Lattice QCD based equation of state at finite baryon density

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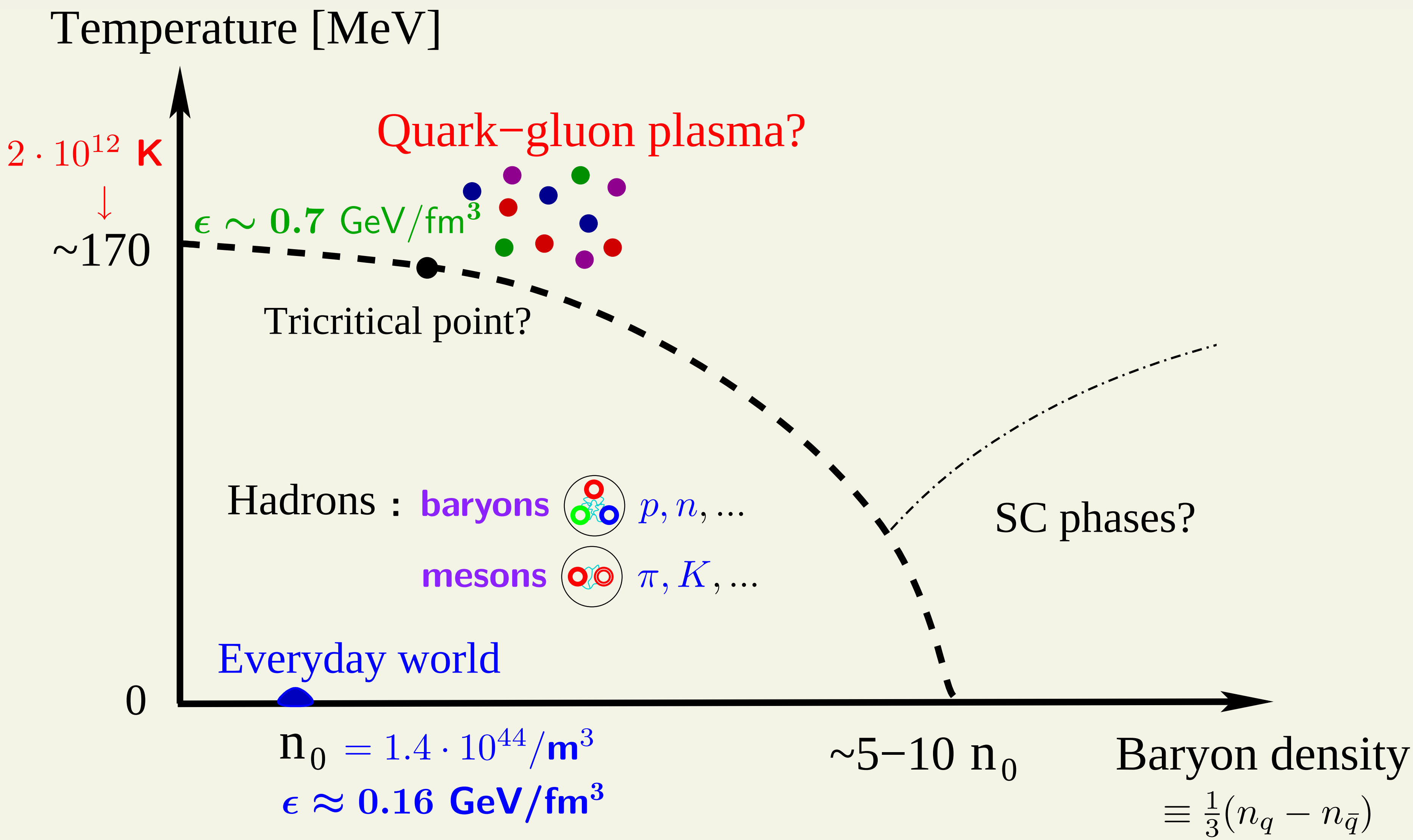
Nuclear phase diagram



Taylor expansion for pressure

$$\frac{P}{T^4} = \Sigma_{i,j}\, c_{ij}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_S}{T}\right)^j ,$$

where

$$c_{ij}(T) = \frac{1}{i!j!} \frac{\partial^i}{\partial(\mu_B/T)^i} \frac{\partial^j}{\partial(\mu_S/T)^j} \frac{P}{T^4},$$

