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Non-equilibrium phase transitions in expanding matter

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- **Introduction: Effects of fast dynamics**
- **Fluctuations of the order parameter**
- **Chiral fluid dynamics with damping and noise**
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This talk is based on two recent publications:

T. Koide, G. Danikol, G. Torrieri and I. Mishustin, Dynamics and stability of chiral fluid, to be submitted;

M. Nahrgang, M. Bleicher, S. Leupold and I. Mishustin, The impact of dissipation and noise on fluctuations in chiral fluid dynamics, arXiv:1105.1962 [nucl-th]

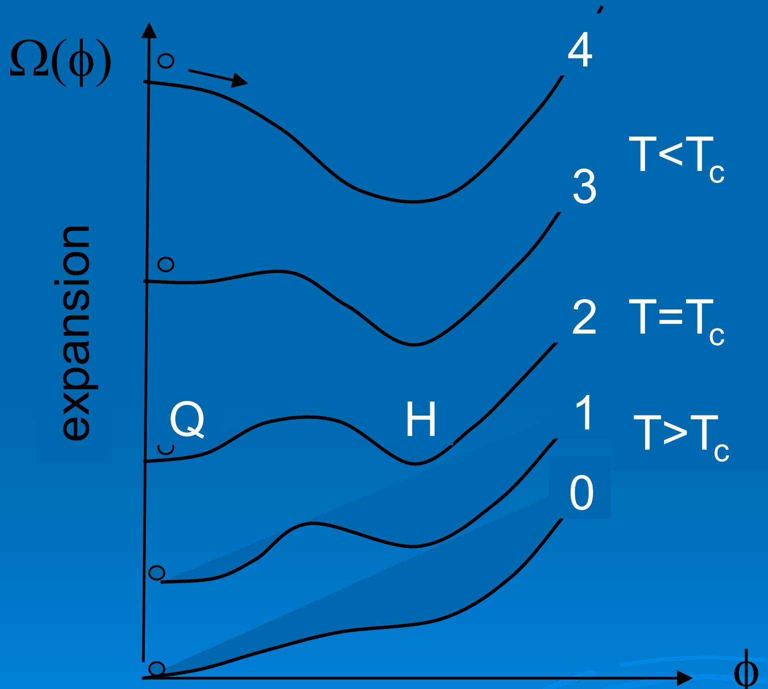
Effects of fast dynamics

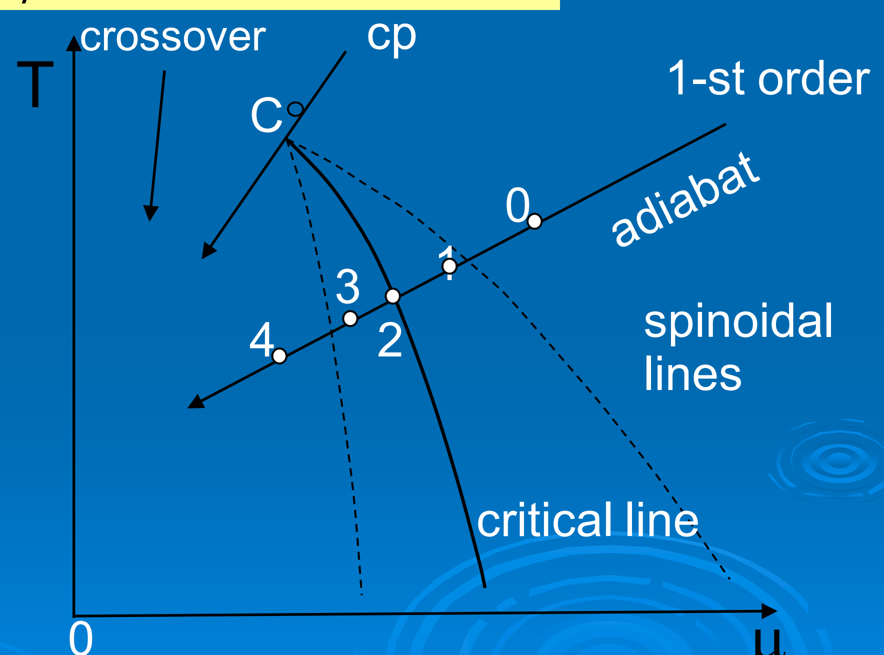
Effective thermodynamic potential for a 1st order transition

$$\Omega(\phi;T,\mu)=\Omega_0(T,\mu)+\frac{a}{2}\phi^2+\frac{b}{4}\phi^4+\frac{c}{6}\phi^6$$

a,b,c are functions of T and μ

Equilibrium ϕ is determined by $$\frac{\partial\Omega}{\partial\phi}=0\Rightarrow P=-\Omega(\phi_{eq})$$

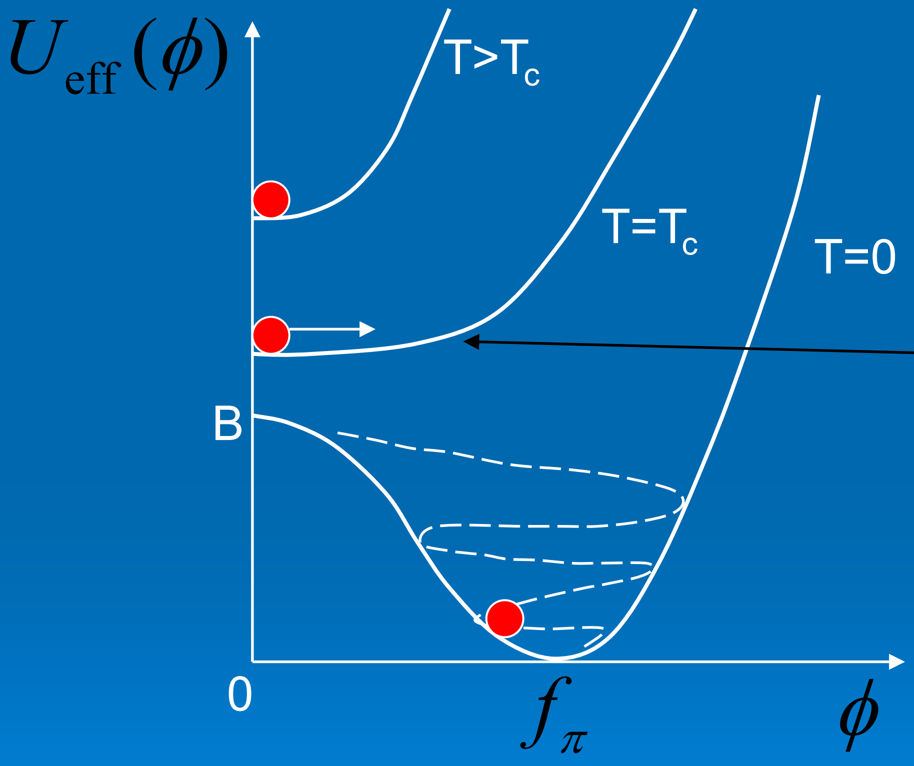




In rapidly expanding system 1-st order transition is delayed until the barrier between two competing phases disappears - spinodal decomposition

I.N. Mishustin, Phys. Rev. Lett. 82 (1999) 4779; Nucl. Phys. A681 (2001) 56

Critical slowing down in the 2nd order phase transition



"Rolling down" from the top of the potential is similar to spinodal decomposition (Csernai&Mishustin 1995)

Fluctuations of the order parameter evolve according to the equation

$$\frac{d\delta\phi}{dt} = -\gamma \frac{\partial \Omega}{\partial \phi} \approx -\frac{\delta\phi}{\tau_{\text{rel}}}$$

