Inhomogeneous chiral symmetry breaking phases

TECHNISCHE UNIVERSITÄT DARMSTADT

Michael Buballa

NICA/JINR-FAIR Bilateral Workshop

“Matter at highest baryon densities in the laboratory and in space”

FIAS, Frankfurt, April 2-4, 2012

Motivation

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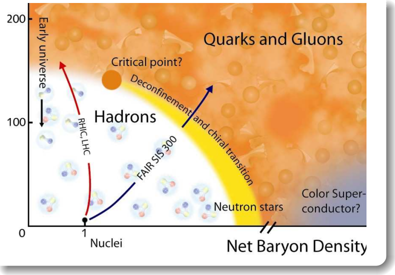

- QCD phase diagram

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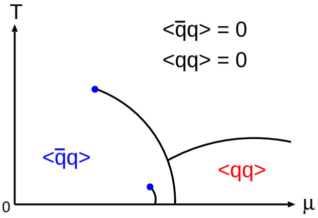

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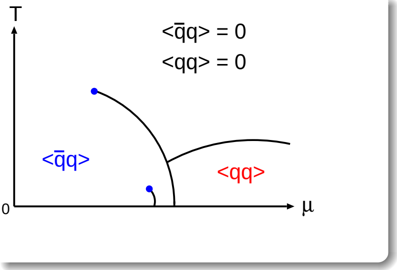

- QCD phase diagram
- frequent assumption:
  $\langle \bar{q}q \rangle$, $\langle qq \rangle$ constant in space

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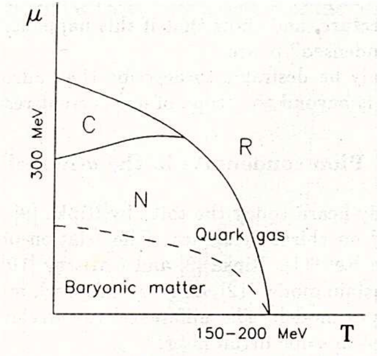

Broniowski et al., Acta Phys. Pol. B (1991)

- QCD phase diagram
- frequent assumption:
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- inhomogeneous phases:
  - pion condensates
  - chiral density wave
  - Skyrme crystals
  - crystalline (color) superconductors
  - 1+1 D Gross-Neveu model

Motivation

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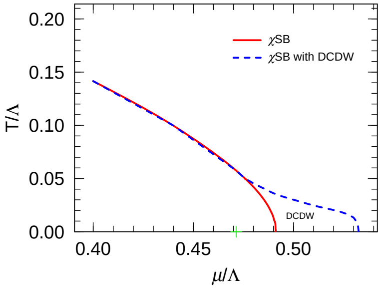

Nakano, Tatsumi, PRD (2005)

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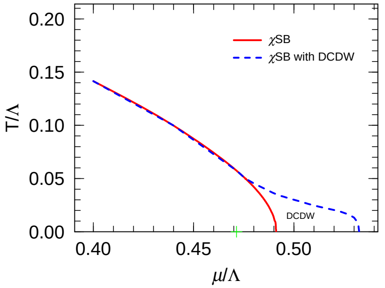

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- This talk:
  inhomogeneous χSB in the NJL model

Model

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- NJL model:

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + G_S\left((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right)$$

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$$\Rightarrow \quad \mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - m + 2G_S(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\right)\psi - G_S\left(\sigma^2 + \vec{\pi}^{\,2}\right)$$

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- mean-field approximation:

$$\sigma(x) \to \langle\sigma(x)\rangle \equiv S(\vec{x}), \quad \pi_a(x) \to \langle\pi_a(x)\rangle \equiv P(\vec{x})\,\delta_{a3}$$

  - $S(\vec{x})$, $P(\vec{x})$ time independent classical fields
  - retain space dependence !

Mean-field model

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- mean-field Lagrangian: $\mathcal{L}_{MF} = \bar{\psi}(x)\mathcal{S}^{-1}(x)\psi(x) - G_S\left(S^2(\vec{x}) + P^2(\vec{x})\right)$
  - inverse dressed propagator:

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- thermodynamic potential:

$$\Omega_{MF}(T,\mu;S,P) = -\frac{T}{V}\mathbf{Tr}\ln\left(\frac{1}{T}(i\partial_0 - \mathcal{H}_{MF} + \mu)\right) + \frac{G_S}{V}\int_V d^3x\,\left(S^2(\vec{x}) + P^2(\vec{x})\right)$$

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$$= -\frac{1}{V}\sum_\lambda\left[\frac{E_\lambda - \mu}{2} + T\ln\left(1 + e^{\frac{E_\lambda - \mu}{T}}\right)\right] + \frac{1}{V}\int_V d^3x\,\frac{|M(\vec{x}) - m|^2}{4G_s}$$

  - mass function: $M(\vec{x}) = m - 2G_S(S(\vec{x}) + iP(\vec{x}))$
  - $E_\lambda = E_\lambda[M(\vec{x})]$ = eigenvalues of $\mathcal{H}_{MF}$

One dimensional modulations

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- remaining tasks:
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  - minimize w.r.t. $M(\vec{x})$

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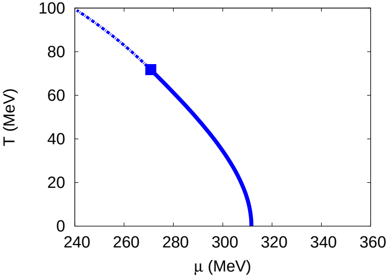
  - remaining task:

    minimize w.r.t. 2 parameters ($m \neq 0$: 3 parameters) much easier!