i.e. moments of baryon number and strangeness fluctuations and correlations

- **an EoS based on lattice calculations of these?**

But: Most extensive study using p4 action with $N_\tau = 4$

$\Rightarrow$ **large discretization effects?**

Hadrons on lattice

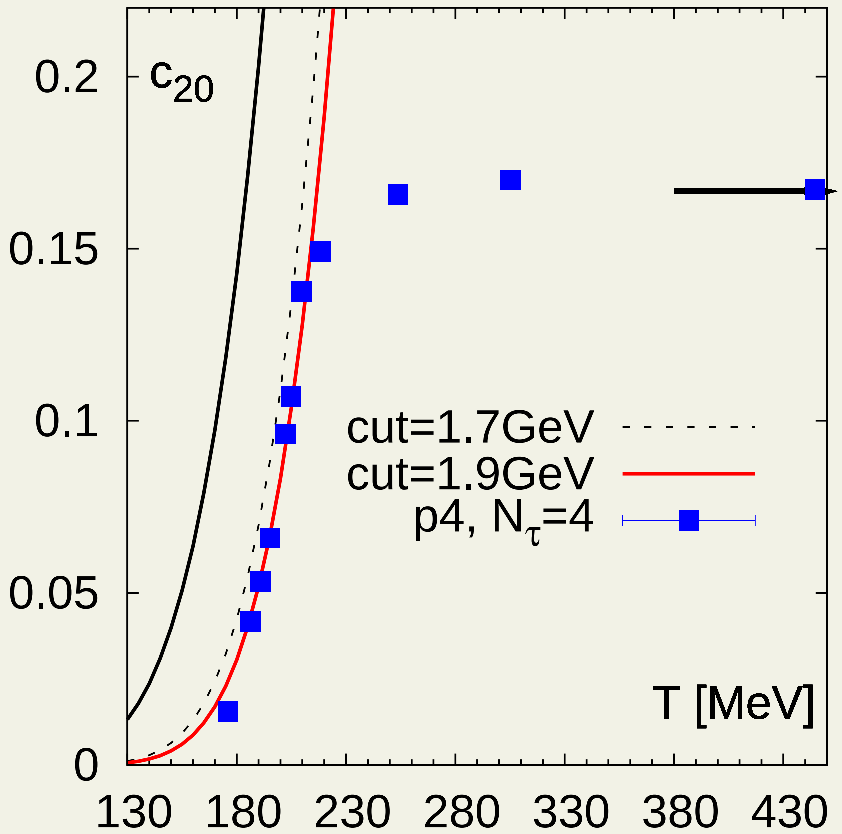
- 16 pseudoscalar mesons on lattice

- Hadron masses depend on lattice cutoff

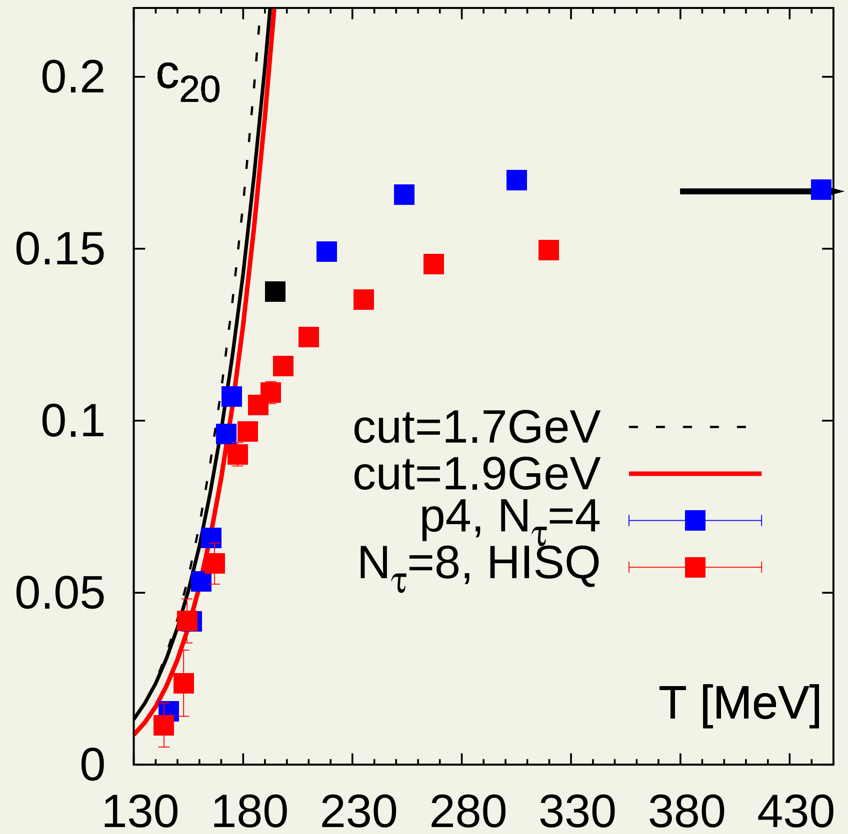
$\Rightarrow$ i.e. on temperature:
E.g. for pseudoscalar mesons on asqtad calculations

$$\begin{aligned} m^2_{\mathrm{ps}_i} &= m^2_{\mathrm{ps}_0} + \frac{1}{r_1^2}\frac{a^i_{\mathrm{ps}}x + b^i_{\mathrm{ps}}x^2}{(1 + c^i_{\mathrm{ps}}x)^{\beta_i}} \\ x &= (a/r_1)^2 \\ a &= \frac{1}{N_\tau T} \end{aligned}$$

30 MeV shift



$\Rightarrow$



Parametrization

$$c_{ij}(T) = \frac{a_{1ij}}{\hat{T}^{n_{1ij}}} + \frac{a_{2ij}}{\hat{T}^{n_{2ij}}} + \frac{a_{3ij}}{\hat{T}^{n_{3ij}}} + \frac{a_{4ij}}{\hat{T}^{n_{4ij}}} + \frac{a_{5ij}}{\hat{T}^{n_{5ij}}} + \frac{a_{6ij}}{\hat{T}^{n_{6ij}}} + c_{ij}^{SB},$$

where n_{kij} **are integers with** $1 < n_{kij} < 42$**,**
and

$$\hat{T} = \frac{T - T_s}{R},$$

with $T_s = 0.1$ **or** 0 **GeV, and** $R = 0.05$ **or** 0.15 **GeV.**

Constraints:

$$
\begin{aligned}
c_{ij}(T_{\rm sw}) &= c_{ij}^{\rm HRG}(T_{\rm sw}) \\
\frac{\mathrm{d}}{\mathrm{d}T}c_{ij}(T_{\rm sw}) &= \frac{\mathrm{d}}{\mathrm{d}T}c_{ij}^{\rm HRG}(T_{\rm sw}) \\
\frac{\mathrm{d}^2}{\mathrm{d}T^2}c_{ij}(T_{\rm sw}) &= \frac{\mathrm{d}^2}{\mathrm{d}T^2}c_{ij}^{\rm HRG}(T_{\rm sw}) \\
\frac{\mathrm{d}^3}{\mathrm{d}T^3}c_{ij}(T_{\rm sw}) &= \frac{\mathrm{d}^3}{\mathrm{d}T^3}c_{ij}^{\rm HRG}(T_{\rm sw})
\end{aligned}
$$

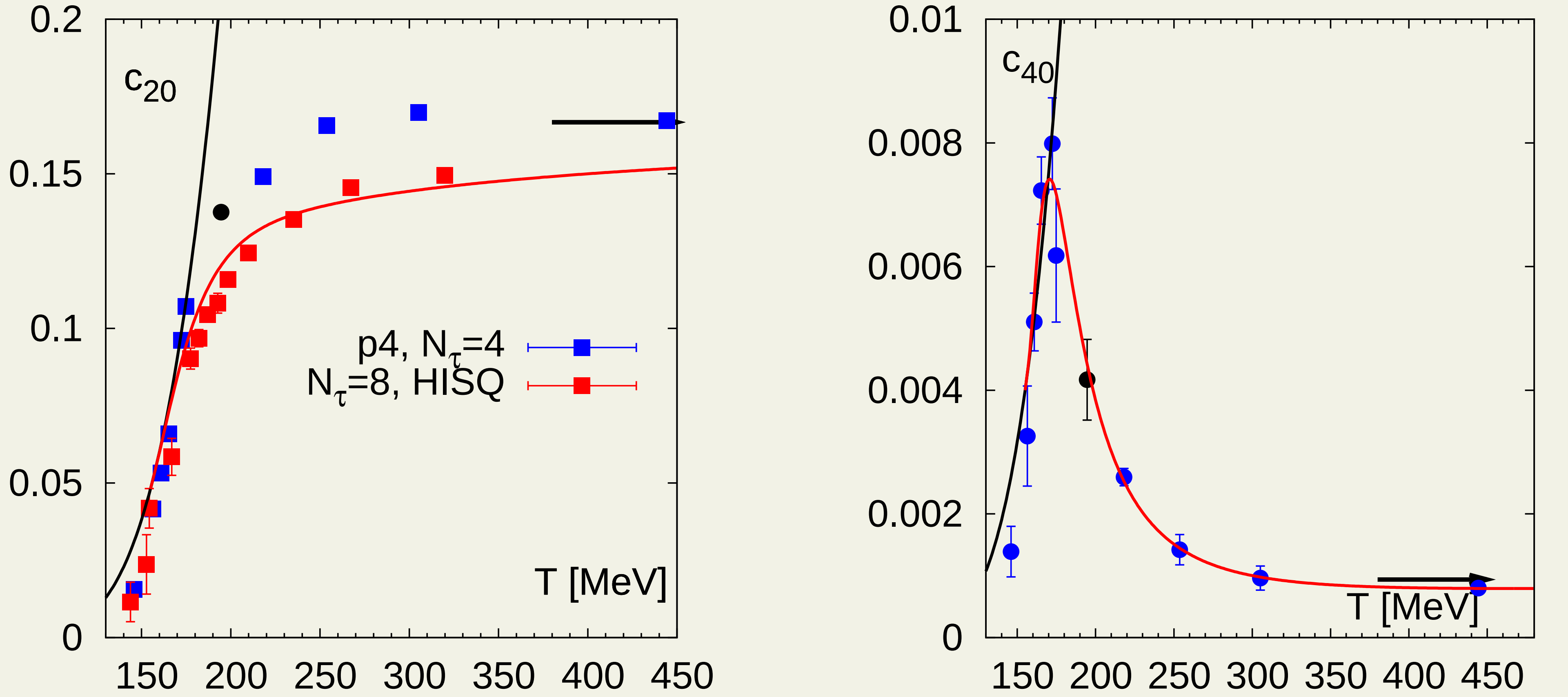
at $T_{\rm sw} = 155$ MeV

3rd derivative to quarantee smooth behaviour of speed of sound:

