Phase diagram in a non-local PNJL model calibrated with lattice QCD

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Outline

- Motivation.
- The model and its parameterizations.
- Thermodynamical properties and phase transitions.
- Phase Diagrams.
- CEP determination and critical exponents.
- Outlooks & Conclusions.

Cosmology

Motivation





Heavy ion collisions (RHIC, LHC, FAIR, NICA ...)



Compact stars astrophysics



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QCD Phase Diagram?

Non-local extended NJL model with WFR

$$S_{E} = \int d^{4}x \left\{ \bar{\psi}(x)(i\not\partial + m_{q})\psi(x) - \frac{G_{s}}{2} \left[j_{a}(x)j_{a}(x) + j_{P}(x)j_{P}(x) \right] \right\}$$

$$j_a(x) = \int d^4 z \ g(z)\overline{\psi}(x+\frac{z}{2})\Gamma_a \psi(x-\frac{z}{2})$$
$$j_p(x) = \int d^4 z \ f(z)\overline{\psi}(x+\frac{z}{2})\frac{i\partial}{2\varkappa_p}\psi(x-\frac{z}{2})$$

$$\Gamma_a = (\mathbf{1}, i\gamma_5 \, \vec{\tau})$$

$$\mathcal{Z} = \int \mathcal{D} \, \overline{\psi} \, \mathcal{D} \psi \, \exp\left(-S_E\right)$$

After the bozonization and taking Mean Field Aproximation, we have

$$S_{E}^{(MFA)} = -4N_{c}\int \frac{d^{4}p}{(2\pi)^{4}} \ln\left[\frac{(p)^{2} + M^{2}(p)}{Z^{2}(p)}\right] + \frac{\sigma_{1}^{2}}{2G_{s}} + \frac{\varkappa_{p}^{2}\sigma_{2}^{2}}{2G_{s}}$$

$$Z(p) = (1 - \sigma_{c}f(p))^{-1}$$

$$Z(p) = (1 - \sigma_2 f(p))^{-1}$$

$$M(p) = Z(p)(m_q + \sigma_1 g(p))$$

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S. Noguera, N. N. Scoccola, Phys.Rev. D78 (2008) 114002.

Non-local extended NJL model with WFR

Parameterization without WFR

Exponential (Set A)

$$g(p) = e^{-(p^2/\Lambda_0^2)}$$

 $f(p) = 0$, $\sigma_2 = 0$



Parameterizations with WFR

Exponential (Set B)

$$f(p) = e^{-(p^2 / \Lambda_1^2)}$$
$$g(p) = e^{-(p^2 / \Lambda_0^2)}$$

Lattice adjusted lorentzian (Set C)

$$f(p) = \frac{1+\alpha_z}{1+\alpha_z f_z(p)} f_z(p)$$

$$g(p) = \frac{1+\alpha_z}{1+\alpha_z f_z(p)} \frac{\alpha_m f_m(p) - m_q \alpha_z f_z(p)}{\alpha_m - m_q \alpha_z}$$

$$f_m(p) = [1 + (p^2 / \Lambda_0^2)^{3/2}]^{-1} , \quad f_z(p) = [1 + (p^2 / \Lambda_1^2)]^{-5}$$

$$\alpha_z = -0.3$$
 , $\alpha_m = 309 \, MeV$

S. Noguera, N. N. Scoccola, Phys.Rev. D78 (2008) 114002.

We incorporate the Polyakov loop using covariant derivative

$$D_{\mu} \equiv \partial_{\mu} - i \mathbf{A}_{\mu}$$

Assuming that quarks move into a color gauge field given by

$$\phi = iA_0 = ig\delta_{\mu 0}G_a^{\mu}\lambda^a / 2$$

And the traced Polyakov loop (parameter of the confinement) results: $\Phi = \frac{1}{3} \operatorname{Tr} \exp(i\phi)$

