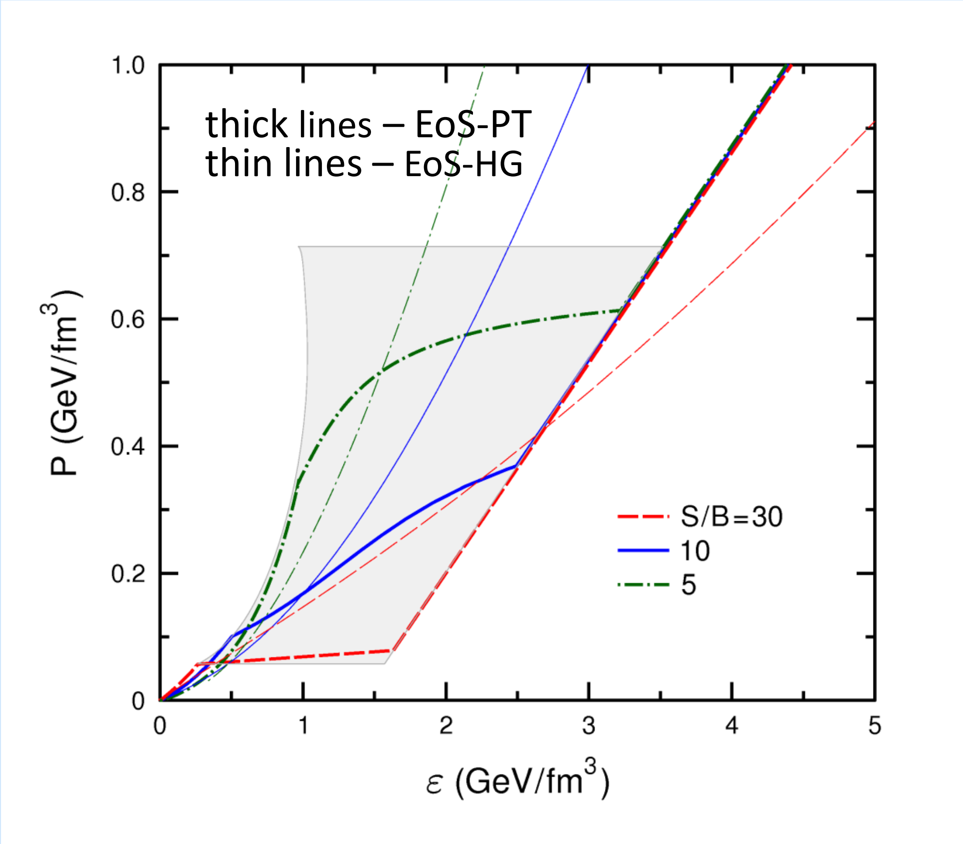


Isentropes in $\mu - T$ plane (EoS-PT)



zigzag-type behavior of adiabatic trajectories (reheating)

Isentropes in $\varepsilon - P$ plane



$$10\,\mathrm{AGeV\ Au{+}Au} \to S/B \simeq 9 \to \Delta P \simeq -0.3\,\mathrm{GeV/fm^3}$$

$$\sigma \equiv S/B = \mathrm{const}$$

⇒ jump of sound velocity

$$c_s = \sqrt{(\partial P/\partial \varepsilon)_\sigma}$$

at MP boundary

⇓

crossing of flow characteristics

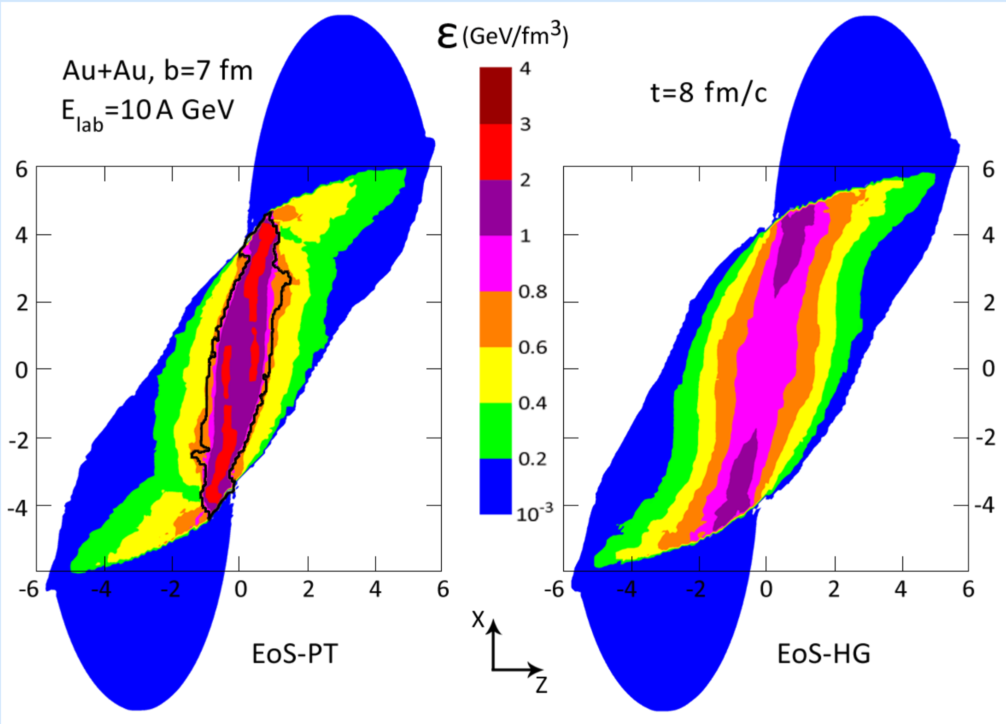
⇓

possibility of rarefaction shocks
(deflagration/rapid hadronization)

⇓

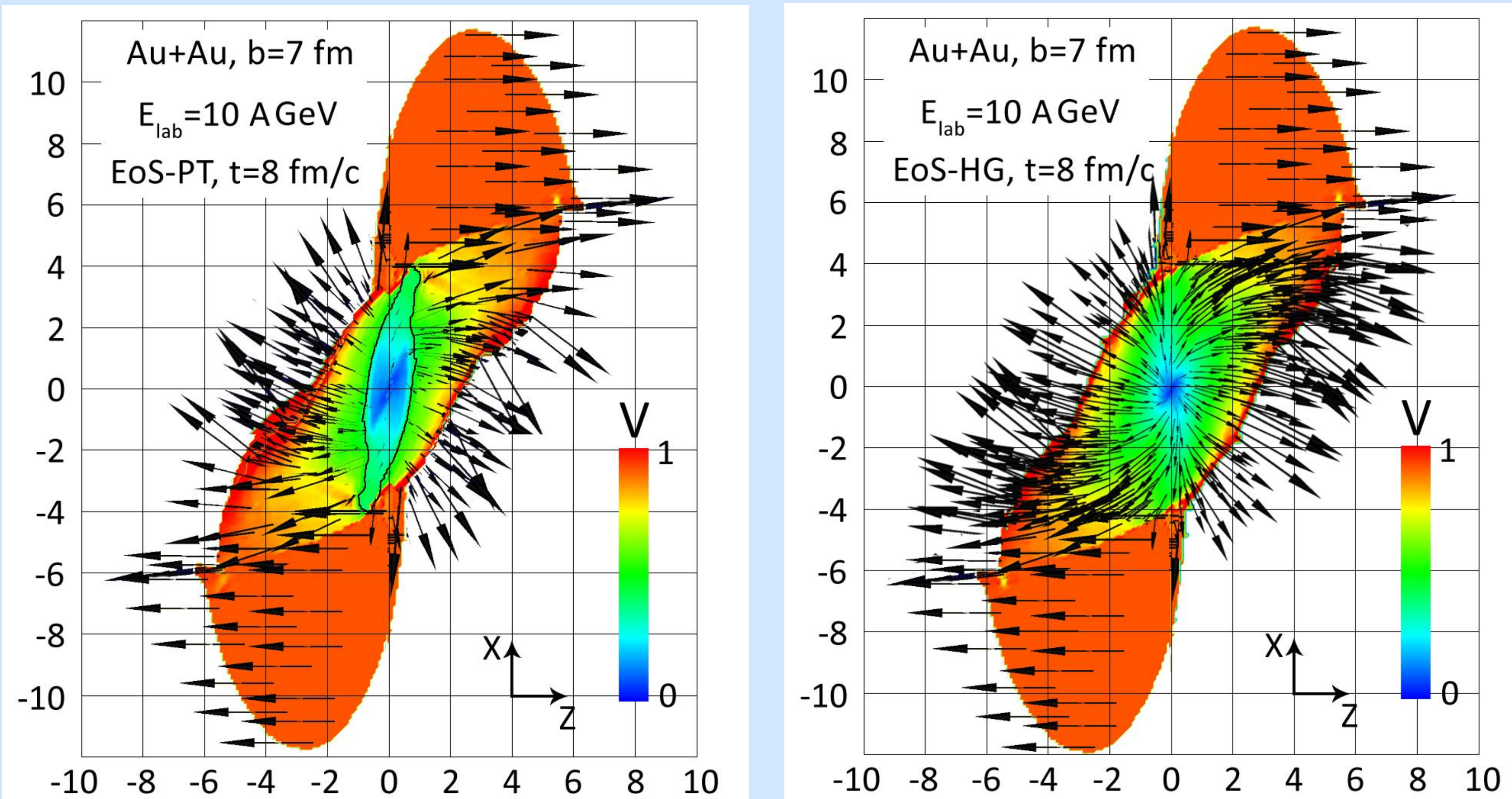
jump of collective velocity
(more pronounced at FAIR/NICA energies)

Contours of energy density in reaction plane



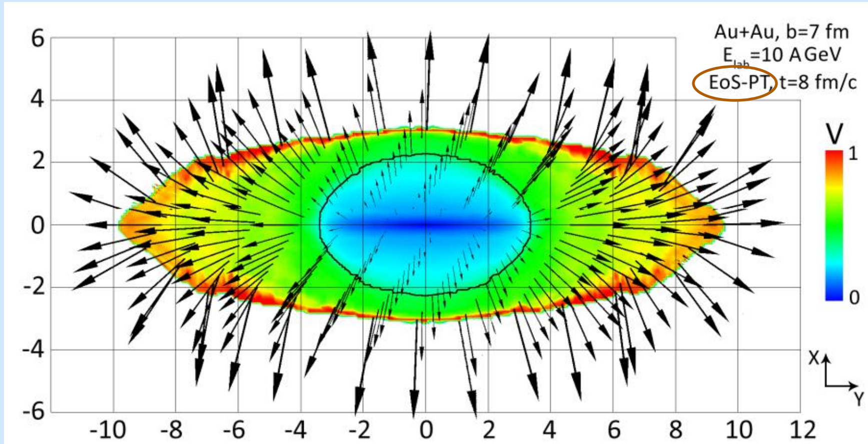
➡ larger gradients of energy densities for EoS-PT