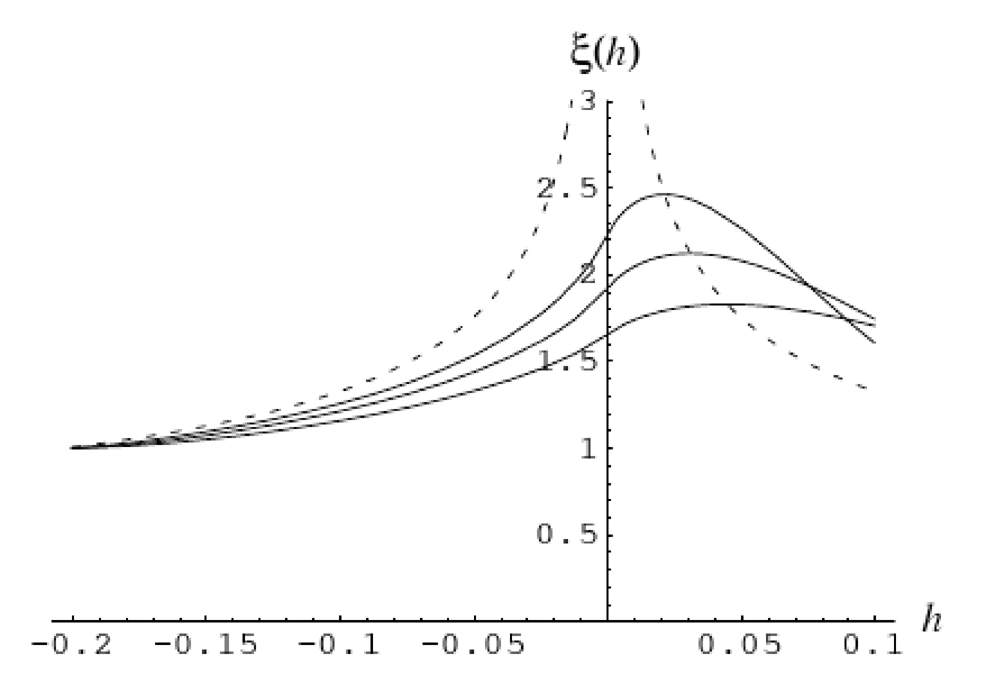
In the vicinity of the critical point the relaxation time for the order parameter diverges - no restoring force

$$\tau_{\text{rel}}(T) \sim \frac{1}{|T - T_c|} \simeq$$

(Landau&Lifshitz, vol. X, Physical kinetics)

Critical slowing down 2

B. Berdnikov, K. Rajagopal, Phys. Rec. D61 (2000)



Critical fluctuations have not enough time to build up. One can expect only a factor 2 enhancement in the correlation length (even for slow cooling rate, dT/dt=10 MeV/fm).

Dynamical model of chiral phase transition

Linear sigma model (LσM) with constituent quarks

$$L = \bar{q}[i\gamma\partial - g(\sigma + i\gamma_5 \boldsymbol{\tau\pi})]q + \frac{1}{2}[\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}] - U(\sigma,\boldsymbol{\pi}),$$

$$U(\sigma,\vec{\pi}) = \frac{\;}{4}(\sigma^2 + \boldsymbol{\pi}^2 - v^2)^2 - H\sigma, \;\; <\sigma>_{\text{vac}} = f_\pi \to H = f_\pi m_\pi^2$$

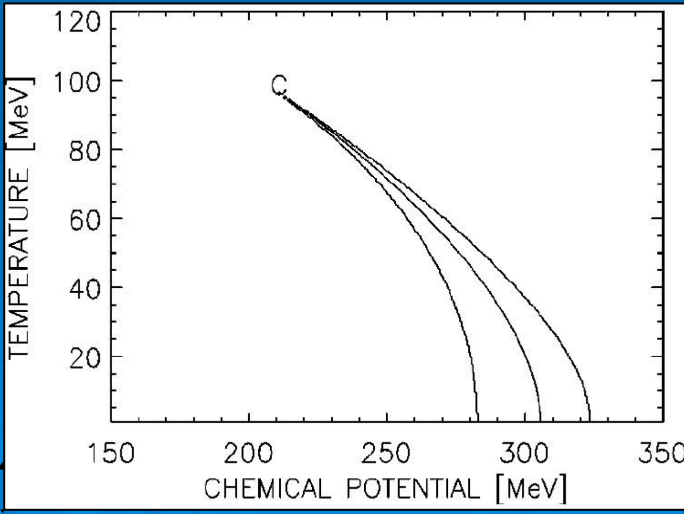
Thermodynamics of LσM on the mean-field level was studied in

Scavenius, Mocsy, Mishustin&Rischke, Phys. Rev. C64 (2001) 045202

Effective thermodynamic potential:

$$U_{\text{eff}}(\sigma;T,\mu) = U(\sigma,\boldsymbol{\pi}) + \Omega_q(m;T,\mu)$$

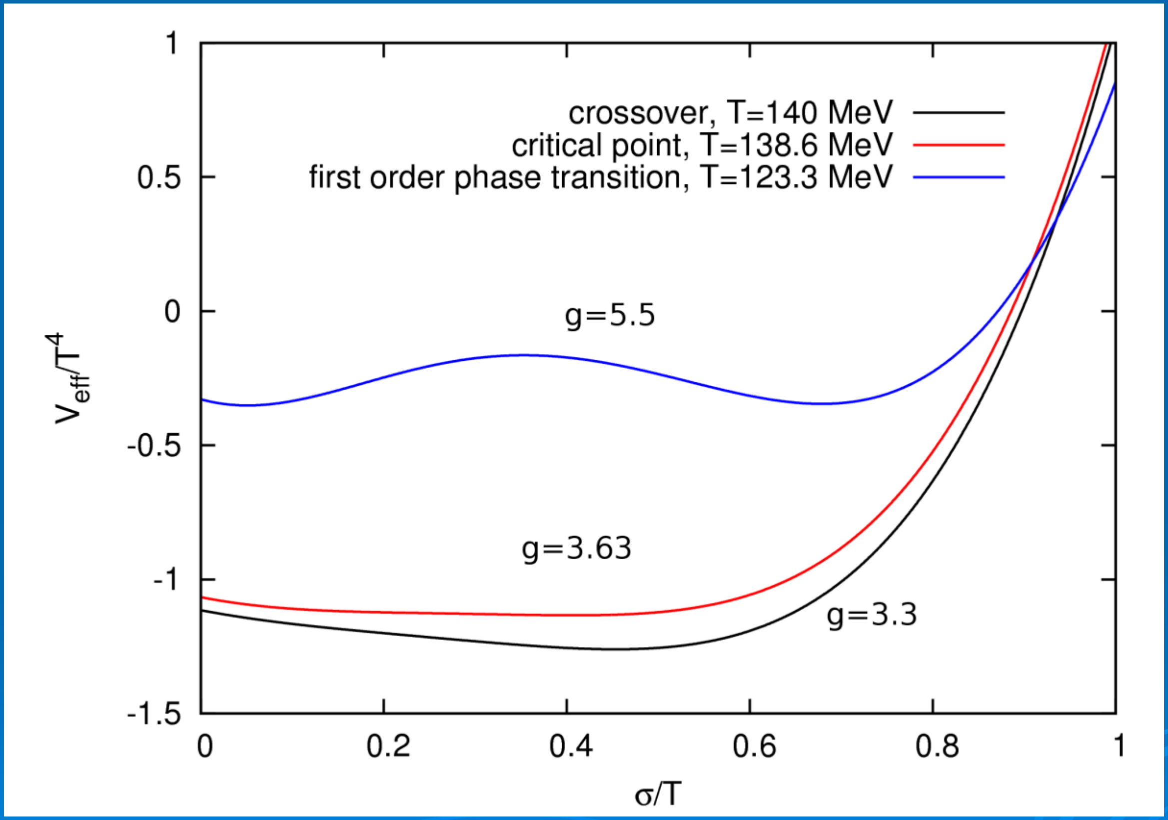
$$m^2 = g^2(\sigma^2 + \boldsymbol{\pi}^2)$$



CO, 2nd and 1st order chiral transitions are obtained in T-μ plane.