Using the named Polyakov gauge, the matrix ϕ has diagonal representation: $\phi = \phi_3 \lambda_3 + \phi_8 \lambda_8$ In order to keep Ω_{MFA} real valued :

$$\phi_8 = 0$$
; $\Phi = \frac{1}{3} [1 + 2\cos(\phi_3/T)]$



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Non-local extended PNJL model at finite T and µ

Matsubara and imaginary time formalisms $\begin{cases} p^2 \to \rho_n^2 = (\omega_n - i\mu)^2 + \vec{p}^2 & \text{with} \quad \omega_n = (2n+1)\pi T \\ \int \frac{d^4p}{(2\pi)^4} g(p) \to 2T \sum_{n=0}^{\infty} \int \frac{d^3\vec{p}}{(2\pi)^3} g(\omega_n - i\mu, \vec{p}) \end{cases}$ Polyakov loop $\begin{cases} \mu \to \mu_c = \mu - i\phi_c \\ N_c \to \sum \quad \text{with} \quad \phi_c = +\phi_3, 0, -\phi_3 \quad \text{for } r, g, b \end{cases}$ $\Omega_{MFA}(T,\mu) = -4T \sum_{a} \sum_{m} \int \frac{d^{3}\vec{p}}{(2\pi)^{3}} \ln \left[\frac{\rho_{nc}^{2} + M^{2}(\rho_{nc}^{2})}{Z^{2}(\rho_{nc}^{2})} \right] + \frac{\sigma_{1}^{2}}{2G_{s}} + \frac{\varkappa_{p}^{2}\sigma_{2}^{2}}{2G_{s}} + \mathcal{U}(\Phi,T)$

$$\rho_{nc}^{2} = \left[(2n+1)\pi T - i\mu + \phi_{c} \right]^{2} + \vec{p}^{2}$$

$$a(T), b(T)$$

$$\mathcal{U}(\Phi, T) = \left[-\frac{1}{2}a(T)\Phi^{2} + b(T)\ln(1 - 6\Phi^{2} + 8\Phi^{3} - 3\Phi^{4}) \right]T^{4}$$

$$g(T) = \left[-\frac{1}{2}a(T)\Phi^{2} + b(T)\ln(1 - 6\Phi^{2} + 8\Phi^{3} - 3\Phi^{4}) \right]T^{4}$$

$$g(T) = \left[-\frac{1}{2}a(T)\Phi^{2} + b(T)\ln(1 - 6\Phi^{2} + 8\Phi^{3} - 3\Phi^{4}) \right]T^{4}$$

S. Roessner, C. Ratti and W. Weise, Phys. Rev. D 75 (2007) 034007.
 GAC, M. Orsaria, N.N. Scoccola, Phys. Rev. D 82 (2010) 054026.

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Regularization

 Ω_{MFA} turns to be divergent and needs to be regularized. We used

$$\Omega_{MFA}^{reg} = \Omega_{MFA} - \Omega_{free} + \Omega_{free}^{reg} - \Omega_0$$

where: • Ω_{free} is obtained from Ω_{MFA} by setting $\sigma_1 = \sigma_2 = 0$.

- Ω_0 is a constant fixed by the condition $\Omega_{MFA}^{reg} = 0$ at $T = \mu = 0$
- Ω_{free}^{reg} is the regularized expression for the thermodynamical potential in the absence of fermion interactions. It is given by

$$\Omega_{free}^{reg} = -4T \int \frac{d^3 \vec{p}}{(2\pi)^3} \sum_{c} \left[\ln \left(1 + \exp \left[-\frac{E_p + \mu + i\phi_c}{T} \right] \right) + \ln \left(1 + \exp \left[-\frac{E_p - \mu - i\phi_c}{T} \right] \right) \right]$$

with
$$E_p = \sqrt{\vec{p}^2 + m^2}$$

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D.Gómez Dumm and N.N. Scoccola, Phys. Rev. C 72 (2005) 014909.

