Influence of a magnetic field on the chiral/deconfining phase transition

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Outline:

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4. How to couple an external constant magnetic field B to the non-Abelian gauge field
5. The influence of an external magnetic field on the chiral condensate and on the critical temperature
6. The chiral condensate in the chiral limit
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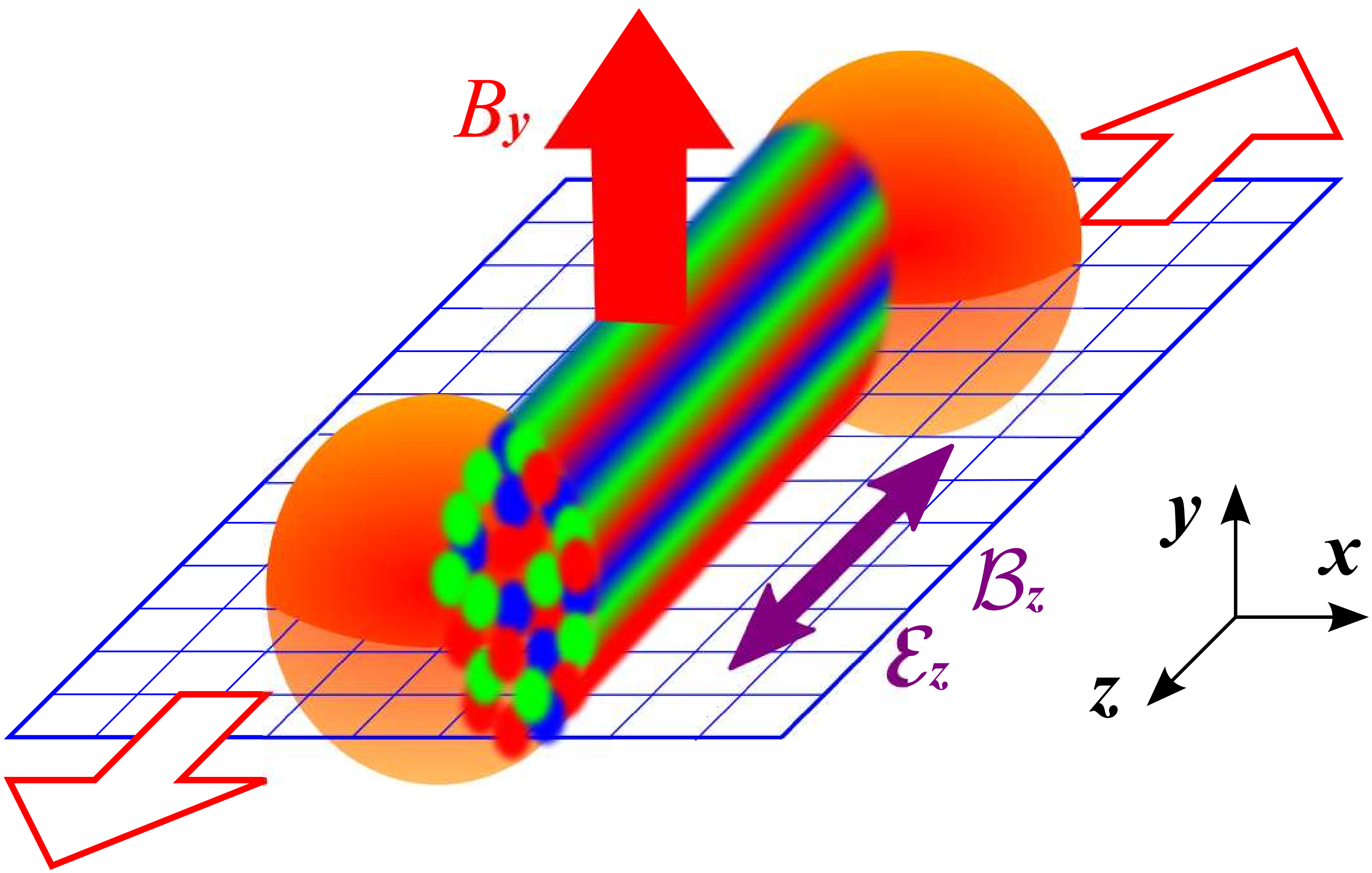
1. Introduction

Very strong magnetic fields may exist (or have existed)

- **during the electroweak phase transition** ($\sqrt{eB} \sim 1 - 2$ **GeV**)
- **in the interior of dense neutron stars (magnetars)** ($\sqrt{eB} \sim 1$ **MeV**)
- **in noncentral heavy ion collisions at RHIC** ($\sqrt{eB} \sim 100$ **MeV**)
  and LHC ($\sqrt{eB} \sim 500$ **MeV**),
  because antiparallel currents of the spectators create
  a strong magnetic field

Non-central heavy ion collision Kharzeev, McLerran, Warringa, '08



Such strong magnetic fields may lead to

- **a strengthening of the chiral symmetry breaking at low temperature (increase of the chiral condensate, increase of F_π, decrease of M_π)**

- **a change of the finite temperature chiral transition both in temperature (T_c) and in strength (eventually changing order)**

- **the chiral magnetic effect (CME): induced by a background of definite-sign topological density, an event-by-event charge asymmetry could be generated in non-central heavy ion collisions**

Chiral model at $T = 0$ (Shushpanov, Smilga, '97)

$$< \bar{\psi}\psi >_B = < \bar{\psi}\psi >_0 \left(1 + \frac{1}{F_\pi^2} \frac{(eB)^2}{96\pi^2 M_\pi^2} + \mathcal{O}\left(\frac{(eB)^4}{F_\pi^4 M_\pi^4}\right)\right)$$

In the chiral limit, $M_\pi \ll \sqrt{eB} \ll 2\pi F_\pi \sim \Lambda_{hadr}$**:**

from J. Schwinger's ('51) solution

$$< \bar{\psi}\psi >_B = < \bar{\psi}\psi >_0 \left(1 + \frac{1}{F_\pi^2} \frac{(eB) \log 2}{16\pi^2} + \mathcal{O}\left(\frac{(eB)^2}{F_\pi^4}, \frac{(eB)^2}{\Lambda_{hadr}^4}\right)\right)$$

$$M_{\pi^0}(B) = M_{\pi^0}(0) \left(1 - \frac{1}{F_\pi^2} \frac{(eB) \log 2}{16\pi^2} + \ldots\right)$$

$$F_\pi(B) = F_\pi(0) \left(1 + \frac{1}{F_\pi^2} \frac{(eB) \log 2}{8\pi^2} + \ldots\right)$$

$$M_{\pi^+}(B) = M_{\pi^-}(B) \propto \sqrt{eB}$$

Strong fields $\sqrt{eB} \gg F_\pi, M_\pi, \Lambda_{hadr}$

or in deconfined phase ($T > T_c$)

$< \bar{\psi}\psi >_B \sim |eB|^{3/2} \quad \Longrightarrow \quad$ **e**B **the only scale**

Dyson-Schwinger equations suggest a selfconsistent quark mass:

$$m_q(B) \sim \sqrt{|eB|} \exp\left[-\sqrt{\pi/(\alpha_s c_F)}\right]$$

$$< \bar{\psi}\psi >_B \sim |eB|^{3/2} \exp\left[-\frac{\pi}{2}\sqrt{\pi/(2\alpha_s c_F)}\right]$$

where $\alpha_s \equiv \alpha_s(|eB|)$

Effective models on the influence of eB on the transition ?