$$c_s^2 \propto \frac{\mathrm{d}^2}{\mathrm{d}T^2}c_{ij}$$

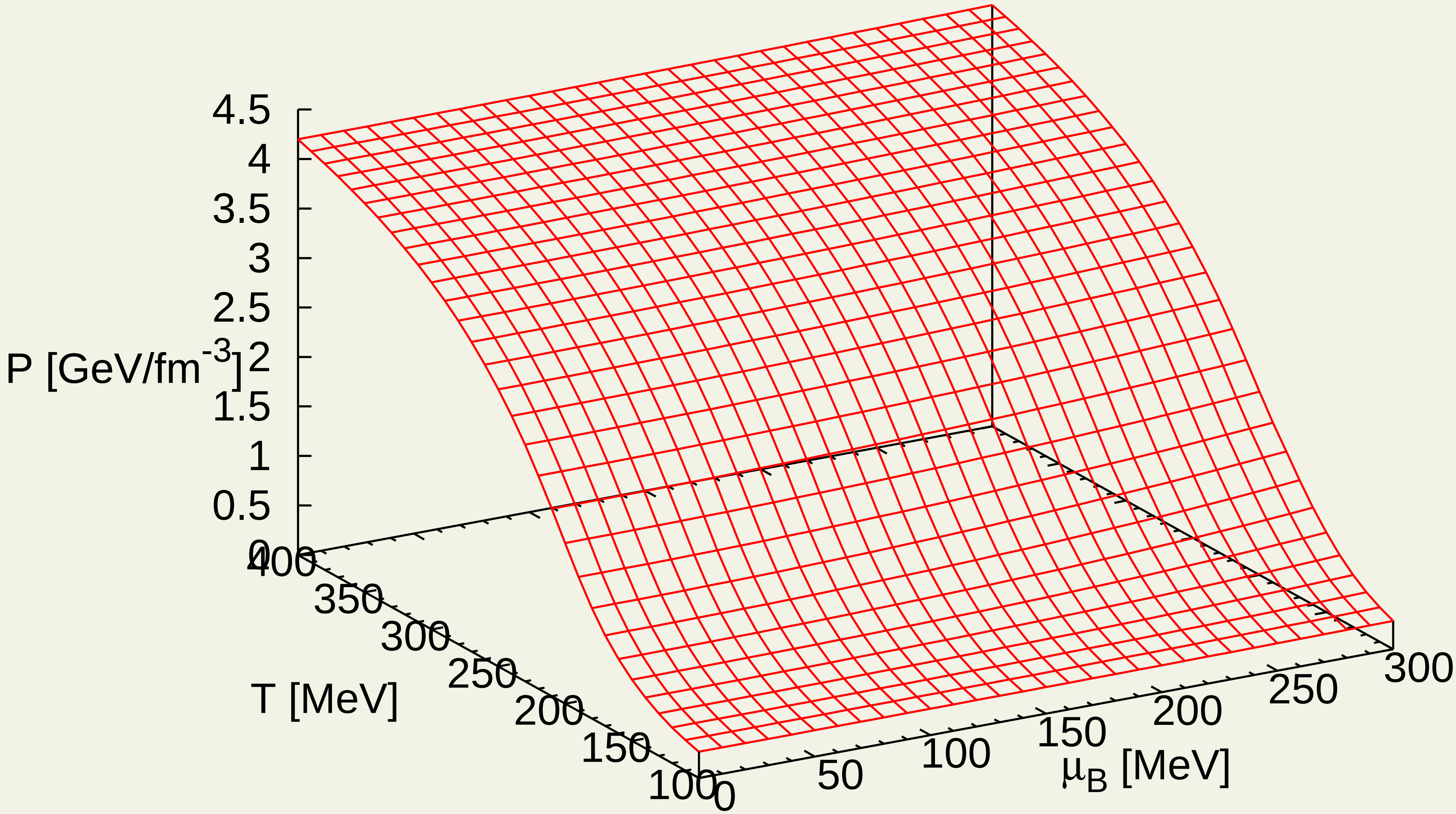
For second order terms also

$$c_{ij}(T = 800\,\mathrm{MeV}) = 0.95 \cdot SB_{ij}$$



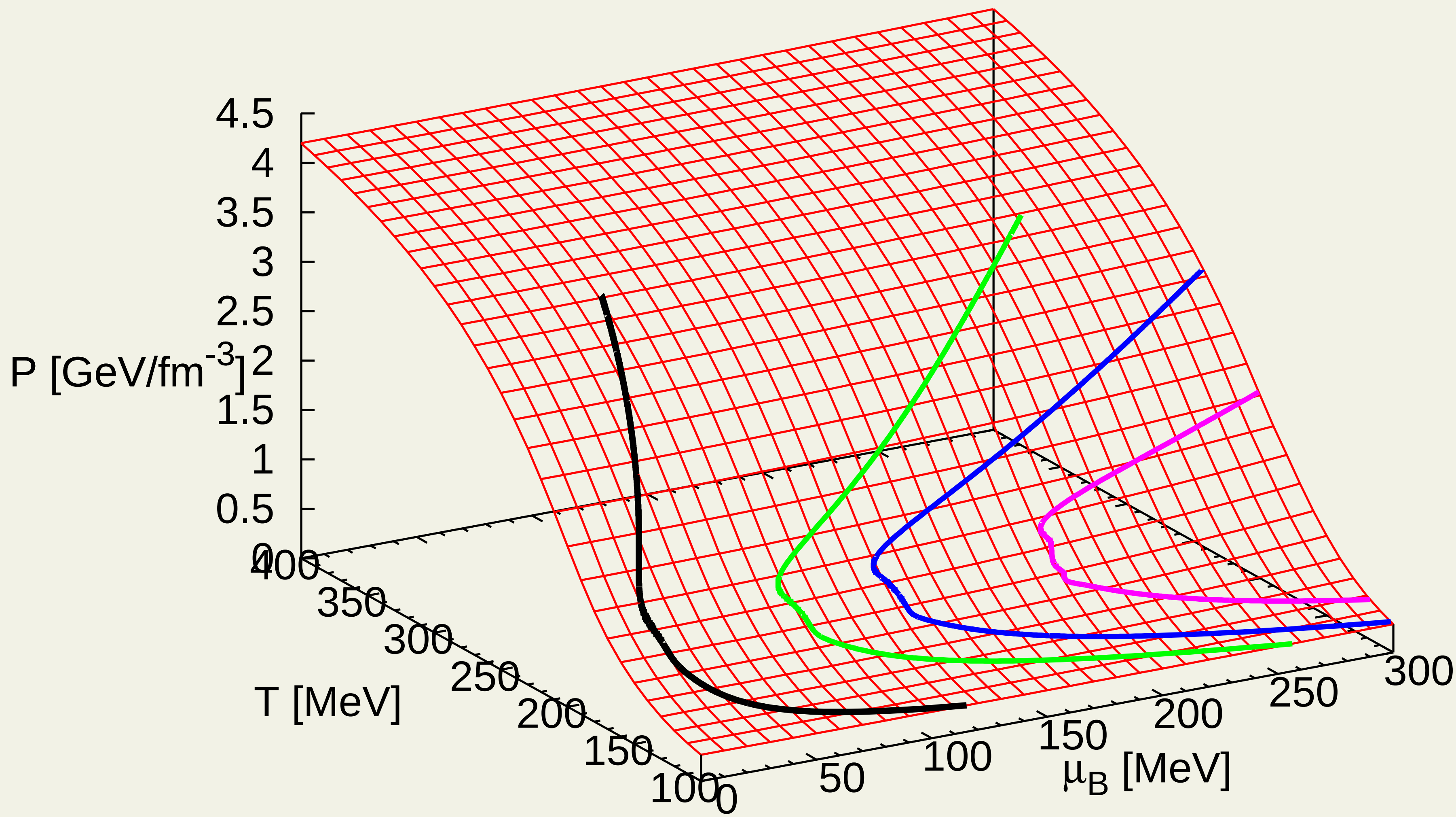
c20
p4, Nτ=4
Nτ=8, HISQ
T [MeV]
0.2
0.15
0.1
0.05
0
150 200 250 300 350 400 450
c40
0.01
0.008
0.006
0.004
0.002
0
150 200 250 300 350 400 450
T [MeV]