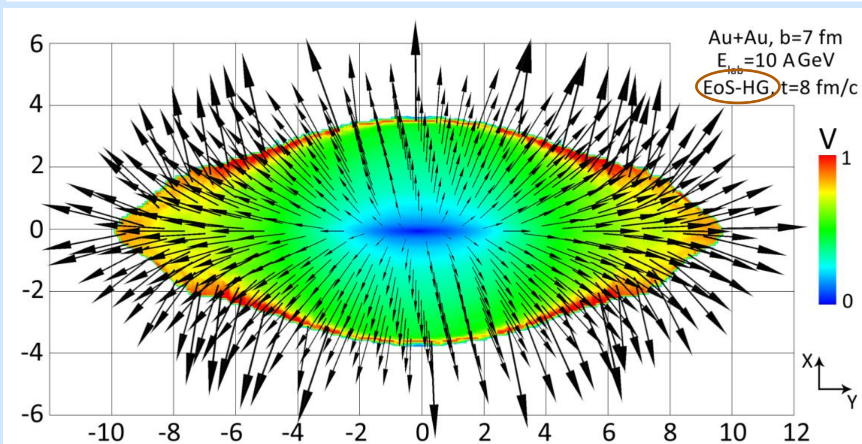
Collective velocity field in reaction plane



- jump of velocity at MP boundary (solid line in left panel)
- more pronounced "antiflow" $(v_x/v_z < 0)$ for EoS-PT

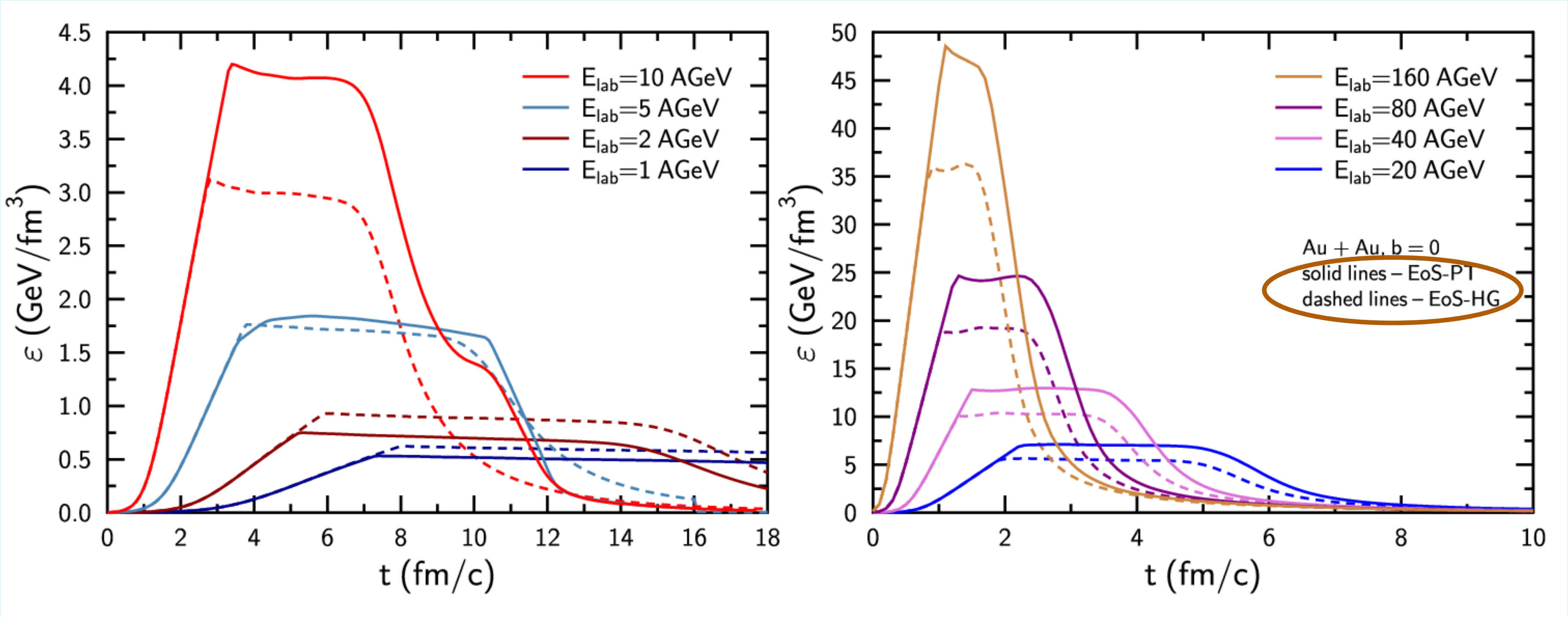
Collective velocity field in transverse plane





- larger azimuthal anisotropy of transverse velocity
- large gradients of v_x, v_y at MP boundary
- smooth velocity field

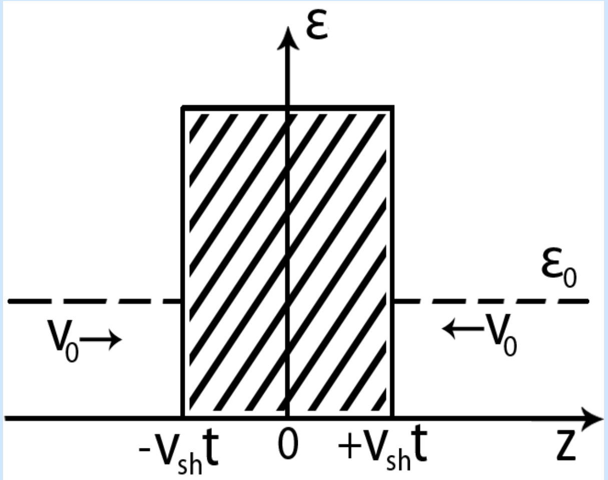
Energy density evolution in central box $(|x|, |y|, \gamma_0|z| < 1\,\mathrm{fm}/c)$

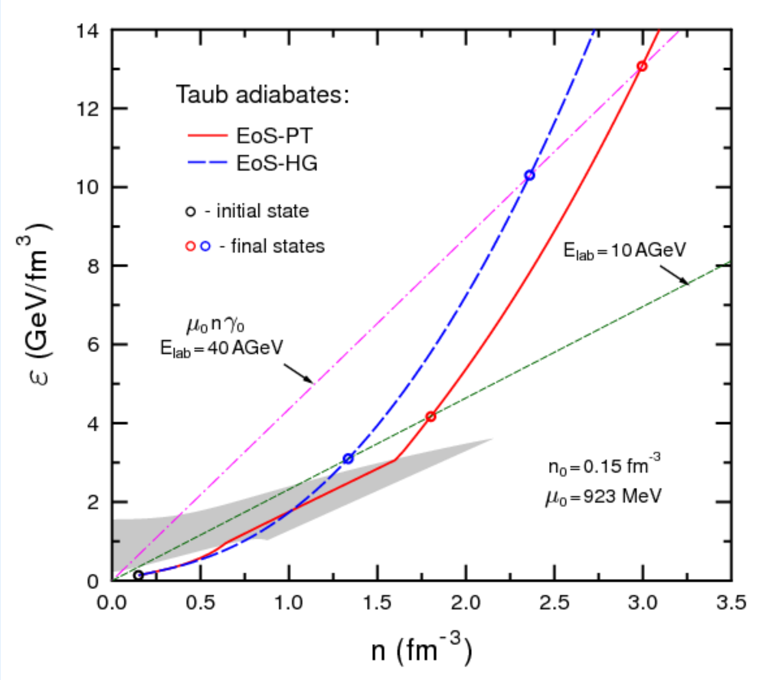


- larger energy densities for EoS-PT at $E_{\mathrm{lab}} \gtrsim 5\,\mathrm{A\,GeV}$
- $\varepsilon_{\max}$ and $n_{\max}$ values are well reproduced in 1D shock model

One-dimensional shock wave

collision of two slabs





T^{0z}, T^{zz}, nU^z continuous in shock front rest frame

Taub adiabate $(P_0 = 0, \varepsilon_0 = \mu_0 n_0)$:

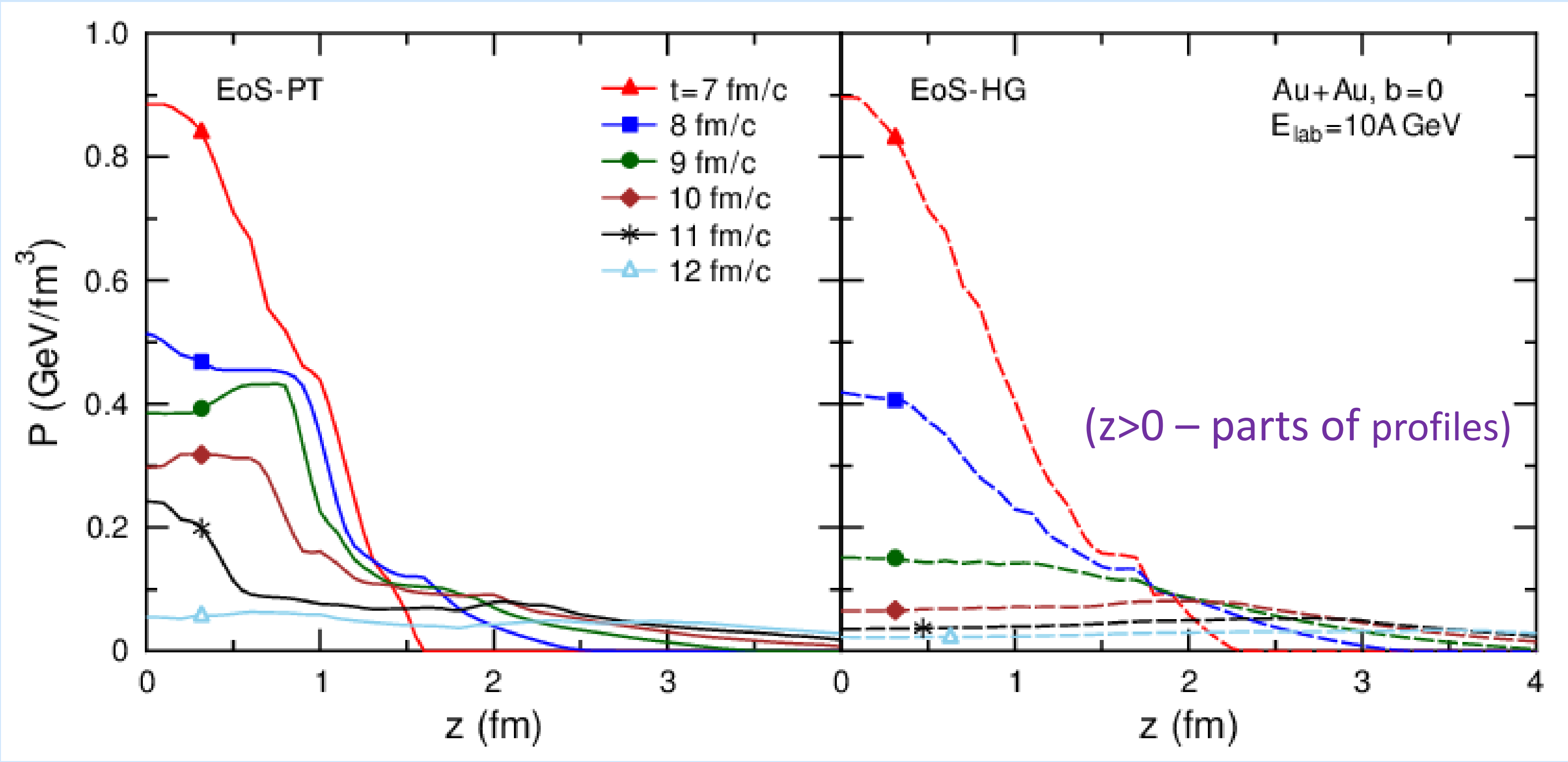
$$\varepsilon_0 (P + \varepsilon_0)\, n^2 = \varepsilon (P + \varepsilon)\, n_0^2$$