What could be determined with $\Omega_{MFA}^{reg}(T,\mu)$?

• Mean field values $\sigma_{1,2}$ and Φ at a given *T* and μ

$$\frac{\partial \Omega_{MFA}^{reg}}{\partial \sigma_1}\Big|_{T,\mu} = \frac{\partial \Omega_{MFA}^{reg}}{\partial \sigma_2}\Big|_{T,\mu} = \frac{\partial \Omega_{MFA}^{reg}}{\partial \Phi}\Big|_{T,\mu} = 0 \quad \text{"gap" equation}$$



• Chiral quark condensate $\langle \overline{q}q \rangle$ and quark density ρ

$$\left\langle \overline{q}q \right\rangle = rac{\partial \Omega_{MFA}^{reg}}{\partial m} \qquad ; \qquad
ho = - \; rac{\partial \Omega_{MFA}^{reg}}{\partial \mu}$$

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 \mathbf{S}

What could be determined with $\Omega_{MFA}^{reg}(T,\mu)$?

• Specific heat:

$$c_{v} = -T \frac{\partial^{2} \Omega_{MFA}^{reg}}{\partial T^{2}}$$

• Chiral susceptibility:

$$\chi_{ch} = \frac{\partial \langle \overline{q}q \rangle}{\partial m}$$

• Quark number susceptibility:

$$\chi_q = \frac{\partial \rho}{\partial \mu}$$



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Phase diagrams







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Phase diagrams: Spinodal differencies







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CEP verification: Critical exponents (first attempts)



$$\chi_q = \left|\frac{h-h_c}{h_c}\right|^{-\gamma_q}$$

$$C_v = \left|\frac{h-h_c}{h_c}\right|^{-\alpha}$$

where $|h - h_c|$ is the distance to the critical point in the (μ, T) plane and γ_{ch} , γ_q and α are the corresponding critical exponents.



For trajectories which are not tangential to the critical line, they should be $\gamma_{ch} = \gamma_q = \alpha = 2/3$.

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CEP determination: Set of equations

In the finite quark mass regime the CEP determination can be performed by solving the following set of coupled equations:

$$\frac{\partial \Omega_{MFA}^{reg}}{\partial \sigma_1}\Big|_{T,\mu} = \frac{\partial \Omega_{MFA}^{reg}}{\partial \sigma_2}\Big|_{T,\mu} = \frac{\partial \Omega_{MFA}^{reg}}{\partial \Phi}\Big|_{T,\mu} = 0$$

$$\frac{d^2 \Omega_{MFA}^{reg}(\mu, T, \Phi, \sigma_2)}{d \sigma_1^2} = 0$$

$$\frac{d^3 \Omega_{MFA}^{reg}(\mu, T, \Phi, \sigma_2)}{d \sigma_1^3} = 0$$

$$\frac{d^3 \Omega_{MFA}^{reg}(\mu, T, \Phi, \sigma_2)}{d \sigma_1^3} = 0$$

with

$$\sigma_{2}' = \frac{\partial \sigma_{2}}{\partial \sigma_{1}} = \frac{\frac{\partial^{2}\Omega}{\partial \Phi^{2}} \frac{\partial^{2}\Omega}{\partial \sigma_{1} \partial \sigma_{2}} - \frac{\partial^{2}\Omega}{\partial \sigma_{1} \partial \Phi} \frac{\partial^{2}\Omega}{\partial \sigma_{2} \partial \Phi}}{\left(\frac{\partial^{2}\Omega}{\partial \sigma_{2} \partial \Phi}\right)^{2} - \frac{\partial^{2}\Omega}{\partial \sigma_{2}^{2}} \frac{\partial^{2}\Omega}{\partial \Phi^{2}}} \qquad \Phi' = \frac{\partial \Phi}{\partial \sigma_{1}} = \frac{\frac{\partial^{2}\Omega}{\partial \sigma_{2}^{2}} \frac{\partial^{2}\Omega}{\partial \sigma_{1} \partial \Phi} - \frac{\partial^{2}\Omega}{\partial \sigma_{2} \partial \Phi} \frac{\partial^{2}\Omega}{\partial \sigma_{1} \partial \sigma_{2}}}{\left(\frac{\partial^{2}\Omega}{\partial \sigma_{2} \partial \Phi}\right)^{2} - \frac{\partial^{2}\Omega}{\partial \sigma_{2}^{2}} \frac{\partial^{2}\Omega}{\partial \Phi^{2}}}$$

