Neutrino Cosmology

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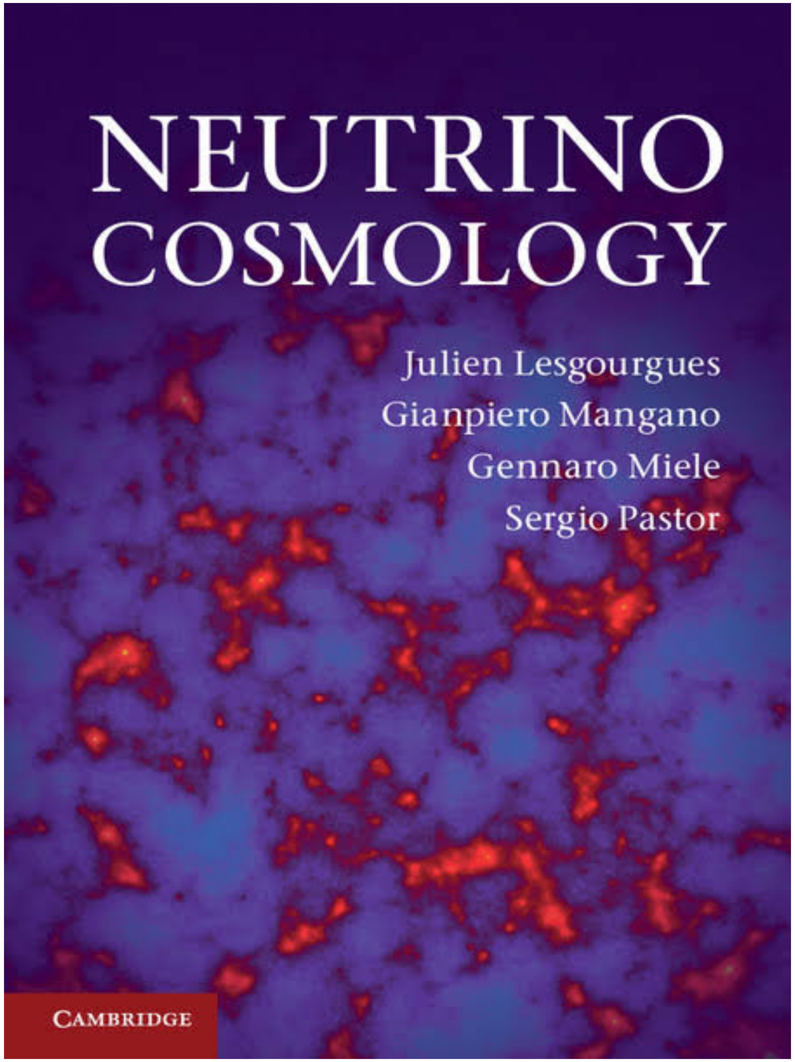
Plan of Lecture

- Standard cosmological model & CMB
- Cosmic Neutrino Background (CNB)
- Sterile neutrinos and extra relativistic dof
- Impact of neutrinos on observables
- Present status of constraints : Planck 2018
- Future prospects

Neutrino Cosmology : a good reference



Natural units

$$1\,\mathrm{GeV} = 1.6 \times 10^{-10}\,\mathrm{J}$$

$$1\,\mathrm{GeV} = 1.16 \times 10^{13}\,\mathrm{K}$$

$$1\,\mathrm{GeV} = 1.78 \times 10^{-27}\,\mathrm{Kg}$$

$$1\,\mathrm{GeV}^{-1} = 6.58 \times 10^{-25}\,\mathrm{sec}$$

$$1\,\mathrm{GeV}^{-1} = 1.97 \times 10^{-16}\,\mathrm{m}$$

Section 1

Standard cosmology model &

Cosmic Microwave Background (CMB)

FRW universe

- Homogenous and isotropic spacetimes : k=0,±1

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - \frac{kr^2}{R^2}} + r^2 d\Omega^2 \right]$$

R = curvature radius

- Einstein equations and conservation equations

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \qquad \nabla_\mu T^{\mu\nu} = 0$$

$$\begin{aligned} H^2 = \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G}{3}\rho - \frac{k}{R^2 a^2} + \frac{1}{3}\Lambda \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho + 3P) + \frac{1}{3}\Lambda \\ 0 &= \dot{\rho} + 3H(\rho + P) \end{aligned}$$

Basic equation to solve !

$$\dot{a} = H_0 \left[\frac{\Omega_r}{a^2} + \frac{\Omega_m}{a} + \Omega_k + \Omega_\Lambda a^2 \right]^{\frac{1}{2}}$$

Ω_r = Radiation density

Ω_m = Matter density

Ω_k = Curvature density

Ω_Λ = Dark energy density

Solution for matter and radiation

Conformal time : dt = adη allows a parametric solution

$$a(\eta) = \sqrt{\Omega_\mathrm{r}} H_0 \eta + \frac{1}{4}\Omega_\mathrm{m}(H_0\eta)^2$$

$$H_0 t = \frac{1}{2}\sqrt{\Omega_\mathrm{r}}(H_0\eta)^2 + \frac{1}{12}\Omega_\mathrm{m}(H_0\eta)^3$$

$$H_0\eta \ll 1 \quad \rightarrow \quad a(t) \propto t^{1/2}$$

- Radiation era

$$H_0\eta \gg 1 \quad \rightarrow \quad a(t) \propto t^{2/3}$$

- Matter era

Time of equal-matter radiation

$$
\begin{aligned}
t_{\mathrm{eq}} &= \frac{2(2-\sqrt{2})\Omega_{\mathrm{r}}^{3/2}}{3H_0\Omega_{\mathrm{m}}^2} \\
&\approx 3.19 \times 10^{10}\,\mathrm{sec}\,(\Omega_{\mathrm{m}}h^2)^{-2} \\
&\approx 10^3\,\mathrm{yr}\,(\Omega_{\mathrm{m}}h^2)^{-2} \\
a_{\mathrm{eq}} &\approx 4.2 \times 10^{-5}\,(\Omega_{\mathrm{m}}h^2)^{-1} \\
1+z_{\mathrm{eq}} &\approx 24000\,(\Omega_{\mathrm{m}}h^2) \quad \approx 3000 \text{ for } \Omega_{\mathrm{m}} \approx 0.12 \\
T_{\mathrm{eq}} &\approx 6.5 \times 10^4\,\mathrm{K}\,(\Omega_{\mathrm{m}}h^2)
\end{aligned}
$$

calculated by setting $a_{eq} = \Omega_r/\Omega_m$ in the previous expressions

Solution for matter and Λ

$$H = H_0\left(\frac{\Omega_m}{a^3} + \Omega_\Lambda\right)^{\frac{1}{2}}$$

$$\rightarrow a(t) = \left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{\frac{1}{3}} \sinh^{\frac{2}{3}}\left[\frac{3\Omega_\Lambda^{\frac{1}{2}} H_0 t}{2}\right]$$

matches onto the matter radiation solution for $H_0 t$ small

Initial perturbations

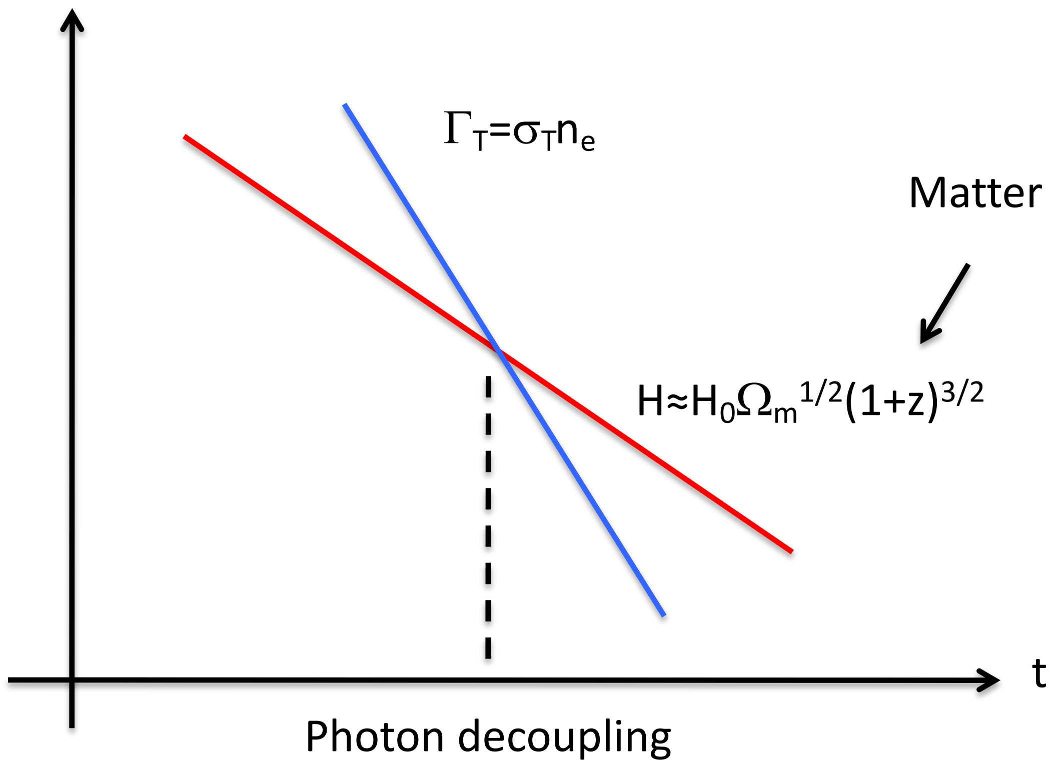
- Assumed to be generated during inflation
  - exponential expansion due to a hypothetical scalar field
- Adiabatic/Curvature and nearly scale invariant

$$P(k) = A_{\mathrm{S}} \left(\frac{k}{k_{\mathrm{pivot}}} \right)^{n_{\mathrm{S}}}$$

$n_s \approx 1$, $k_{pivot} = 0.015 h^{-1}$ Mpc

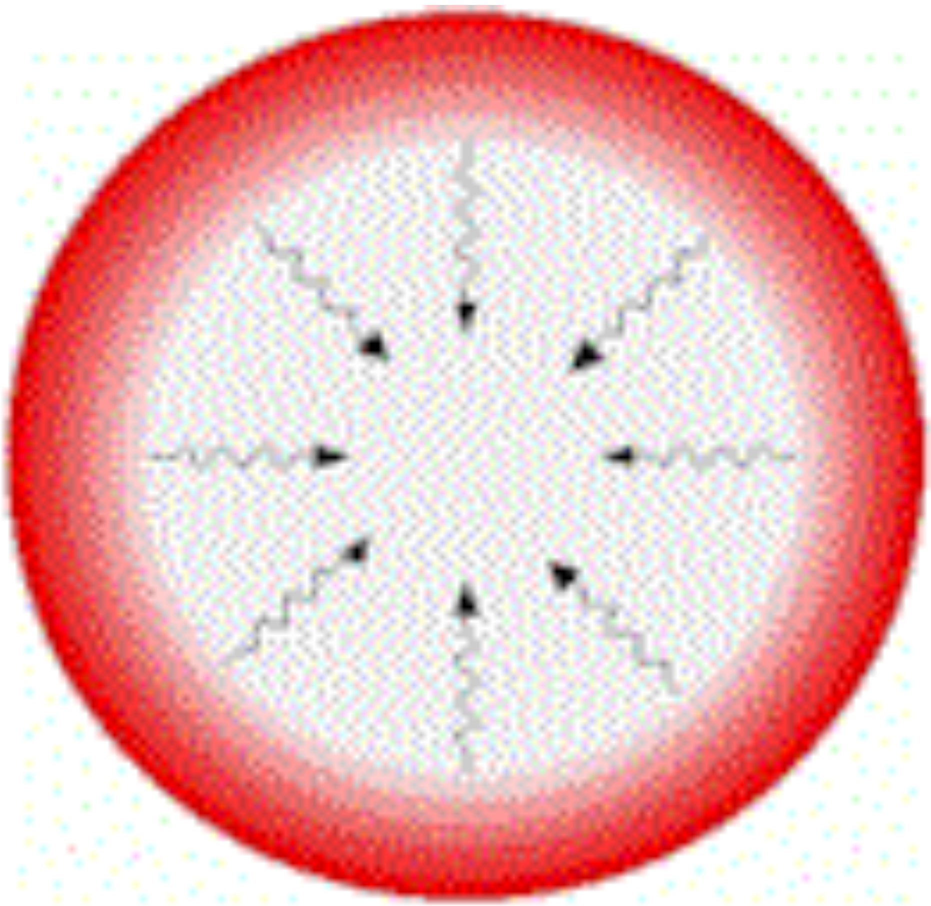
- Inflation expected to flatten the universe
  - $\Omega_k \approx 0$

Photon decoupling



$T_{dec} \approx 0.3\text{eV}$, $z_{dec} \approx 1089$ and $t_{dec} \approx 380{,}000$ years