- **Splitting of chiral and deconfining transition with increasing magnetic field is in different effective models predicted by**
  K. Fukushima, M. Ruggieri, R. Gatto, Phys. Rev. D 81 (2010) 114031 (PNJL-model)
  A. J. Mizher, M. N. Chernodub, E. S. Fraga, Phys. Rev. D 82 (2010) 105016 (quark-meson model)
  R. Gatto, M. Ruggieri, Phys. Rev. D 82 (2010) 054027
  Both transitions enhanced by the magnetic field, chiral transition temperature rises with increasing eB !

- **R. Gatto, M. Ruggieri [arXiv:1012.1291]**
  improved non-local Polyakov-NJL models (fitting lattice data at zero and imaginary chemical potential) predict:
  Both transitions remain coupled to each other !

- **K. Fukushima, J. M. Pawlowski [arXiv:1203.4331]**
  Chiral transition temperature is increasing with increasing magnetic field; influence of quantum fluctuations is studied in FRG approach.

2. Previous non-quenched lattice studies (controversial)

All with staggered fermions. All with $N_c = 3$ colors.

- **M. D'Elia, S. Mukherjee, F. Sanfilippo, Phys. Rev. D 82 (2010) 051501(R)**
  $N_f = 2$ **flavours, unimproved fermion action. At fixed lattice spacing** $a = 0.3$ fm.
  Different quark masses corresponding to $m_\pi = 200...480$ fm.
  $\Rightarrow$ **slightly rising transition temperature** $\frac{T_c(B)}{T_c(0)} = 1 + A\left(\frac{|eB|}{T^2}\right)^{1.45}$
- **G. S. Bali, F. Bruckmann, G. Endrödi, Z. Fodor, S. D.Katz, S. Krieg, A. Schäfer, K. K. Szabo, JHEP 1202 (2011) 044**
  $N_f = 2 + 1$ **flavours, stout-link improved action.**
  Continuum limit probed with $N_\tau = 6$**, 8, 10**
  Finite volume effects probed at $N_\tau = 6$
  Different quark masses for u, d **and** s **quarks**
  $\Rightarrow$ **significantly decreasing transition temperature,**
  transition strength increasing with the magnetic field strength.

3. Our $SU(2)$ lattice model (arXiv:1203.3360)

Our simplified quark-gluon matter:

- colour $SU(2)$,
- staggered fermions without rooting of fermionic determinant, i.e. $N_f = 4$ flavours,
- unique e.-m. charge.

Why this model?

- Very similar chiral behaviour as in $SU(3)$ colour.
- Can be extended to finite baryon chemical potential without sign problem.
- Topology (important also for the CME) can be studied in a more simple case.
- Dyons (as caloron constituents) under magnetic field.
- Much faster to simulate. Can easily take the chiral limit. Use a farm of PC's (and recently GPU's).
- Educational aspect: nice model to be proposed for master students.

Pioneering calculations with magnetic field have been done in quenched $SU(2)$ working with - chirally optimal - overlap fermions (and their eigenvalues):

Braguta, Buividovich, Chernodub, Lushchevskaya, Polikarpov,...

We have studied the respective unquenched case.

Lattice gauge action: **from elementary closed (Wilson) loops ("plaquettes")**

$$U_{n,\mu\nu} \equiv U_{n,\mu}\, U_{n+\hat{\mu},\nu}\, U^{\dagger}_{n+\hat{\nu},\mu}\, U^{\dagger}_{n,\nu}\,, \qquad U_{n,\mu} \in SU(N_c)$$

$$\begin{aligned} S_G^W &= \beta \sum_{n,\mu<\nu} \left(1 - \frac{1}{N_c}\, \mathrm{Re\ tr}\ U_{n,\mu\nu}\right), \qquad \beta = \frac{2N_c}{g_0^2} \\ &= \frac{1}{2} \sum_n a^4\ \mathrm{tr}\ G^{\mu\nu} G_{\mu\nu} + O(a^2), \\ &\to \frac{1}{2} \int d^4x\ \mathrm{tr}\ G^{\mu\nu} G_{\mu\nu}. \end{aligned}$$

Continuum limit:

$$a(g_0) = \frac{1}{\Lambda_{Latt}} (\beta_0 g_0^2)^{-\frac{\beta_1}{2\beta_0^2}} \exp\left(-\frac{1}{2\beta_0 g_0^2}\right) (1 + O(g_0^2)).$$

$\Longrightarrow$ $a \to 0$ **for** $g_0 \to 0$ **(or** $\beta \to \infty$**),** *asymptotic freedom*.

For $SU(N_c)$ **and** N_f **massless fermions, independent on renormalization scheme:**

$$\beta_0 = \frac{1}{(4\pi)^2} \left(\frac{11}{3} N_c - \frac{2}{3} N_f\right), \qquad \beta_1 = \frac{1}{(4\pi)^4} \left(\frac{34}{3} N_c^2 - \frac{10}{3} N_c N_f - \frac{N_c^2 - 1}{N_c} N_f\right).$$

Staggered fermion action

Kogut, Susskind, '75

their steps towards staggered quarks:

- Use naive discretization and diagonalize action with respect to spinor degrees of freedom.
- Neglect three of four degenerate Dirac components.
- Attribute the 16 fermionic degrees of freedom, localized around one elementary hypercube, to four tastes.

Chiral symmetry restored $\quad\Longleftrightarrow\quad$ flavor symmetry broken.

Naturally the mass-degenerated four-flavor case is described.

Path integral quantization for Euclidean time $\Longrightarrow$ 'statistical averages'.

Fermions as anticommuting Grassmann variables

$\Longrightarrow$ analytically integrated $\Rightarrow$ non-local effective action $S^{eff}(U)$.

'Partition function' describing $N_f = 4$ mass-degenerate staggered flavors:

$$\begin{aligned} Z &= \int [dU][d\psi]\,[d\bar{\psi}]\; e^{-S^G(U)\; +\; \bar{\psi}M(U)\psi} \\ &= \int [dU]\; e^{-S^G(U)}\; \mathrm{Det}M(U) \\ &= \int [dU]\; e^{-S^{eff}(U)}, \qquad S^{eff}(U) = S^G(U) - \log(\mathrm{Det}M(U)) \end{aligned}$$

with $M(U) \equiv D_{\mathrm{Latt}}(U) + m$.

Simulation on a finite lattice $N_t \times N_s^3$,

with (anti-) periodic boundary conditions for gluons (quarks).

Most simulations are using the rooting prescription:

for $N_f = 2+1\ (+1)$ 4th-root of the fermionic determinant is taken

for each flavor $\Longrightarrow$ Locality violated (??)

$N_f = 4$ without rooting $\Longrightarrow$ standard Hybrid Monte Carlo algorithm applicable !