Y. Hatta, T. Ikeda . Phys. Rev. D **67** (2003) 014028. GAC, M. Orsaria, N.N. Scoccola, Phys. Rev. D **82** (2010) 054026.

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$$\frac{d^{3}\Omega}{d\sigma_{1}^{3}} = 3\sigma_{2}^{\prime\prime}\frac{\partial^{2}\Omega}{\partial\sigma_{1}\partial\sigma_{2}} + 3(\sigma_{2}^{\prime})^{2}\frac{\partial^{3}\Omega}{\partial\sigma_{1}\partial\sigma_{2}^{2}} + 6\sigma_{2}^{\prime}\Phi^{\prime}\frac{\partial^{3}\Omega}{\partial\sigma_{1}\partial\sigma_{2}\partial\Phi} + 3\Phi^{\prime\prime}\frac{\partial^{2}\Omega}{\partial\sigma_{1}\partial\Phi} + 3(\Phi^{\prime})^{2}\frac{\partial^{3}\Omega}{\partial\sigma_{1}\partial\Phi^{2}} + 3\sigma_{2}^{\prime}\frac{\partial^{3}\Omega}{\partial\sigma_{1}^{2}\partial\sigma_{2}} + 3\Phi^{\prime}\frac{\partial^{3}\Omega}{\partial\sigma_{1}^{2}\partial\Phi} + \frac{\partial^{3}\Omega}{\partial\sigma_{1}^{3}} + 3\sigma_{2}^{\prime}\Phi^{\prime}\frac{\partial^{2}\Omega}{\partial\sigma_{2}\partial\Phi} + 3(\sigma_{2}^{\prime})^{2}\Phi^{\prime}\frac{\partial^{3}\Omega}{\partial\sigma_{2}^{2}\partial\Phi} + (\sigma_{2}^{\prime})^{3}\frac{\partial^{3}\Omega}{\partial\sigma_{2}^{3}} + 3\sigma_{2}^{\prime}\Phi^{\prime\prime}\frac{\partial^{2}\Omega}{\partial\sigma_{2}\partial\Phi} + \sigma_{2}^{\prime}(\Phi^{\prime})^{2}\frac{\partial^{3}\Omega}{\partial\sigma_{2}\partial\Phi^{2}} + 3\sigma_{2}^{\prime}\sigma_{2}^{\prime}\frac{\partial^{2}\Omega}{\partial\sigma_{2}^{2}} + 3\Phi^{\prime}\Phi^{\prime\prime}\frac{\partial^{2}\Omega}{\partial\Phi^{2}} + (\Phi^{\prime})^{3}\frac{\partial^{3}\Omega}{\partial\Phi^{3}} = 0$$