Surface of last scattering



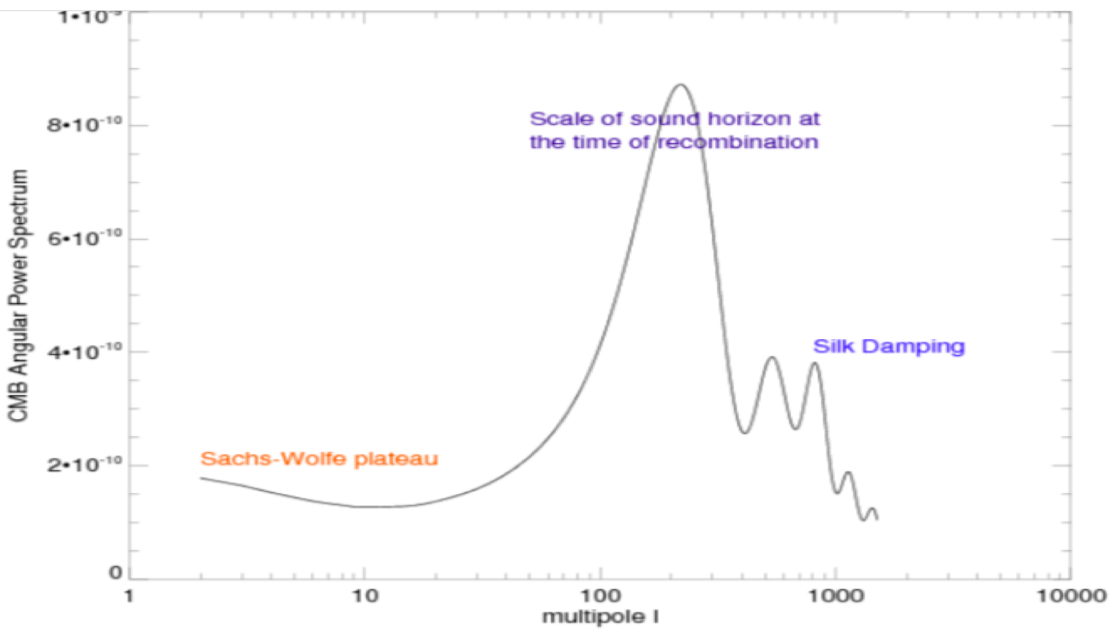
CMB anisotropies

$$\frac{\Delta T}{T}|_{LSS} = \frac{1}{4}\delta\gamma + v_{||}$$

Peaks/troughs

$$\frac{\Delta T}{T}|_{LOS} = \Delta\Phi + 2\int d\ell\dot{\Phi}$$

Plateau



Standard Cosmological Model : Summary

GEOMETRY & GRAVITY

$$ds^2 = dt^2 - a^2(dr^2 + r^2 d\Omega^2)$$

$$\left(\frac{\dot a}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho$$

$$\dot\rho = -3H(\rho + P)$$

MATTER CONTENT

$$\rho = \rho_{\mathrm{r}} + \rho_{\mathrm{b}} + \rho_{\mathrm{c}} + \rho_d$$

$$P = \frac{1}{3}\rho_{\mathrm{r}} - \rho_{\mathrm{d}}$$

COLD DM

BARYONS

RAD

COSMOLOGICAL CONSTANT

INITIAL PERTURBATIONS

$$P_i(k) = A_{\mathrm{S}}\left(\frac{k}{k_{\mathrm{p}}}\right)^{n_{\mathrm{S}}}$$

ASSUMED SCALAR
CURVATURE PERTS

7 PARAMETERS

H_0	HUBBLE'S CONSTANT
$\rho_{\mathrm{r}}, \rho_{\mathrm{b}}, \rho_{\mathrm{c}}$	MATTER DENSITIES TODAY
$A_{\mathrm{S}}, n_{\mathrm{S}}$	INITIAL PERTURBATIONS
τ	OPTICAL DEPTH

Section 2

Cosmic Neutrino Background (CNB)

Fermi-Dirac Statistics : g_i dof

- Relativistic and zero chemical potential

$$\rho_i = \frac{7}{8} \times \frac{\pi^2}{30} g_i T_i^4$$

Density

$$n_i = \frac{3}{4} \times \frac{\zeta(3)}{\pi^2} g_i T_i^3$$

Number density

$$s_i = \frac{7}{8} \times \frac{2\pi^2}{45} g_i T_i^3$$

Entropy density

- Relativistic and non-zero chemical potential

$$n_i - n_{\bar{i}} = \frac{1}{6} g_i T_i^3 \left[\frac{\mu_i}{T_i} + \frac{1}{\pi^2} \left(\frac{\mu_i}{T_i} \right)^3 \right]$$

Thermal equilibrium

- At high temperature all particles in eqm

$$g = g_{\mathrm{B}} + \frac{7}{8} g_{\mathrm{F}} = 106.75$$

- for the standard model

- Key neutrino reactions

$$e^{+} + e^{-} \longleftrightarrow \nu_i + \bar{\nu}_i$$

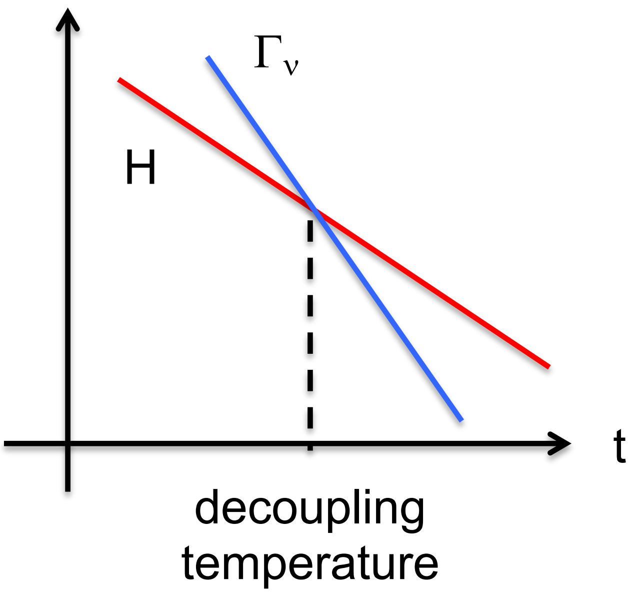
for i = e, μ, τ with cross section

$$\langle \sigma v \rangle \sim G_{\mathrm{F}}^2 T^2$$

Fermi constant = 1.16 x 10^{-5} GeV^{-2}

Instantaneous neutrino decoupling

- Particle Interactions decouple when $\Gamma_\nu \sim$ H



$$\Gamma_\nu = H \quad \rightarrow \quad \sqrt{g}\frac{T^2}{m_{\mathrm{pl}}} \sim G_{\mathrm{F}}^2 T^5$$

$$\rightarrow \quad T^3 \sim \frac{\sqrt{g}}{m_{\mathrm{pl}} G_{\mathrm{F}}^2}$$

$$\rightarrow \quad T \sim 1\,\mathrm{MeV}$$

- Subsequently, no interactions with thermal bath comprising of photons and $e^\pm$

Electron-positron annihilation

- At T=0.5MeV $e^{\pm}$ annihilation takes place
- Entropy from $e^{\pm}$ left in the thermal bath

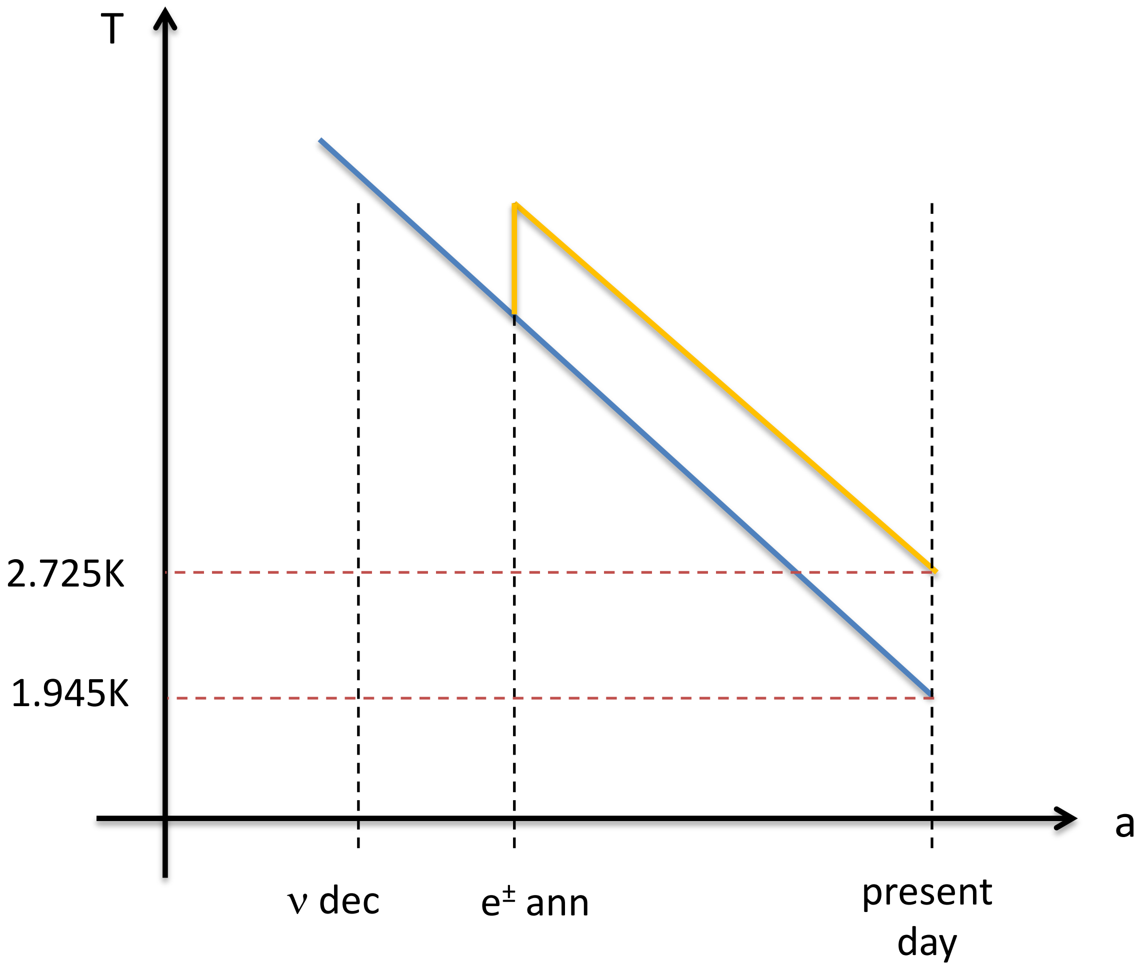
  -> increase in γ temp relative to ν

$$g_S T^3|_{\text{before}} = g_S T^3|_{\text{after}}$$

$$\rightarrow \frac{11}{2} T_\nu^3 = 2T_\gamma^3$$

$$\rightarrow T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma$$

Increase in temp for CMB



Cosmic radiation density

$$\rho_{\mathrm{r}} \;=\; \frac{\pi^2}{30}\left(2T_\gamma^4 + \frac{7}{8}\times 6T_\nu^4\right) \approx 1.681\rho_\gamma$$

$$\rightarrow\;\; \Omega_{\mathrm{r}} = \frac{1.681 \times \frac{\pi^2}{15}T_\gamma^4}{\frac{3H_0^2}{8\pi G}}$$

$$\rightarrow\;\; \Omega_{\mathrm{r}} h^2 \approx 4.2\times 10^{-5}$$

Density of massive neutrinos

$$\rho(T) \;=\; \sum_i m_i n_i = \left(\sum_i m_i\right) \times \frac{3}{4} \times \frac{2\zeta(3)}{\pi^2} T^3$$

$$\rightarrow \;\; \Omega_\nu = \frac{\rho(T_{\mathsf{CMB}})}{\frac{3H_0^2}{8\pi G}}$$

$$\rightarrow \;\; \Omega_\nu h^2 = \frac{\sum_i m_i}{94.1\,\mathrm{eV}}$$

Non-instantaneous decoupling

- and QED corrections

- Decoupling and e^+/e^- annihilation overlap!

  - NB ν_e decoupling is delayed relative to ν_μ & ν_τ

- Correction to radiation density : N_{eff}=3.046

$$\rho_r = \frac{\pi^2}{30}\left(2T_\gamma^4 + \frac{7}{8}\times 2N_{\text{eff}}T_\nu^4\right) \approx (1+0.227N_{\text{eff}})\rho_\gamma$$

- Matter density

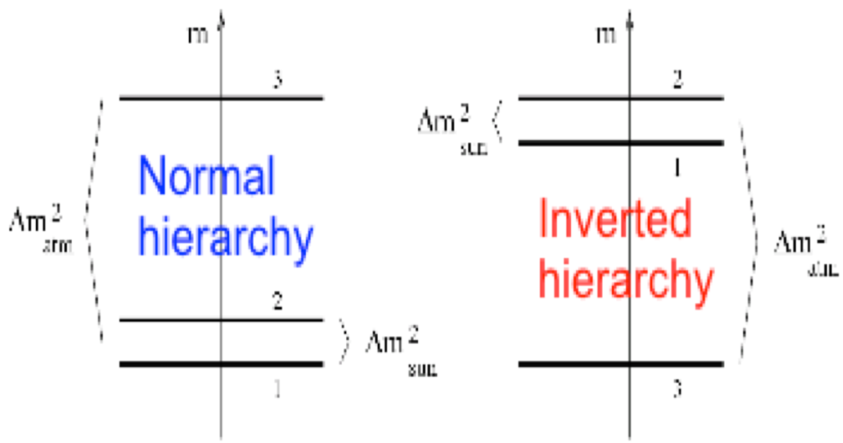
94.1 eV -> 93.1 eV

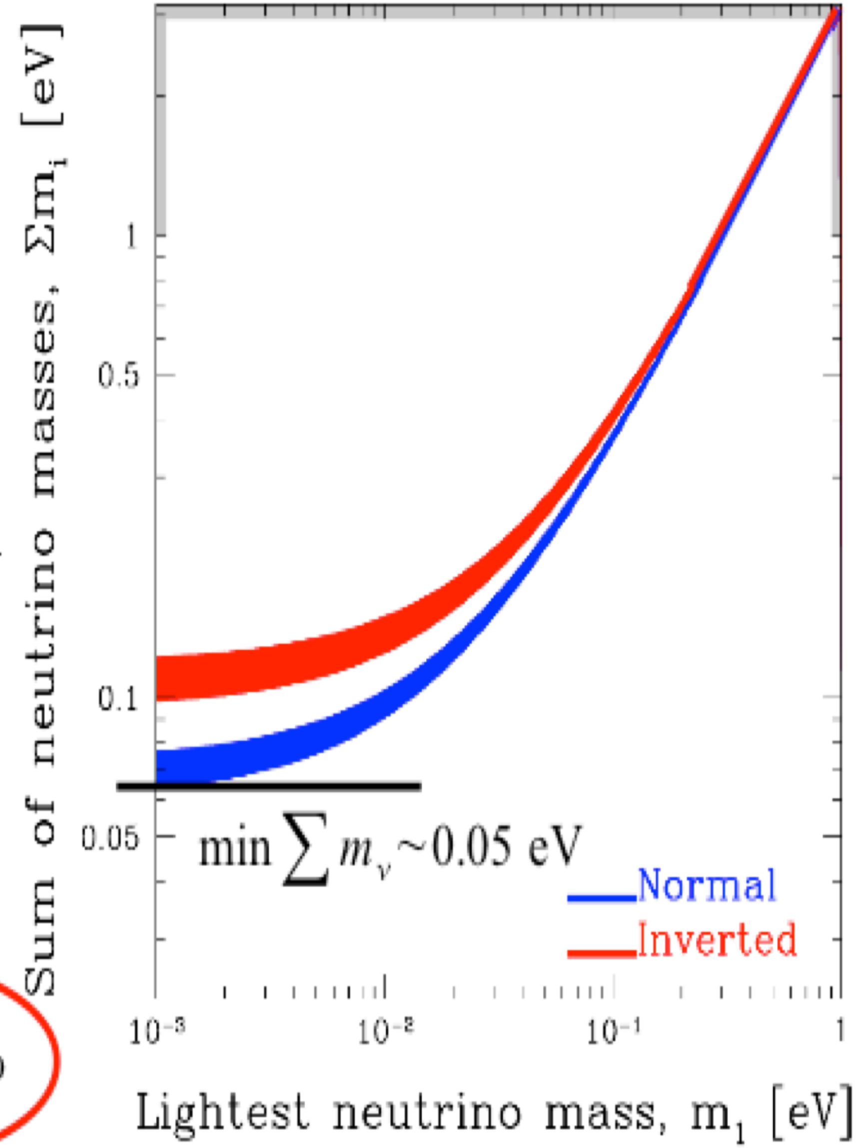
$$\Omega_\nu h^2 = \frac{\sum_i m_i}{93.1\,\text{eV}}$$