Non-zero temperature $T \equiv 1/L_t = 1/(N_t \; a(\beta))$:

T **varied by changing** β **at fixed** N_t **(fixed-scale approach: changing** N_t **at fixed** β**).**

Order parameters:

Polyakov loop: $L(\vec{x}) \equiv \frac{1}{N_c} \mathrm{tr} \prod_{x_4=1}^{N_t} U_4(\vec{x}, x_4), \qquad \langle L(\vec{x}) \rangle = \exp(-\beta F_Q),$

$F_Q \;=\;$ **free energy of an isolated infinitely heavy quark.**

$\Longrightarrow \quad F_Q \to \infty$, **i.e.** $\langle L(\vec{x}) \rangle \to 0$ **within the confinement phase** ($T < T_c$)**.**

$\Longrightarrow \quad \langle L(\vec{x}) \rangle$ **order parameter for the deconfinement transition** ($T = T_c$)**.**

Chiral condensate: $\langle \bar{\psi} \psi \rangle$ **(here from a stochastic estimator)**

order parameter for chiral symmetry breaking ($T < T_c$) **and restoration** ($T > T_c$)

Find critical T_c (or β_c) from maxima of susceptibilities of $L(\vec{x})$ and/or $\bar{\psi}\psi$. This is possible in our model.

In real QCD (assuming, say $O(4)$ universality) from a fit of the condensate to the "magnetic equation of state" (the scaling function of J. Engels et al.) the transition temperature can be determined.

Fixing the physical scale:

$T > 0$ **calculations done on lattices of size:** $16^3 \times 6$ $(24^3 \times 6)$

$T = 0$ **calculations for calibration for each** β **:** $16^3 \times 32$

The lattice unit scale $a(\beta)$ **fixed via scale parameter** r_0 **(R. Sommer, '94), assumed to be the same as in real QCD:**

Compute static force $F(r) = dV/dr$ **phenomenologically well-known from** $\bar{c}c$**- or** $\bar{b}b$**-potential** $V_{\bar{Q}Q}$**:**

$$F(r_0)\; r_0^2 \;\equiv\; 1.65 \quad \leftrightarrow \quad r_0 \simeq 0.468(4)\ \mathrm{fm}$$

Then determine e.g. the pion mass m_π**.**

For $T = 0, \quad ma = 0.01, \quad B = 0$ **we obtain at** $\beta = 1.80$ **(this is** $\simeq \beta_c$ **for** $N_t = 6$**).**

$$\begin{aligned} a &= 0.170(5)\ \mathrm{fm} \\ m_\pi &= 330(10)\ \mathrm{MeV} \\ T_c &= 193(6)\ \mathrm{MeV} \end{aligned}$$

4. How to couple an external constant magnetic field B to the non-Abelian gauge field

$\bar{B} = (0, 0, B)$ $\qquad\qquad$ $\bar{A}(\bar{r}) = \frac{B}{2}(-y, x, 0)$

On the lattice we use the compact formulation. Constant magnetic field $\equiv$ constant magnetic flux $\phi = a^2(eB)$ through all (x, y) plaquettes.

On the links, in addition to the non-Abelian transporters, define $U(1)$ elements both coupled to quark fields in lattice covariant derivative.

$$V_x(\bar{r}, \tau) = e^{-i\phi y/2}$$
$$V_y(\bar{r}, \tau) = e^{i\phi x/2}$$
$$V_x(N_s, y, z, \tau) = e^{-i\phi(N_s+1)y/2}$$
$$V_y(x, N_s, z, \tau) = e^{i\phi(N_s+1)x/2}$$

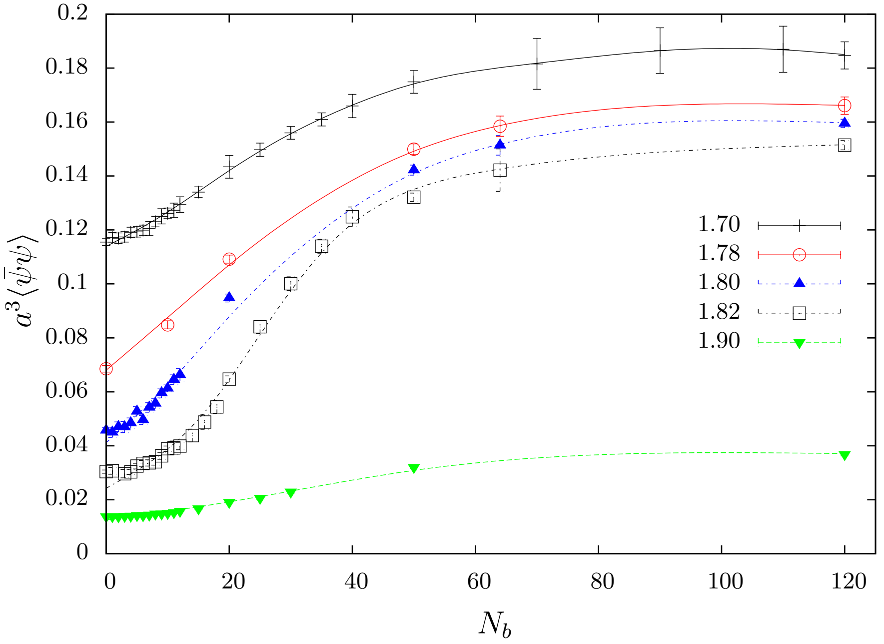
Flux will be quantized: $\qquad \phi = \frac{2\pi N_b}{N_S^2} \qquad N_b = 1, 2, \ldots$ $\qquad$ **DeGrand, Toussaint '80**

Typical field strength for $\beta = 1.80 \simeq \beta_c$ **,** $\quad N_b = 50 \quad \Longleftrightarrow \quad \sqrt{(eB)} \simeq O(1 \text{ GeV})$

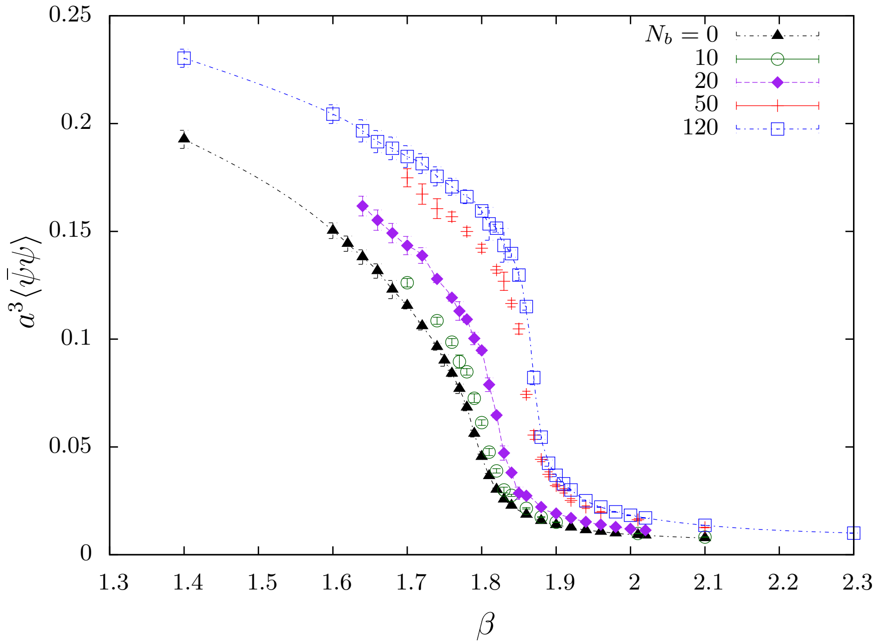
Coupling between electromagnetic and non-Abelian field is indirect, via fermions.