"stopping" condition: $\dfrac{\varepsilon}{n} = \gamma_0 \dfrac{\varepsilon_0}{n_0}$

final states in mixed phase region at Elab=3.7-7.2 AGeV

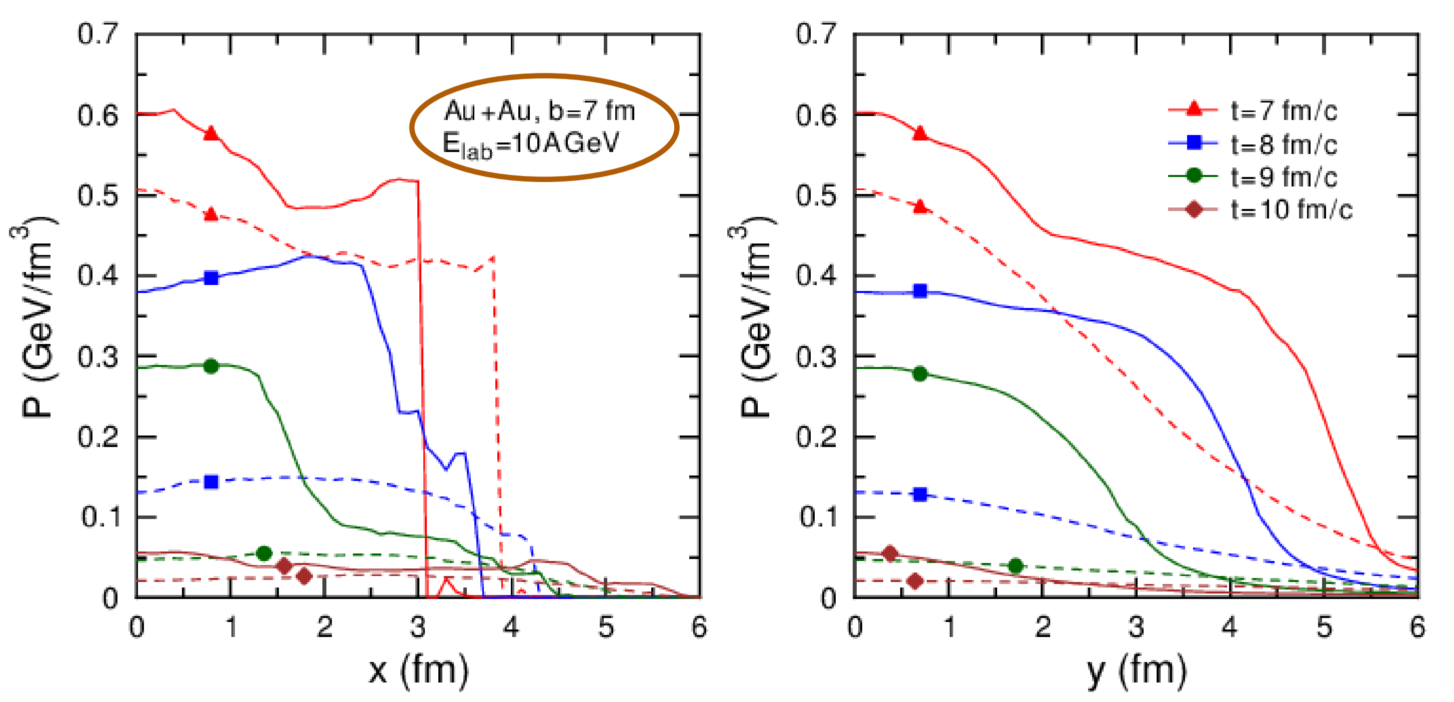
maximal compression is larger for EoS-PT (at fixed Elab)

Pressure profiles in central 10 AGeV Au+Au collision



- EoS-PT calculation: presence of pressure jumps at $t \simeq 7.5 - 10.5\,\mathrm{fm}/c$
- "extrapush" of matter in longitudinal direction -> broadening of dN/dy

Pressure profiles in transverse plane



thick lines: EoS-PT

dashed lines: EoS-HG

- EoS-PT: larger pressure gradients at intermediate stage (7.5-9.5 fm/c)
- larger transverse flows

Excitation function of momentum anisotropy

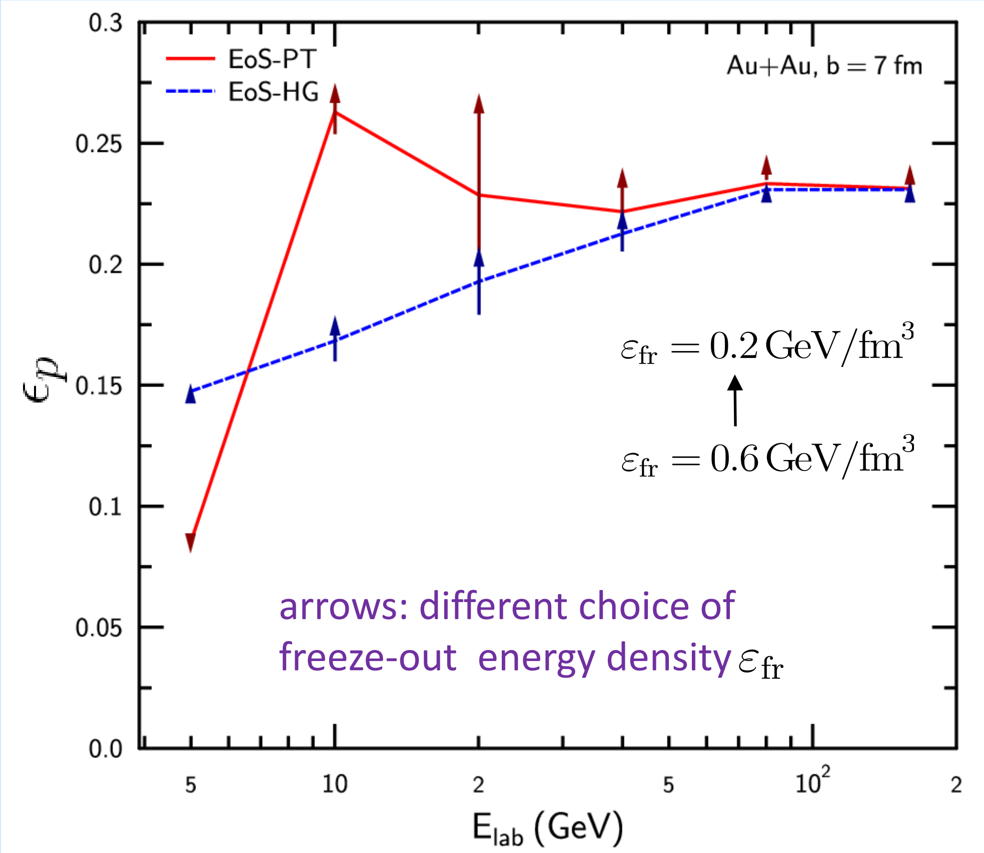
$$\epsilon_p = \frac{\int dxdy\,(T^{xx} - T^{yy})}{\int dxdy\,(T^{xx} + T^{yy})}$$

$$T^{xx} = (\varepsilon + P)\gamma^2 v_x^2 + P$$

$$T^{yy} = (\varepsilon + P)\gamma^2 v_y^2 + P$$

ϵ_p - analogue of elliptic flow

(Kolb, Sollfrank & Heinz, 1999)



Eos-PT: maximum of ϵ_p at $E_{\rm lab} \simeq 10\,{\rm A\,GeV}$

see Petersen & Bleicher, 2010

Hadronic momentum spectra

$$E\frac{d^3N_i}{d^3p} = \frac{d^3N_i}{dy\,d^2p_T} = \frac{g_i}{(2\pi)^3}\int d\sigma_\mu p^\mu \left\{\exp\left(\frac{p_\nu u^\nu - \mu_i}{T}\right) \pm 1\right\}^{-1}$$

instantaneous freeze-out (Cooper & Frye, 1974)

we assume isochronous freeze-out hypersurface ($t = t_{\mathrm{fr}} = \mathrm{const}$)

Contribution of resonance decays:

$$d\sigma_\mu = d^3x \cdot \delta_{\mu,0}$$

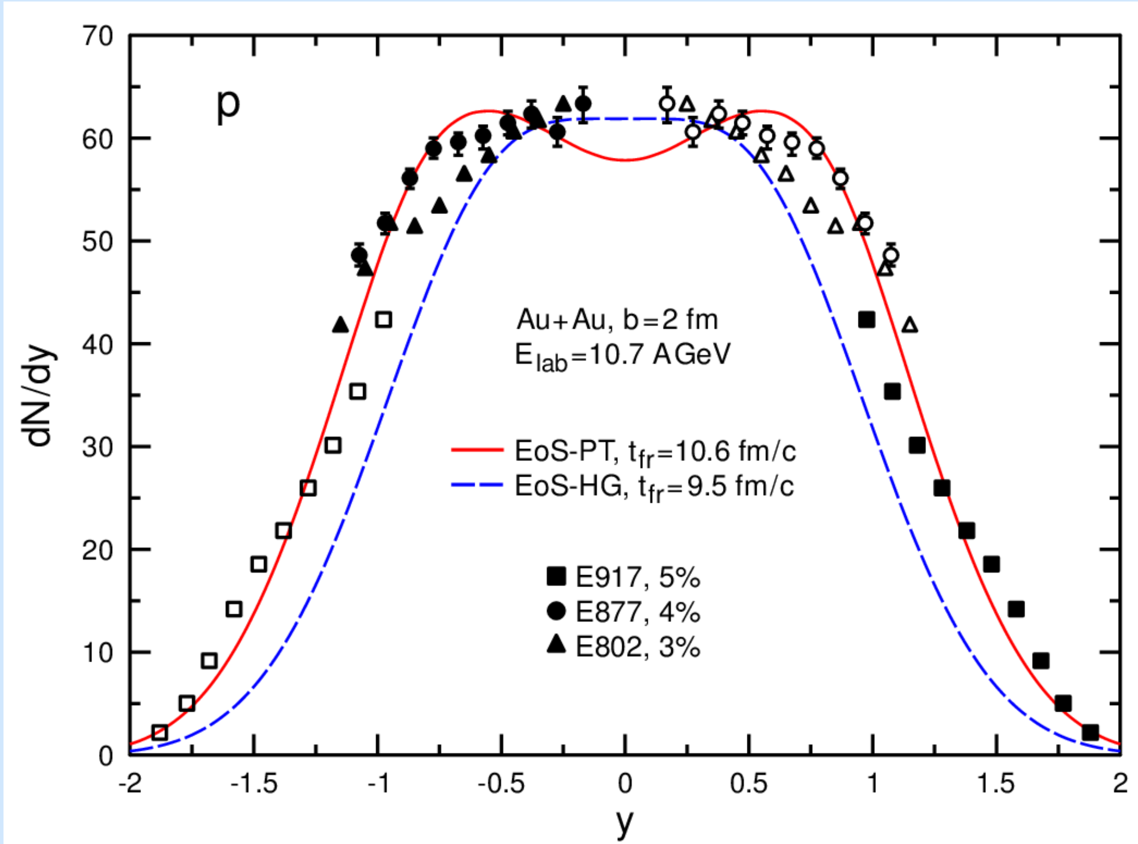
$$E\frac{d^3N_{R\to iX}}{d^3p} = \frac{1}{4\pi q_0}\int dN_R\,\delta\left(\frac{pp_R}{m_R} - E_0\right)$$