Limit on the sum of neutrino masses

- Neutrino oscillations:

$$\Delta m^2_{\mathrm{atm}} \sim 10^{-3}\ \mathrm{eV}^2 \quad \Delta m^2_{\mathrm{sun}} \sim 10^{-5}\ \mathrm{eV}^2$$





Mininum amount of
neutrino dark matter

$$\min \sum m_\nu \sim 0.05\ \mathrm{eV} \rightarrow \min \Omega_\nu \sim 0.1\%$$

Summary of basic facts

- Decoupling

$$T_{\text{dec}} \approx 1-3\,\text{MeV}$$

- Fermi-Dirac Temperature

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_{\text{CMB}} = 1.945\,\text{K} = 1.676\times 10^{-4}\,\text{eV}/k_{\text{B}}$$

- Number Density for each flavour

$$n_{\nu_i} = n_{\bar{\nu}_i} \approx 56\,\text{cm}^{-3}$$

- Contribution to the energy density budget

$$\Omega_\nu h^2 = \frac{\sum_i m_i}{93.1\,\text{eV}}$$

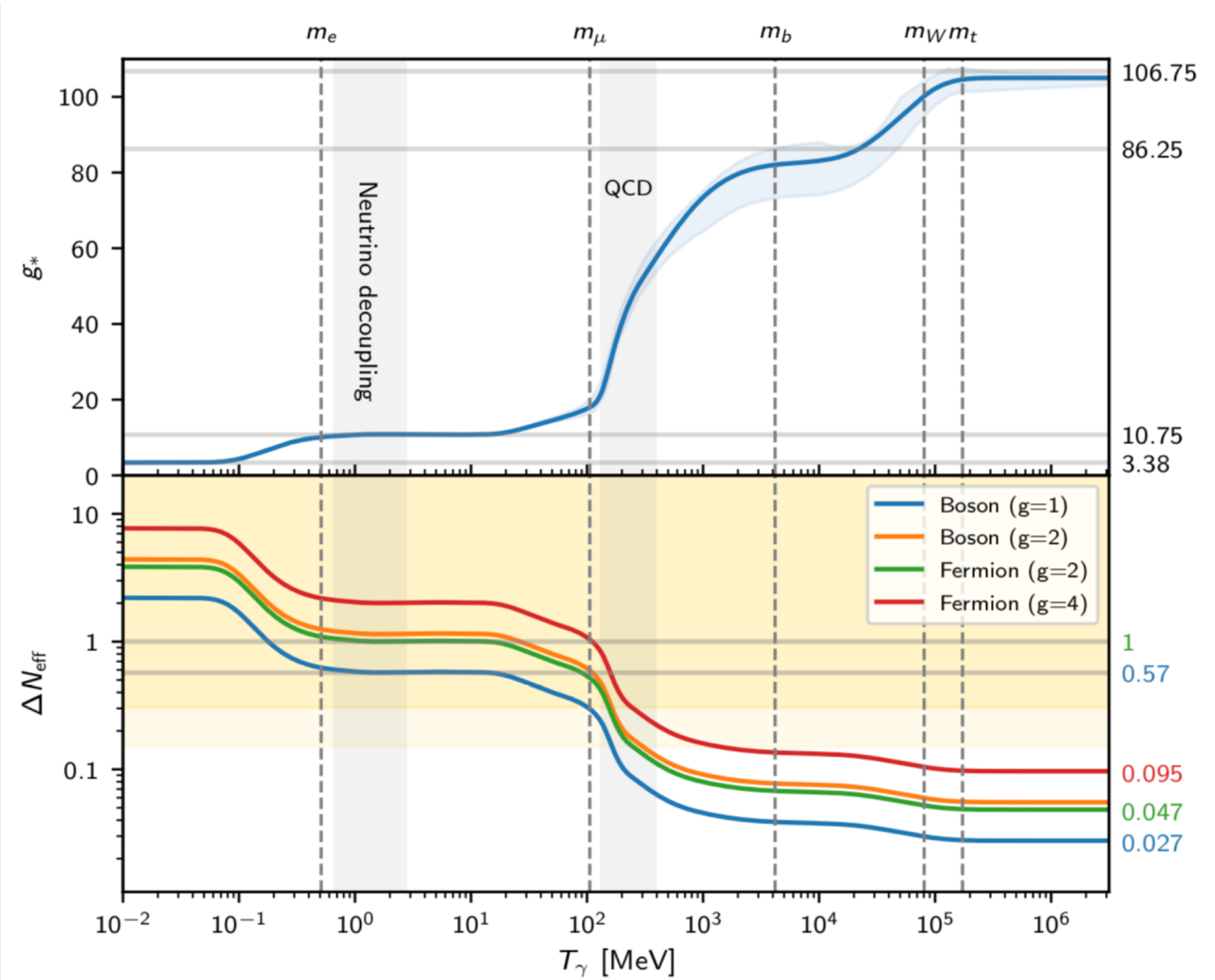
- “Standard” value

$$\sum_i m_i = 0.06\,\text{eV}$$

Section 3

Sterile neutrinos and extra relativistic degrees of freedom

Extra relativistic degrees of freedom



$$\rho_{\rm r} = \frac{\pi^2}{30}\left(2T_\gamma^4 + \frac{7}{8}\times 2N_{\rm eff}T_\nu^4\right) \approx (1+0.227N_{\rm eff})\rho_\gamma$$

Sterile Neutrinos

- Effects governed by two parameters
  - N_{eff} and m_{eff}
- N_{eff}>3 - effective number of neutrinos
  - not necessarily an integer: partial thermalization
  - governs contribution to energy density
- m_{eff} - impacts perturbations via rel v non-rel
- Specific models link the two parameters

Sterile neutrino models

- FD distribution with $T=T_S$
  - $m = \Delta N^{-3/4}\, m_{eff}$

- Dodelson-Widrow model
  - $m = \Delta N^{-1}\, m_{eff}$

- …

Section 4

Impact of cosmological observables

Power spectrum norm : σ_8

- R.M.S. density contrast in $R=8h^{-1}$Mpc

$$\sigma_R^2 = 4\pi \int \frac{dk}{k} k^3 P(k) [W(kR)]^2$$

Filter function for a sphere of radius R

- Value ~1 – in fact approx 0.8

- Related to A_S

  - plus other cosmological parameters relating large and small scales

Impact on cosmological observables

- Main parameter : $\sum m_\nu$

Sensitivity to individual masses is limited but not zero !

- Contribution to the energy density

$\longrightarrow$ Changes the time of equal-matter radiation

- Evolution of perturbations

$\longrightarrow$ Neutrinos are “hot” and therefore free stream

$\longrightarrow$ They do not fall into potential wells until they become non-relativistic

Transition from relativistic to non-relativisitic

- Early times

  - behave like photons (without Thomson scattering)

- Late time

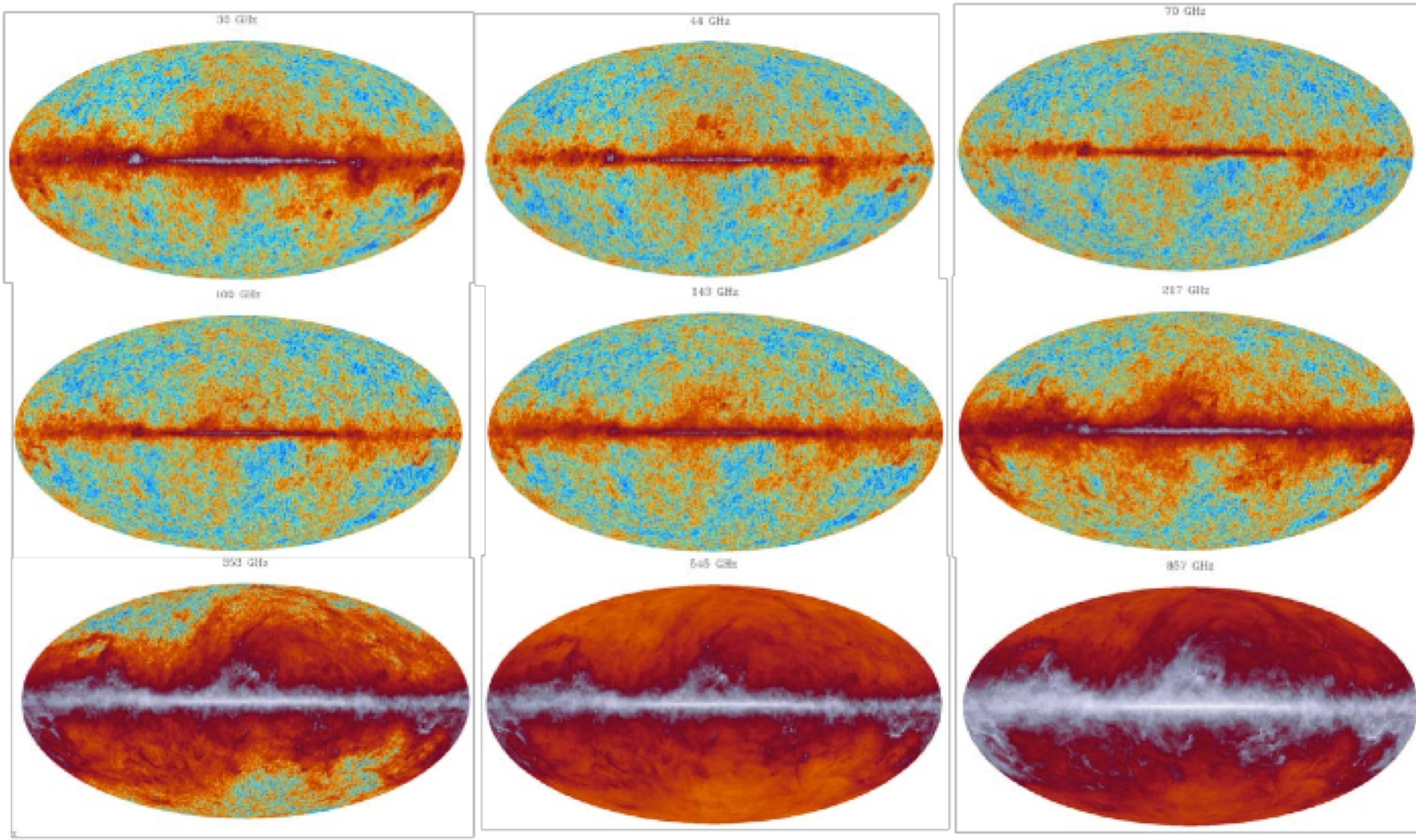
  - behave like CDM

- Time of transition

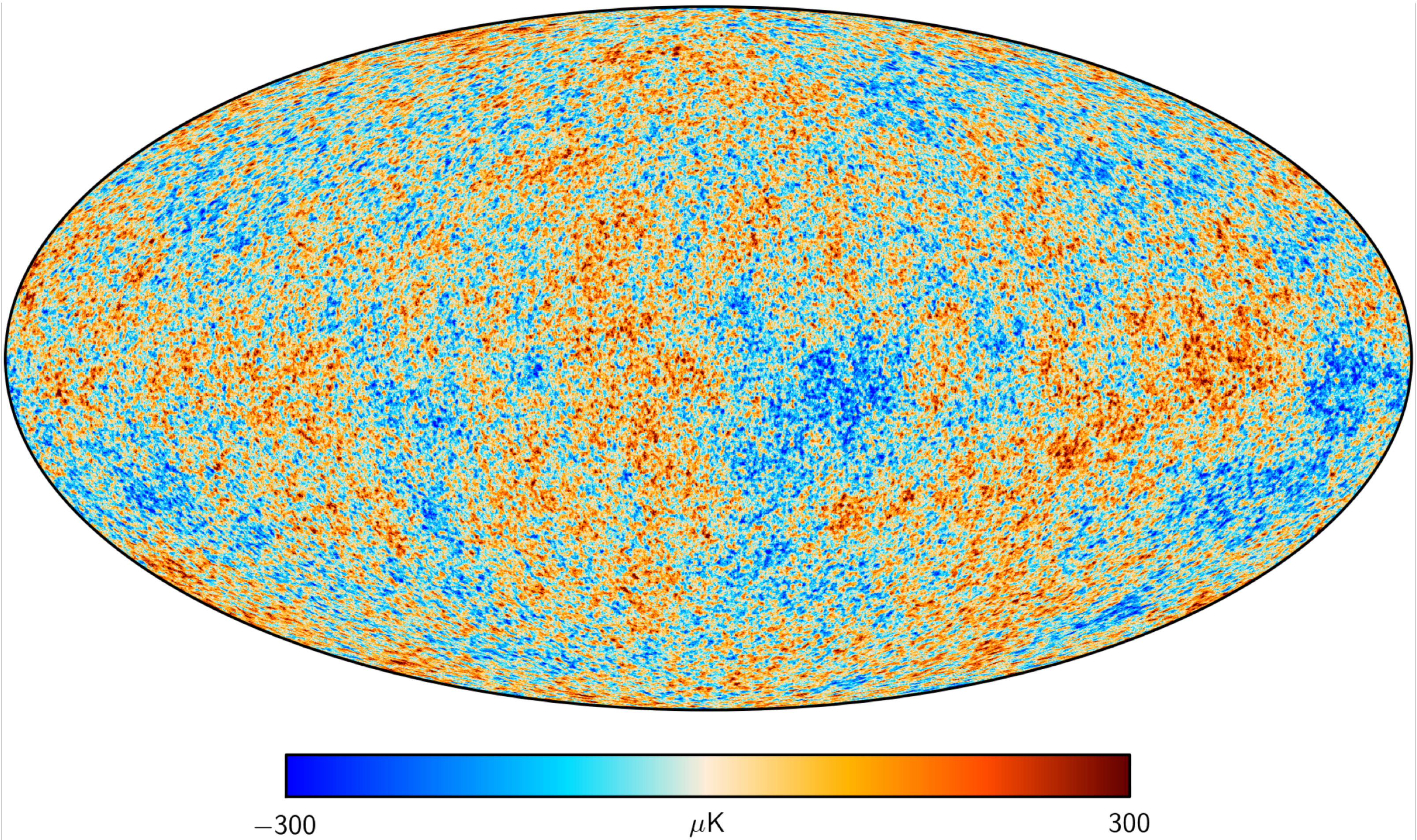
$$3\left(\frac{4}{11}\right)^{\frac{1}{3}}(1+z_{\mathrm{nr}})T_{\mathrm{CMB}} \approx 3T_\nu \approx p \approx m$$

$$\rightarrow z_{\mathrm{nr}} \approx 1500\left(\frac{m}{1\,\mathrm{eV}}\right)$$