Phase diagram (chiral limit)

[D. Nickel, PRD (2009)]

homogeneous phases only



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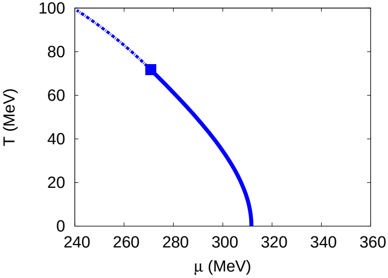
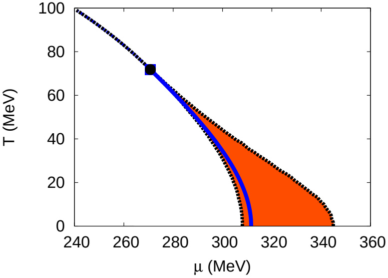


including inhomogeneous phase



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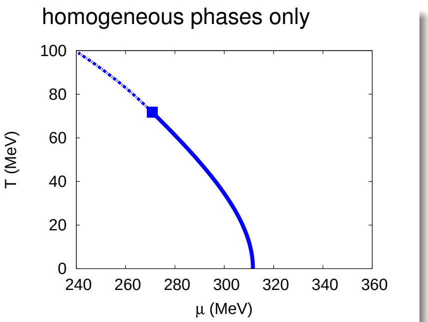

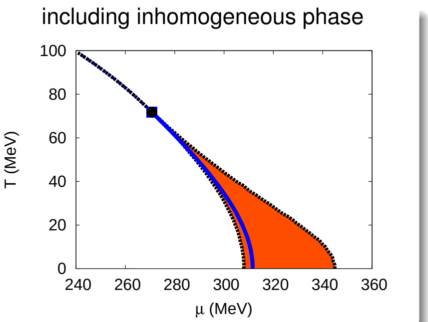

- 1st-order line completely covered by the inhomogeneous phase!
- all phase boundaries 2nd order
- critical point coincides with Lifshitz point

Mass functions and density profiles ($T = 0$)

- $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z|\nu) \quad \to \quad \begin{cases} \Delta\tanh(\Delta z) & \text{for} \quad \nu \to 1 \\ \sqrt{\nu}\Delta\sin(\Delta z) & \text{for} \quad \nu \to 0 \end{cases}$

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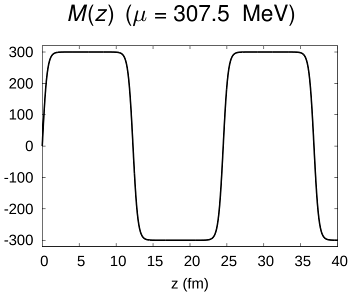

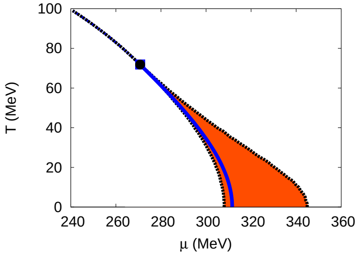

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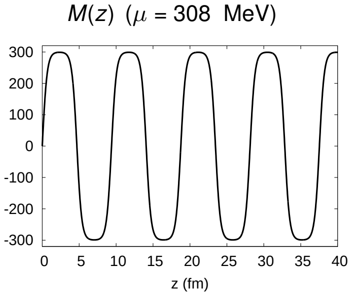

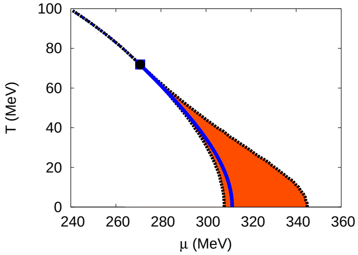

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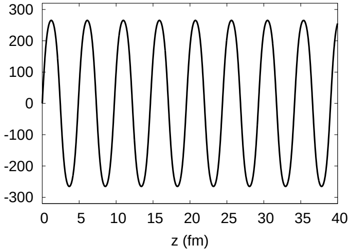
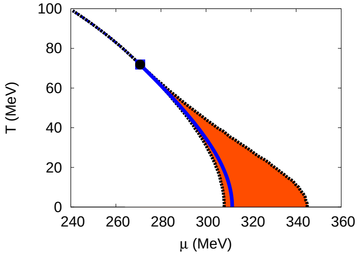
$M(z)$ ($\mu = 309$ MeV)