P/T^4



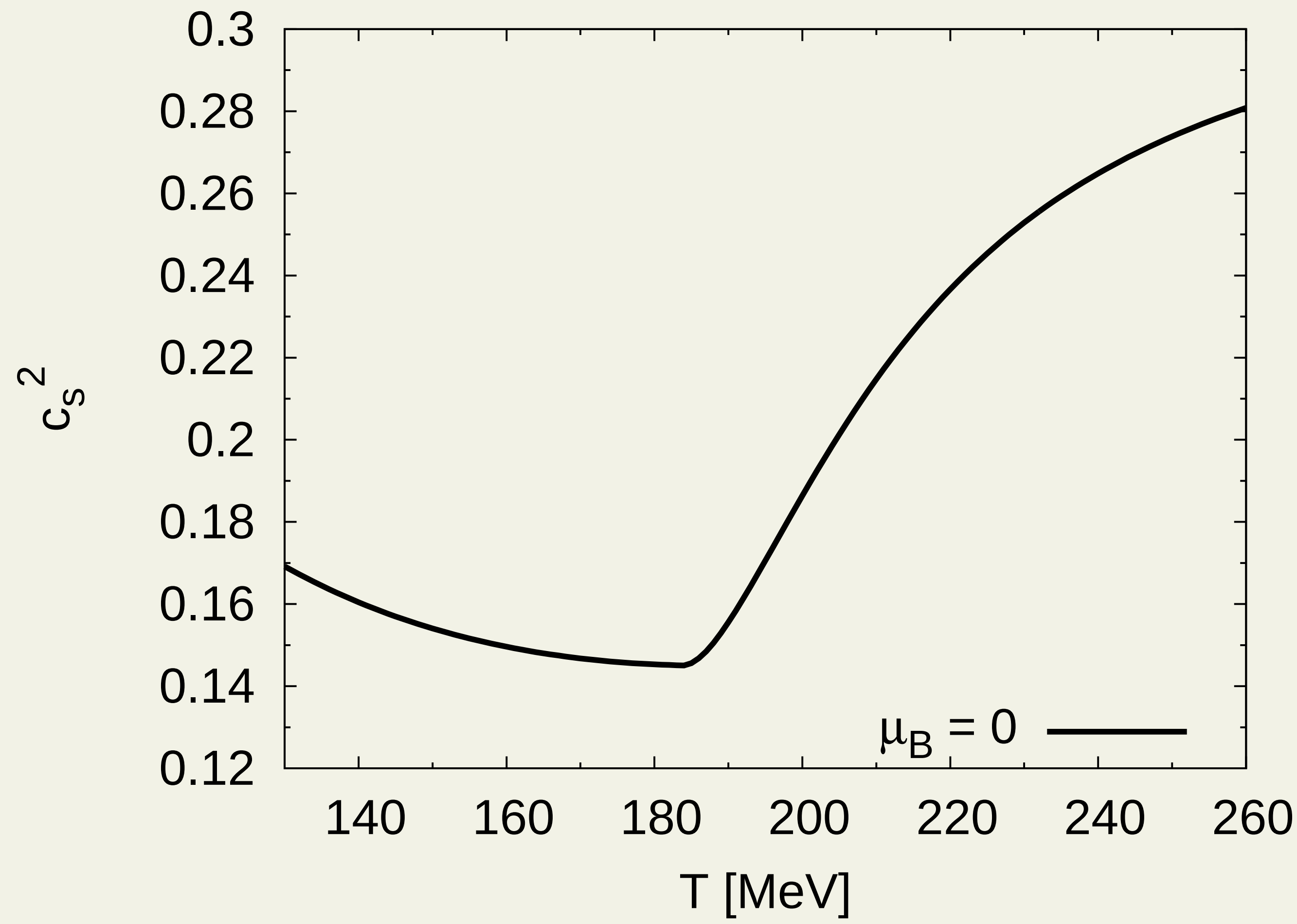
P [GeV/fm-3]
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
400
350
300
250
200
150
100
T [MeV]
0
50
100
150
200
250
300
μB [MeV]