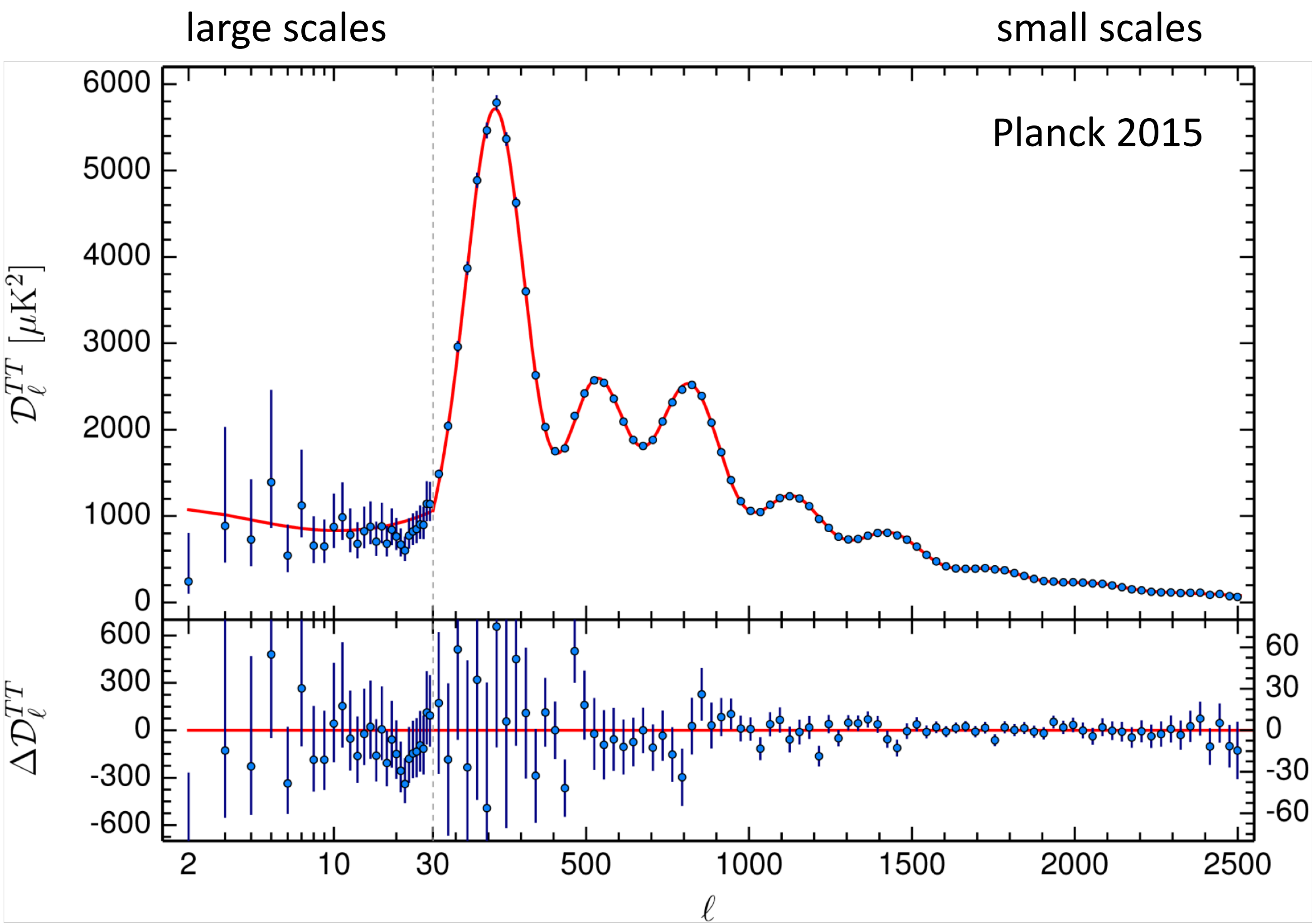
Observables : CMB



Observables : CMB



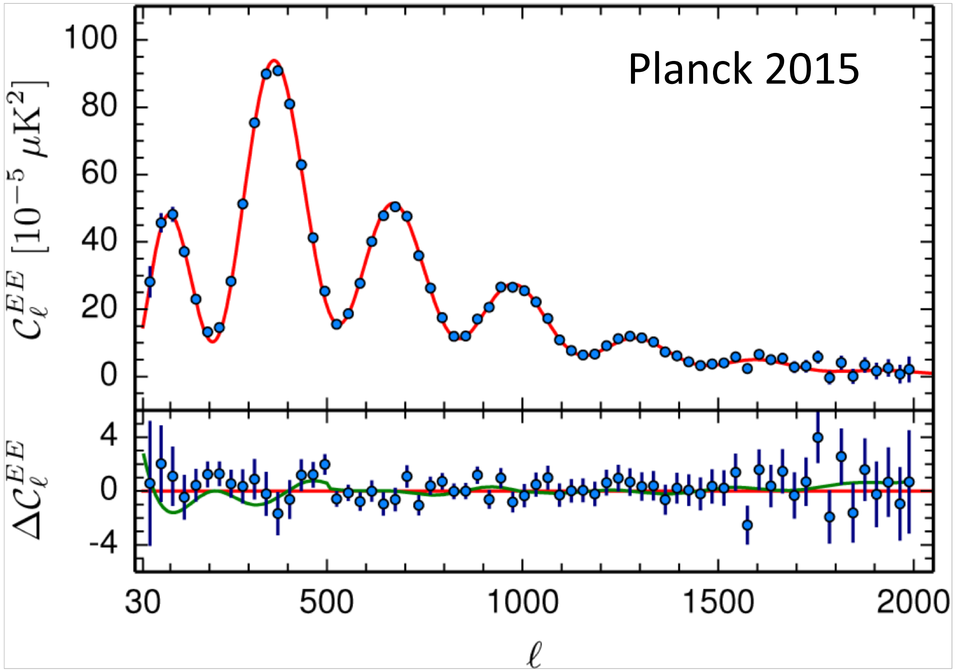
Observables : CMB



Temperature power spectrum

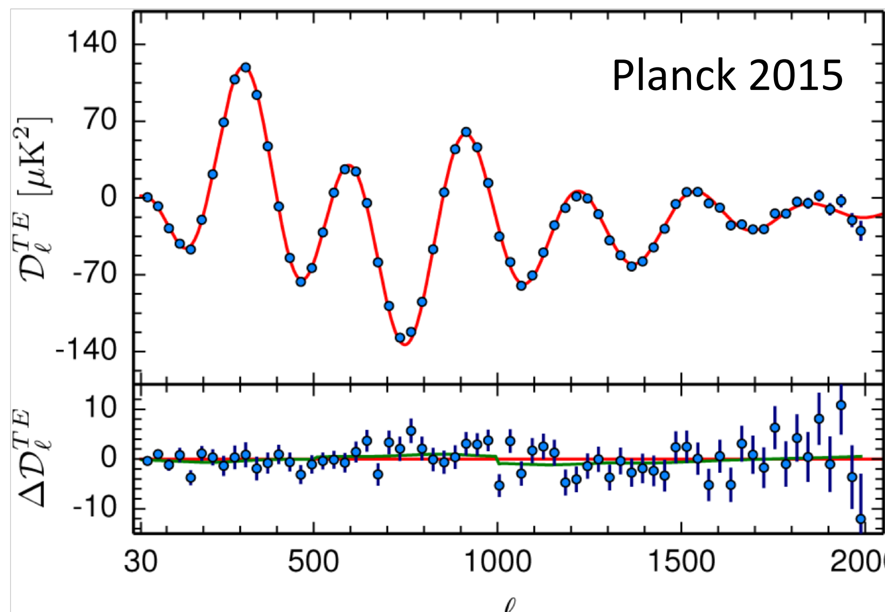
– where the most reliable constraints come from