5. The influence of an external magnetic field on the chiral condensate and on the critical temperature

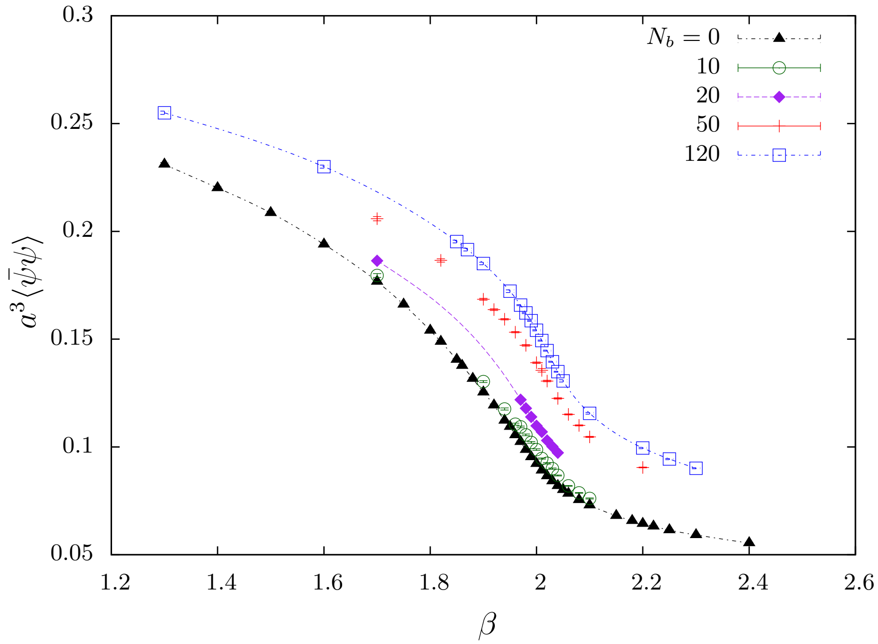
Saturation behavior for various β (V_μ periodic in ϕ):