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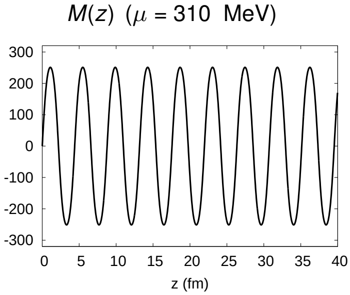

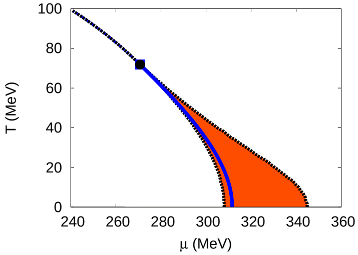

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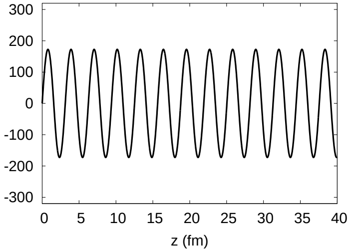
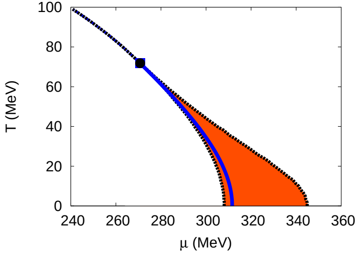
$M(z)$ (μ = 320 MeV)





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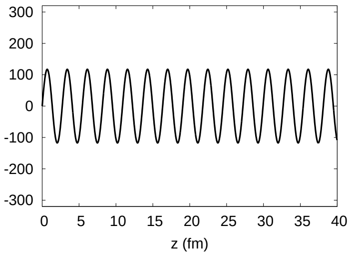

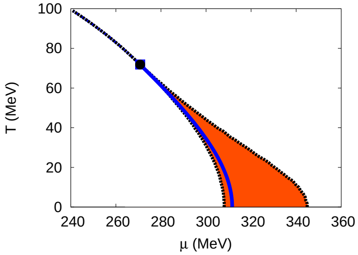

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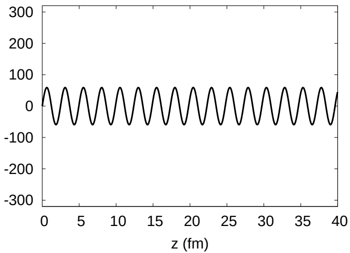
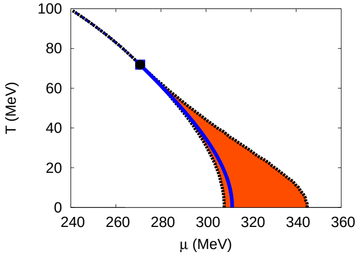
$M(z)$ (μ = 340 MeV)





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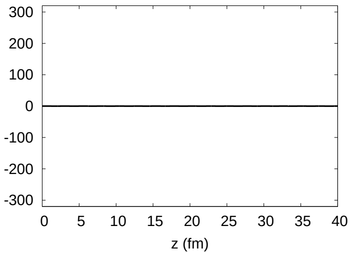
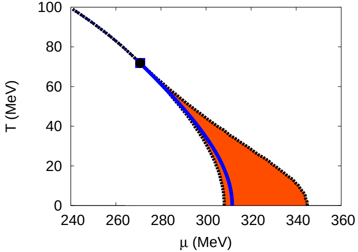
$M(z)$ (μ = 345 MeV)





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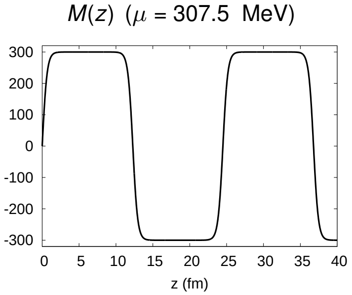

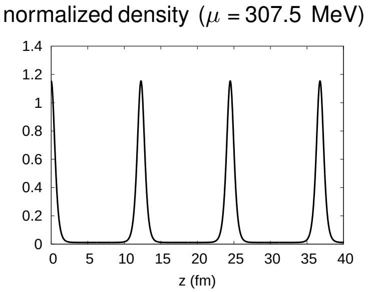

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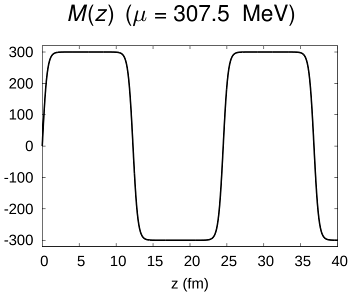

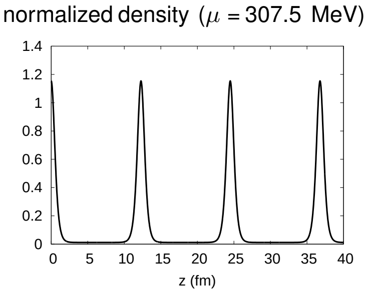

- Quarks reside in the chirally restored regions.