Observables : CMB

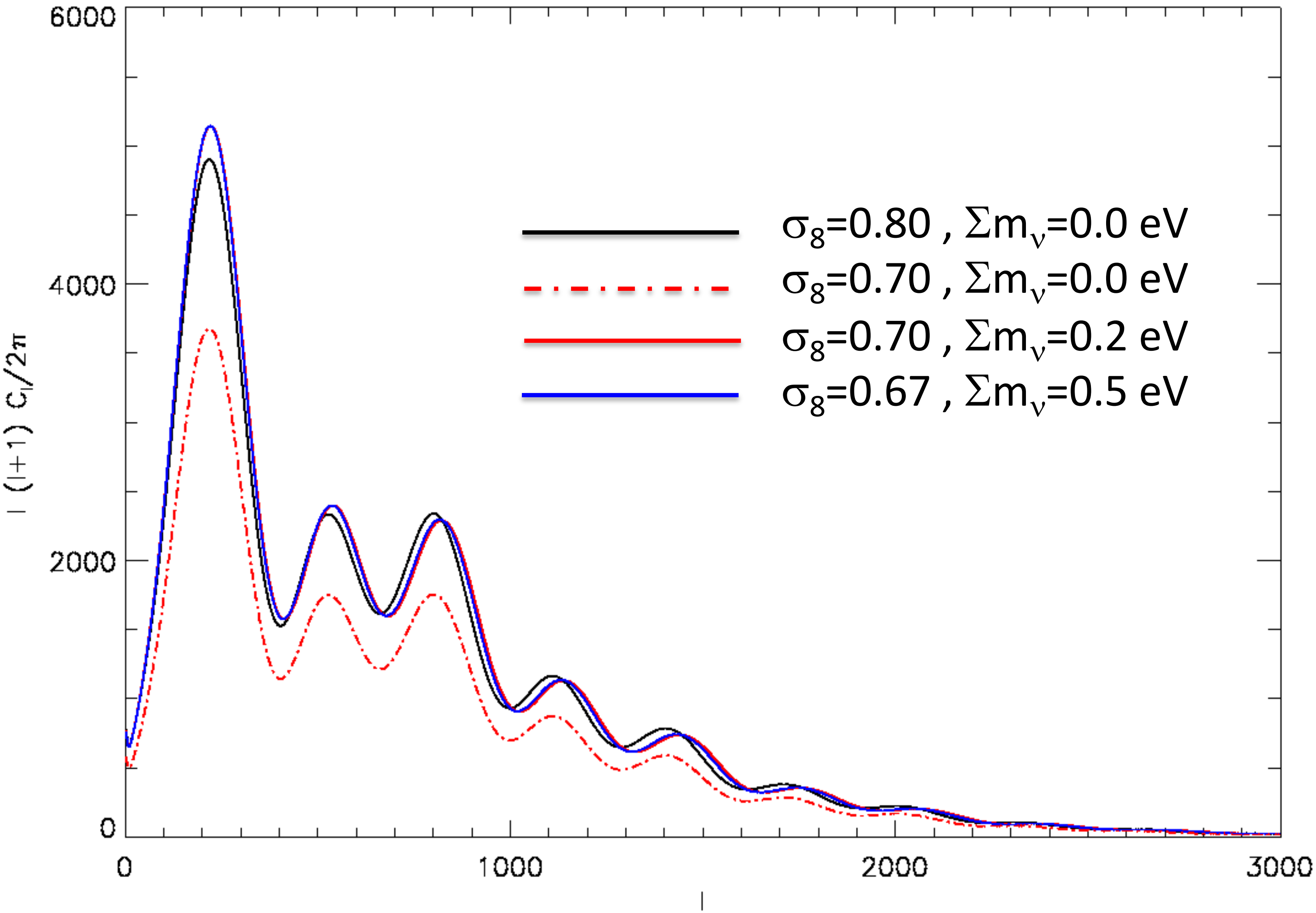


E-mode polarization power spectrum

Temperature-polarization cross power spectrum



Sensitivity of CMB to neutrinos



Two main effects on the CMB

- Change in angular diameter

Degeneracy with H_0

$$d_A(z_{LSS}) = \frac{1}{(1+z_{LSS})H_0}\int_0^{z_{LSS}} \frac{dz}{E(z)}$$

$$E(z) = \sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda}$$

Change in peak positions

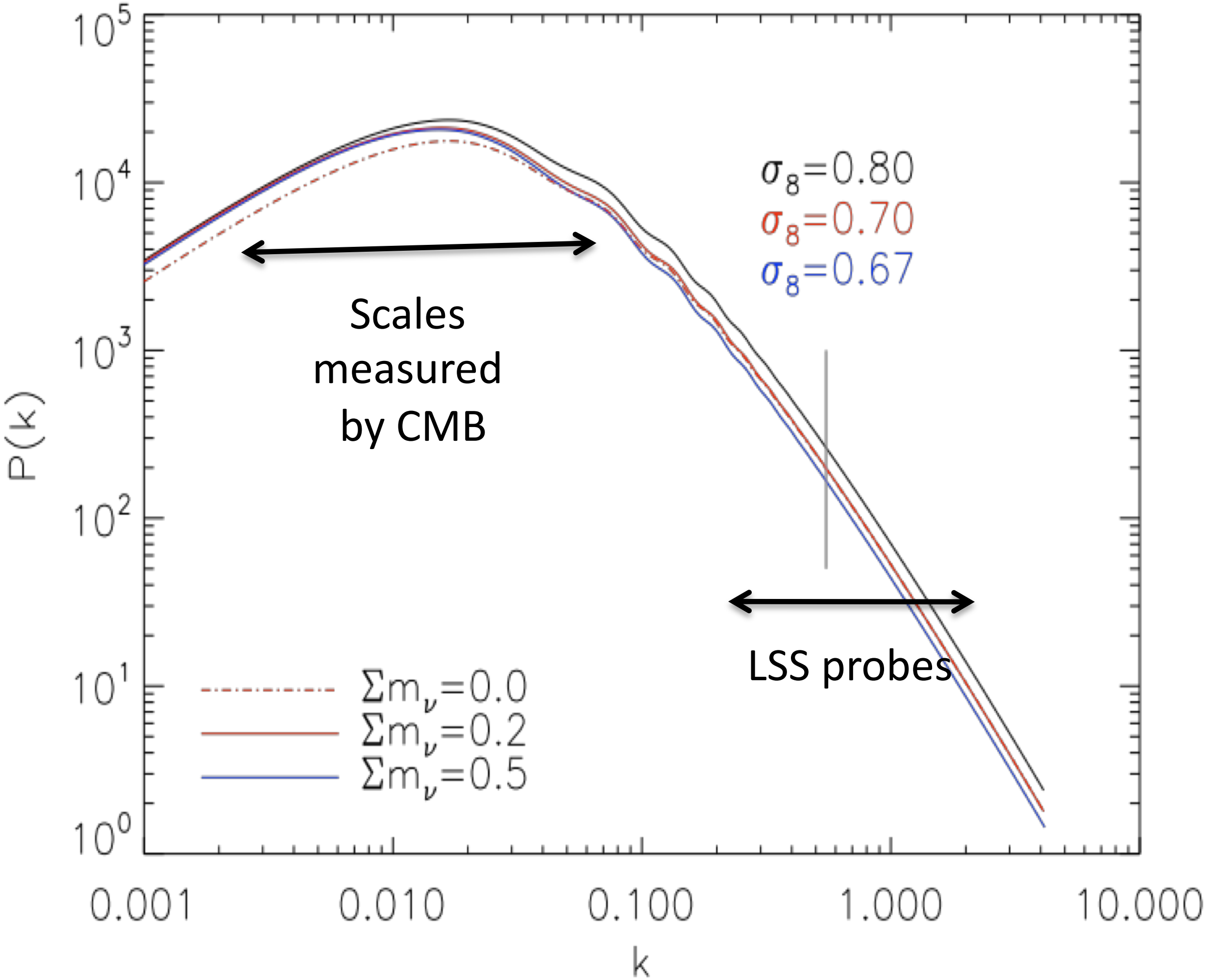
- ISW effect

$$\frac{\Delta T}{T}|_{ISW} = 2\int d\ell\dot{\Phi}$$

Starts to vary

Impact on large-scales

Observables : matter power spectrum



large scales

small scales

Free streaming length and effect on matter power spectrum

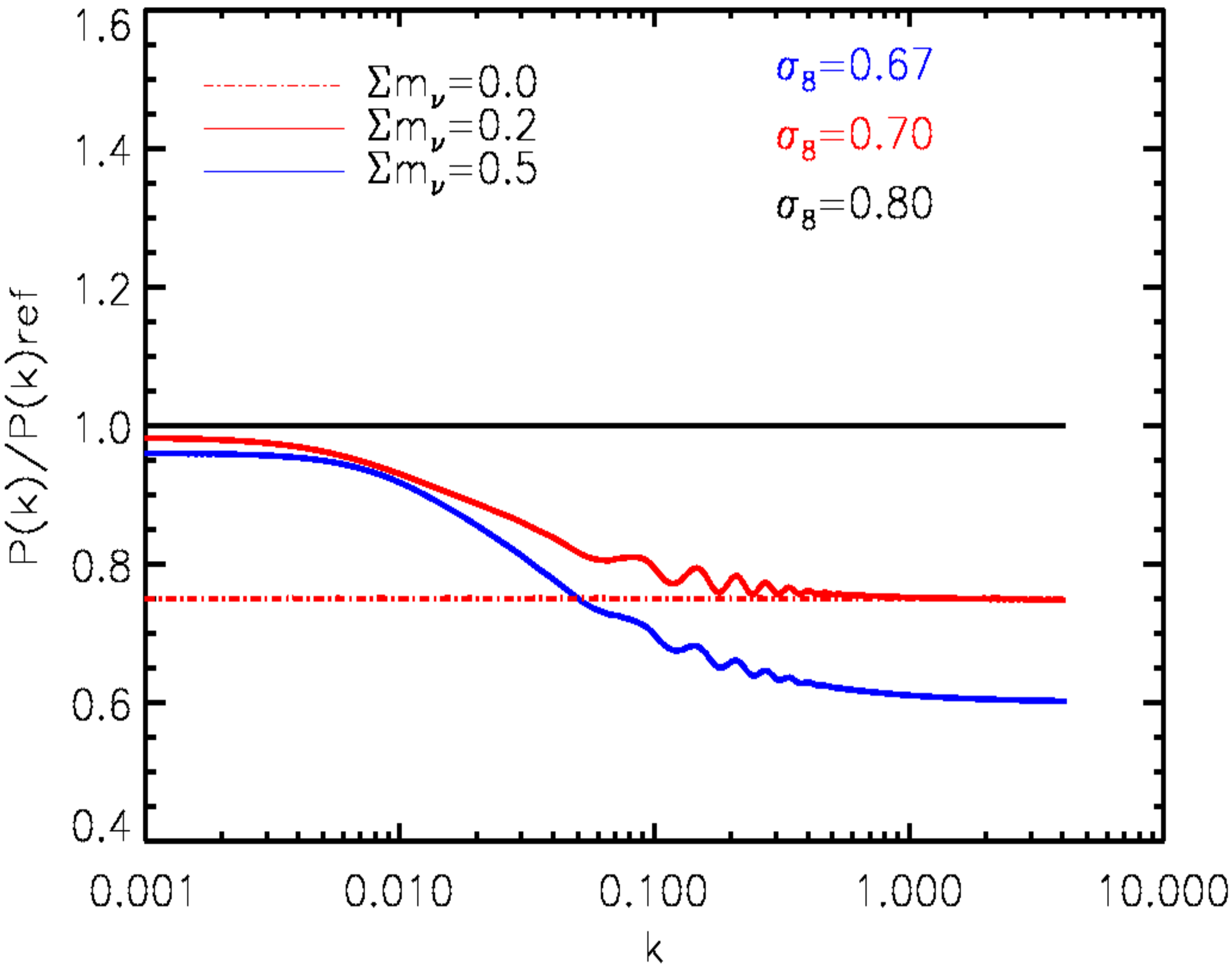
$$k_{\rm FS} \approx 0.82 h^{-1}\,{\rm Mpc} \left(\frac{m}{1\,{\rm eV}}\right) \frac{\sqrt{\Omega_\Lambda + \Omega_{\rm m}(1+z)^3}}{(1+z)^2}$$

For $k > k_{FS}$ ⇨

$$\frac{\Delta P}{P} = -8 \frac{\Omega_\nu}{\Omega_{\rm m}}$$

Impact on small-scales

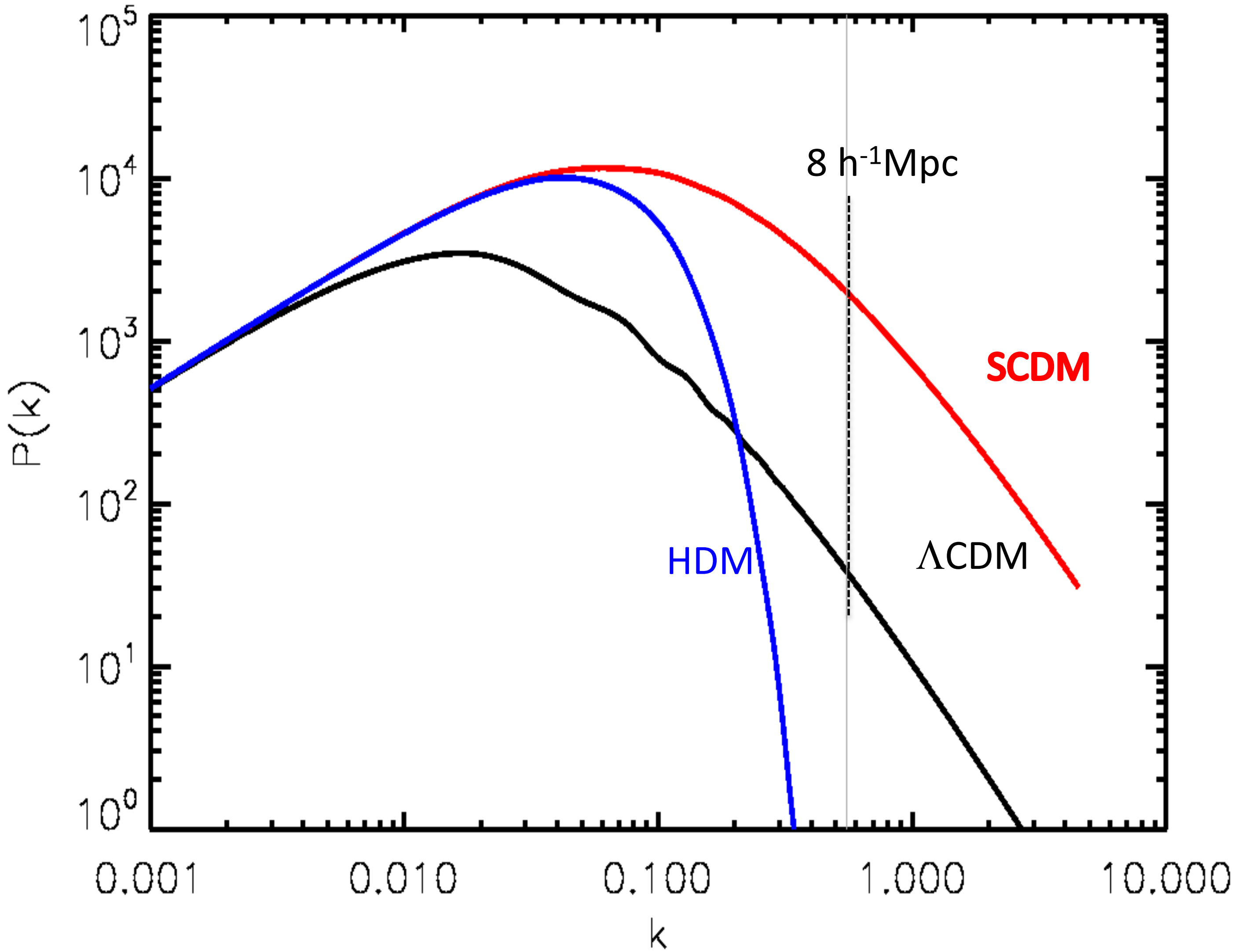
Impact of massive neutrinos



Probing the matter power spectrum

- Galaxy redshift surveys
- Lensing effects
  - CMB temperature anisotropies lensing
  - Cosmic shear (weak lensing of galaxy shapes)
  - CMB polarization lensing
- Cluster counts
  - Using the SZ effect, X-ray or optical
- Redshift space distortions
- …

Ruling out of pure HDM & pure CDM



Section 5

Present status of constraints :

Planck 2018

!!! Health Warning !!!

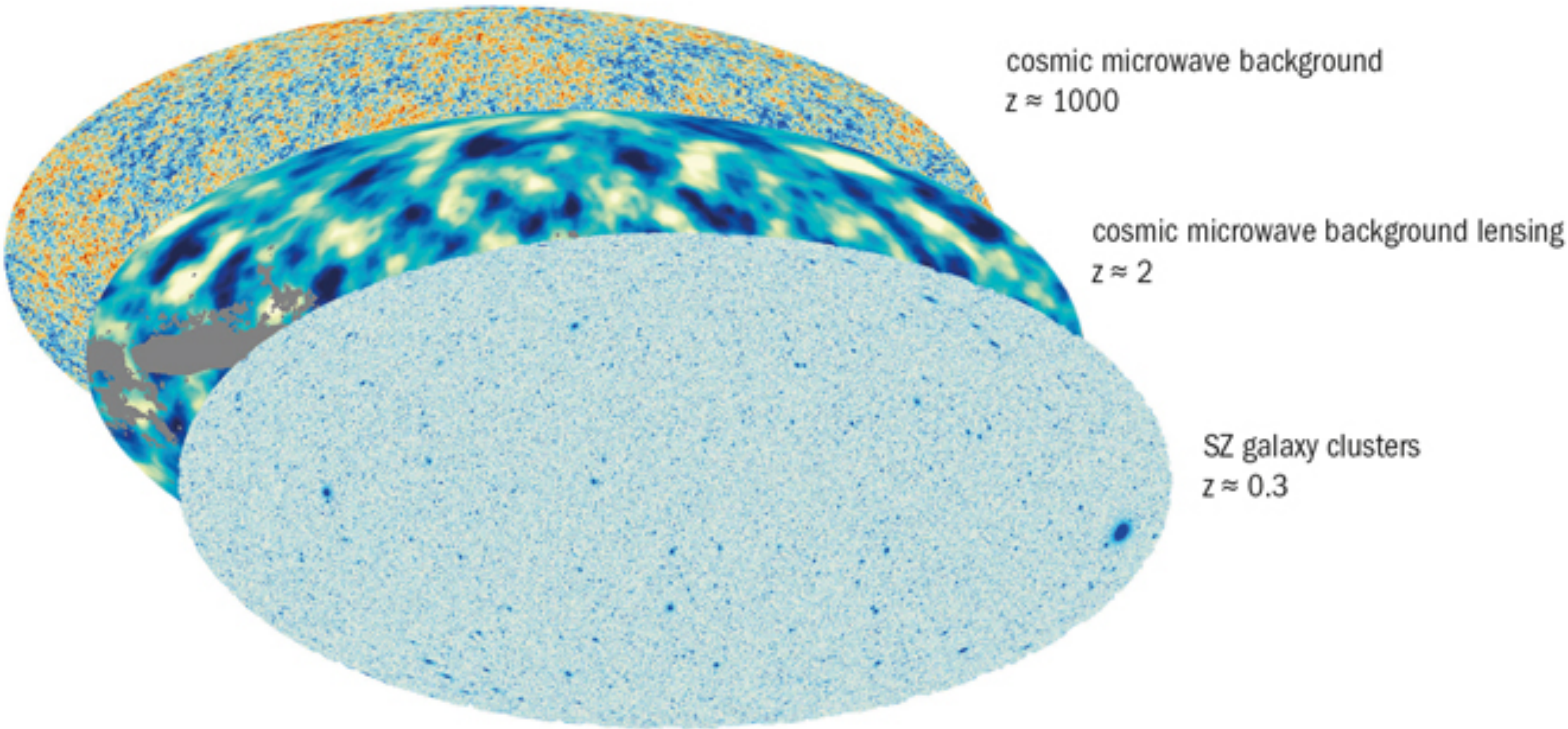
- All cosmological limits on neutrino masses are model dependent

- Standard cosmological limits on all other parameters are sensitive to neutrino history

- At the moment $\sum m_\nu = 0.06\,\mathrm{eV}$ is used with 3 equal masses – degenerate case