β-dependence ($\equiv$ T dependence) of the bare chiral condensate



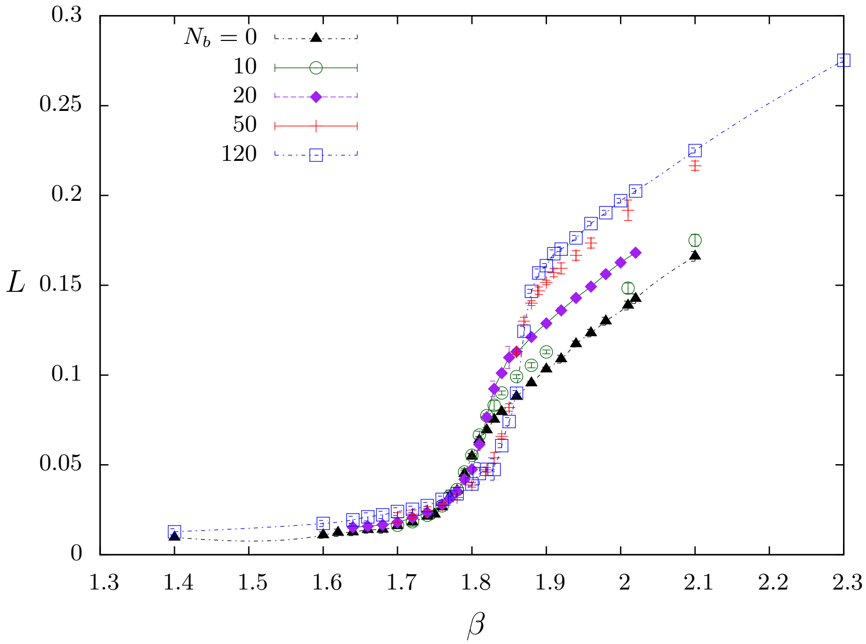
$ma = 0.01$



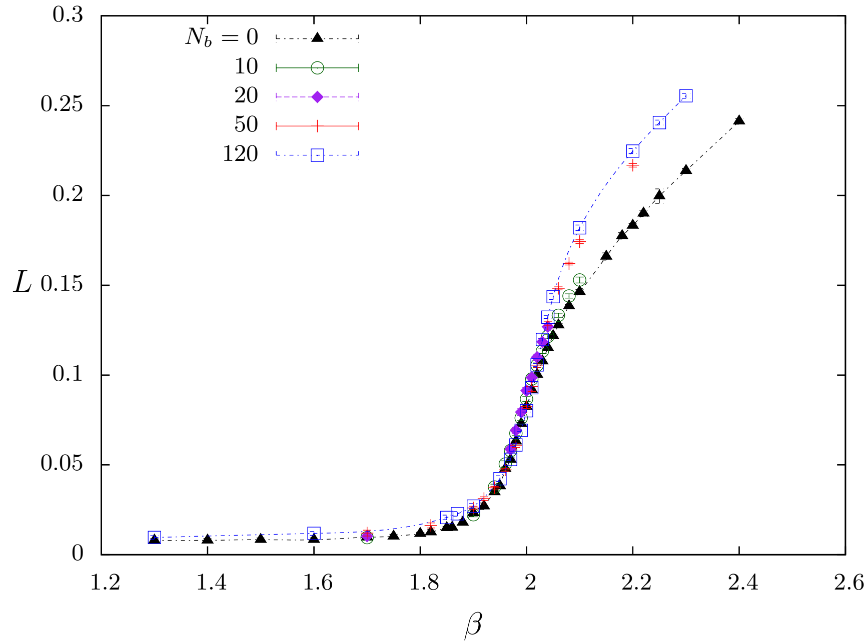
$ma = 0.1$

$<\bar{\psi}\psi>$ increases with B for all β $\Longrightarrow T_c$ increases

Polyakov loop

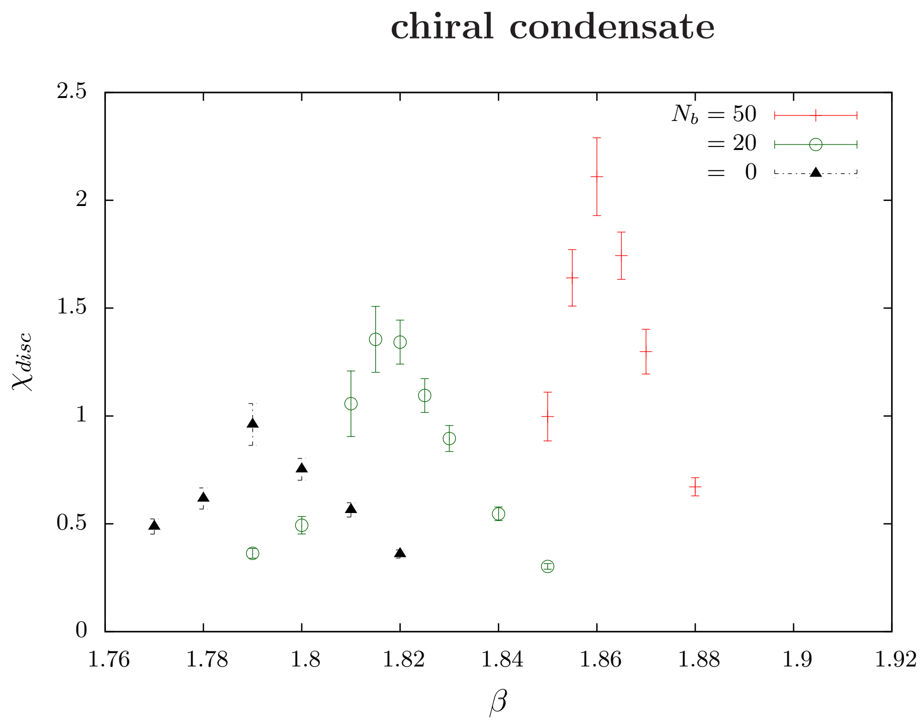


$ma = 0.01$

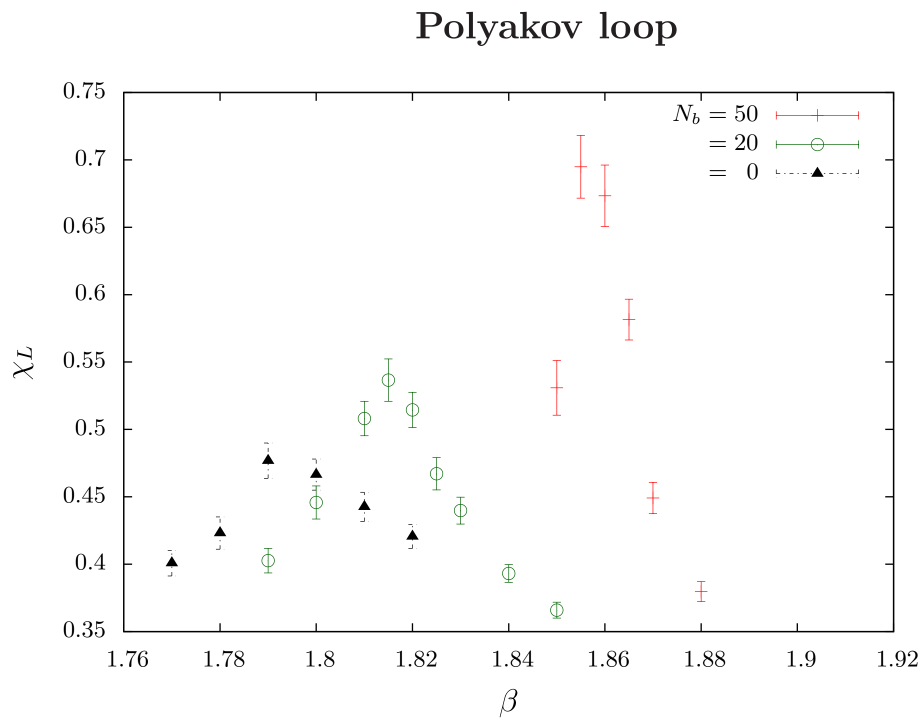


$ma = 0.1$

Susceptibilities



$ma = 0.01$

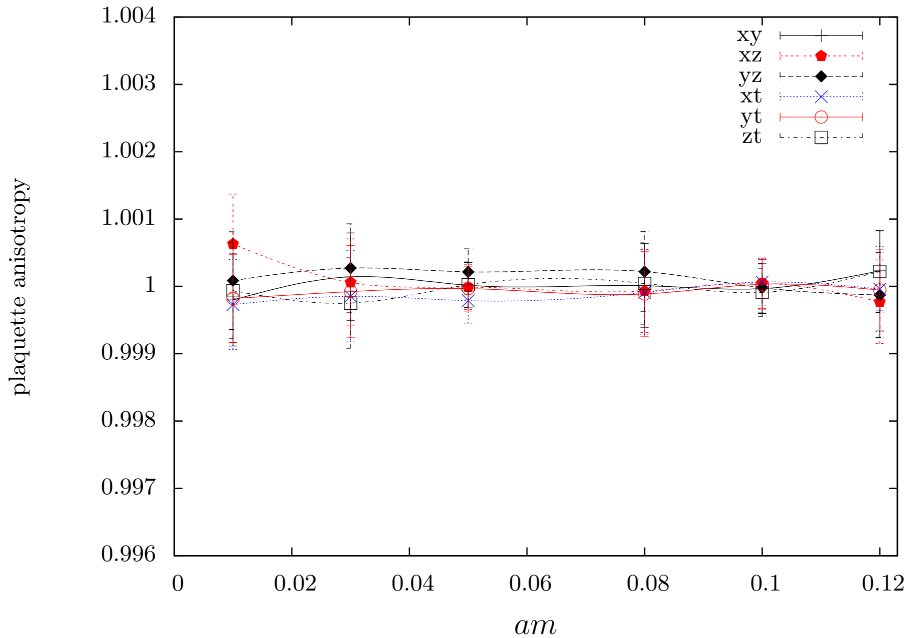


$ma = 0.01$

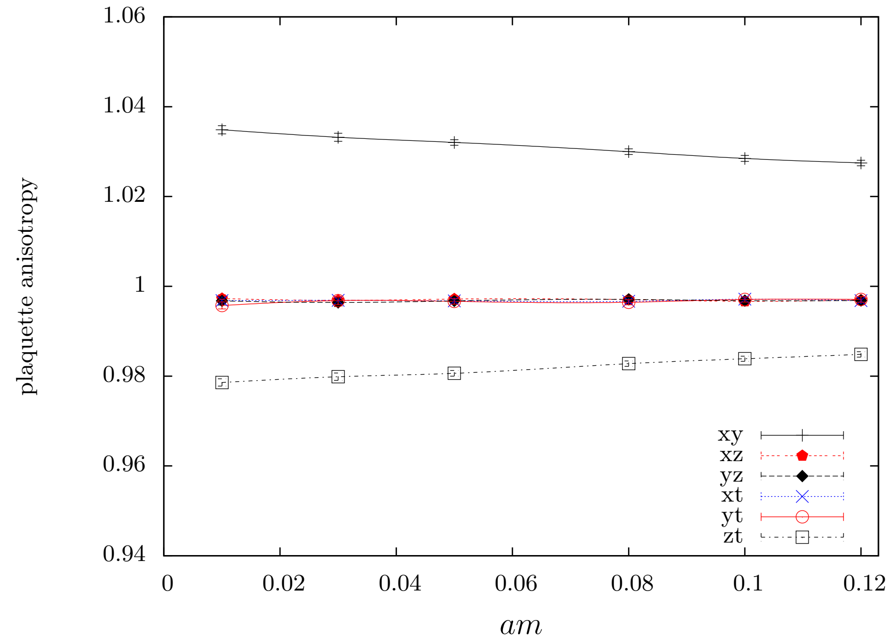
$B \nearrow \;\Rightarrow\; T_c \nearrow$ **coherently shifted, no splitting into two transitions**