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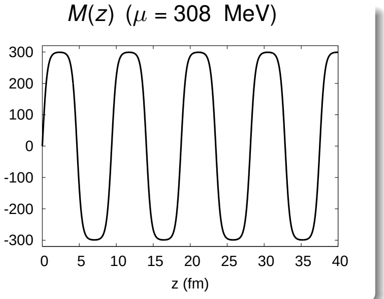

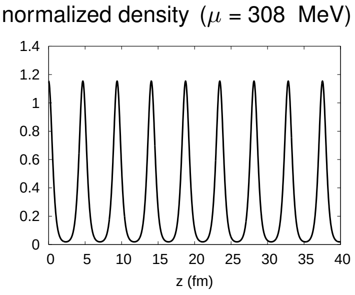

- Quarks reside in the chirally restored regions.

Mass functions and density profiles ($T = 0$)

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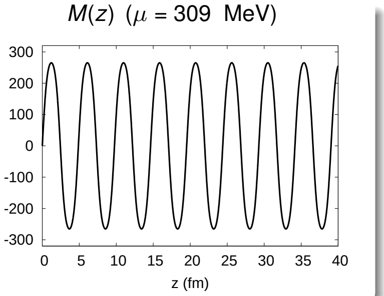

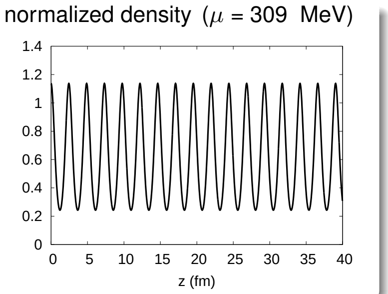

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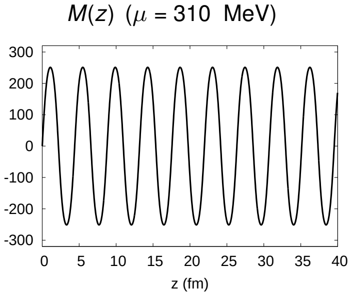

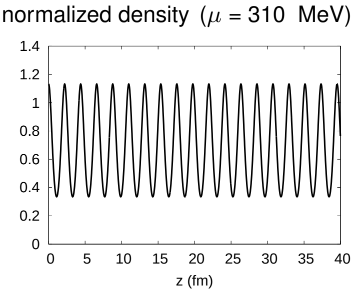

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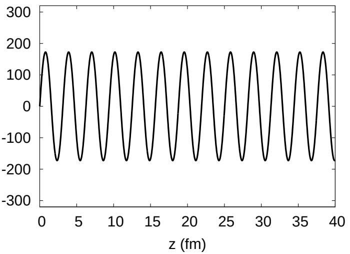
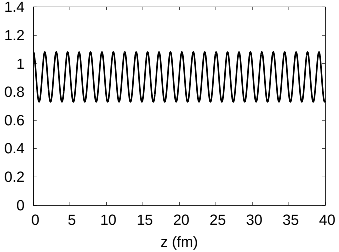
$M(z)$ (μ = 320 MeV)



normalized density (μ = 320 MeV)



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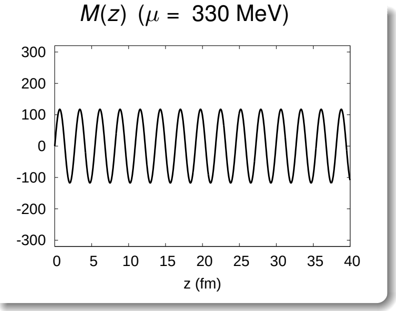

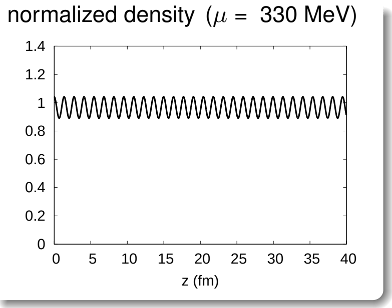

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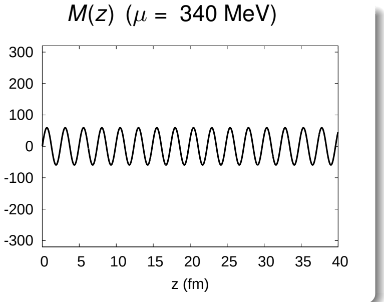

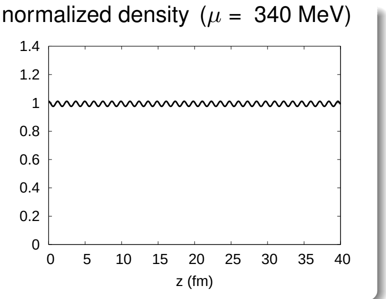

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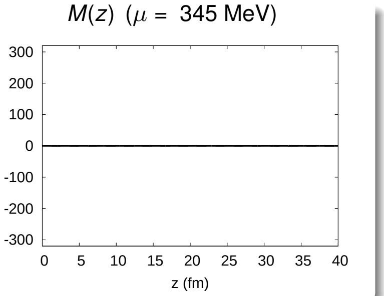