in zero-width approximation

$$E_0 = \sqrt{m_i^2 + q_0^2} = \frac{m_R^2 + m_i^2 - m_X^2}{2m_R}$$

parameter t_{fr} is chosen to achieve the best fit of experimental data

Rapidity distribution of protons



broader distribution for EoS-PT (better agreement)

Parameters of collective flow

Directed flow

(of i-th hadrons)

$$v_1^{(i)}(y) = \frac{\int d^2p_T \, \cos\varphi \, E \, d^3N_i/d^3p}{\int d^2p_T E \, d^3N_i/d^3p}$$

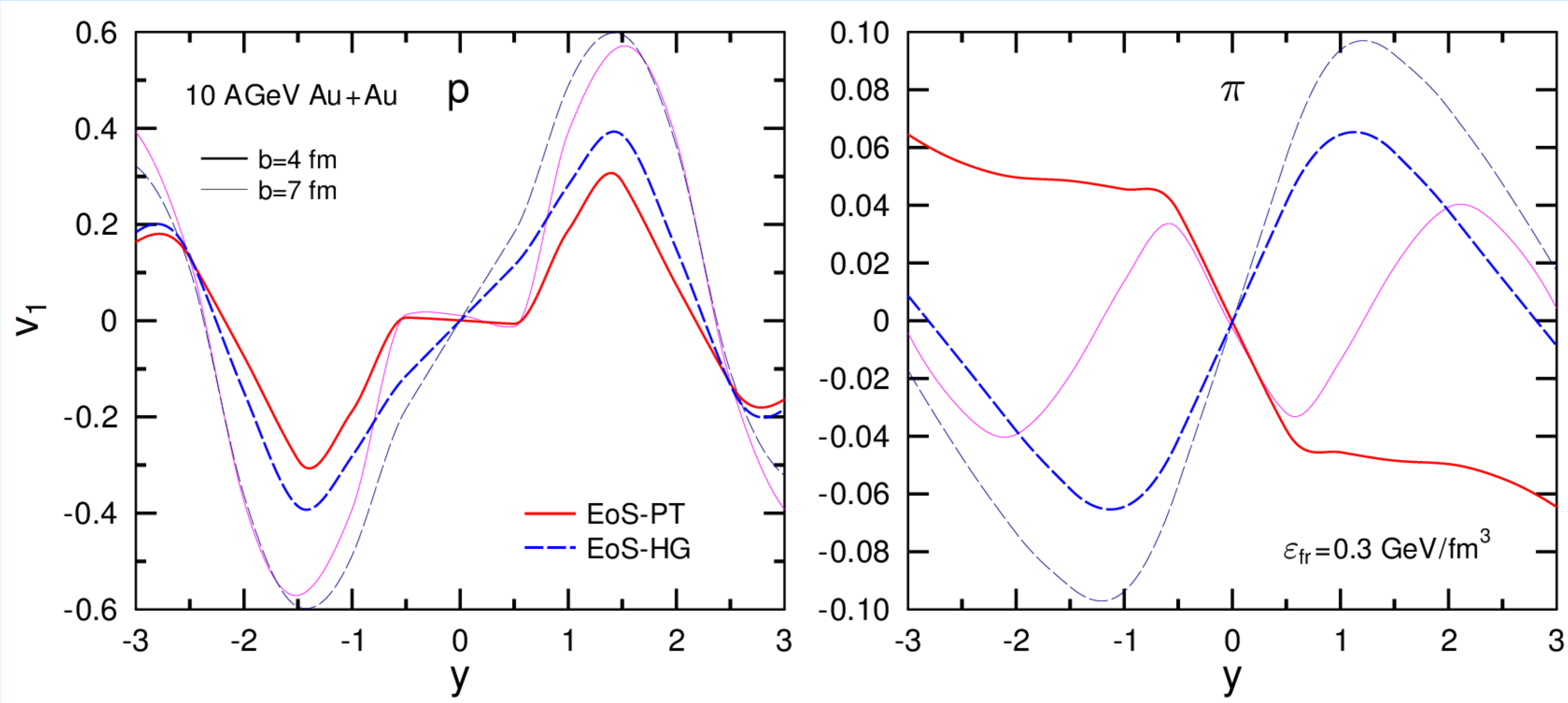
Elliptic flow

$$v_2^{(i)}(y) = \frac{\int d^2p_T \, \cos(2\varphi) \, E \, d^3N_i/d^3p}{\int d^2p_T E \, d^3N_i/d^3p}$$

$$p_T = (p_T \cos\varphi, p_T \sin\varphi)$$

resonance decays are included in d^3N_i/d^3p

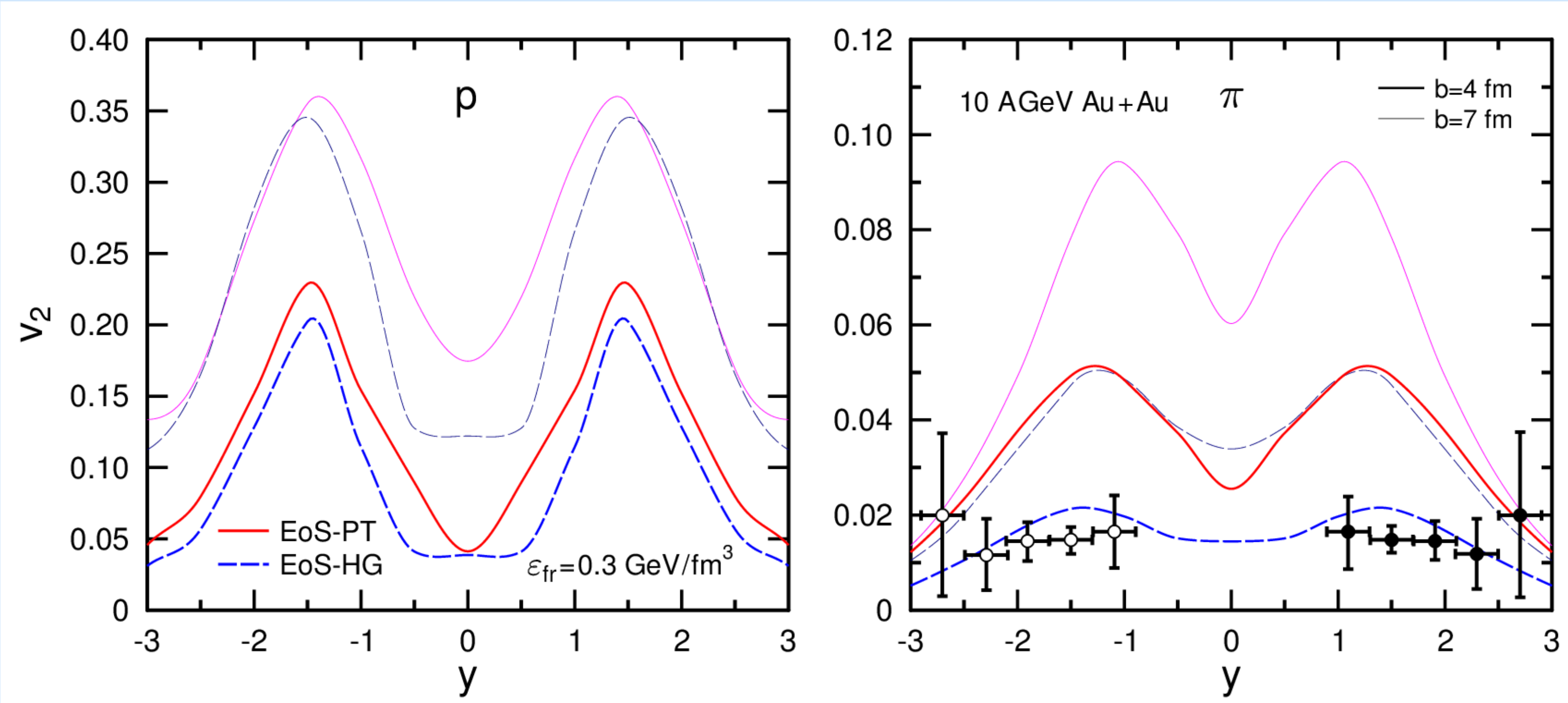
Directed flow in 10 AGeV Au+Au



EoS-PT: formantion of “antiflow” $(v_1'(y) < 0)$ in central rapidity region

Rischke et al. (1995), Csernai & Röhrich (1999) - antiflow as PT-signature

Elliptic flow in 10 AGeV Au+Au



- strong dependence on centrality
- larger elliptic flow for EoS-PT

Conclusions

- Calculations with Eos-PT as compared with EoS-HG leads to:

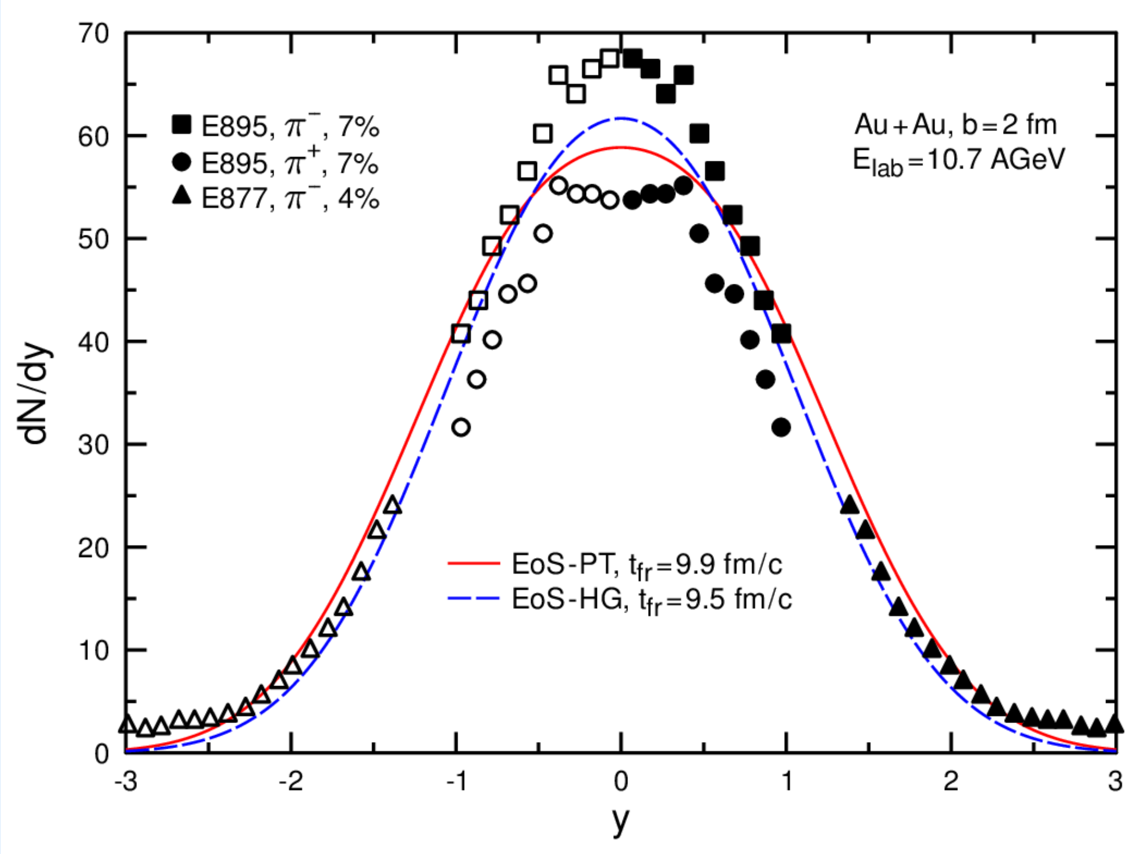
  - broader proton rapidity distributions
  - formation of antiflow for pions and protons
  - enhancement of elliptic flow

- These effects are mainly due to formation of a deflagration shock in the mixed phase

- Strongest sensitivity to PT at Elab ~ 10 A GeV (good for FAIR and NICA).

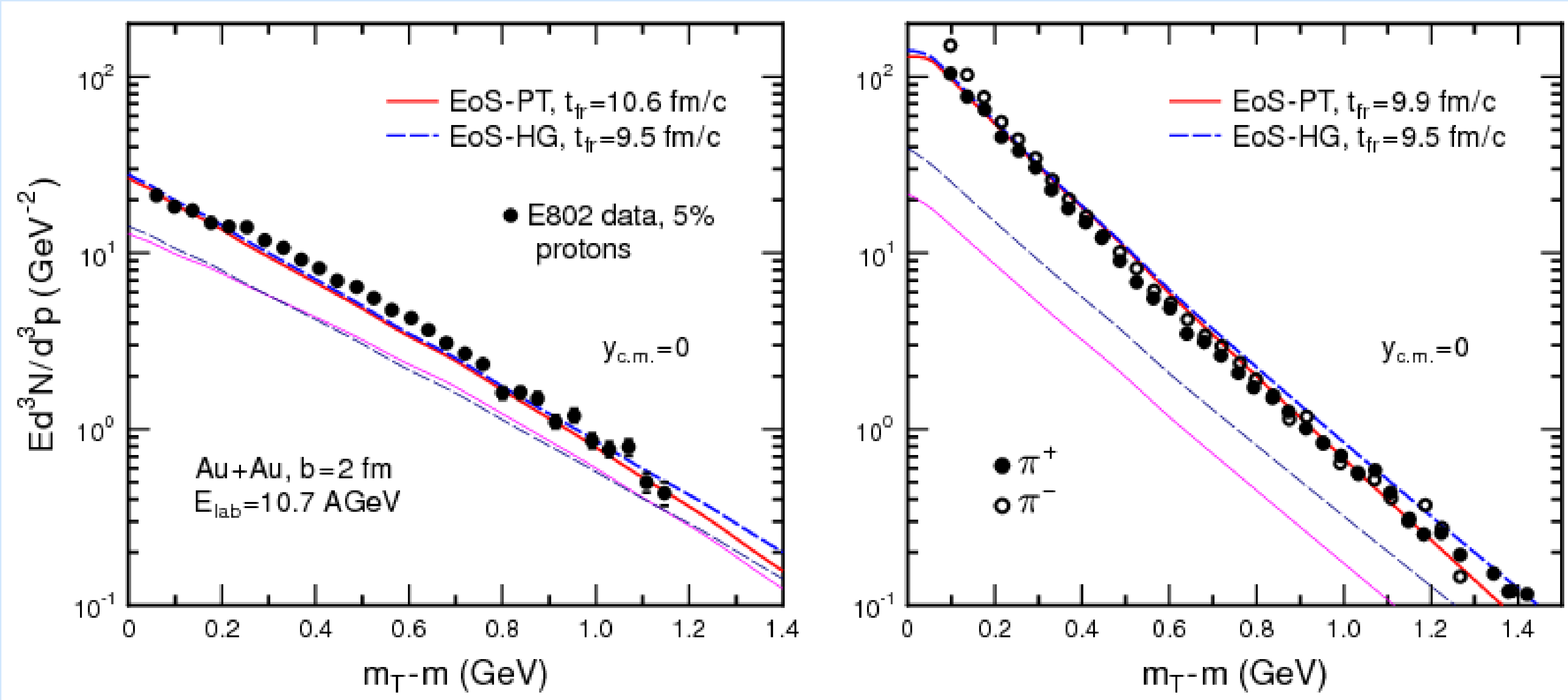
similar conclusion by C. Arsene et al. , Phys. Rev. C75 (2007)

Rapidity distributions of pions



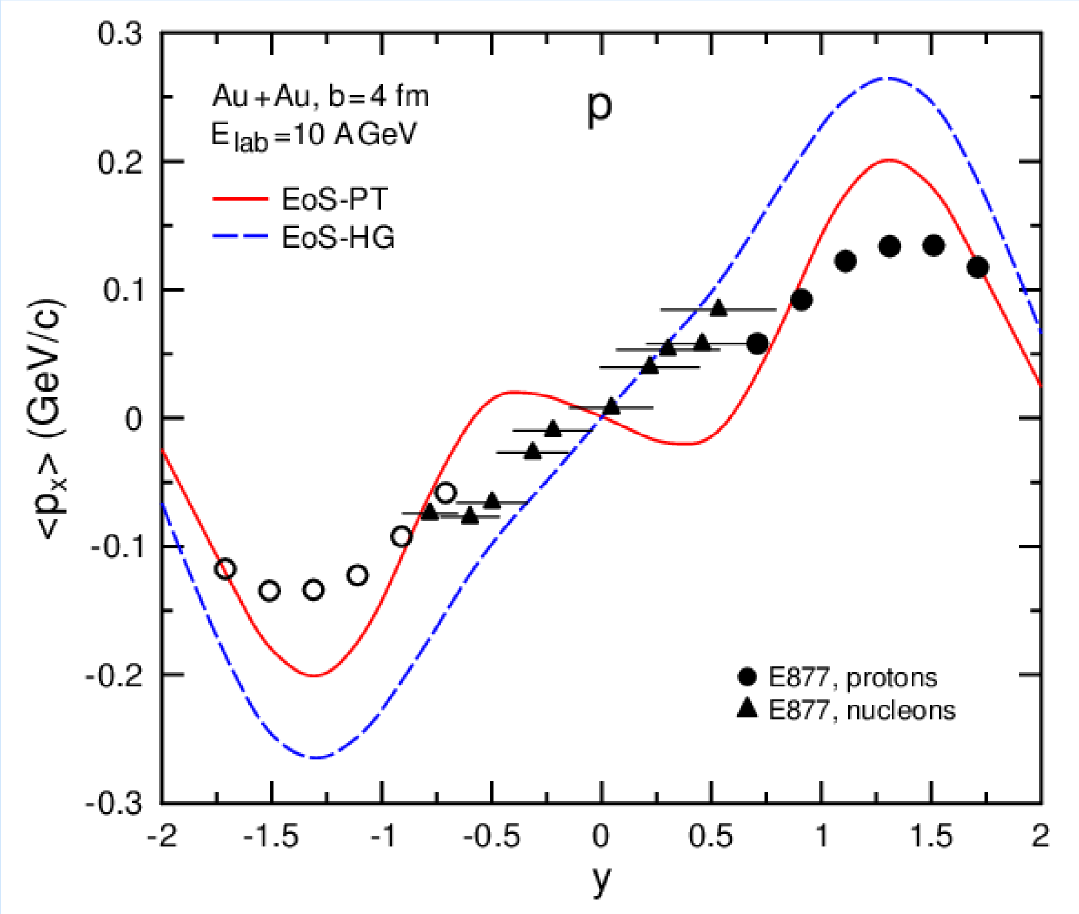
- low sensitivity to EoS
- smaller freeze-out time (as compared to protons in EoS-PT case)