$$\begin{aligned} \sigma_{2}^{\prime\prime} &= \left[\left(\frac{\partial^{2}\Omega}{\partial\sigma_{2}\partial\Phi} \right)^{2} - \frac{\partial^{2}\Omega}{\partial\sigma_{2}^{2}} \frac{\partial^{2}\Omega}{\partial\Phi^{2}} \right]^{-1} \left[2\Phi^{\prime} \left(\frac{\partial^{2}\Omega}{\partial\Phi^{2}} \frac{\partial^{3}\Omega}{\partial\sigma_{1}\partial\sigma_{2}\partial\Phi} - \frac{\partial^{3}\Omega}{\partial\sigma_{1}\partial\Phi^{2}} \frac{\partial^{2}\Omega}{\partial\sigma_{2}\partial\Phi} \right) \\ &+ 2\Phi^{\prime} \sigma_{2}^{\prime} \left(\frac{\partial^{2}\Omega}{\partial\Phi^{2}} \frac{\partial^{3}\Omega}{\partial\sigma_{2}^{2}\partial\Phi} - \frac{\partial^{2}\Omega}{\partial\sigma_{2}\partial\Phi} \frac{\partial^{3}\Omega}{\partial\sigma_{2}\partial\Phi^{2}} \right) + 2\sigma_{2}^{\prime} \left(\frac{\partial^{2}\Omega}{\partial\Phi^{2}} \frac{\partial^{3}\Omega}{\partial\sigma_{1}\partial\sigma_{2}^{2}} - \frac{\partial^{2}\Omega}{\partial\sigma_{2}\partial\Phi} \frac{\partial^{3}\Omega}{\partial\sigma_{1}\partial\sigma_{2}\partial\Phi} \right) \\ &+ \left(\frac{\partial^{2}\Omega}{\partial\Phi^{2}} \frac{\partial^{3}\Omega}{\partial\sigma_{1}^{2}\partial\sigma_{2}} - \frac{\partial^{2}\Omega}{\partial\sigma_{2}\partial\Phi} \frac{\partial^{3}\Omega}{\partial\sigma_{1}^{2}\partial\Phi} \right) + \left(\sigma_{2}^{\prime} \right)^{2} \left(\frac{\partial^{3}\Omega}{\partial\sigma_{2}^{3}} \frac{\partial^{2}\Omega}{\partial\Phi^{2}} - \frac{\partial^{2}\Omega}{\partial\sigma_{2}\partial\Phi} \frac{\partial^{3}\Omega}{\partial\sigma_{2}^{2}\partial\Phi} \right) + \left(\Phi^{\prime} \right)^{2} \left(\frac{\partial^{2}\Omega}{\partial\Phi^{2}} \frac{\partial^{3}\Omega}{\partial\sigma_{2}\partial\Phi^{2}} - \frac{\partial^{3}\Omega}{\partial\Phi^{2}} \frac{\partial^{2}\Omega}{\partial\sigma_{2}\partial\Phi} \right) \\ &+ \left(\frac{\partial^{2}\Omega}{\partial\Phi^{2}} \frac{\partial^{3}\Omega}{\partial\sigma_{1}^{2}\partial\sigma_{2}} - \frac{\partial^{2}\Omega}{\partial\sigma_{2}\partial\Phi} \frac{\partial^{3}\Omega}{\partial\sigma_{1}^{2}\partial\Phi} \right) + \left(\sigma_{2}^{\prime} \right)^{2} \left(\frac{\partial^{3}\Omega}{\partial\sigma_{2}^{3}} \frac{\partial^{2}\Omega}{\partial\Phi^{2}} - \frac{\partial^{2}\Omega}{\partial\sigma_{2}\partial\Phi} \frac{\partial^{3}\Omega}{\partial\sigma_{2}^{2}\partial\Phi} \right) \\ &+ \left(\frac{\partial^{2}\Omega}{\partial\Phi^{2}} \frac{\partial^{3}\Omega}{\partial\sigma_{1}^{2}\partial\sigma_{2}} - \frac{\partial^{2}\Omega}{\partial\sigma_{2}\partial\Phi} \frac{\partial^{3}\Omega}{\partial\sigma_{1}^{2}\partial\Phi} \right) + \left(\sigma_{2}^{\prime} \right)^{2} \left(\frac{\partial^{3}\Omega}{\partial\sigma_{2}^{3}} \frac{\partial^{2}\Omega}{\partial\sigma_{2}^{2}\partial\Phi} - \frac{\partial^{3}\Omega}{\partial\sigma_{2}^{2}\partial\Phi} \right) \\ &+ \left(\frac{\partial^{2}\Omega}{\partial\Phi^{2}} \frac{\partial^{3}\Omega}{\partial\sigma_{1}^{2}\partial\sigma_{2}} - \frac{\partial^{3}\Omega}{\partial\sigma_{2}^{2}\partial\Phi} \right) + \left(\sigma_{2}^{\prime} \right)^{2} \left(\frac{\partial^{3}\Omega}{\partial\sigma_{2}^{3}} \frac{\partial^{2}\Omega}{\partial\sigma_{2}^{2}\partial\Phi} - \frac{\partial^{3}\Omega}{\partial\sigma_{2}^{2}\partial\Phi} \right) \\ &+ \left(\frac{\partial^{2}\Omega}{\partial\Phi^{2}} \frac{\partial^{3}\Omega}{\partial\sigma_{1}^{2}\partial\Phi} - \frac{\partial^{3}\Omega}{\partial\sigma_{1}^{2}\partial\Phi} \right) + \left(\frac{\partial^{3}\Omega}{\partial\sigma_{2}^{2}} \frac{\partial^{3}\Omega}{\partial\sigma_{2}^{2}\partial\Phi} - \frac{\partial^{3}\Omega}{\partial\sigma_{2}^{2}\partial\Phi} \right) \\ &+ \left(\frac{\partial^{2}\Omega}{\partial\Phi^{2}} \frac{\partial^{3}\Omega}{\partial\sigma_{1}^{2}\partial\Phi} - \frac{\partial^{3}\Omega}{\partial\sigma_{1}^{2}\partial\Phi} \right) \\ &+ \left(\frac{\partial^{2}\Omega}{\partial\Phi^{2}} \frac{\partial^{3}\Omega}{\partial\sigma_{1}^{2}\partial\Phi} - \frac{\partial^{3}\Omega}{\partial\sigma_{1}^{2}\partial\Phi} \right) \\ &+ \left(\frac{\partial^{3}\Omega}{\partial\Phi^{2}} \frac{\partial^{3}\Omega}{\partial\sigma_{1}^{2}\partial\Phi} - \frac{\partial^{3}\Omega}{\partial\sigma_{1}^{2}\partial\Phi} \right) \\ &+ \left(\frac{\partial^{3}\Omega}{\partial\Phi^{2}} \frac{\partial^{3}\Omega}{\partial\Phi} - \frac{\partial^{3}\Omega}{\partial\Phi} - \frac{\partial^{3}\Omega}{\partial\Phi} - \frac{\partial^{3}\Omega}{\partial\Phi} \right) \\ &+ \left(\frac{\partial^{3}\Omega}{\partial\Phi^{2}} \frac{\partial^{3}\Omega}{\partial\Phi} - \frac{\partial^{3}\Omega}{\partial$$