P/T^4



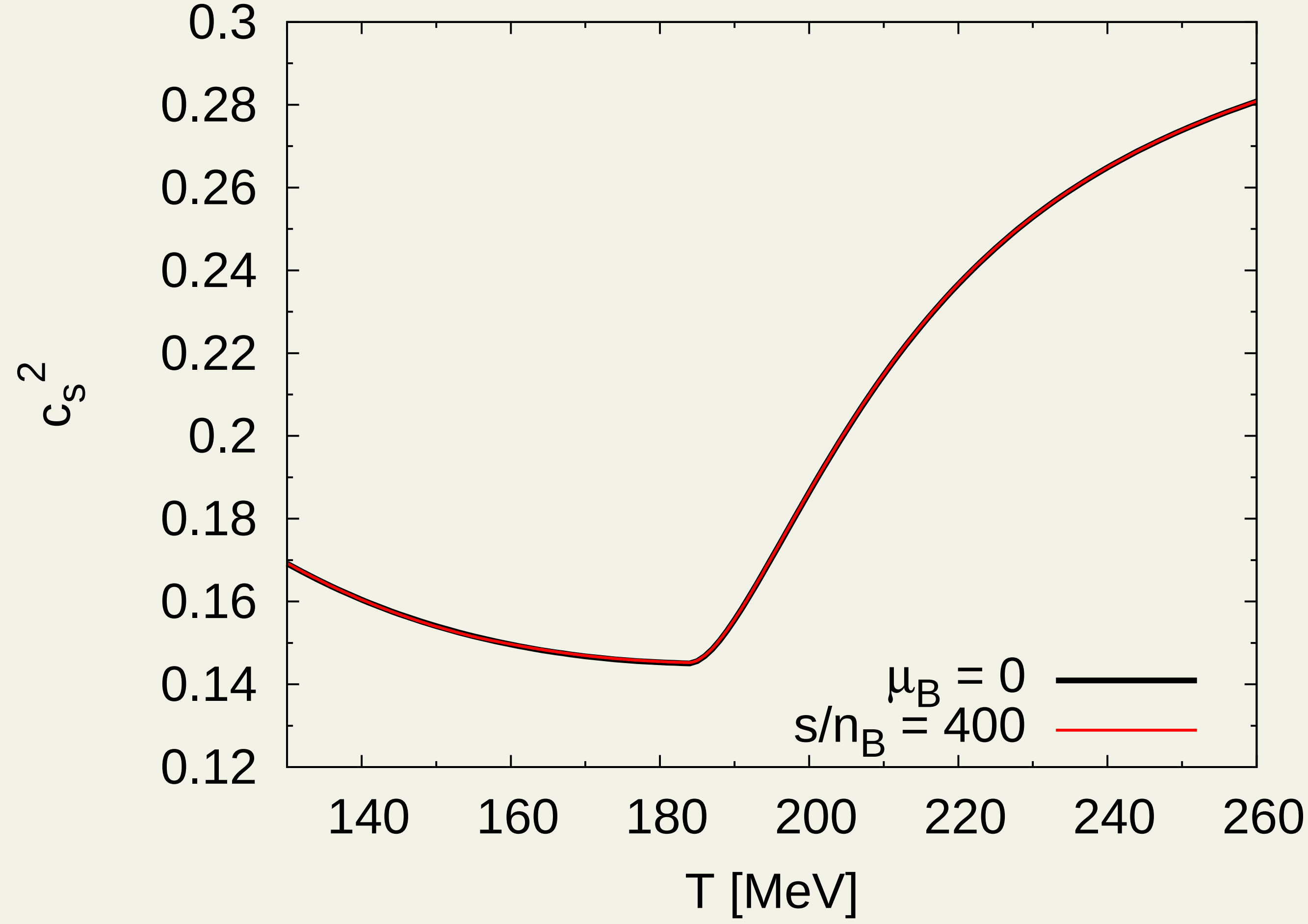
P [GeV/fm^{-3}]
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
T [MeV]
400
350
300
250
200
150
100
μ_B [MeV]
0
50
100
150
200
250
300