p_T - distributions of protons and pions



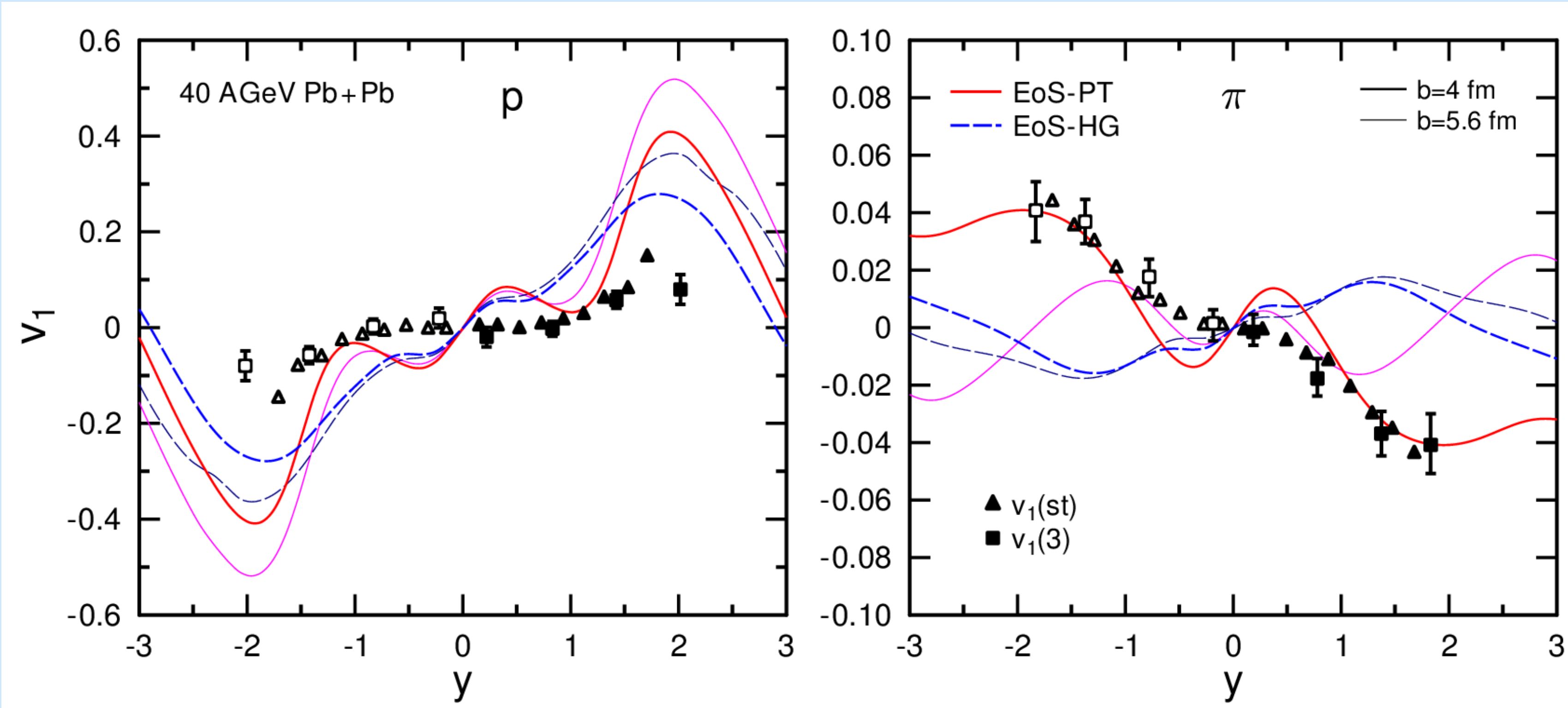
low sensitivity to EoS

Mean transverse momentum (sideflow) of protons



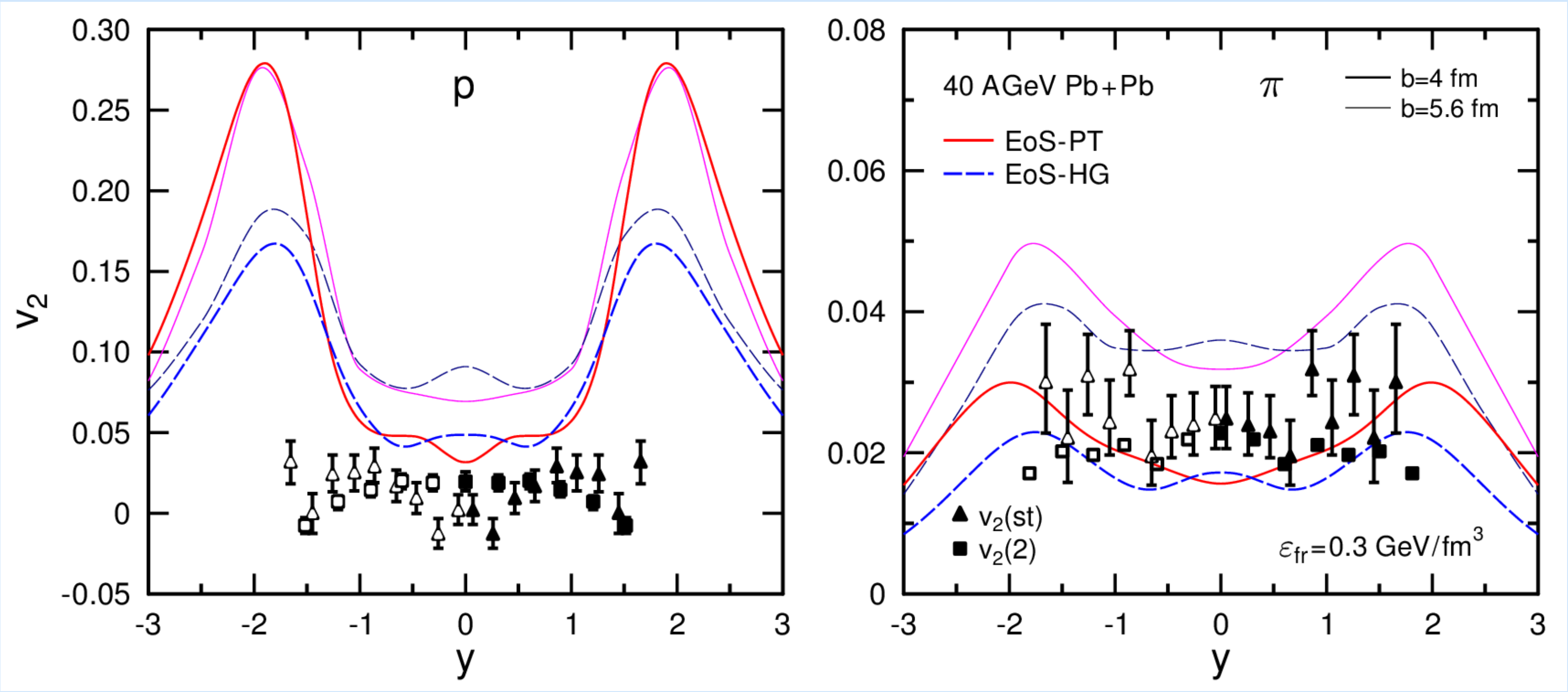
antiflow in the EoS-PT calculation

Directed flow at Elab=40 A GeV



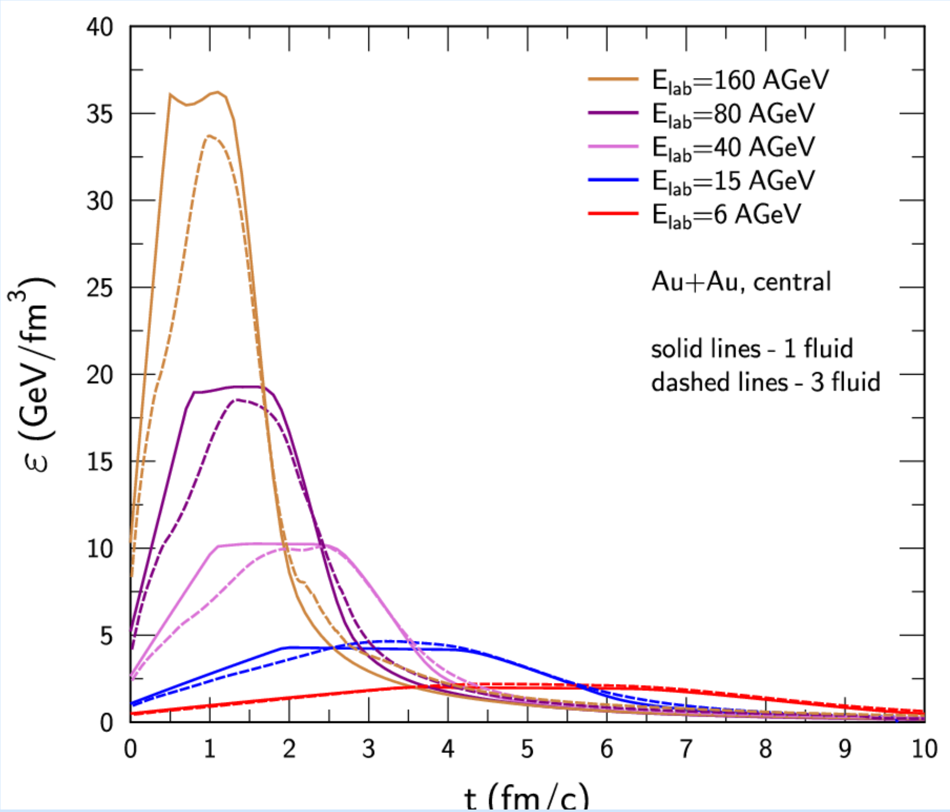
- pion antiflow for EoS-PT
- calculation without PT disagrees with pion data

Elliptic flow in 40 AGeV Pb+Pb



weak sensitivity to EoS

Comparison of one- and three-fluid models

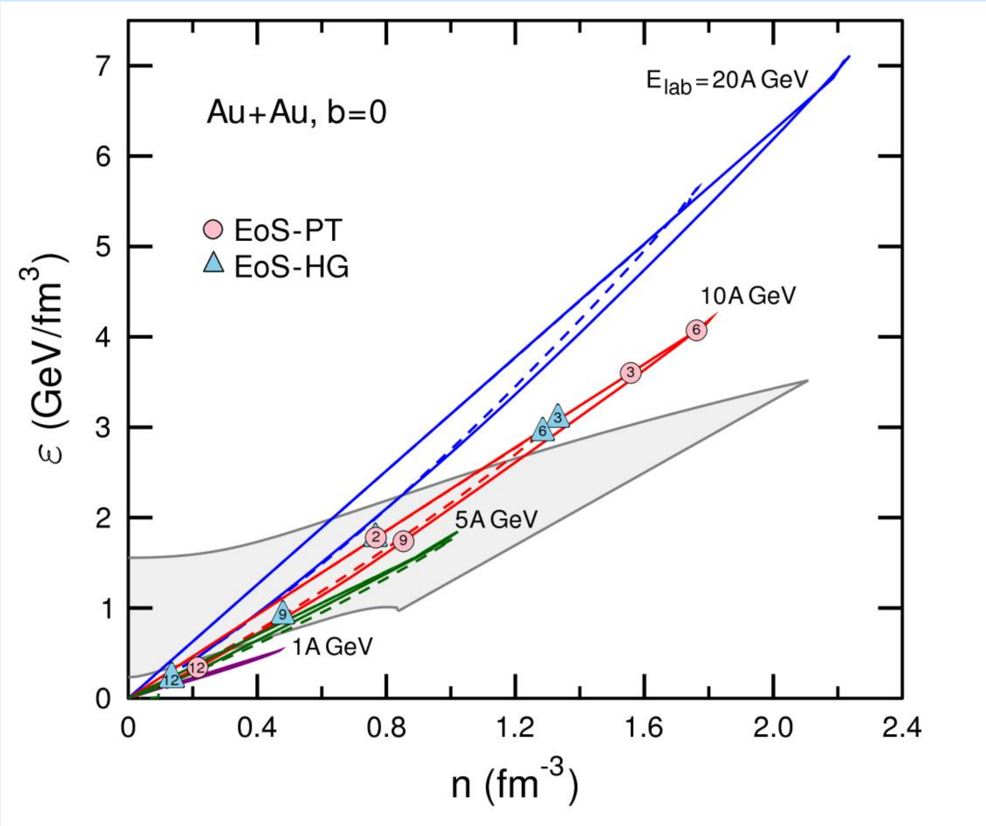


three-fluid model:
Ivanov, Russkikh & Toneev (2006)