Effective thermodynamic potential

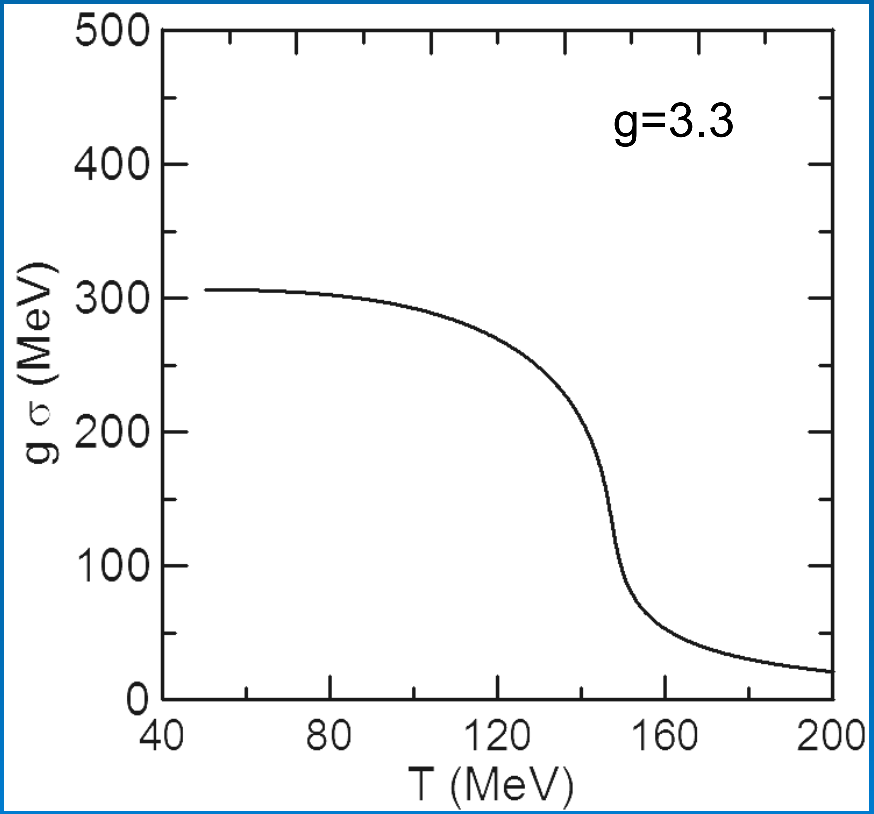
$$\Omega(m;T,\mu) = -\nu_q T \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \left\{ \ln\left[1 + \exp\left(\frac{\mu - \sqrt{m^2 + p^2}}{T} \right) \right] + (\mu \to -\mu) \right\}, \quad \nu = 2 N_f N_c$$



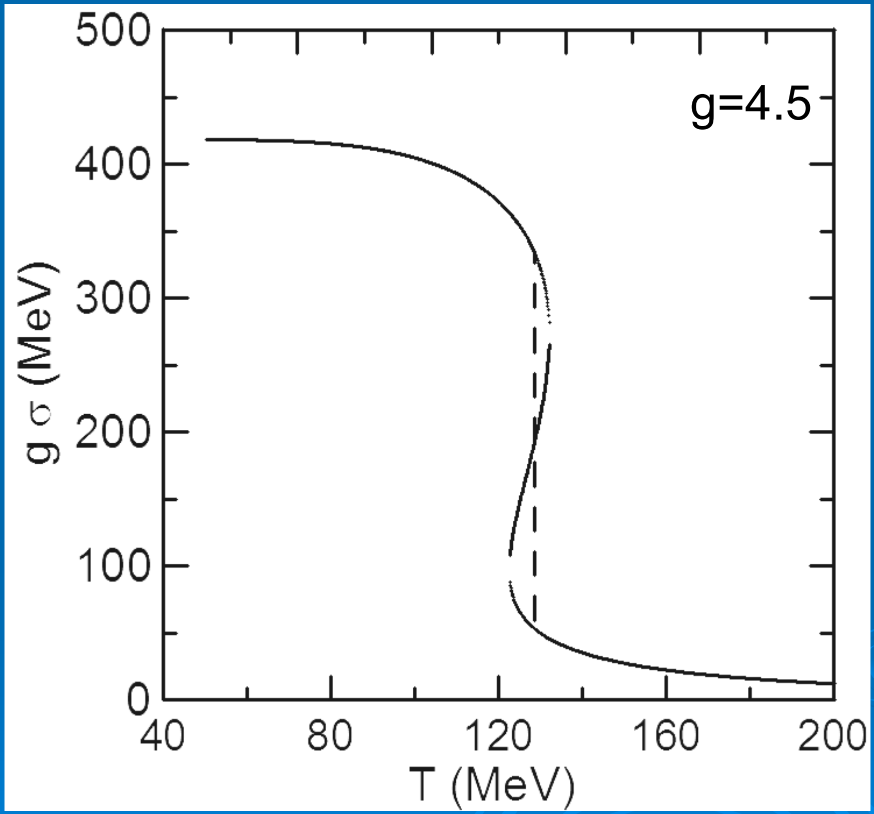
Here we consider μ=0 system but tune the order of the chiral phase transition by changing the coupling g.

Order parameter field vs T

crossover



1-st order

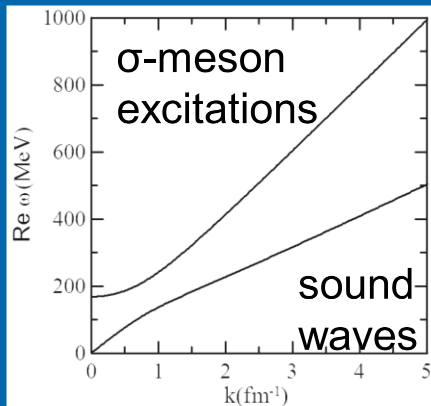


unstable states at 122 MeV<T<132 MeV

$\Rightarrow$ spinodal instability

Spectrum of plane-wave excitations

$$\delta\sigma(x) = \delta\sigma(\omega,k)e^{i\omega t - ikx}$$

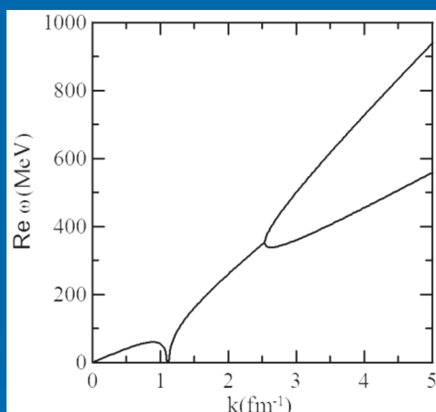


Generally two branches: 1) sound branch $\omega^2 \approx c_s^2 k^2$

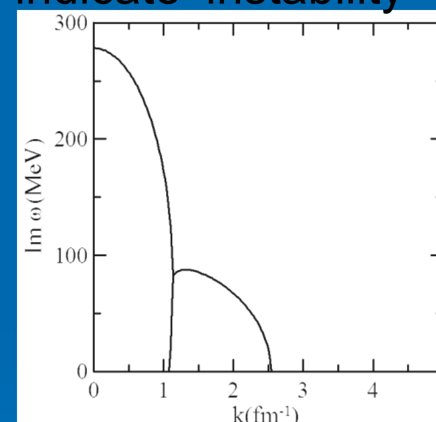
2) sigma branch $\omega^2 \approx m_\sigma^2 + k^2$

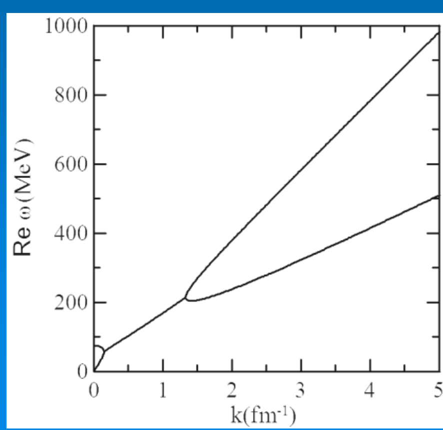
T=120 MeV

Solutions with $\omega^2<0$ indicate instability

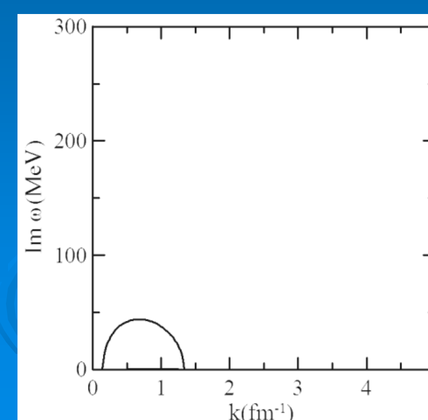


T=125 MeV





T=131 MeV



Chiral fluid dynamics (CFD)