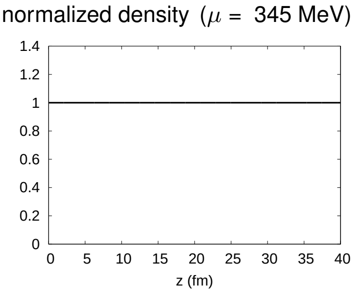

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Including vector interactions

[S. Carignano, D. Nickel, M.B., PRD (2010)]

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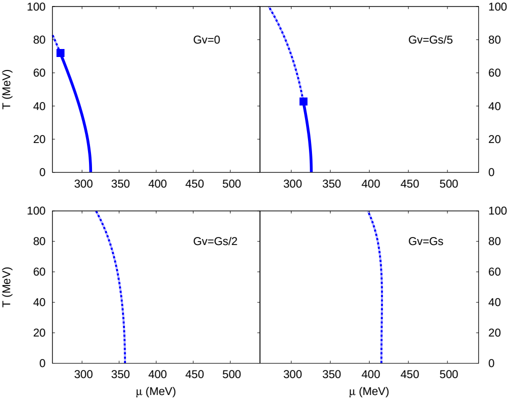

- additional interaction term:

  $\mathcal{L}_V = -G_V(\bar{\psi}\gamma^\mu\psi)^2$

- homogeneous phases: strong G_V-dependence of the critical point

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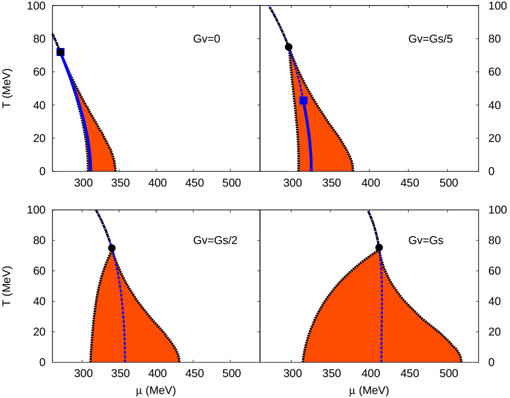

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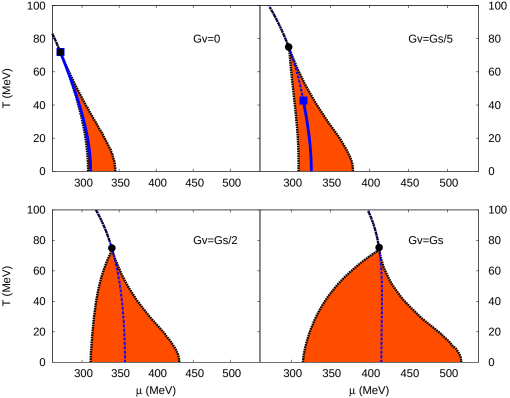

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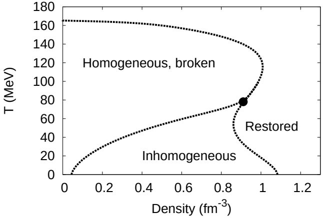
  T-$\langle n \rangle$ phase diagram:



- independent of G_V!

- homogeneous phases: strong G_V-dependence of the critical point
- inhomogeneous regime: stretched in μ direction, Lifshitz point at constant T

Susceptibilities

- signature of the critical point: divergent susceptibilities
- e.g., quark number susceptibility:

$$\chi_{nn} = -\frac{\partial^2 \Omega}{\partial \mu^2} = \frac{\partial n}{\partial \mu}$$

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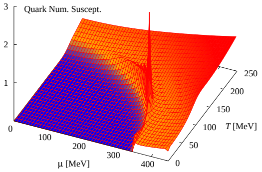

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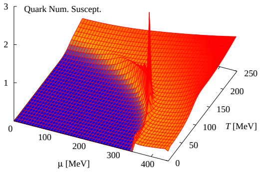

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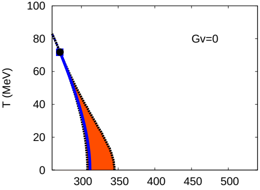

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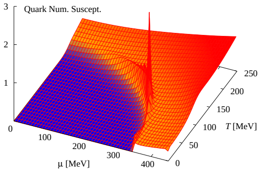

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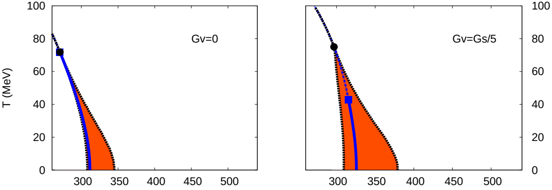