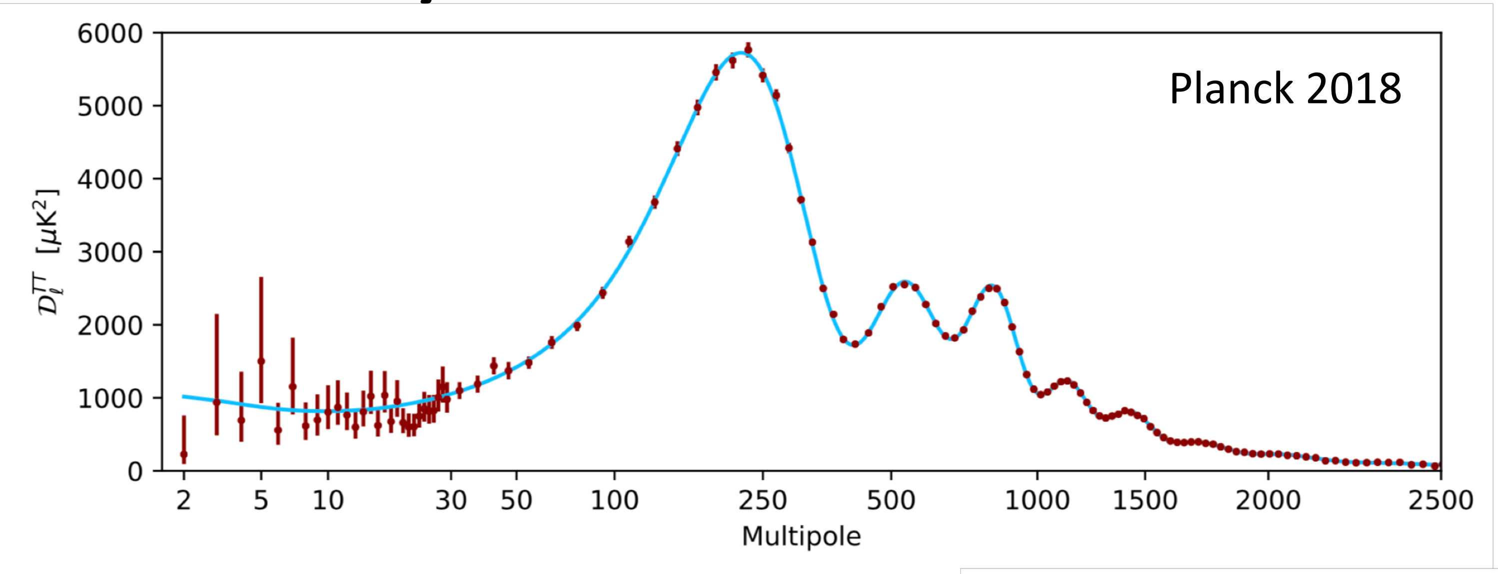
- Probes of LSS are less mature at the moment

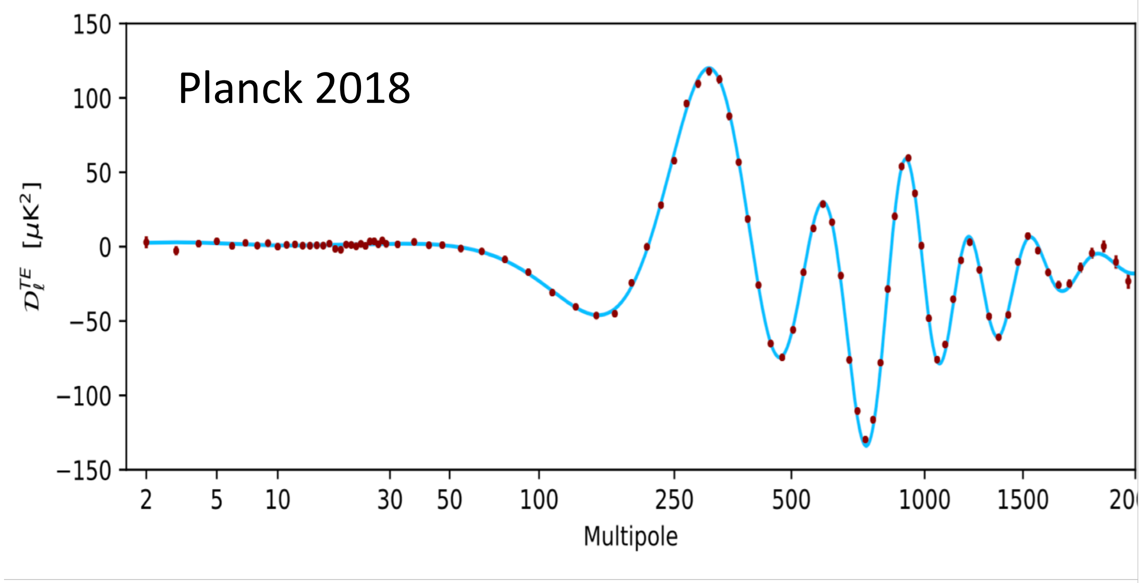
Planck 2018 results

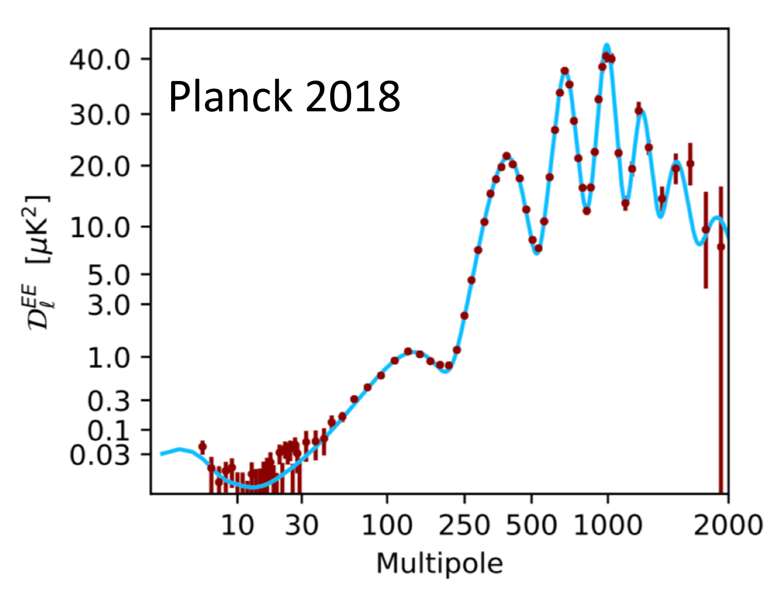


Three separate probes of cosmology !

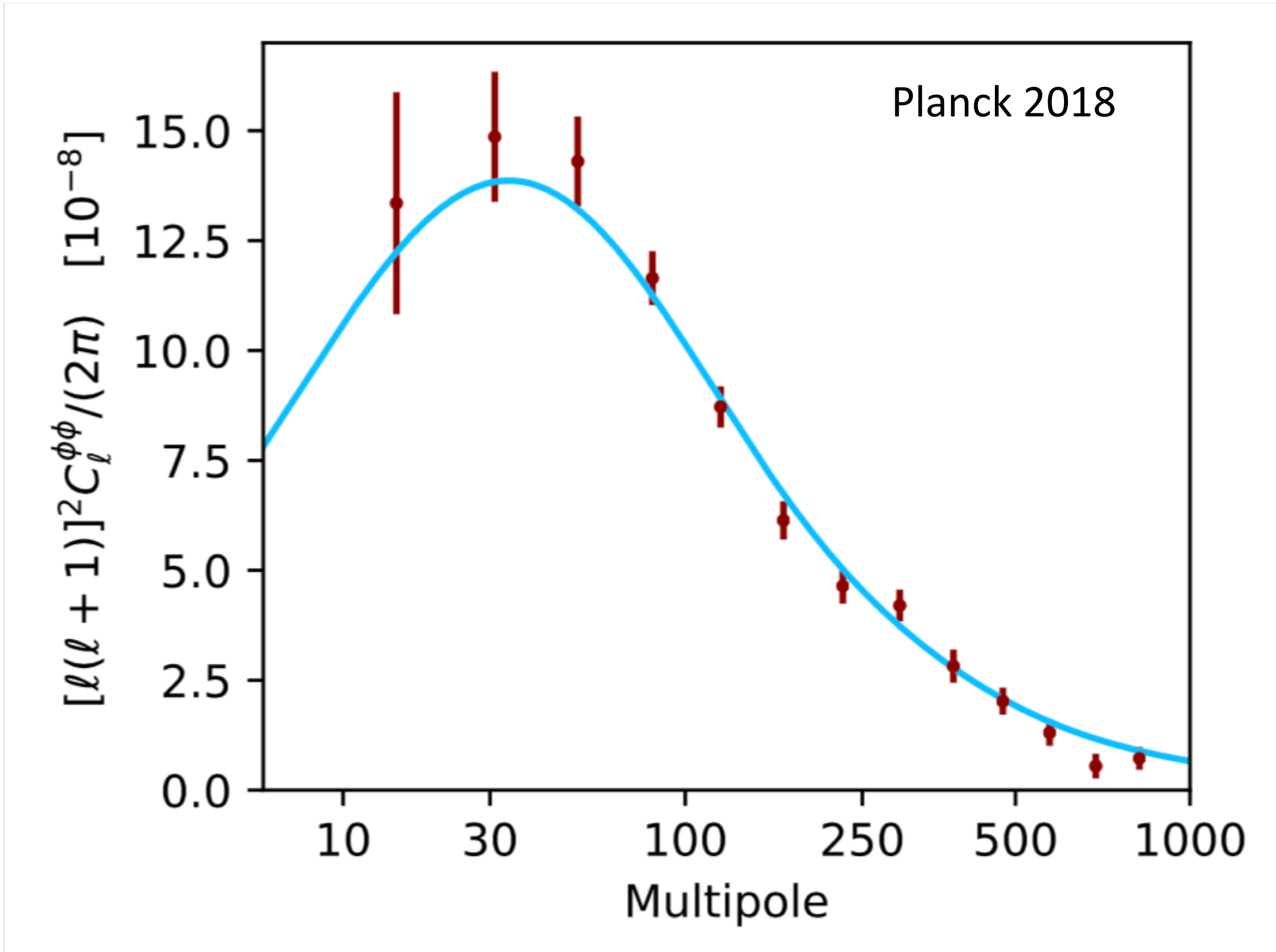
Primary CMB measurements







CMB lensing power spectrum



will focus on TT, EE, TE +low E + lensing results

From Planck Collaboration (2019) : arXiv: 1807.06209v1 to be published in A&A

Constraints on base ΛCDM

Fitted parameters

$$100\Omega_b h^2 = 2.237 \pm 0.015$$
$$\Omega_c h^2 = 0.1200 \pm 0.0012$$
$$100\theta_{MC} = 1.04092 \pm 0.00031$$
$$\boxed{\tau = 0.0544 \pm 0.0073}$$
$$\log(10^{10} A_s) = 3.044 \pm 0.014$$
$$n_s = 0.9649 \pm 0.0042$$

Derived parameters

$$\Omega_m = 0.3153 \pm 0.0073$$
$$\sigma_8 = 0.8111 \pm 0.0060$$
$$10^9 A_s e^{-2\tau} = 1.883 \pm 0.011$$

Planck TT, TE, EE + low E + lensing (ie just CMB) : 68% confidence

From Planck Collaboration (2019) : arXiv: 1807.06209v1 to be published in A&A

Extensions to ΛCDM

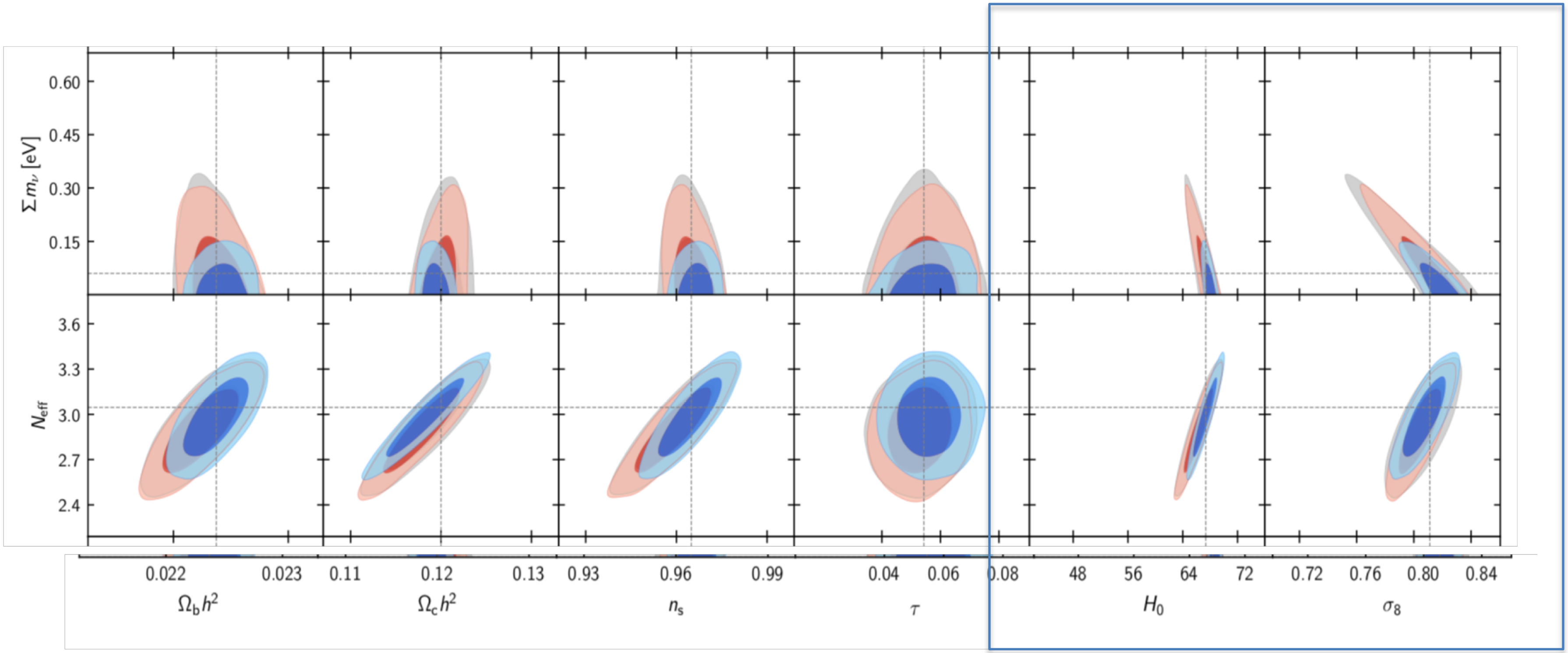
$$
\begin{aligned}
\Omega_{\mathrm{k}} &= -0.01 \pm 0.01 \\
\boxed{\sum m_\nu/\mathrm{eV} < 0.241 \quad N_{\mathrm{eff}} = 2.89 \pm 0.38} \\
Y_{\mathrm{p}} &= 0.239 \pm 0.025 \\
dn_{\mathrm{S}}/d(\log k) &= -0.005 \pm 0.013 \\
r_{0.002} &< 0.1 \\
w &= -1.5 \pm 0.5
\end{aligned}
$$

Our interest here !