$$\Phi^{\prime\prime} = \left[\left(\frac{\partial^{2}\Omega}{\partial\sigma_{2}\partial\Phi} \right)^{2} - \frac{\partial^{2}\Omega}{\partial\sigma_{2}^{2}} \frac{\partial^{2}\Omega}{\partial\Phi^{2}} \right]^{-1} \left[2\Phi^{\prime} \left(\frac{\partial^{2}\Omega}{\partial\sigma_{2}^{2}} \frac{\partial^{3}\Omega}{\partial\sigma_{1}\partial\Phi^{2}} - \frac{\partial^{3}\Omega}{\partial\sigma_{1}\partial\sigma_{2}\partial\Phi} \frac{\partial^{2}\Omega}{\partial\sigma_{2}\partial\Phi} \right) + 2\Phi^{\prime}\sigma^{\prime}_{2} \left(\frac{\partial^{2}\Omega}{\partial\sigma_{2}^{2}} \frac{\partial^{3}\Omega}{\partial\sigma_{2}\partial\Phi^{2}} - \frac{\partial^{2}\Omega}{\partial\sigma_{2}\partial\Phi} \frac{\partial^{3}\Omega}{\partial\sigma_{2}^{2}\partial\Phi} \right) + 2\sigma^{\prime}_{2} \left(\frac{\partial^{2}\Omega}{\partial\sigma_{2}^{2}} \frac{\partial^{3}\Omega}{\partial\sigma_{1}\partial\sigma_{2}\partial\Phi} - \frac{\partial^{2}\Omega}{\partial\sigma_{2}\partial\Phi} \frac{\partial^{3}\Omega}{\partial\sigma_{1}\partial\sigma_{2}^{2}} \right) + \left(\frac{\partial^{2}\Omega}{\partial\sigma_{2}^{2}} \frac{\partial^{3}\Omega}{\partial\sigma_{1}^{2}\partial\Phi} - \frac{\partial^{3}\Omega}{\partial\sigma_{1}^{2}\partial\sigma_{2}} \frac{\partial^{2}\Omega}{\partial\sigma_{2}\partial\Phi} \right) + \left(\sigma^{\prime}_{2} \right)^{2} \left(\frac{\partial^{3}\Omega}{\partial\sigma_{2}^{2}\partial\Phi} \frac{\partial^{2}\Omega}{\partial\sigma_{2}^{2}} - \frac{\partial^{2}\Omega}{\partial\sigma_{2}\partial\Phi} \frac{\partial^{3}\Omega}{\partial\sigma_{2}^{2}} \right) + \left(\Phi^{\prime} \right)^{2} \left(\frac{\partial^{2}\Omega}{\partial\sigma_{2}^{2}} \frac{\partial^{3}\Omega}{\partial\Phi^{3}} - \frac{\partial^{3}\Omega}{\partial\sigma_{2}\partial\Phi^{2}} \frac{\partial^{2}\Omega}{\partial\sigma_{2}\partial\Phi} \right) \right]$$