Speed of sound

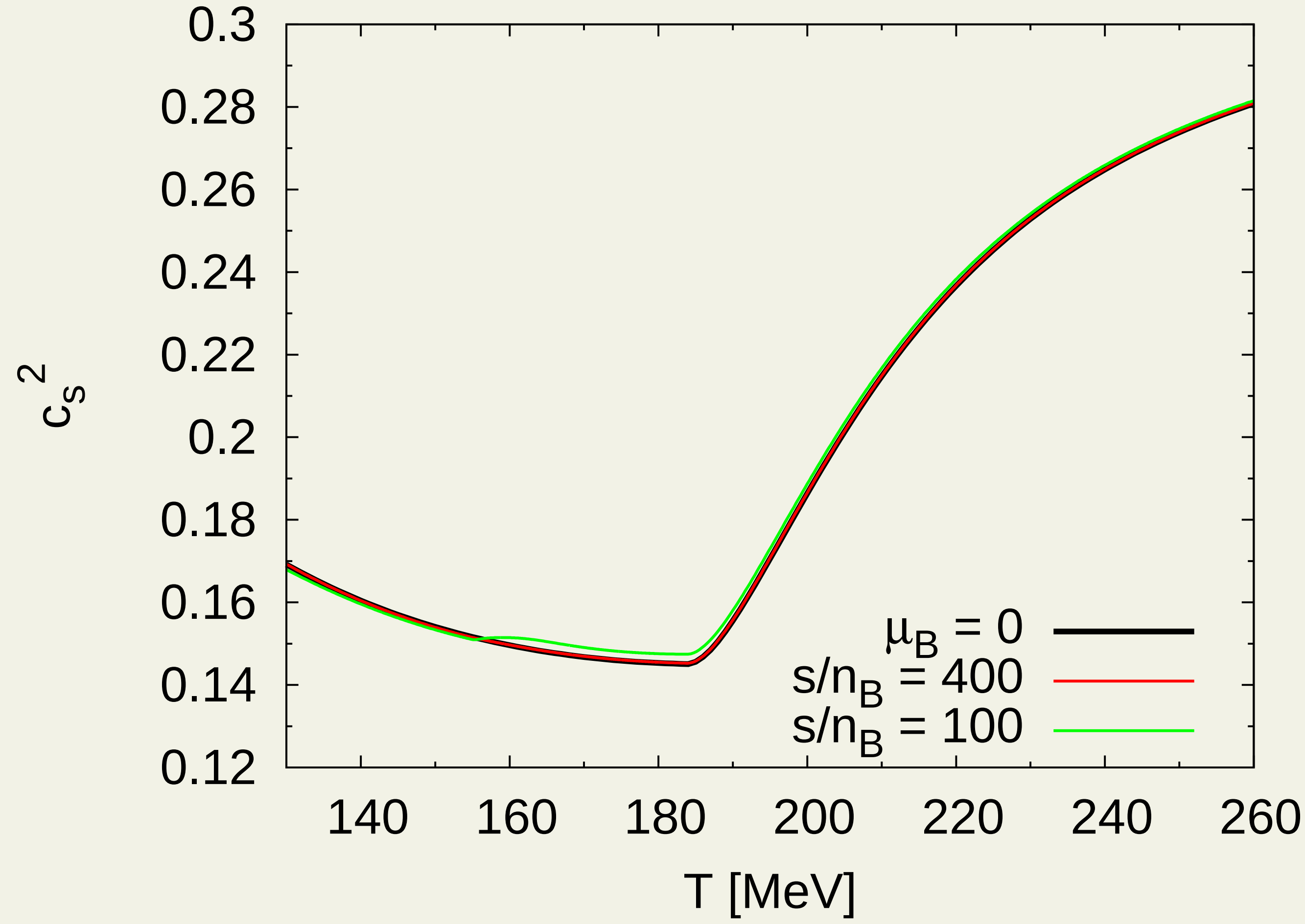


- **s95p-v1 parametrization by P. Petreczky and P.H.**

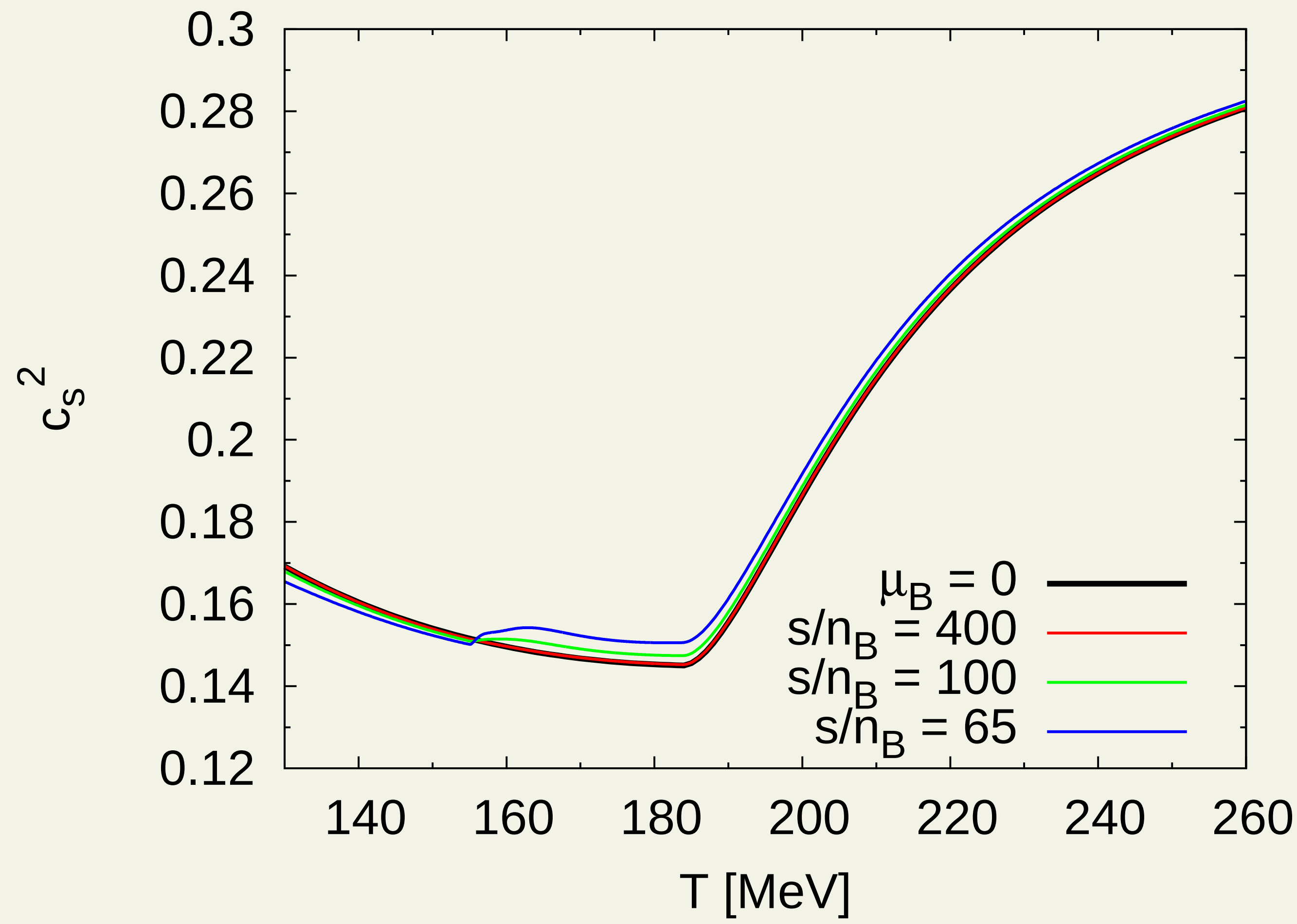
Speed of sound



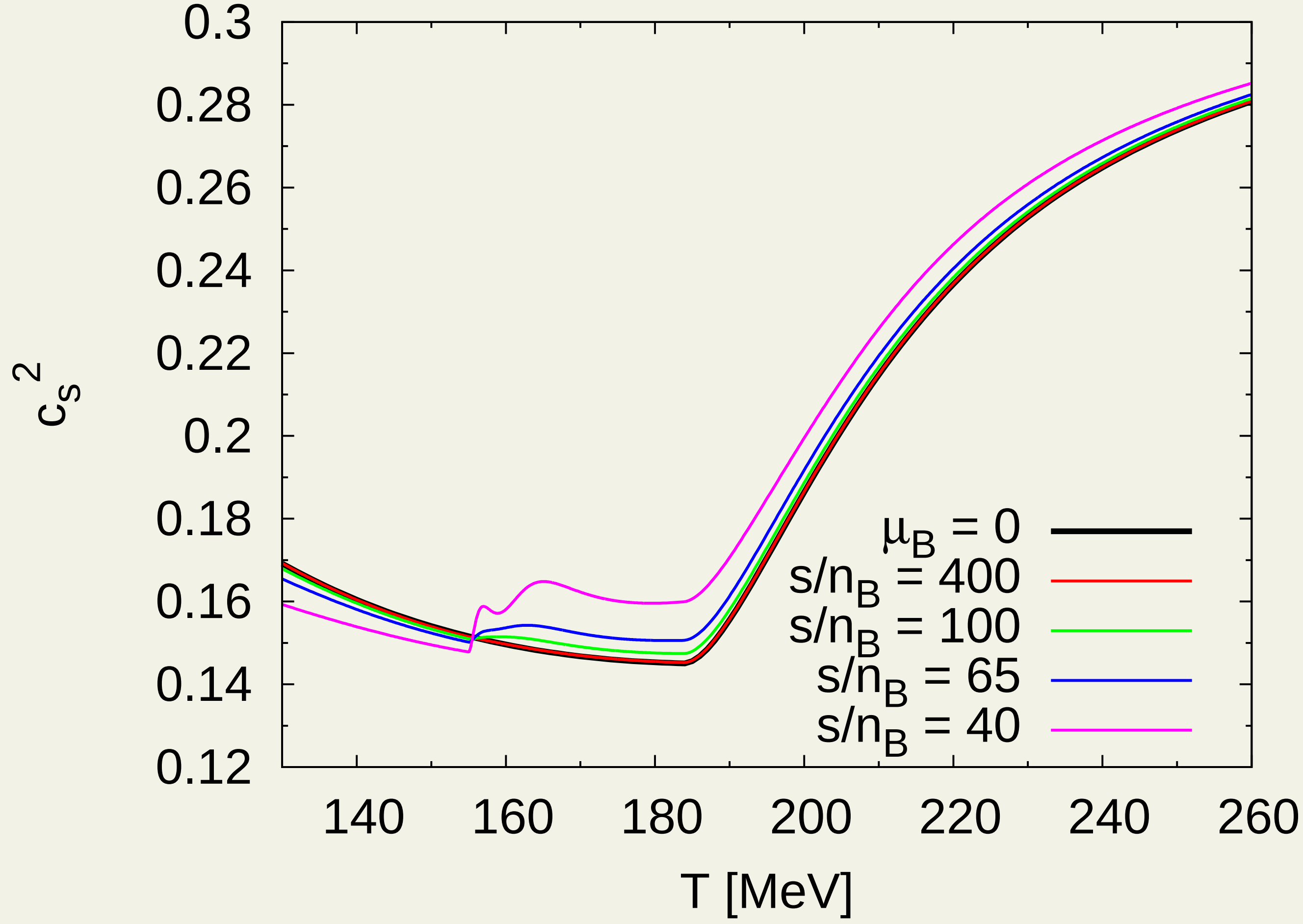
Speed of sound



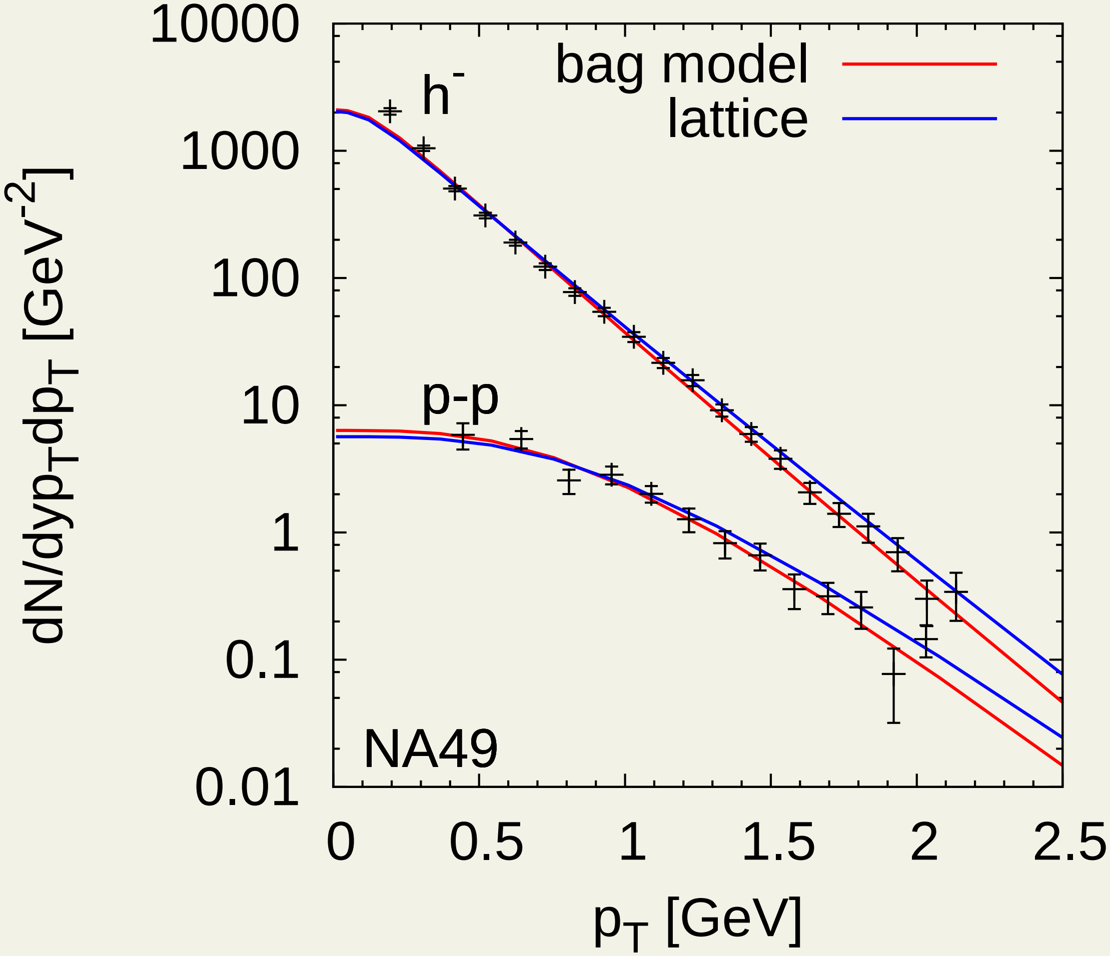
Speed of sound



Speed of sound

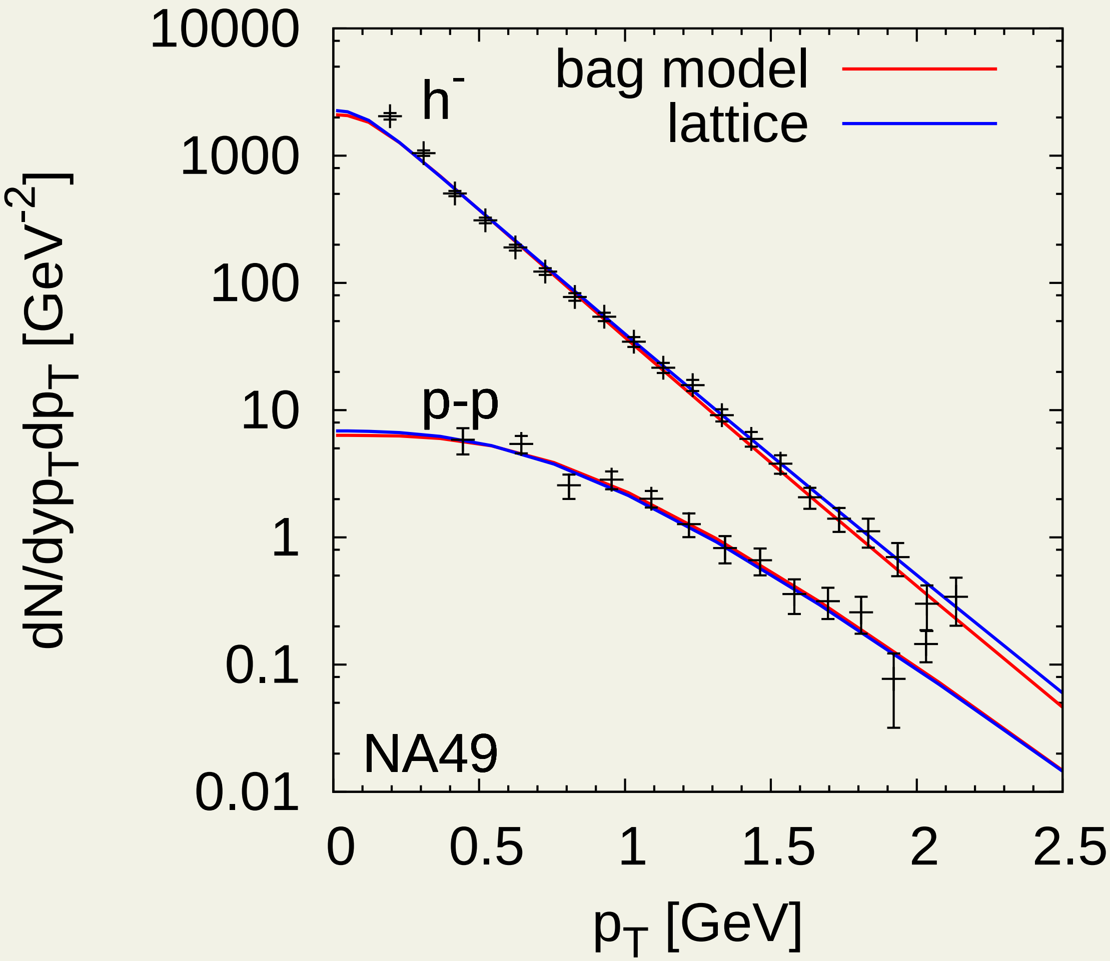


p_T-spectra at SPS



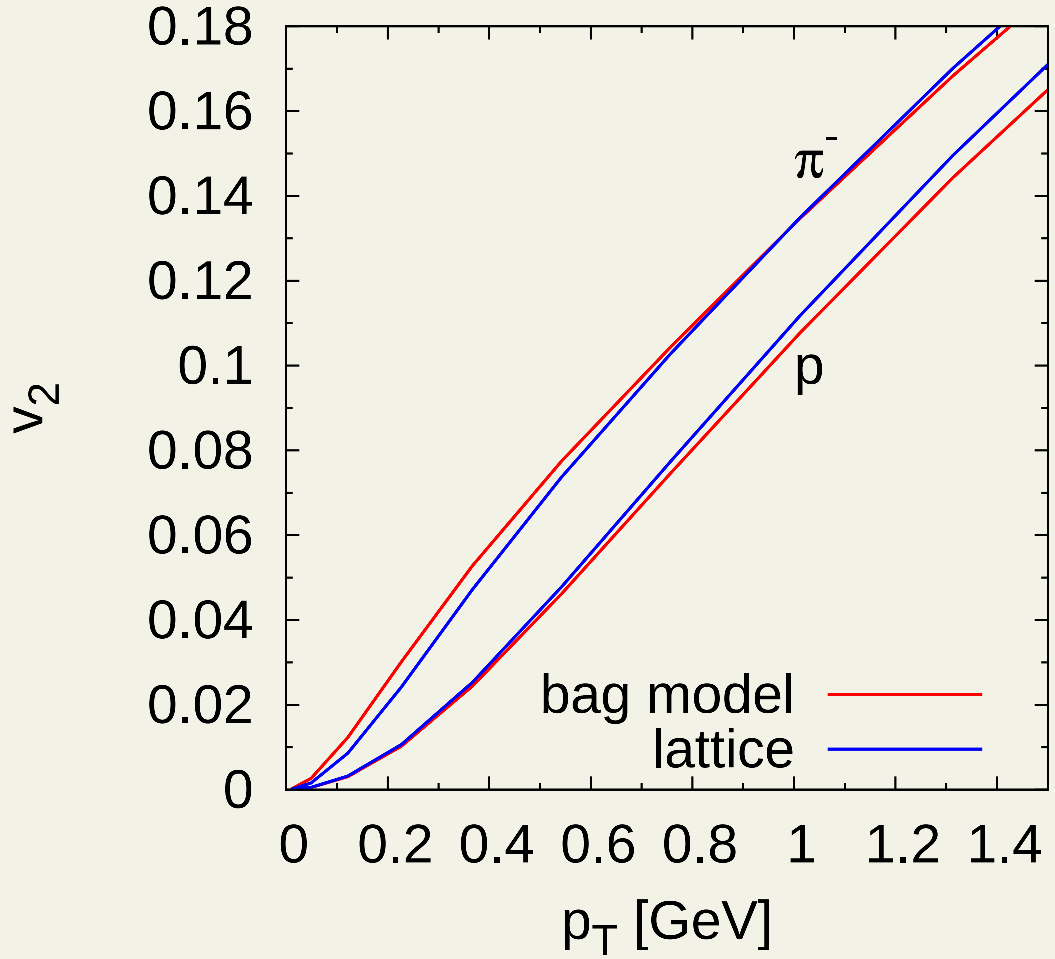