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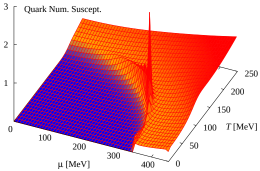

[K. Fukushima, PRD (2008)]

- $G_V = 0$:
  CP = Lifshitz point
  $\rightarrow$ no qualitative change
- $G_V > 0$:
  no CP $\rightarrow$ no divergence

Susceptibilities

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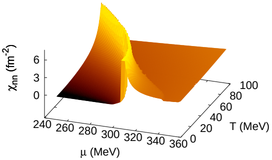
- results:



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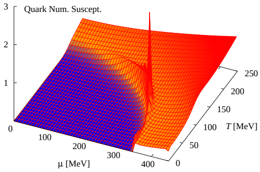

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- $G_V = 0$:

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Susceptibilities

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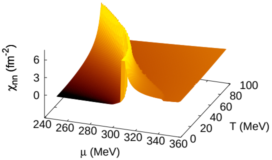

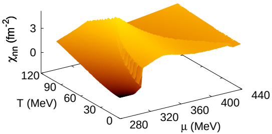

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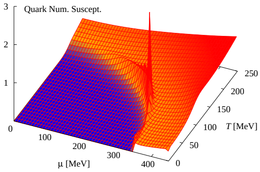

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- $G_V = 0$:
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- $G_V > 0$:
  no divergence

Including Polyakov-loop effects

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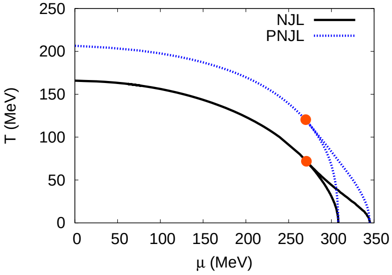
- PNJL model: $\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi + G_S\left((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right) + U(\ell, \bar{\ell})$
- simplifying assumption:
  ℓ, $\bar{\ell}$ space-time independent, even in inhomogeneous phases

phase diagram:



- Polyakov loop:
  suppression of thermal effects
  $\rightarrow$ phase diagram stretched in T direction

Two-dimensional modulations

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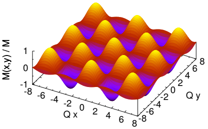
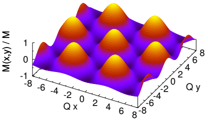
- consider two shapes:
  - square lattice ("egg carton")

    $$M(x,y) = M\cos(Qx)\cos(Qy)$$



  - hexagonal lattice

    $$M(x,y) = \frac{M}{3}\left[2\cos(Qx)\cos\left(\frac{1}{\sqrt{3}}Qy\right) + \cos\left(\frac{2}{\sqrt{3}}Qy\right)\right]$$



- minimize both cases numerically w.r.t. M and Q

Two-dimensional modulations: results

[S. Carignano, M.B., arXiv:1203.5343]

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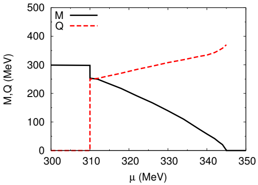
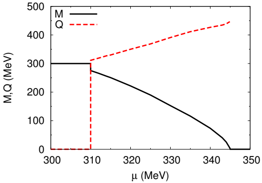
- amplitudes and wave numbers:
  - egg carton:



  - hexagon:



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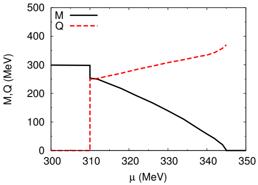

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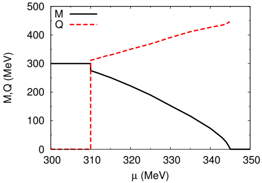
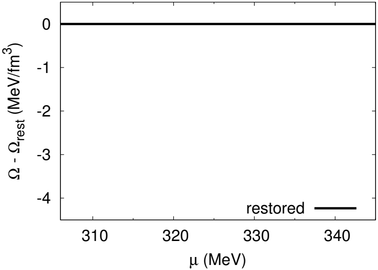


free-energy gain at T = 0:



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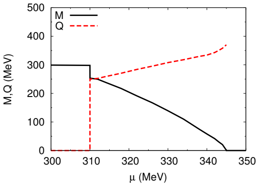

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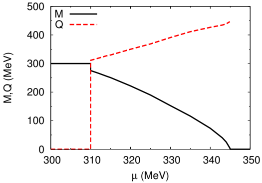

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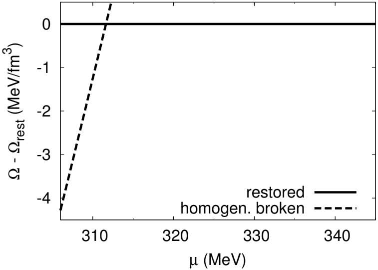

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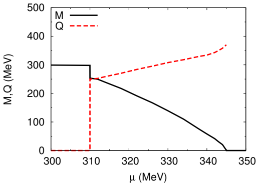