I.N. Mishustin, O. Scavenius, Phys. Rev. Lett. 83 (1999) 3134
K. Paech, H. Stocker and A. Dumitru, Phys. Rev. C 68 (2003) 044907

Fluid is formed by constituent quarks and antiquarks which interact with the chiral field via quark effective mass $m = g\sigma$

CFD equations are obtained from the energy momentum conservation for the coupled system fluid+field

$$\partial_\nu (T_{\text{fluid}}{}^{\mu\nu} + T_{\text{field}}{}^{\mu\nu}) = 0 \Rightarrow \partial_\nu T_{\text{fluid}}{}^{\mu\nu} = -\partial_\mu T_{\text{field}}{}^{\mu\nu} \equiv S^\nu$$

$$S^\nu = -(\partial^2 \sigma + \frac{\partial U_{\text{eff}}}{\partial \sigma})\partial^\nu \sigma = (g\rho_s + \eta \partial_t \sigma)\partial^\nu \sigma$$

We solve generalized e. o. m. with friction (η) and noise (ξ):

$$\partial_\mu \partial^\mu \sigma + \frac{\partial U_{\text{eff}}}{\partial \sigma} + g < \bar{q}q > + \eta \partial_t \sigma = \xi$$

relaxation equation for the order parameter

$$< \xi(t,\vec{r}) > = 0, \quad < \xi(t,r)\xi(t',r') > = \frac{1}{V} m_\sigma \eta \delta(t-t')\delta(r-r') \coth\left(\frac{m_\sigma}{2T}\right)$$

Calculation of damping term

M. Nahrgang, S. Leupold, C. Herold, M. Bleicher, C84, 024912 (2011)

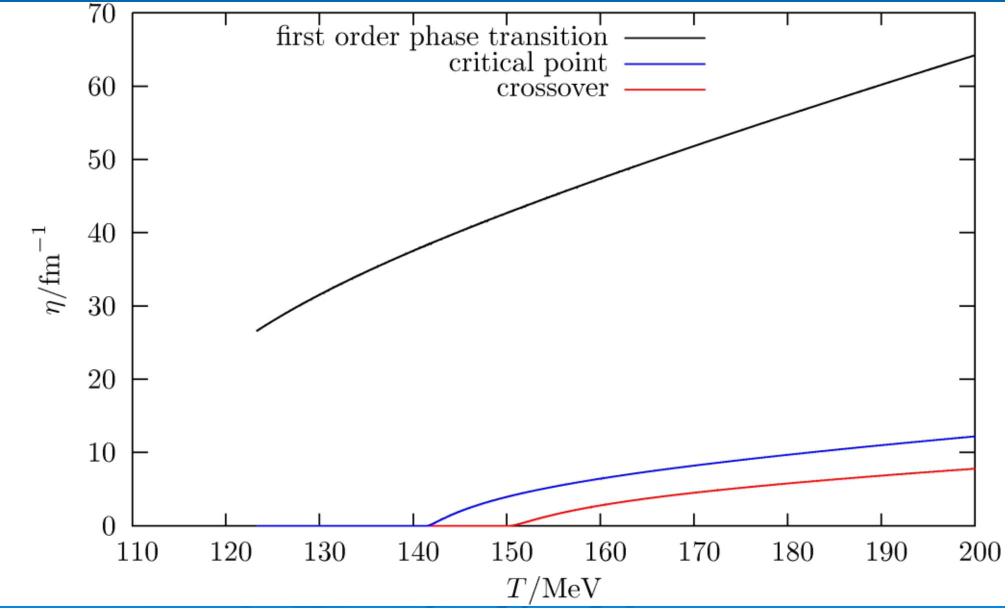
The damping is associated with the processes:

$$\sigma \to \bar{q}q, \ \sigma \to \pi\pi$$

It has been calculated using 2PI effective action

$$\eta = g^2 \frac{\nu_q}{\pi m_\sigma^2}\left[1 - 2n_F\left(\frac{m_\sigma}{2}\right)\right]\left(\frac{m_\sigma^2}{4} - m_q^2\right)^{3/2}$$

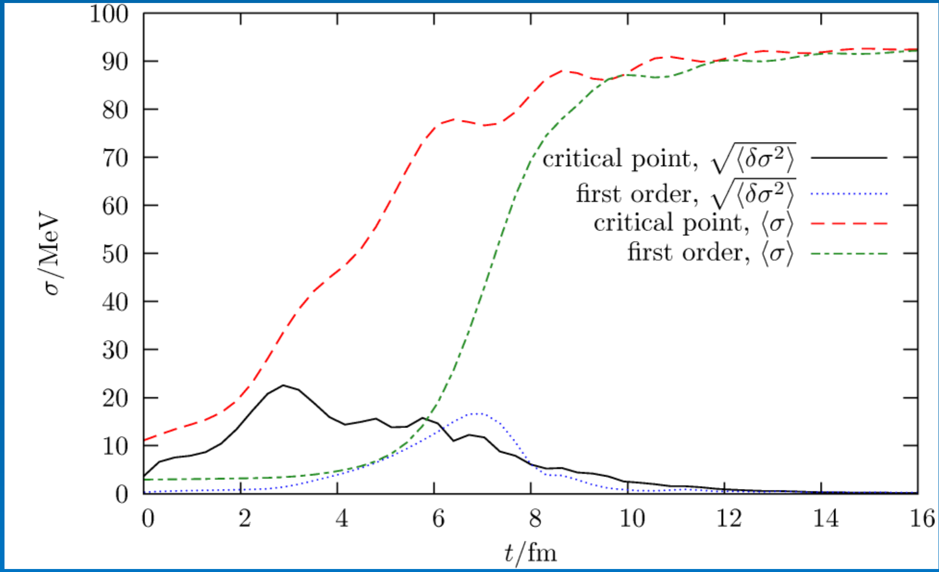
Around Tc the damping is due to the pion modes, η=2.2/fm

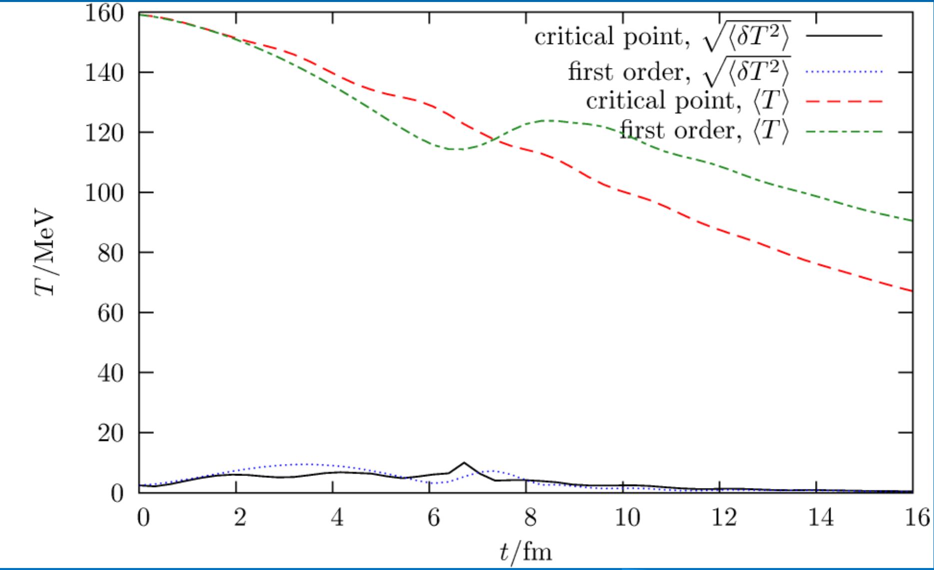


Realistic simulations: Bjorken-like expansion

Initial state: cylinder of length L in z direction, with ellipsoidal cross section in x-y direction

$$\text{At } t=0:\ v(z)=\frac{2z}{L}0.2c,\ \ -\frac{L}{2}<z<\frac{L}{2};\ \ \mathrm{v}_x=v_y=0;\ T=160\ MeV$$