Planck TT, TE, EE + low E + lensing (ie just CMB) : 95% confidence

From Planck Collaboration (2019) : arXiv: 1807.06209v1 to be published in A&A

Degeneracies with standard parameters



Grey : TT , TE, EE + low E

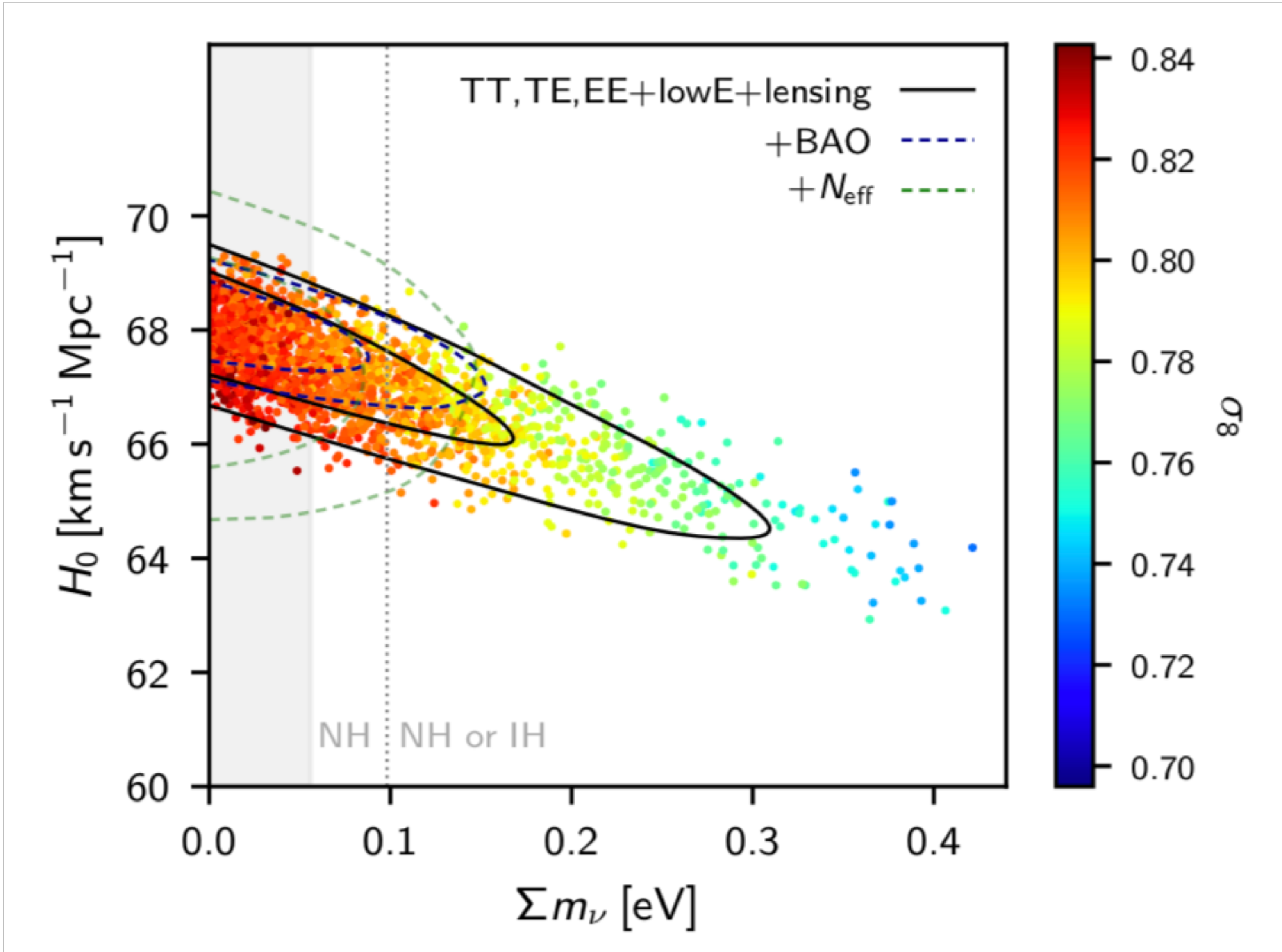
Red : TT, EE, TE + low E + lensing

Blue : TT, EE, TE, low E + lensing + BAO

Key degeneracies

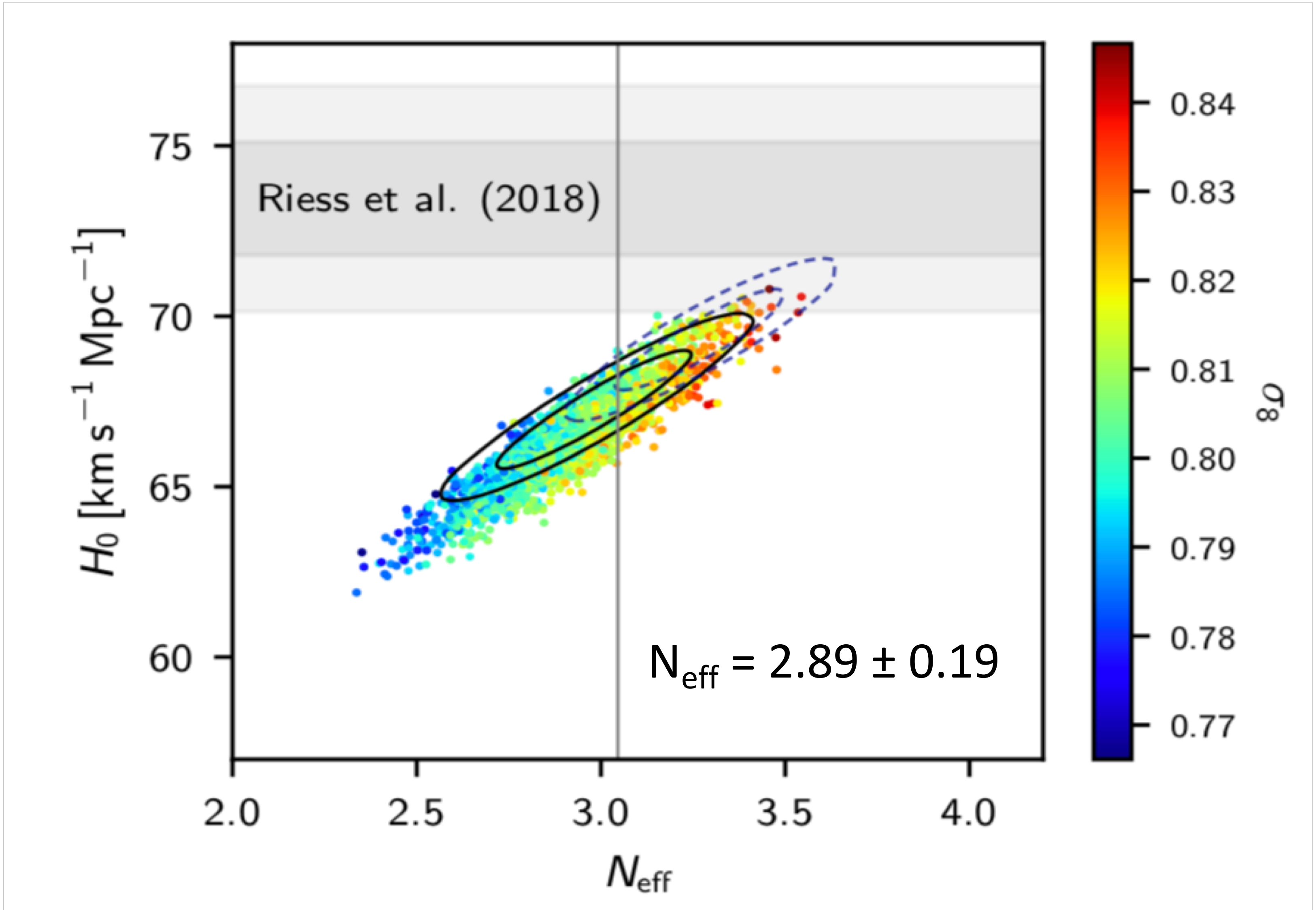
From Planck Collaboration (2019) : arXiv: 1807.06209v1 to be published in A&A

Neutrinos v H_0 and σ_8



From Planck Collaboration (2019) : arXiv: 1807.06209v1 to be published in A&A

N_{eff} v H_0 and σ_8



Limits on active neutrinos 2015

TT, TE, EE $\sum m_\nu < 0.492\,\mathrm{eV}$

TT, TE, EE + Planck lensing $< 0.589\,\mathrm{eV}$

TT, TE, EE + lensing + ext $< 0.194\,\mathrm{eV}$

95% upper limits

Limits on active neutrinos 2018 – factor 2 better !