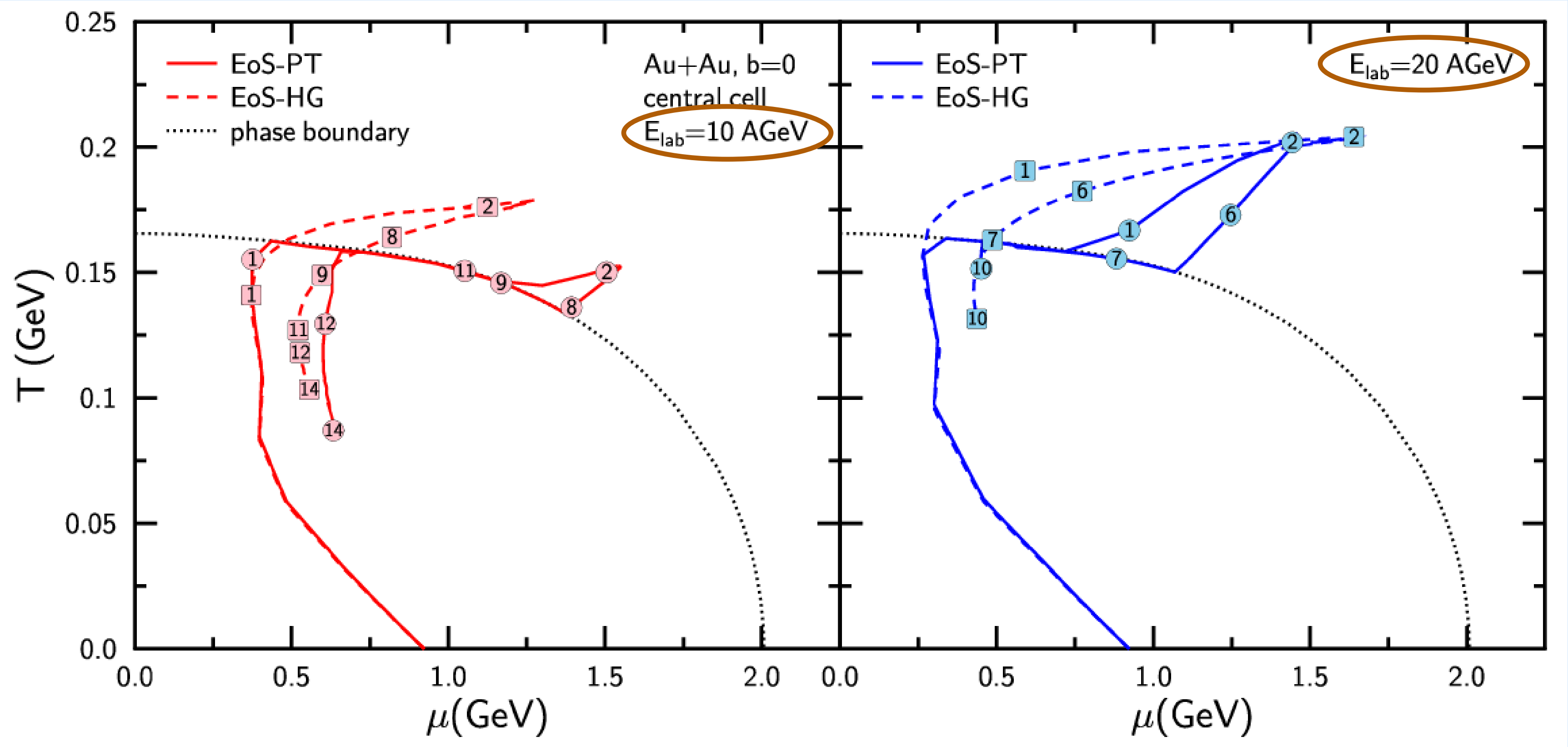
Calculated with EoS-HG and sharp density profiles of initial nuclei

transparency effects are relatively small in central collisions at Elab < 30 A ГэВ

Dynamical trajectories in central box

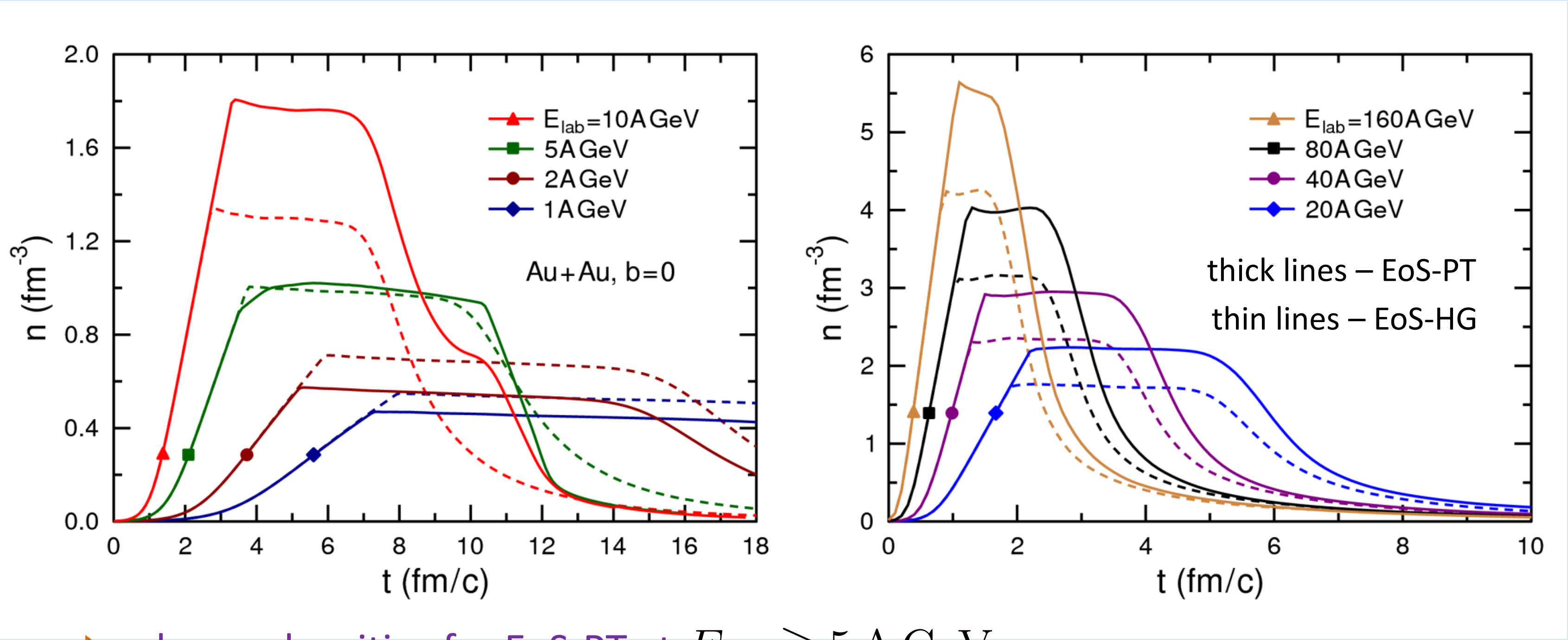


Dynamical trajectories in μ-T plane



trajectories of final states are practically not sensitive to phase transition

Baryon density evolution in central box



- larger densities for EoS-PT at $E_{\rm lab} \gtrsim 5\,\mathrm{A\,GeV}$
- $n_{\max}$ values are well reproduced in 1D shock model

Integrals of motion

total energy

$$E_{tot} = \int \varepsilon dV = const$$

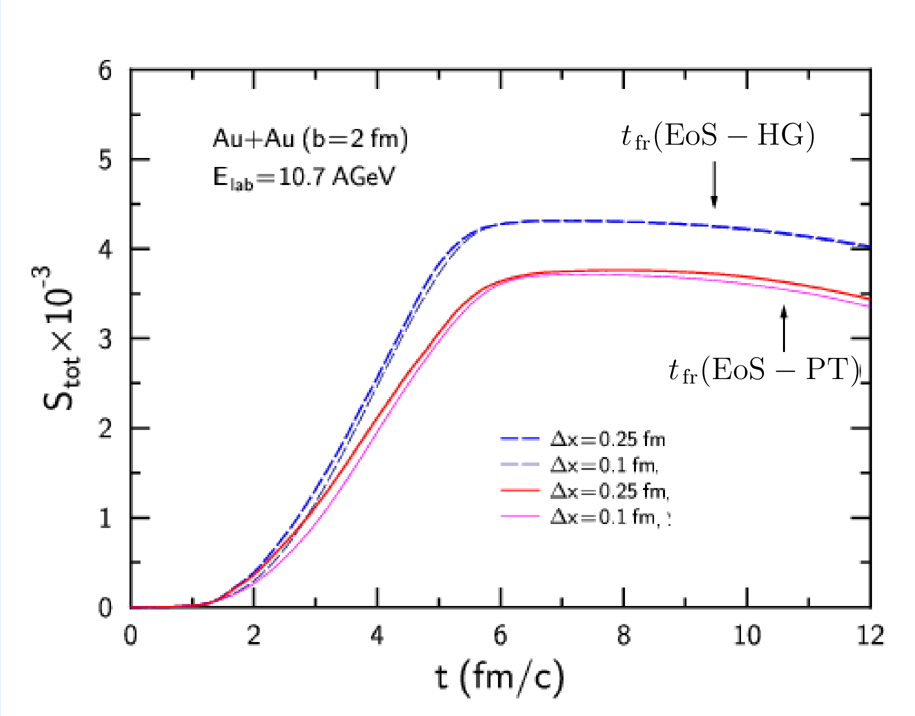
total baryon charge

$$B_{tot} = \int n dV = const$$

total entropy

$$S_{tot} = \int s dV$$

$$s = \frac{\varepsilon + P - \mu n}{T}$$



$$S/B \simeq 9.5\,(\mathrm{EoS-PT})$$

$$S/B \simeq 11.5\,(\mathrm{EoS-HG})$$

EoS-PT: hadron gas with excluded volume corrections

following Rischke, Gorenstein et al., Z. Phys. C51 (1991) 485

$$P = \sum_i P_i^{id}(\mu_i - P v_i, T)$$

P_i^{id} – pressure of ideal gas

$v_i = v = 1\ \mathrm{fm}^3$ excluded volume

Chemical potential for species i

$$\mu_i = \mu_B B_i + \mu_s S_i$$

Baryonic charge

Strangeness

μ_S is determined from the net strangeness neutrality

$$n_S = \sum_i S_i n_i = 0$$

$$n = \sum_i B_i n_i$$

$$n = (\partial_\mu P)_T = \frac{\sum_i n_i^{id}(\mu_i - P v_i, T)}{1 + \sum_i n_i^{id}(\mu_i - P v_i, T) v_i} \qquad s = (\partial_T P)_\mu = \frac{\sum_i s_i^{id}(\mu_i - P v_i, T)}{1 + \sum_i n_i^{id}(\mu_i - P v_i, T) v_i}$$

$$\varepsilon = Ts + n\mu + n_S \mu_S - P \quad \Rightarrow \quad P = P(n, \varepsilon)$$

we include all known hadrons with masses below 2 GeV (in zero width approximation)