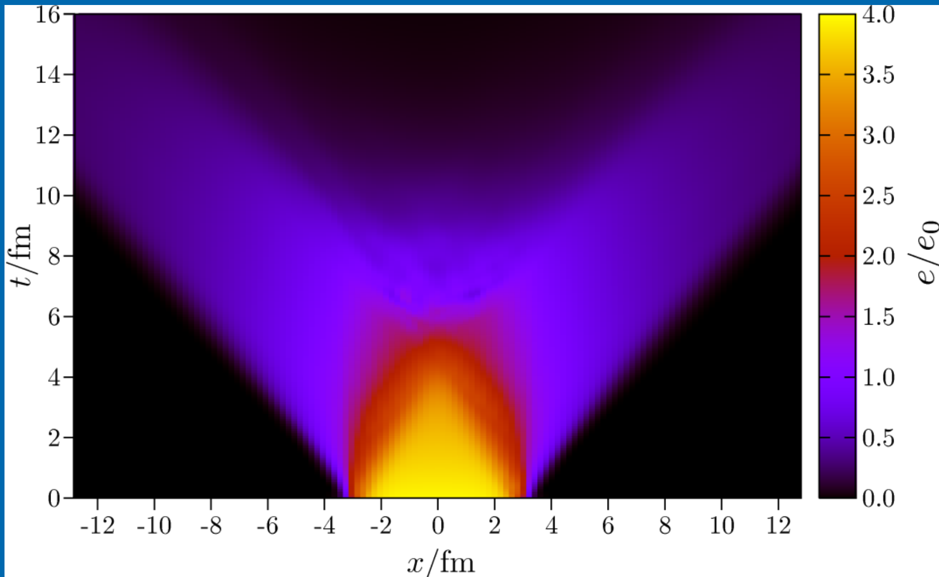
In the vicinity of the c.p. sigma shows pronounced oscillations since damping term vanishes $(m_\sigma < 2m_q)$

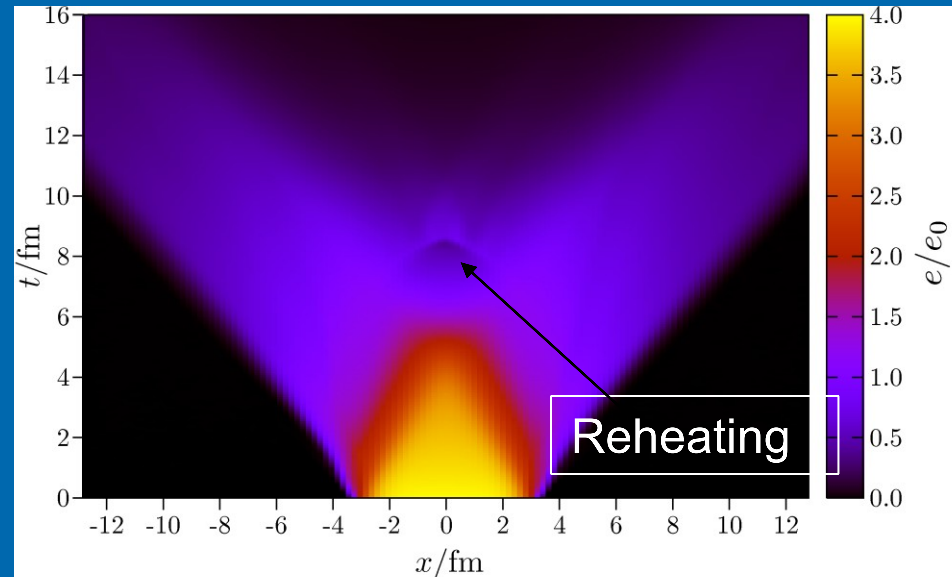
Supercooling and reheating effects are clearly seen in the 1-st order transition.

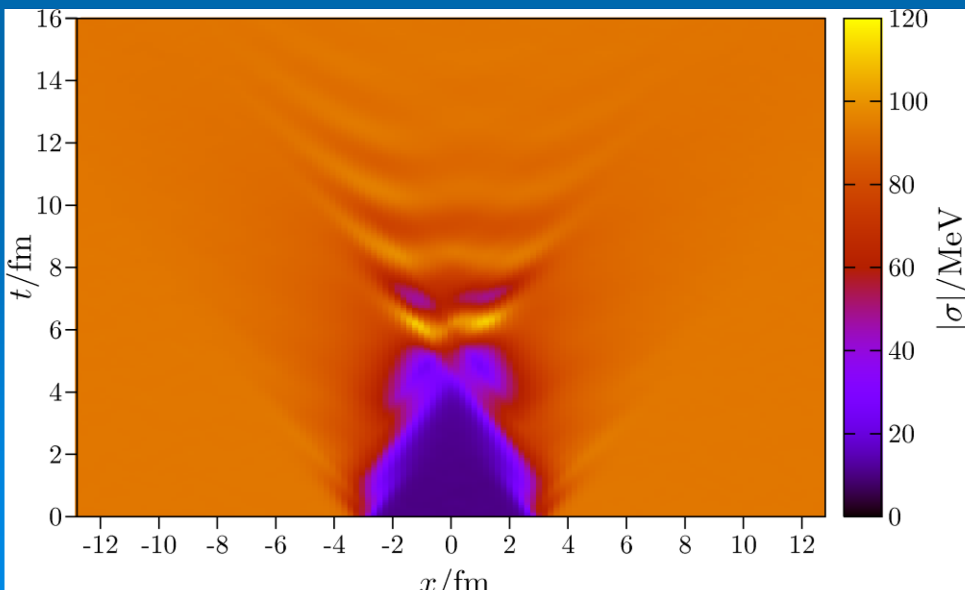
Dynamical evolution of chiral fluid

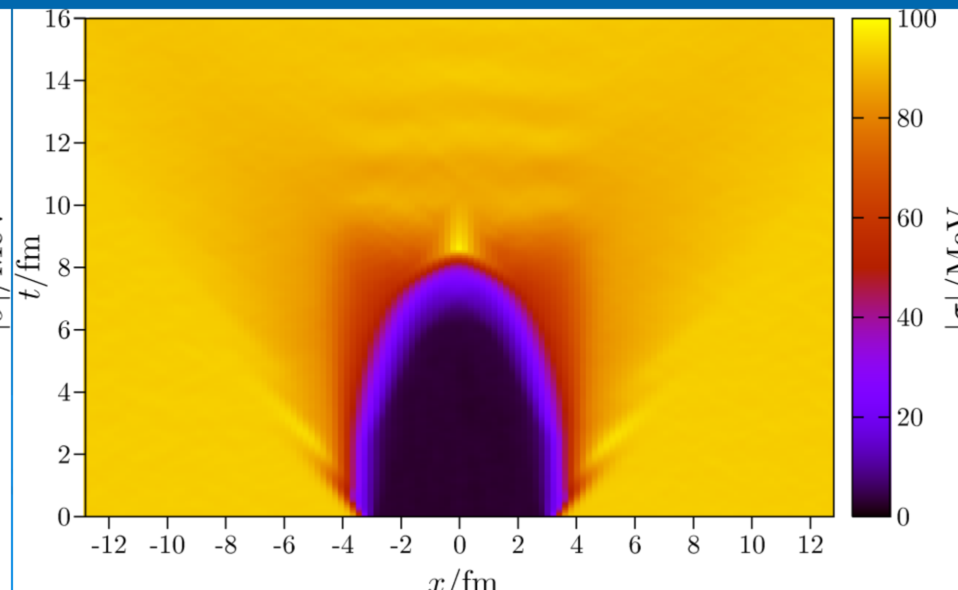
2-nd order transition (g=3.63)

1st order transition (g=5.5)