CEP verification: finite quark mass



	Set A	Set B	Set C
$T_c(0)$	210.0	209.8	214.5
μ_{CEP}	132.5	182.3	234.8
T_{CEP}	197.8	181.6	154.2
$\mu_c(0)$	321.5	311.6	298.1

	γ_{ch}	γ_q	α
$\mu \rightarrow$	0.67(1)	0.67(1)	0.66(1)
$\mu \leftarrow$	0.66(1)	0.66(1)	0.67(1)
$T\uparrow$	0.67(1)	0.67(1)	0.66(2)
$T\downarrow$	0.66(1)	0.66(1)	0.67(1)
MF exponent	2/3	2/3	2/3

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CEP verification: more results !



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Y. Hatta, T. Ikeda . Phys. Rev. D 67 (2003) 014028.

Chemical potential in the Effective potentials

 $T_0 = 208$ MeV which corresponds to 2 flavors case. Then, following Ref [1], we used also a polynomial ansatz for \mathcal{U} given by

$$\mathcal{U}_2(\Phi, T) = \left[-\frac{b_2(T)}{2} \Phi^2 - \frac{b_3}{6} (\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4} \Phi^4 \right] T^4 \tag{10}$$

with the temperature-dependent coefficient

$$b_2(T) = a_0 + a_1 \left(\frac{T_0}{T}\right) + a_2 \left(\frac{T_0}{T}\right)^2 + a_3 \left(\frac{T_0}{T}\right)^3$$
(11)

and the following set of parameters, $a_0 = 6.75$, $a_1 = -1.95$, $a_2 = 2.625$, $a_3 = -7.44$, $b_3 = 0.75$, and $b_4 = 7.5$.

In a more recent work [3] it has been proposed the following μ -dependent logarithmic potential:

$$\mathcal{U}_3(\Phi, T, \mu) = (a_0 T^4 + a_1 \mu^4 + a_2 T^2 \mu^2) \Phi^2 + a_3 T_0^4 \ln\left(1 - 6\Phi^2 + 8\Phi^3 - 3\Phi^4\right), \quad (12)$$

where the parameters are $a_0 = 1.85$, $a_1 = 1.44 \times 10^{-3}$, $a_2 = 0.08$, $a_3 = 0.40$.