EoS-PT: quark-gluon phase in the bag model

$$P_Q(\mu,\mu_S,T) = (\tilde{N}_g + \frac{21}{2}\tilde{N}_f)\frac{\pi^2}{90}T^4 + \tilde{N}_f(\frac{T^2\mu^2}{18} + \frac{\mu^4}{324\pi^2}) +$$
$$+\frac{1-\xi}{\pi^2}\int_{m_s}^{\infty} d\varepsilon(\varepsilon^2 - m_s^2)^{3/2}\left\{\left[e^{\frac{\varepsilon-\mu_s}{T}}+1\right]^{-1} + \left[e^{\frac{\varepsilon+\mu_s}{T}}+1\right]^{-1}\right\} - B$$

$$\widetilde{N}_g = 16(1 - 0.8\xi)$$
$$\widetilde{N}_f = 2(1 - \xi)$$
$$m_u = m_d = 0$$

perturbative correction $(\xi \sim \alpha_s)$

ξ, B, m_s – parameters of the model

$\xi=0.2$ - extracted from lattice data

$B^{1/4}$=230 MeV

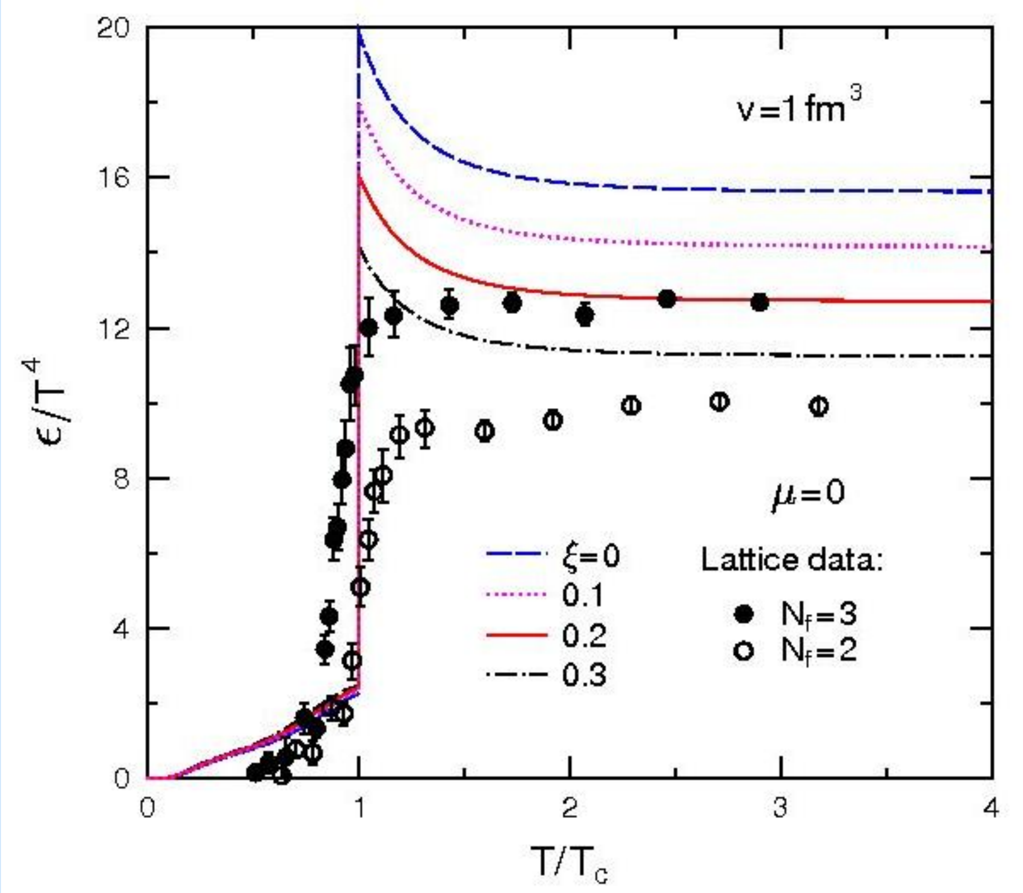
m_s= 150 MeV

$T_c(n=0) \approx 165$ MeV, $\mu_c(T=0) \approx 2$ GeV

chemical potential of s-quarks

$$\mu_s = \frac{\mu}{3} - \mu_S$$

Comparison with lattice data at μ=0



Lattice data - [F. Karsch , et al ; hep-lat / 0305025]

we choose

$\xi = 0.2$

Deconfinement phase transition (HG-QGP)

Gibbs conditions of phase equilibrium: $P_H(\mu_B, T) = P_Q(\mu_B, T)$

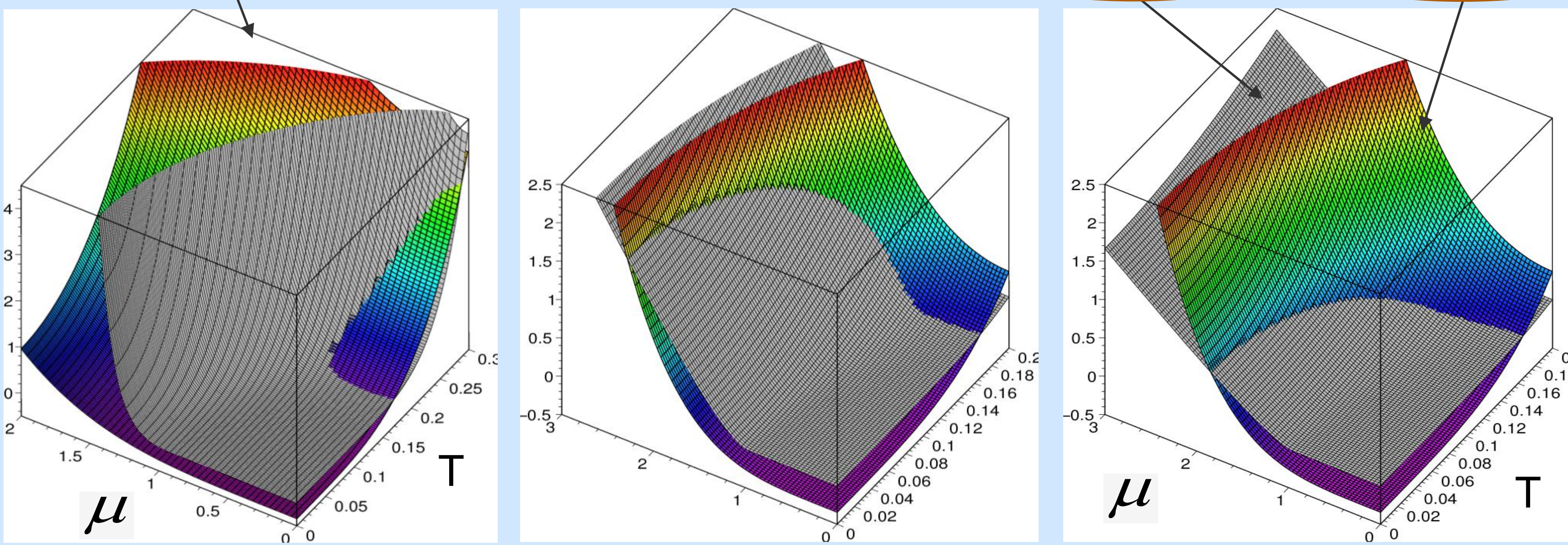
Compare pressures of two phases as functions of T, μ

unphysical phase diagram

let $\mu_S = 0$

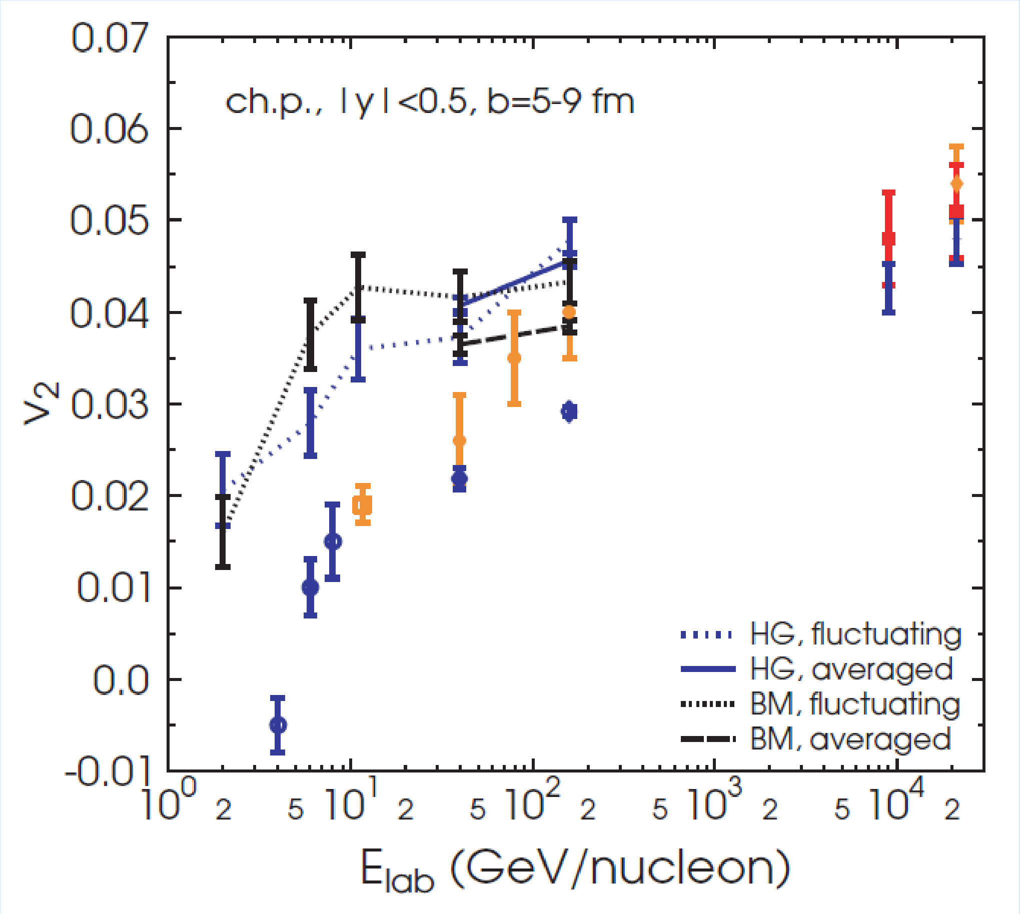
Hadronic phase

Quark phase



$v = 0$ $\quad$ $v = 0.5\, fm^3$ $\quad$ $v = 1\, fm^3$

Elliptic flow for different Elab



Petersen & Bleiher, 2010