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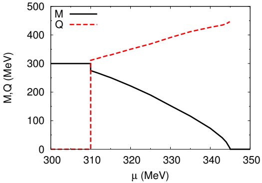

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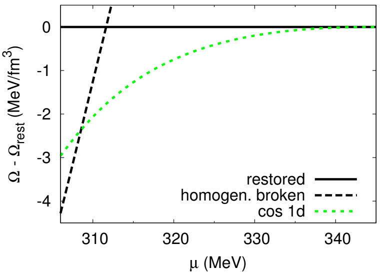

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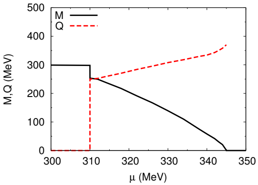

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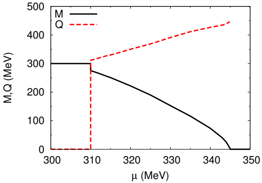

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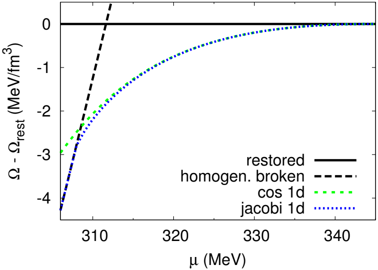

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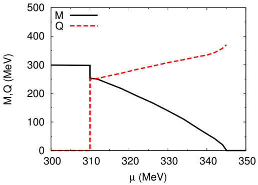

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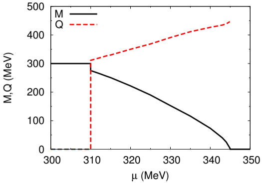

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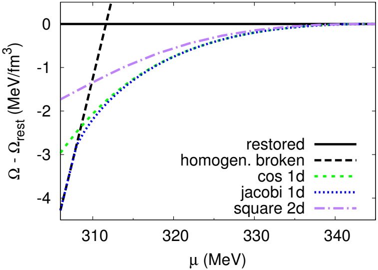

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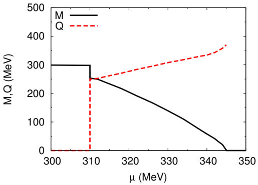

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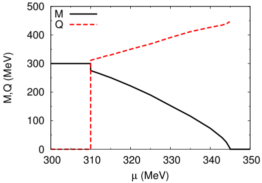

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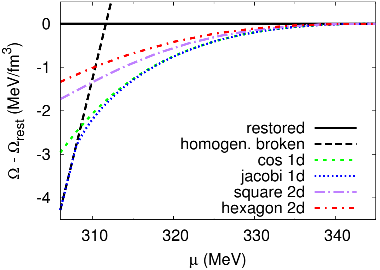
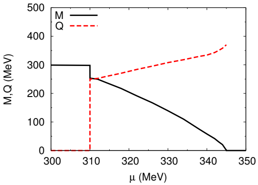


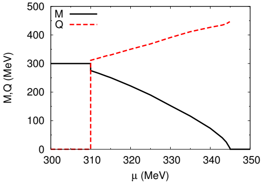
Two-dimensional modulations: results

[S. Carignano, M.B., arXiv:1203.5343]

- amplitudes and wave numbers:
  - egg carton:



  - hexagon:



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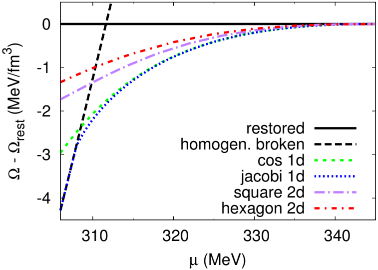

- 2d not favored over 1d in this regime

Two-dimensional modulations: further results

[S. Carignano, M.B., arXiv:1203.5343]

- rectangular lattice:

  $M(x, y) = M \cos(Q_x x) \cos(Q_y y)$

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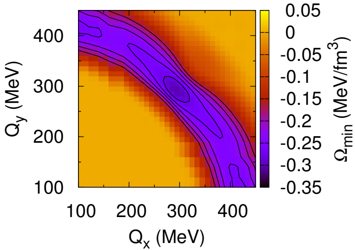

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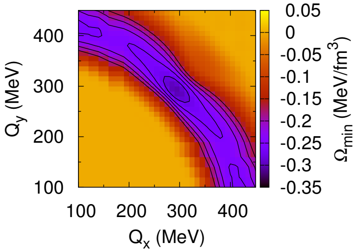

$\Rightarrow$ “egg carton” local minimum

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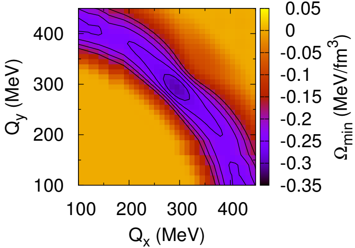
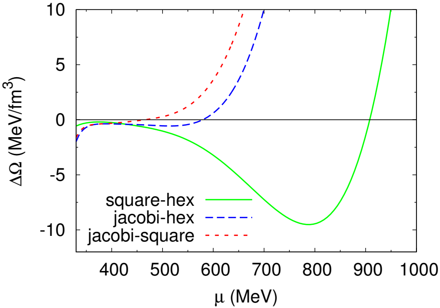