Spatial anisotropy of plaquette averages:

confined phase, $\beta = 1.7$

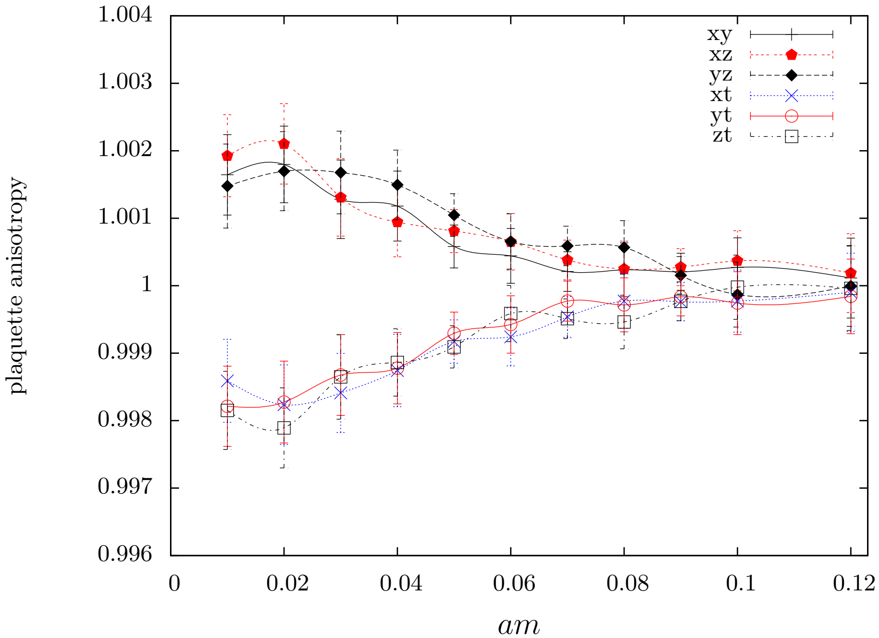


$N_b = 0$

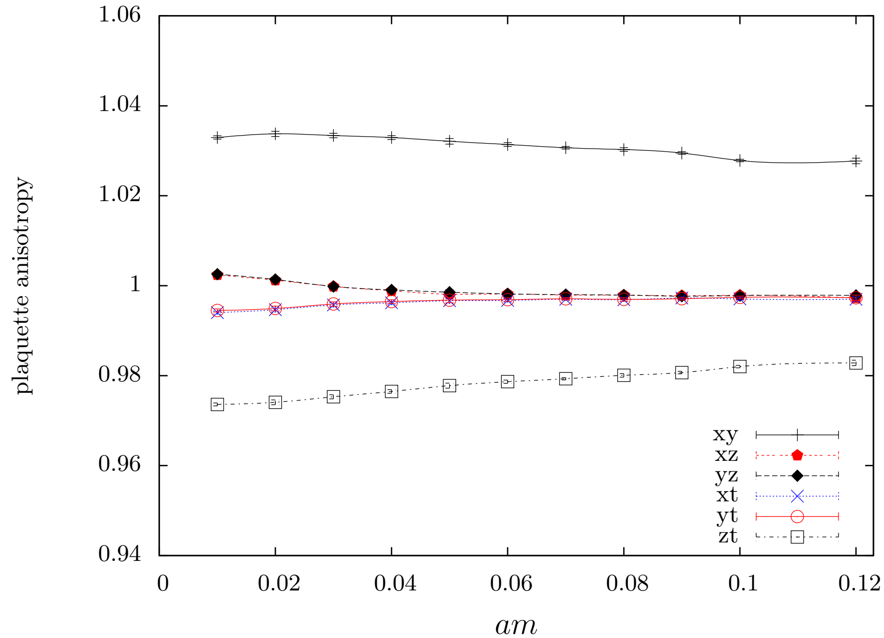


$N_b = 50$

transition region, $\beta = 1.9$



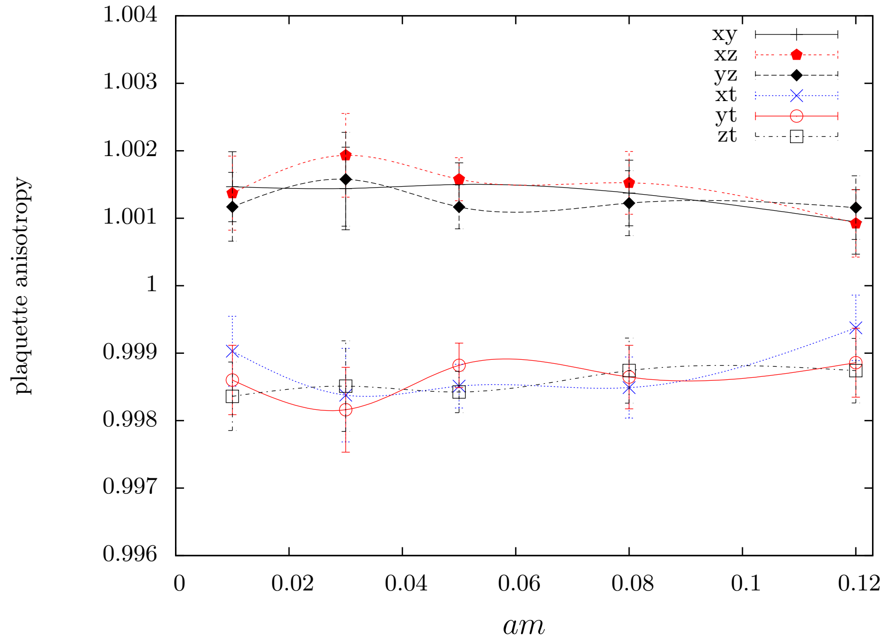
$N_b = 0$



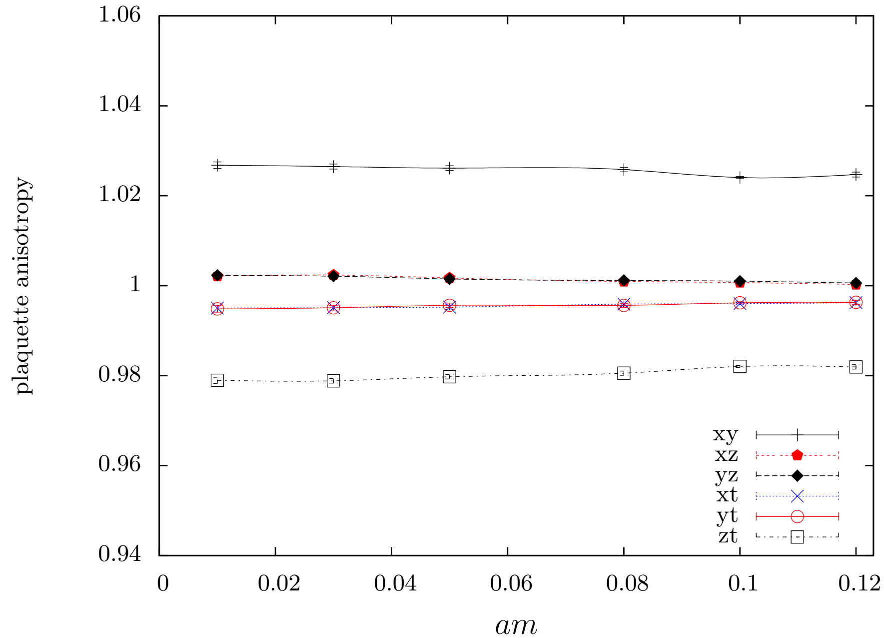
$N_b = 50$

Spacelike-timelike plaquette differences $\propto$ energy density