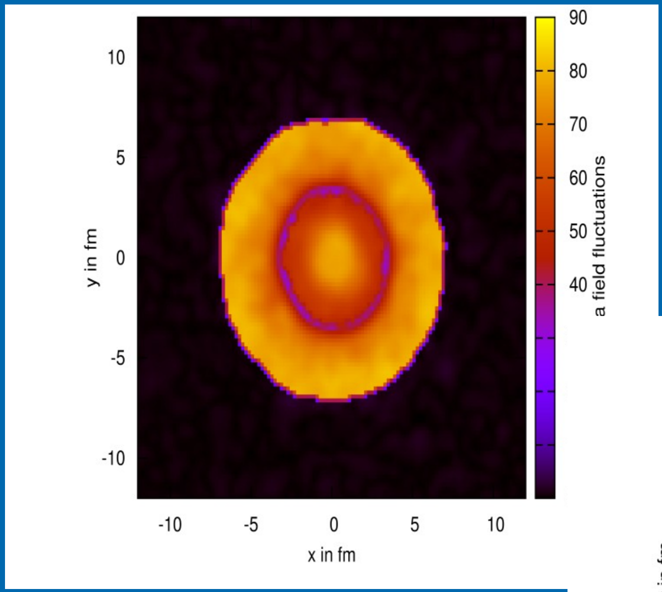




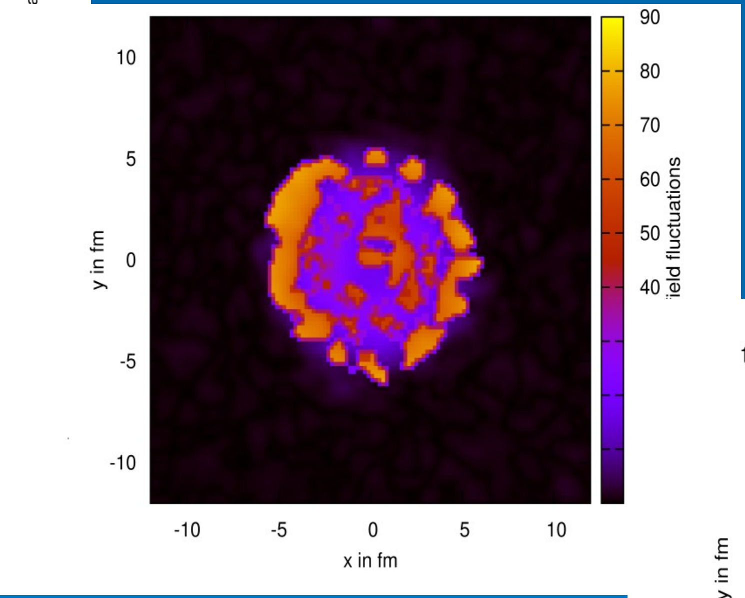




Dynamical evolution of sigma fluctuations In a single event

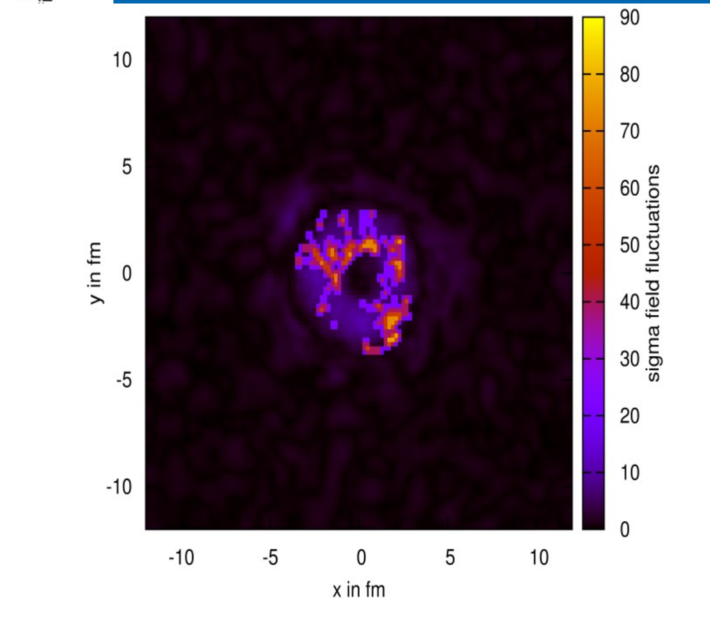


t=5 fm/c



t=7 fm/c

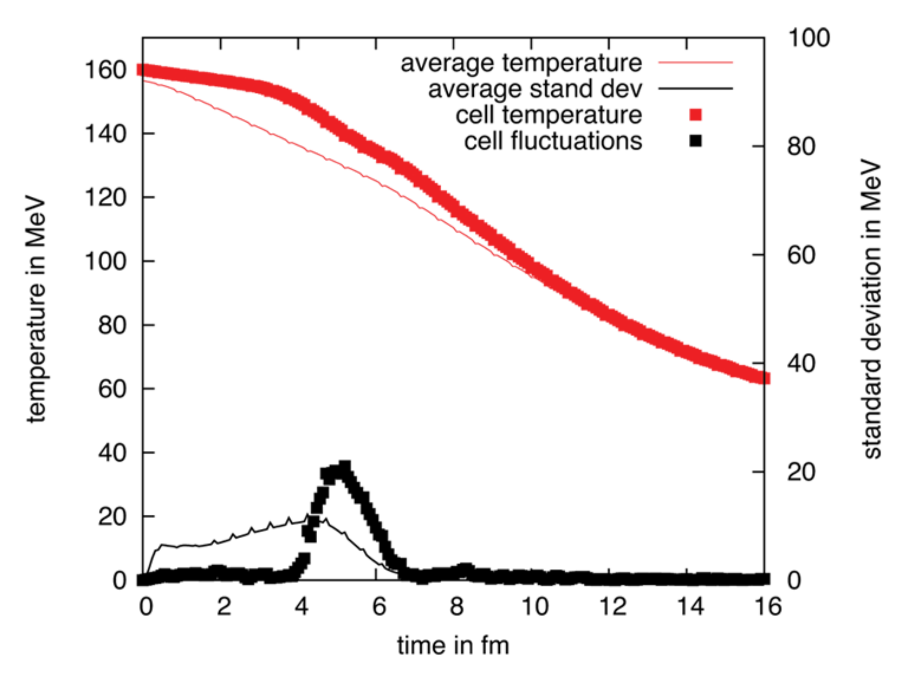
T=9 fm/c

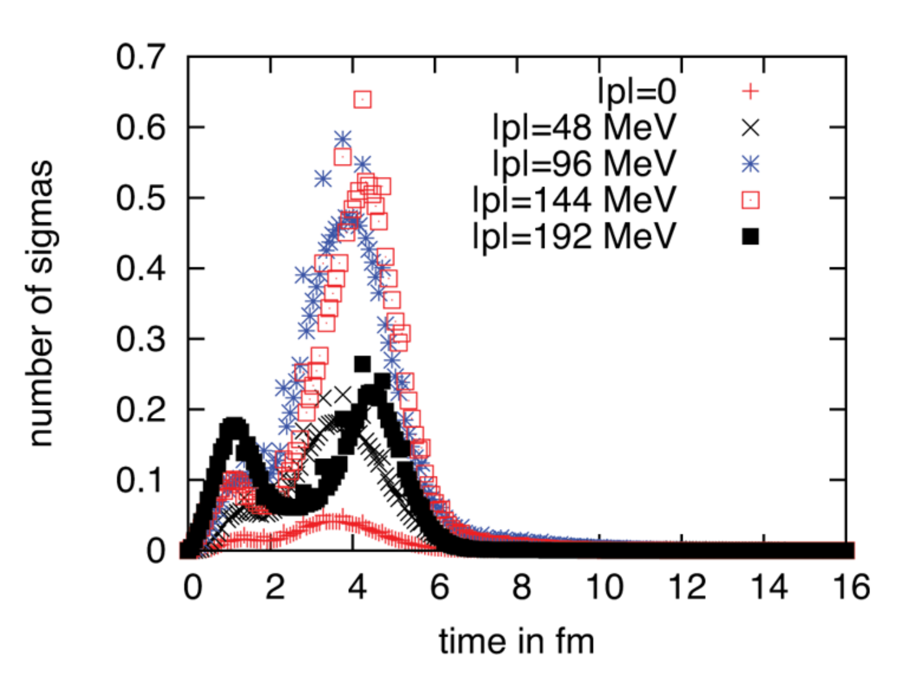


Strength of sigma fluctuations transition

$$\frac{dN_\sigma}{d^3k} = \frac{1}{(2\pi)^3}\frac{1}{2\omega_k}[\omega_k^2 |\sigma_k|^2 + |\dot{\sigma}_k|^2],\ \omega_k = \sqrt{m_\sigma^2 + k^2},\ \ m_\sigma^2 = \frac{\partial^2 U_{\mathrm{eff}}}{\partial \sigma^2}\Big|_{\sigma=\sigma_{\mathrm{eq}}}$$