- **harder EoS, more transverse flow, flatter spectra**

p_T-spectra at SPS



- $T_{\rm fo} \approx 120$ **MeV (bag)** $\Rightarrow$ **130 MeV (lattice)**

v_2 at SPS ($b = 7$ fm)



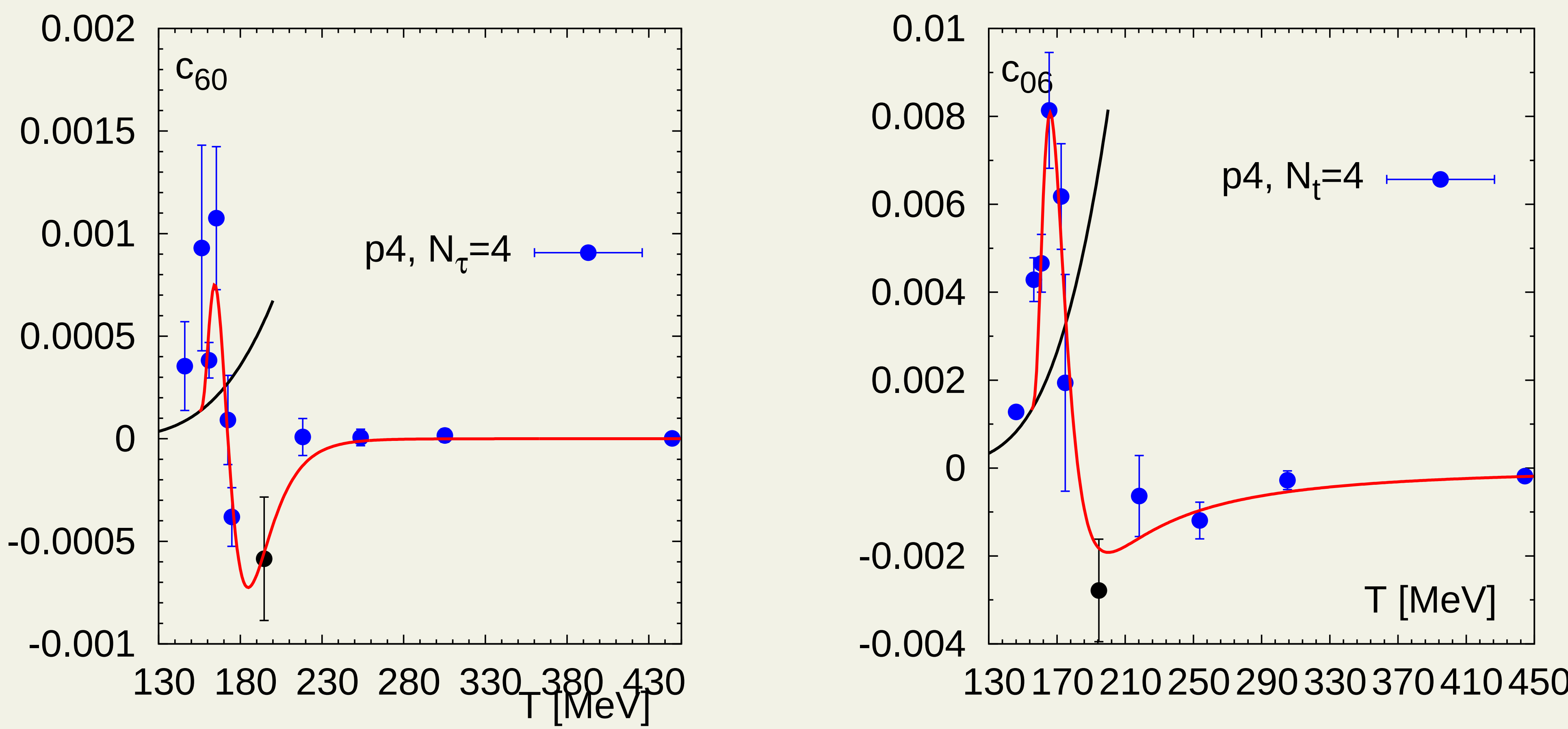
- $T_{\mathrm{fo}} \approx 120$ **MeV (bag)** $\Rightarrow$ **130 MeV (lattice)**

Conclusions

- lattice spacing dependence of hadron masses explains the difference between HRG and lattice QCD
  - 30 MeV shift in temperature
- EoS at finite baryon densities based on lattice QCD calculations of baryon number and strangeness fluctuations and correlations
  - $\sim$10% uncertainty in speed of sound around the transition
- effect on flow when compared to bag model EoS tiny at SPS and (some?) RHIC low energy scan energies

Backups

Some fits are better than others:



Pressure vs. Budapest-Wuppertal lattice