deconfined phase, $\beta = 2.1$



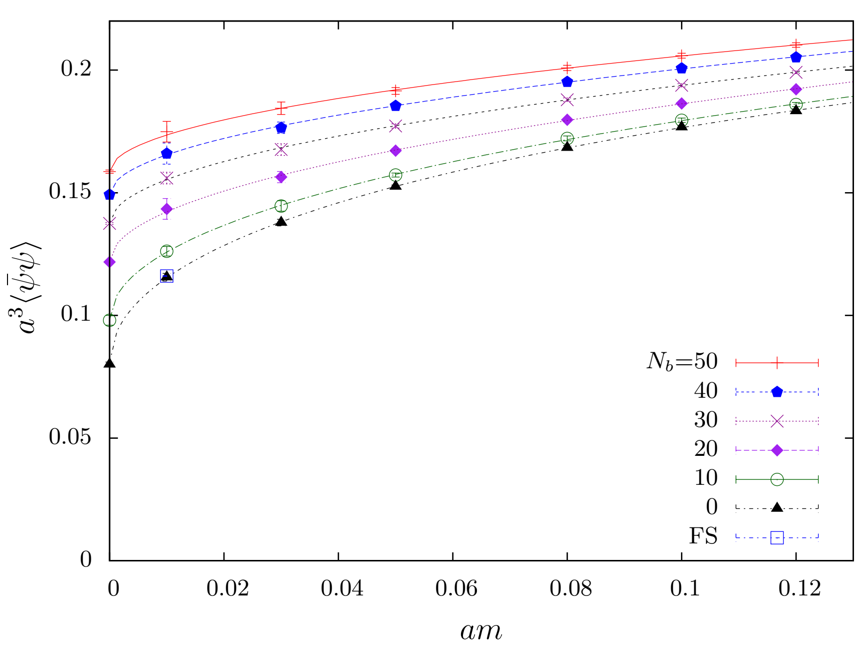
$N_b = 0$

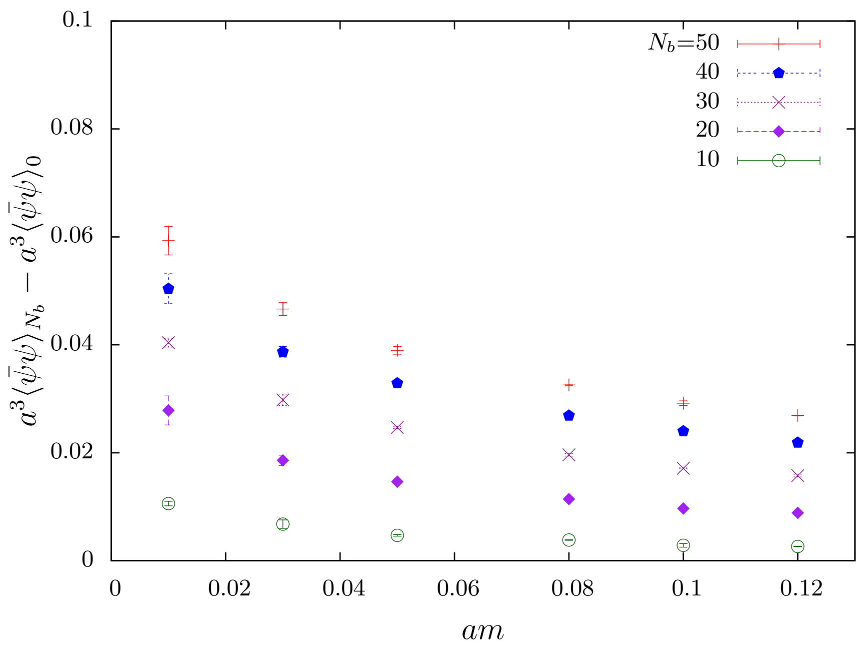


$N_b = 50$

6. The chiral condensate in the chiral limit

Confined phase, $\beta = 1.7$

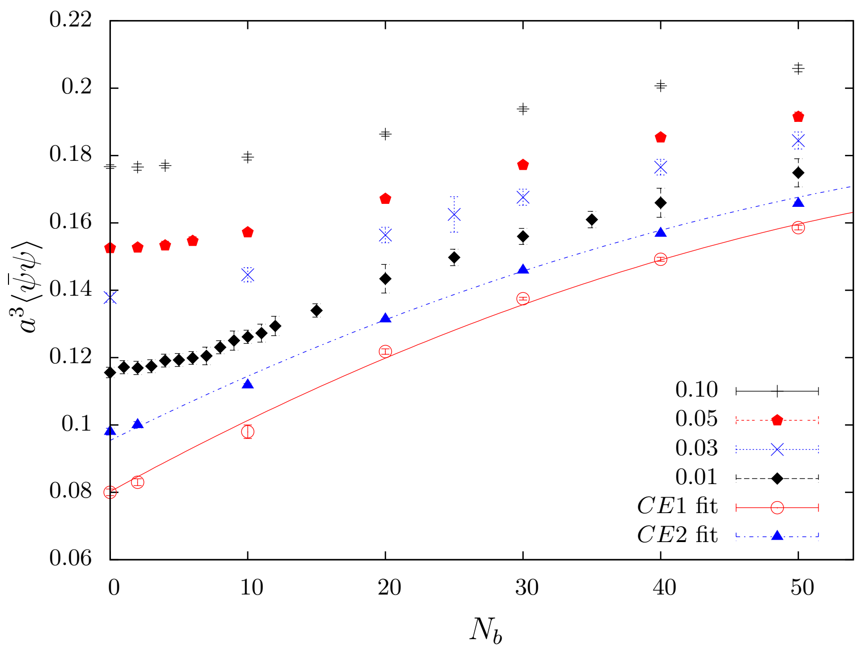




CE1: $a^3 < \bar{\psi}\psi >= a_0 + a_1\sqrt{ma} + a_2 ma$ **CE2:** $a^3 < \bar{\psi}\psi >= b_0 + b_1 ma \log ma + b_2 ma)$