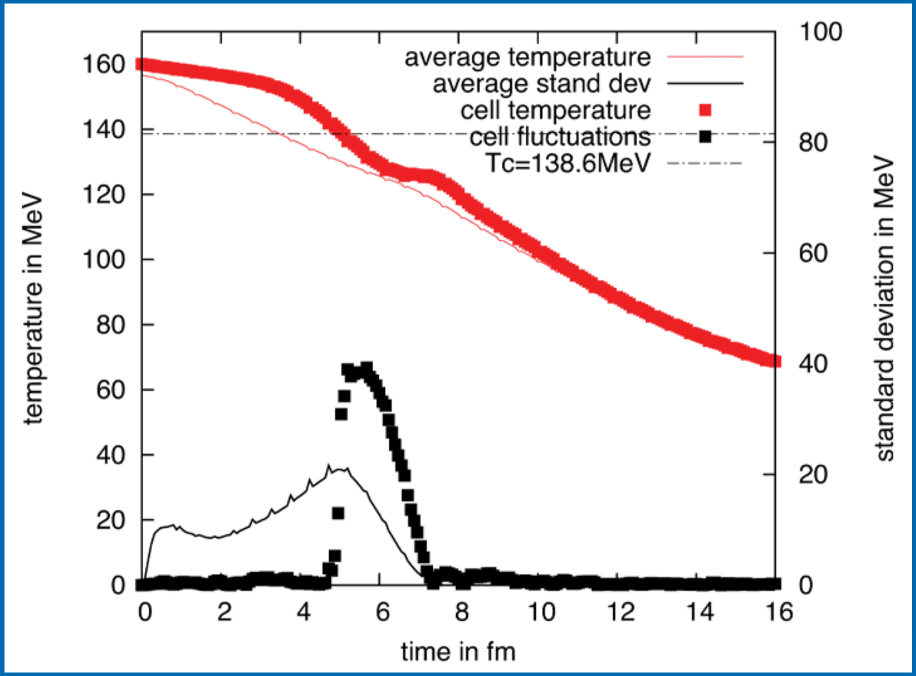
Crossover transition (g-3.3)

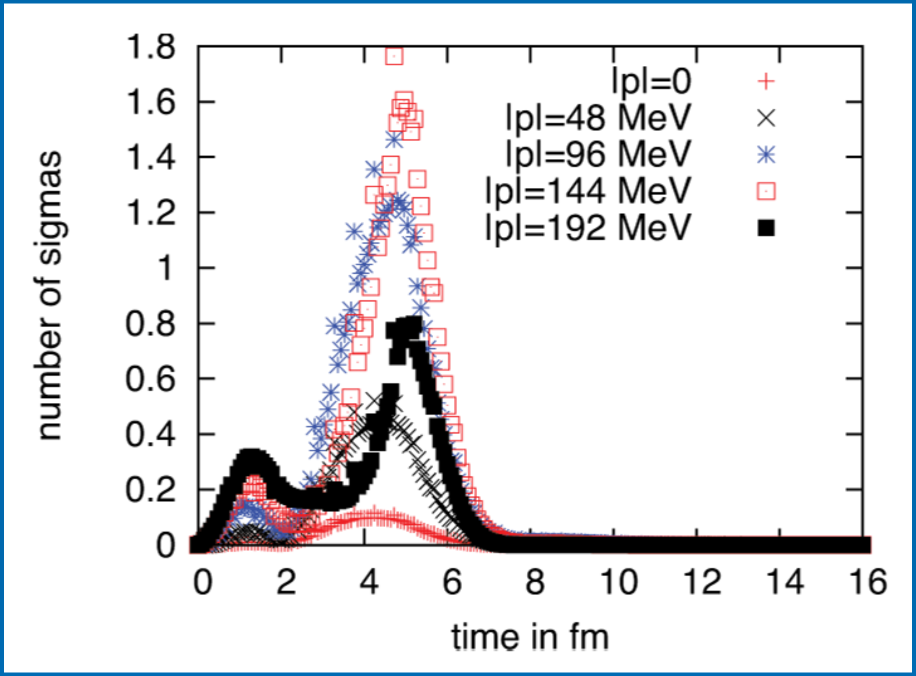




Fluctuations are rather weak, effective number of sigmas is small

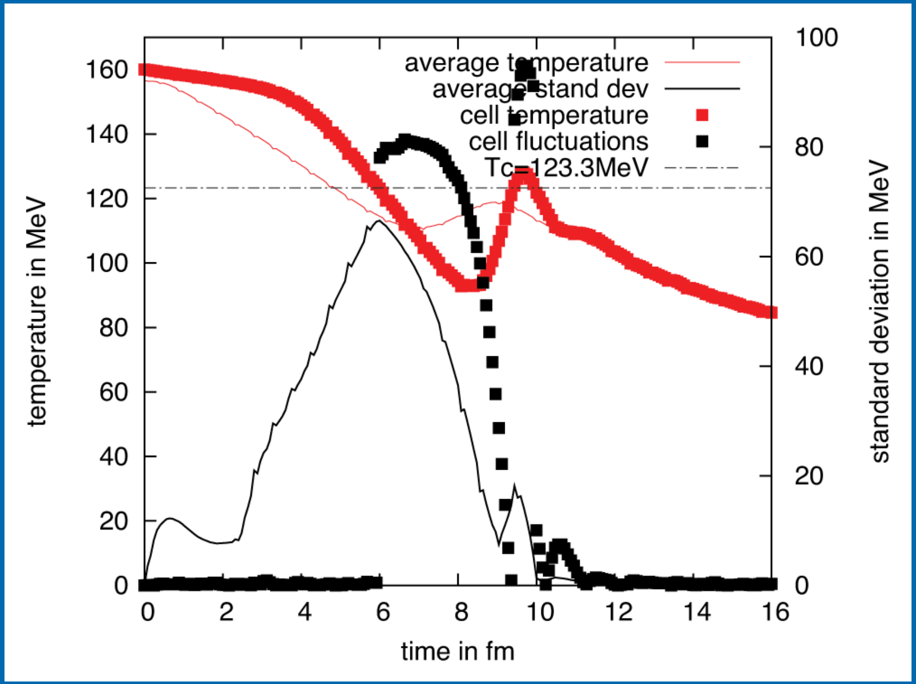
Second order transition with critical point (g=3.63)