In the same way, we propose a polynomial potential which include this μ dependence. So we have:

$$\mathcal{U}_4(\Phi, T) = \left[-\frac{b_2(T)}{2} \Phi^2 - \frac{b_3}{6} (\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4} \Phi^4 \right] T^4$$
(13)

with the T- μ -dependent <u>coefficient</u>.

$$b_2(T,\mu) = \left[a_0 + c_2(\mu/T)^2 + c_4(\mu/T)^4\right] + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3 \quad (14)$$

and the same set of parameters as in the polynomial case, but adding the μ related ones $c_2 = 30/(7\pi^2) = 0.4342$ and $c_4 = 15/(7\pi^4) = 0.02199$

[1] C. Ratti, M. A. Thaler and W. Weise, Phys. Rev. D 73, 014019 (2006)
[2] V. A. Dexheimer and S. Schramm. Phys.Rev.C 81, 045201 (2010)

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Preliminary results: Phase diagrams & deconfinement lines

T₀=270 MeV and RRW potential







T₀=208 MeV and μ-dependent polynomial potential

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Preliminary results: Pressure



[1] C. R. Allton, S. Ejiri, S. J. Hands, O. Kaczmarek, F. Karsch, E. Laermann, C. Schmidt, Phys. Rev. D 68, 014507 (2003).

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Outlooks: Vector interactions + color neutrality

$$\Omega_{MFA}(T,\mu) = -4T\sum_{c} \sum_{n} \int \frac{d^{3}\vec{p}}{(2\pi)^{3}} \ln \left|$$

$$\rho_{nc}^2 = \left[\omega_n - i\mu_c + \phi_c\right]^2 + \vec{p}^2$$

$$q_{nc}^{2} = \left[\omega_{n} - i\mu_{c}^{r} + \phi_{c}\right]^{2} + \vec{p}^{2}$$

$$\phi_c = \lambda_3 \phi_3 + \lambda_8 \phi_8$$
$$\mu_c = \mu I + \lambda_3 \mu_3 + \lambda_8 \mu_8$$

 $\frac{q_{nc}^2 + M^2(\rho_{nc}^2)}{Z^2(\rho_{nc}^2)} + \frac{\sigma_1^2}{2G_S} + \frac{\varkappa_p^2 \sigma_2^2}{2G_S} \left(-\frac{\omega_V^2}{2G_V}\right) + \mathcal{U}\left(\Phi, \Phi^*, T\right)$

$$\mu_c^r = \mu_c - \omega_V g(\rho_{nc}^2)$$



[1] D. Gómez Dumm, D.B. Blaschke, A.G. Grunfeld, N.N. Scoccola, Phys.Rev.D 78, 114021 (2008).
[2] C. Sasaki, B. Friman, K. Redlich, Phys.Rev.D 75, 054026(2007).
[2] K. Evlavskime, Phys. Rev. D 77, 114028 (2008).

Frankfurt 2012 18/18 [3] K. Fukushima, Phys.Rev.D 77, 114028 (2008).

Conclusions

- The main effect of the Polyakov loop is the increment of the critical temperature T_c for all the μ values.
- The position of the CEP and the width of the first order transition are strongly dependent on the election of the form factors in the quark propagator. The better initial adjustment to lattice results, the lower T_{CEP} and higher μ_{CEP} , and the closer are the spinodal lines.
- The position of the CEP can be determined with very good precision by solving a coupled equation system which includes the gap equations and the vanishing of the second and third *total* derivatives of the grand potential.
- By including μ dependence in the effective potential the chiral and deconfinement transitions can be fitted along all the phase diagram.
- By using lattice QCD calibrated form factors can be obtained better fits with thermodynamical lattice results.

Tank you very much for your attention !!

Gustavo A. Contrera