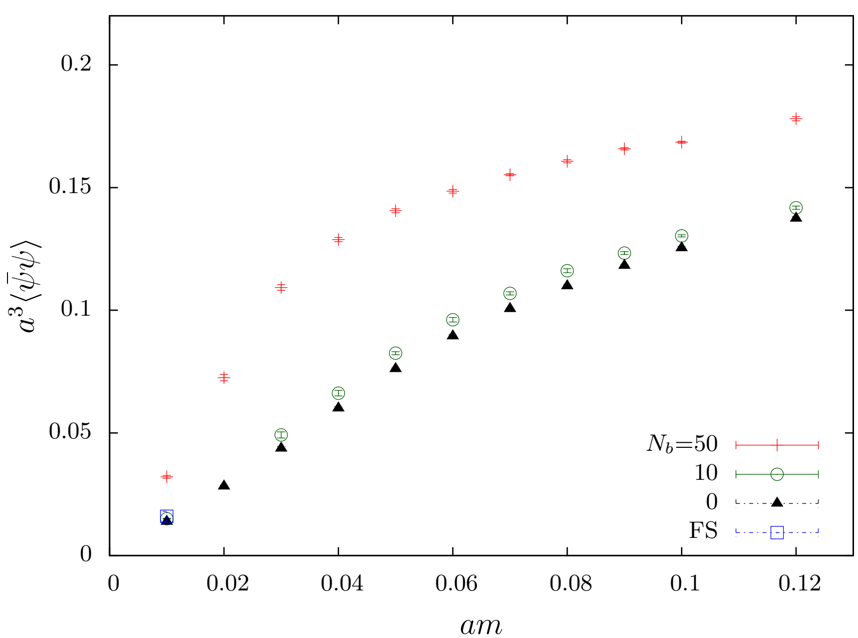
FS = check for finite-size effects with $24^3 \times 6$.

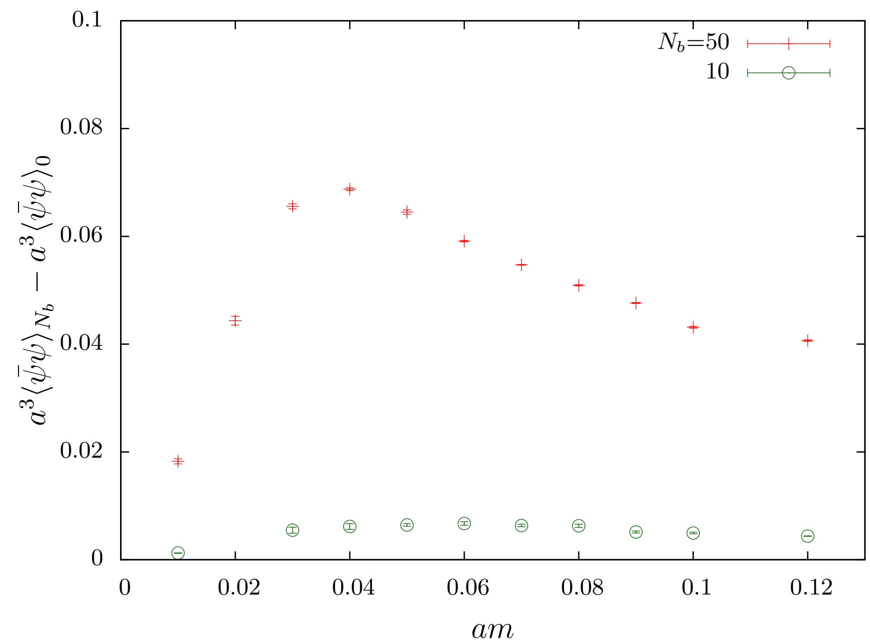
The chiral condensate as a function of the flux for various values of ma and with two chiral extrapolations



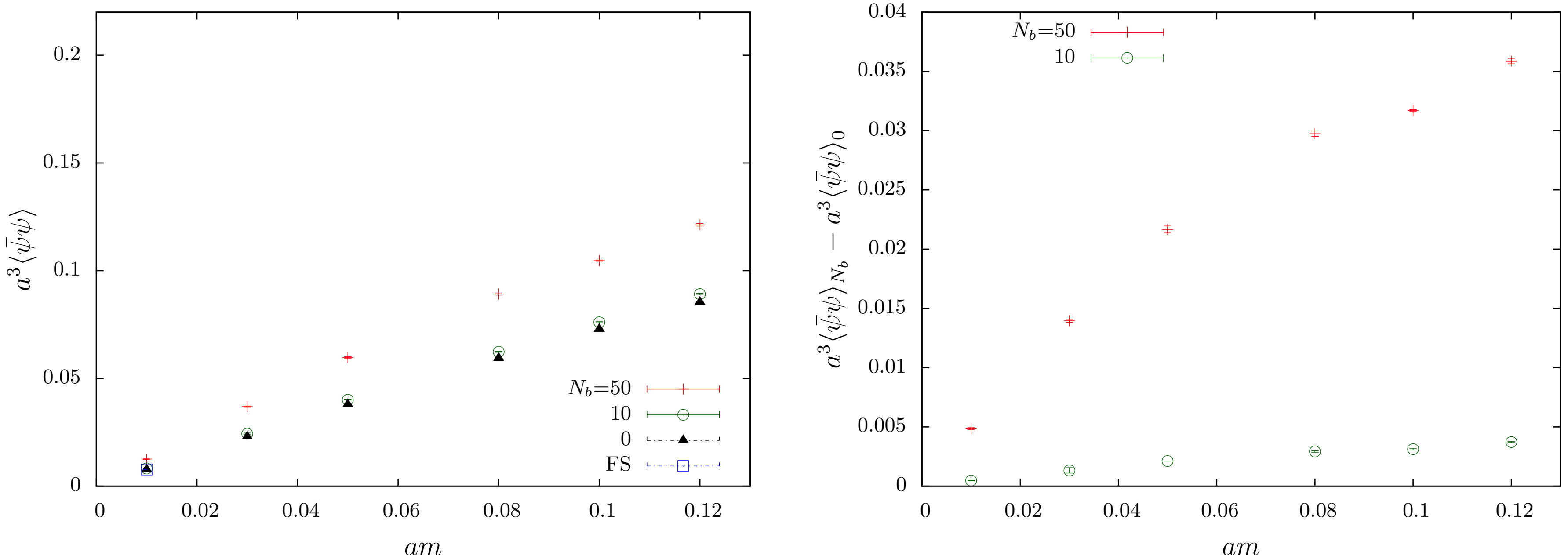
The slope at $ma = 0$ can be compared with chiral model $\Rightarrow F_\pi \approx 60$ MeV

The chiral condensate, transition region, $\beta = 1.9$





The chiral condensate, deconfined phase, $\beta = 2.1$



7. Conclusions and outlook

- We have investigated how a finite temperature system reacts to a constant external magnetic field, in two-colour QCD.

- In the confined phase the chiral condensate increases with the magnetic field strength as predicted by a chiral model, even a semi-quantitative agreement achieved.

- The transition temperature increases with the magnetic field strength.

- The chiral condensate goes to zero in the deconfined region for all values of the magnetic field.

- Simulations in the fixed-scale approach are running on GPU.

Next project

- **Toy model/strong coupling simulations for non-vanishing chemical potential(s) and in the canonical ensemble.**