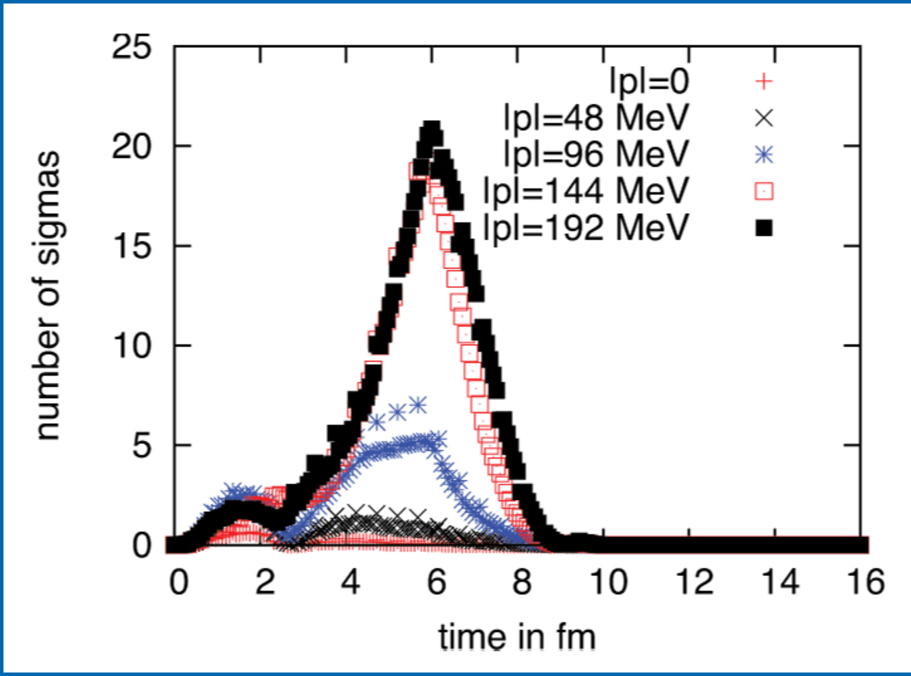




Fluctuations are stronger, but no traces of divergence are seen

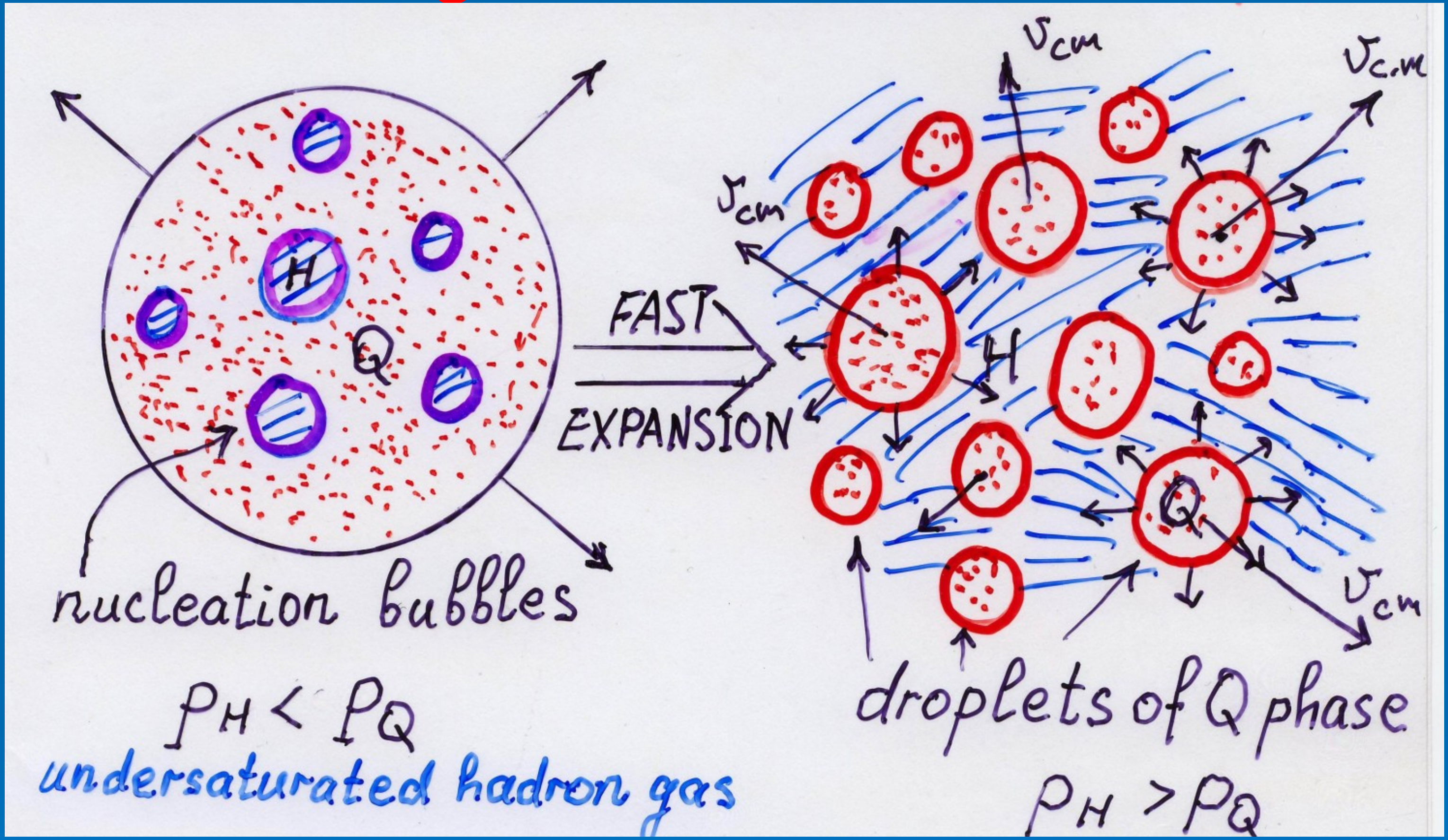
Strong first order transition (g=5.5)





- Strong supercooling and reheating effects are clearly seen.
- Sharp rise of fluctuations after 6 fm/c, when the barrier in thermodynamic potential disappears. Effective number of sigmas increases by two orders of magnitude!

Rapid expansion should lead to dynamical fragmentation of QGP

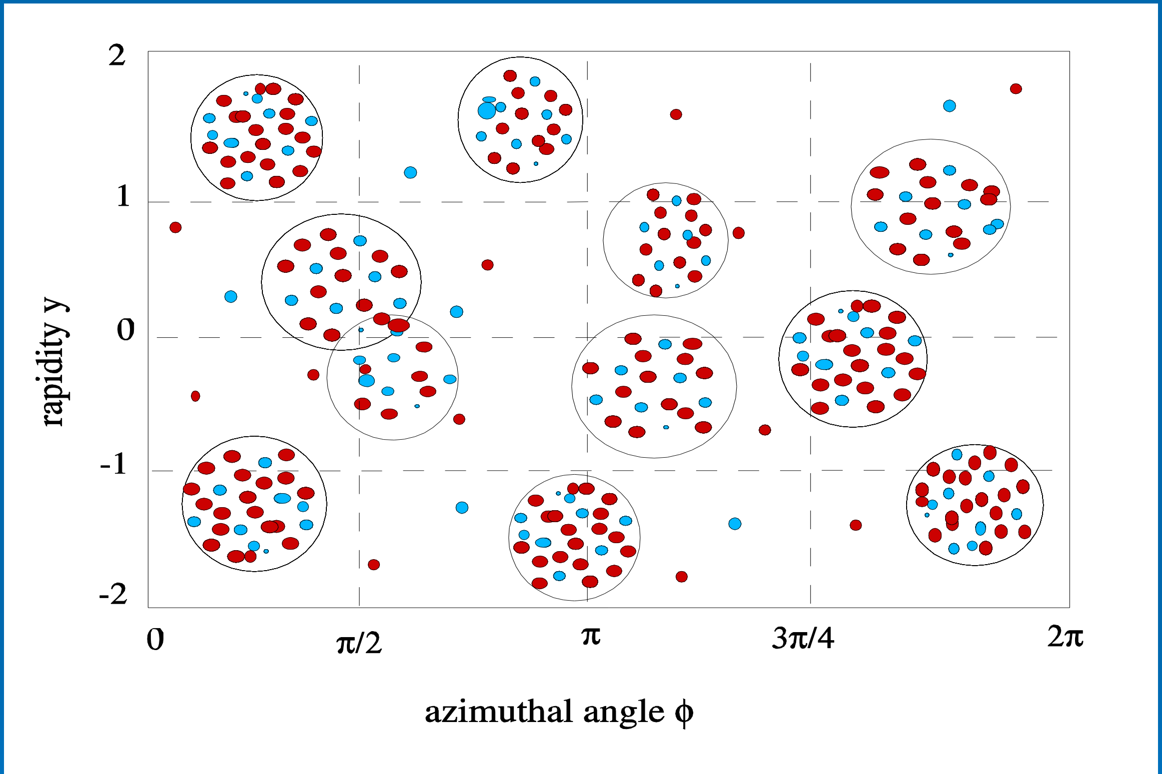


In the course of fast expansion the system enters spinodal instability when Q phase becomes unstable and splits into QGP droplets/hadron resonances

Csernai&Mishustin 1995, Mishustin 1999, Rafelski et al. 2000, Koch&Randrup, 2003

Extreme possibility - direct transition from quarks to hadrons without mixed phase

Experimental signal of droplets in the rapidity-azimuthal angle plane



Look for event-by-event distributions of hadron multiplicities in momentum space associated with emission from QGP droplets. Such measurerments should be done in the broad energy range!

Conclusions

- Phase transitions in relativistic heavy-ion collisions will most likely proceed out of equilibrium
- 2^{nd} order phase transitions (with CEP) are too weak to produce significant observable effects
- Non-equilibrium effects in a1^{st} order transition (spinodal decomposition, strong fluctuations of order parameter) may help to identify the phase transition
- If large QGP droplets are produced in the 1-st order phase transition they will show up in large non-statistical multiplicity fluctuations in single events