$\Rightarrow$ "egg carton" local minimum

- higher chemical potentials



- 450 MeV $< \mu <$ 900 MeV: egg carton favored
- $\mu >$ 900 MeV: hexagon favored

Competition with color superconductivity

[D. Nowakowski et al., MSc thesis]

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- additional quark-quark interaction: $\mathcal{L}_{qq} = H(q^T C i\gamma_5 \tau_A \lambda_{A'} q)(\bar{q} i\gamma_5 \tau_A \lambda_{A'} C \bar{q}^T)$
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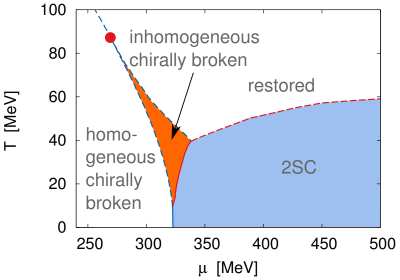

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  2SC phase favored at low T,
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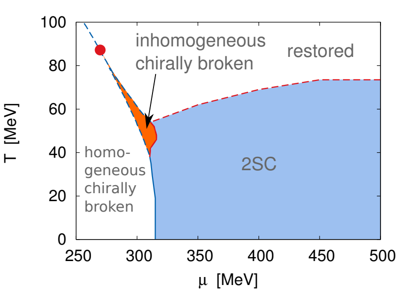

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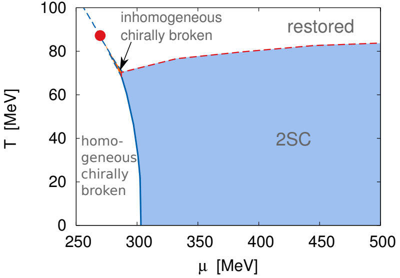

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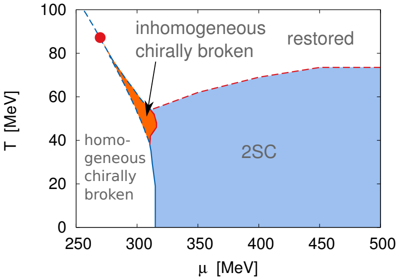

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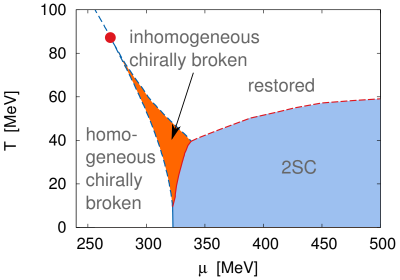

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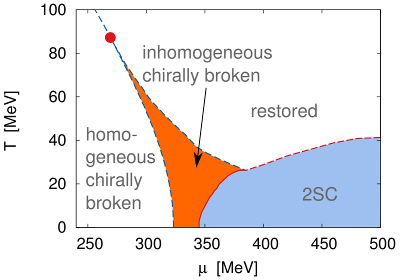

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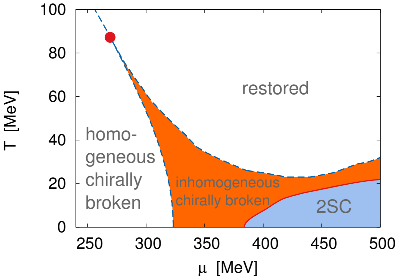

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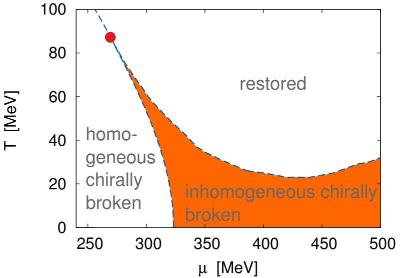

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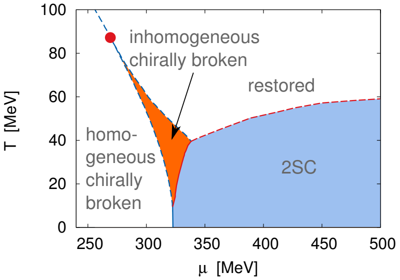

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- depends strongly on diquark coupling constant

Conclusions

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- NJL model with one- and two-dimensional modulations of $\langle \bar{q}q \rangle$:
  - 1st-order line and critical point covered by an inhomogeneous region
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  - number susceptibility always finite (for $G_V > 0$)
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- experimental signatures?
  - theory: calculate mesonic correlations ($\rightarrow$ dilepton spectra)

Collaborators

Dominik Nickel
(INT Seattle → Siemens)

Stefano Carignano
(TU Darmstadt)

Daniel Nowakowski
(TU Darmstadt)