TT, TE, EE, low E $\quad \sum m_\nu < 0.26\,\mathrm{eV}$

TT, TE, EE, low E + Planck lensing $\quad < 0.24\,\mathrm{eV}$

TT, TE, EE, low E + lensing + BAO $\quad < 0.12\,\mathrm{eV}$

95% upper limits

Choudhury & Hannestad 2019

For different hierarchies

- Degenerate

$$\sum m_\nu < 0.12\,\mathrm{eV}$$

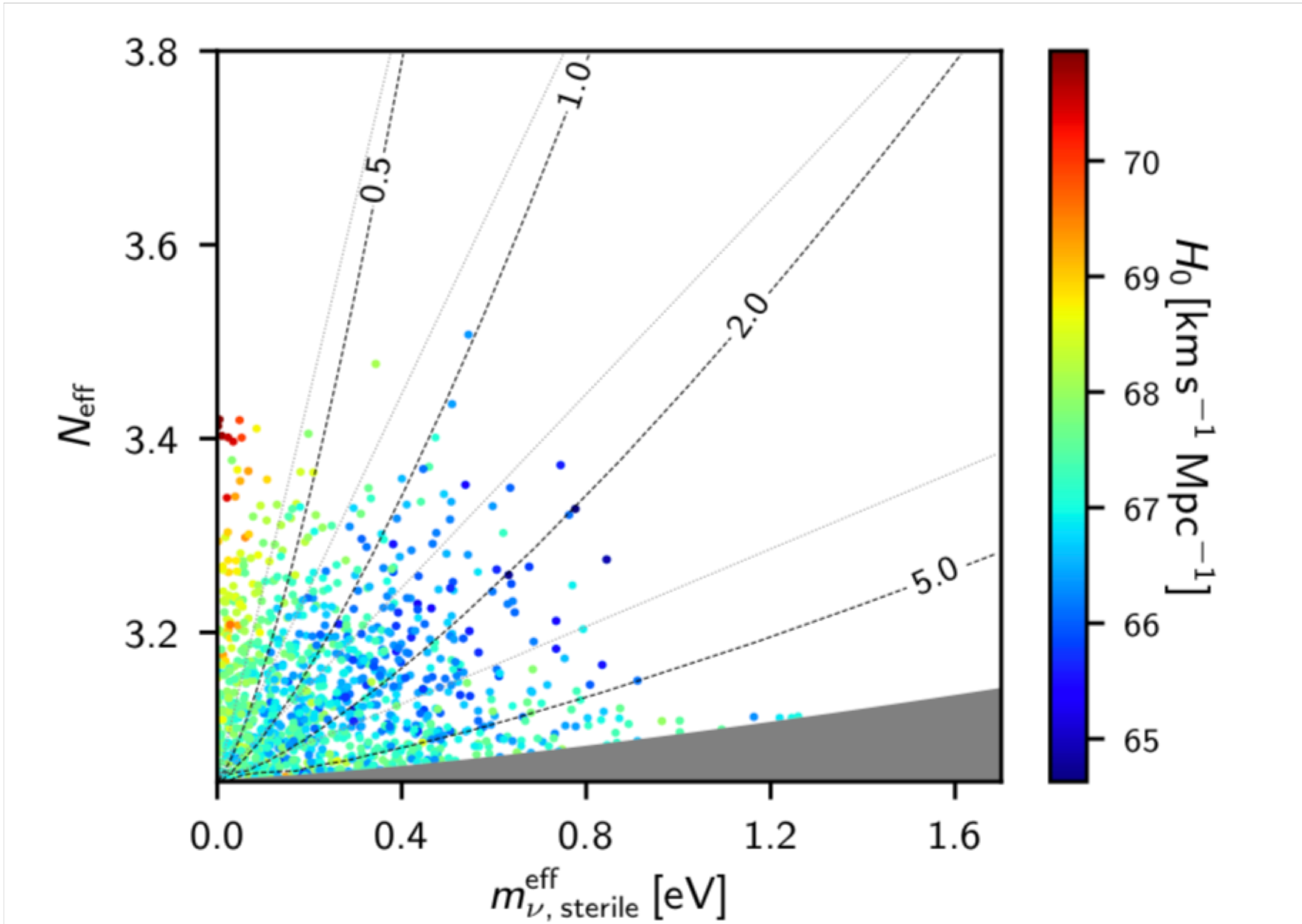
- Normal

$$< 0.15\,\mathrm{eV}$$

- Inverted

$$< 0.17\,\mathrm{eV}$$

Sterile Neutrinos



$N_{\rm eff} < 3.34 \quad m_{\nu,\rm sterile}^{\rm eff} < 0.23\,\rm eV$

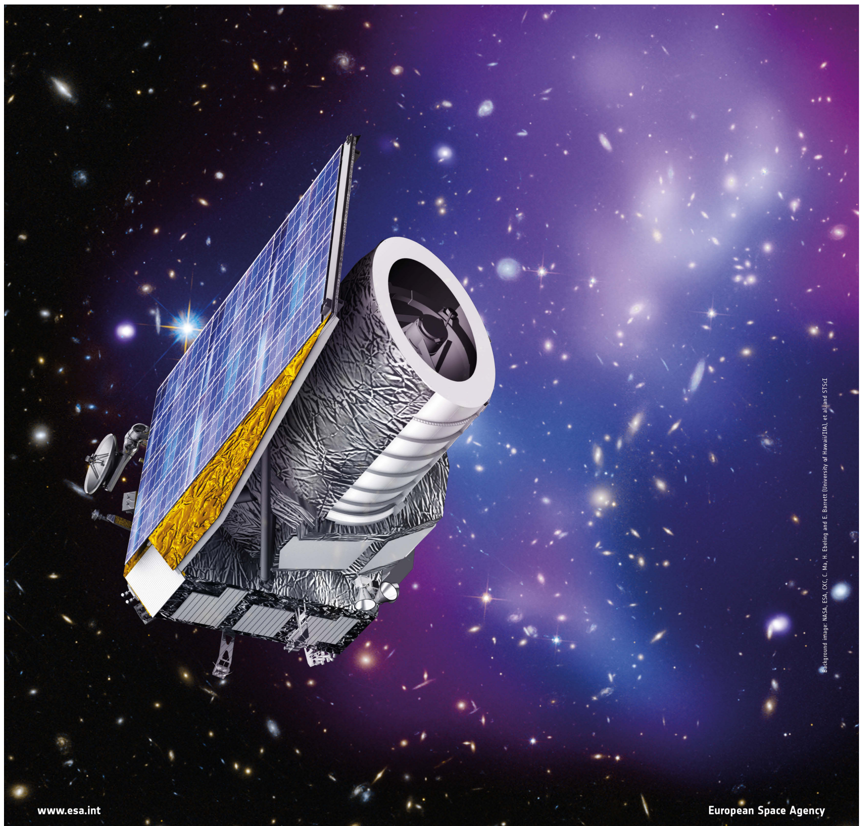
Section 6

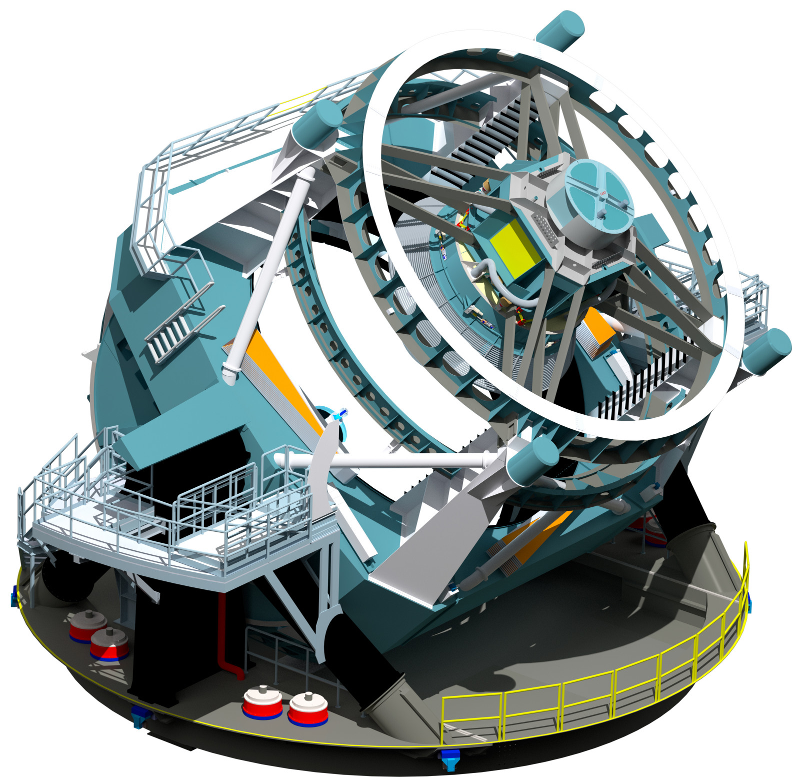
Future prospects

Stage 4 Cosmology Probes

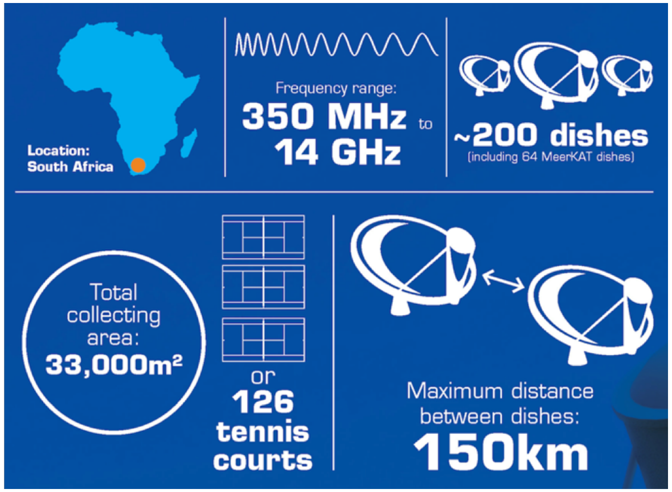
Euclid consortium

LSST
Large Synoptic Survey Telescope

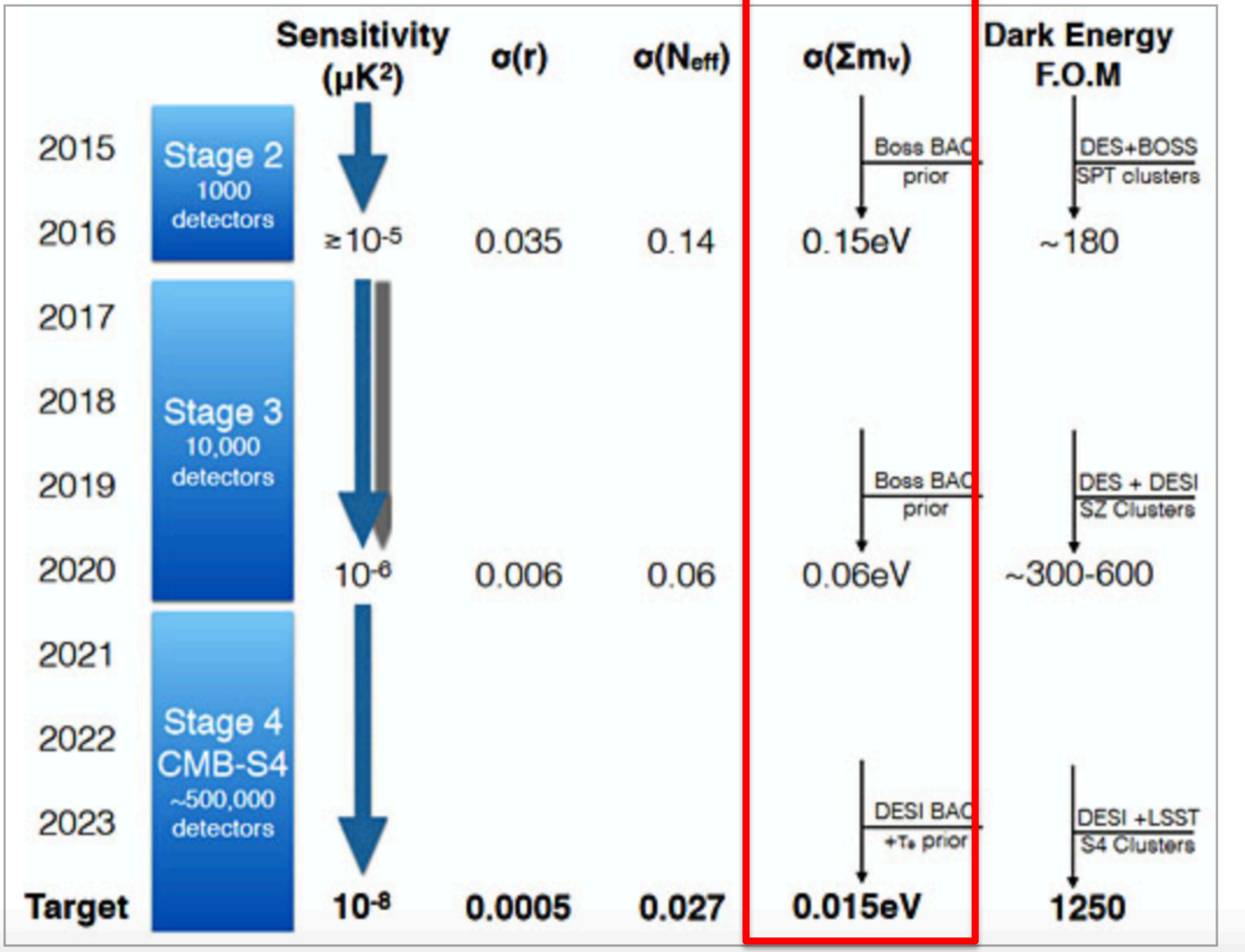




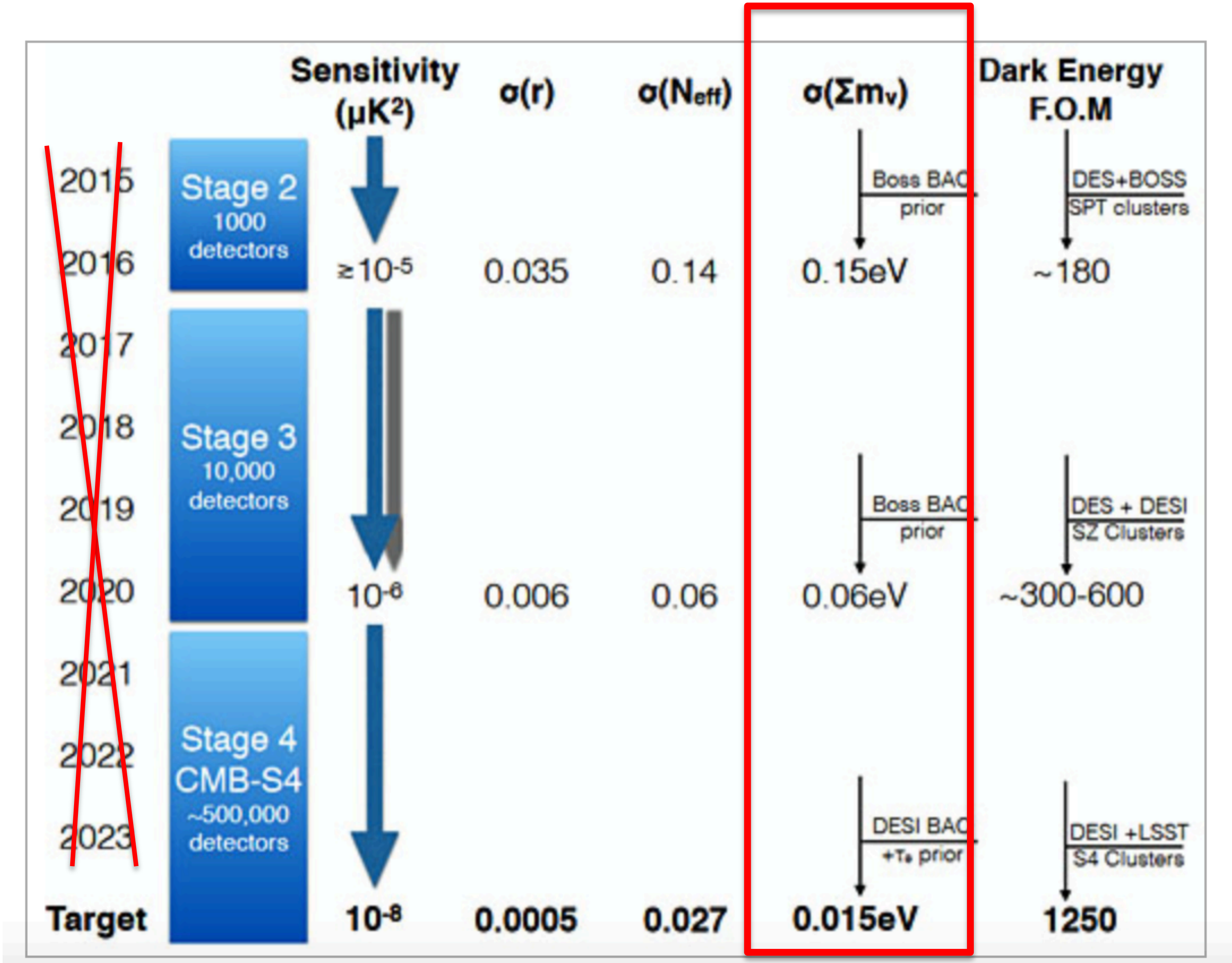
Stage 4 Cosmology Probes



Future Probes



Future CMB Probes



2026

~2035

Simons Observatory : 2021-2026 (60000 detectors, 2 telescope sizes)

	Parameter	SO-Baseline[b] (no syst)	**SO-Baseline[c]**	SO-Goal[d]	Current[e]	Method
Primordial perturbations	r	0.0024	**0.003**	0.002	0.03	BB + ext delens
	$e^{-2\tau}\mathcal{P}(k=0.2/\mathrm{Mpc})$	0.4%	**0.5%**	0.4%	3%	$TT/TE/EE$
	$f_{\mathrm{NL}}^{\mathrm{local}}$	1.8	**3**	1	5	$\kappa\kappa \times$ LSST-LSS + 3-pt
		1	**2**	1		kSZ + LSST-LSS
Relativistic species	N_{eff}	0.055	**0.07**	0.05	0.2	$TT/TE/EE + \kappa\kappa$
Neutrino mass	Σm_ν	0.033	**0.04**	0.03	0.1	$\kappa\kappa$ + DESI-BAO
		0.035	**0.04**	0.03		tSZ-N $\times$ LSST-WL
		0.036	**0.05**	0.04		tSZ-Y + DESI-BAO
Deviations from Λ	$\sigma_8(z=1-2)$	1.2%	**2%**	1%	7%	$\kappa\kappa$ + LSST-LSS
		1.2%	**2%**	1%		tSZ-N $\times$ LSST-WL
	H_0 (ΛCDM)	0.3	**0.4**	0.3	0.5	$TT/TE/EE + \kappa\kappa$
Galaxy evolution	η_{feedback}	2%	**3%**	2%	50-100%	kSZ + tSZ + DESI
	p_{nt}	6%	**8%**	5%	50-100%	kSZ + tSZ + DESI
Reionization	Δz	0.4	**0.6**	0.3	1.4	TT (kSZ)

Conclusions

- Present safe limit : $\Sigma m_\nu < 0.24$ eV from CMB+lensing
- Most stringent limit : $\Sigma m_\nu < 0.12$ eV
  - needs input from other measurements eg BAO
- Future : $\Sigma m_\nu < 60$ meV (2026)
  $\Sigma m_\nu < 15$ meV (~2035)

At this level of precision we can start to think about separating the masses