Neutrinoless $\beta\beta$ decay nuclear matrix elements

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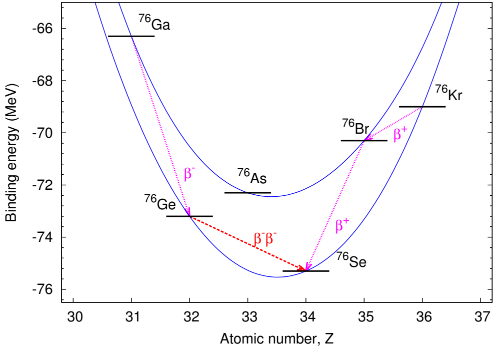
$\beta\beta$ decay

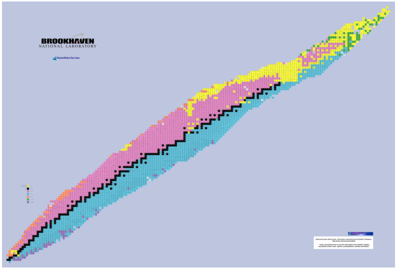
Second order process in the weak interaction

Only observable in nuclei where (much faster) β-decay is forbidden energetically due to nuclear pairing interaction

$$BE(A) = -a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + \frac{(A-2Z)^2}{A} + \begin{cases} -\delta_{pairing} & \text{N,Z even} \\ 0 & \text{A odd} \\ \delta_{pairing} & \text{N,Z odd} \end{cases}$$

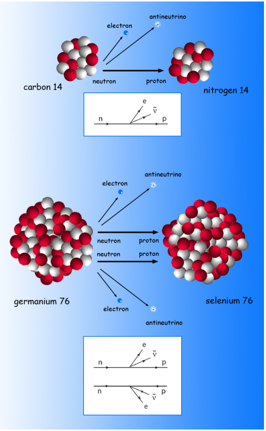
or where β-decay is very suppressed by ΔJ (total angular momentum) difference between mother and daughter nuclei





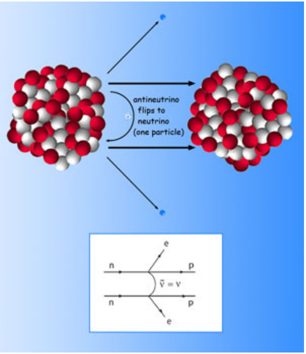
$\beta\beta$ decays: lepton-number conservation

Lepton number is conserved
in all physical processes
observed to date



β decay, $\beta\beta$ decay...

Uncharged massive particles
like Majorana neutrinos ($\nu = \bar{\nu}$)
theoretically allow lepton number violation
connected to matter dominance
over antimatter in universe



Neutrinoless $\beta\beta$ ($0\nu\beta\beta$) decay

Double-beta decay emitters

Only decay candidates with $Q_{\beta\beta} > 2$ MeV
experimentally interesting due to extremely long lifetimes

ECEC, ECβ^+ and $\beta^+\beta^+$ also more suppressed

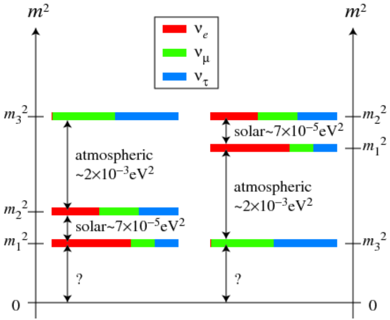
Transition	$T^{2\nu\beta\beta}$ (y)	$Q_{\beta\beta}$ (MeV)	Ab. (%)	
^{48}Ca $\rightarrow$ ^{48}Ti	$4.4 \cdot 10^{19}$	4.274	0.2	CANDLES
^{76}Ge $\rightarrow$ ^{76}Se	$1.7 \cdot 10^{21}$	2.039	8	GERDA, MAJORANA, LEGEND
^{82}Se $\rightarrow$ ^{82}Kr	$9.2 \cdot 10^{19}$	2.996	9	SuperNEMO
^{96}Zr $\rightarrow$ ^{96}Mo	$2.3 \cdot 10^{19}$	3.350	3	
^{100}Mo $\rightarrow$ ^{100}Ru	$7.1 \cdot 10^{18}$	3.034	10	AMoRE, CUPID
^{110}Pd $\rightarrow$ ^{110}Cd		2.013	12	
^{116}Cd $\rightarrow$ ^{116}Sn	$2.9 \cdot 10^{19}$	2.802	7	COBRA
^{124}Sn $\rightarrow$ ^{124}Te		2.288	6	
^{130}Te $\rightarrow$ ^{130}Xe	$6.9 \cdot 10^{20}$	2.530	34	CUORE, SNO+
^{136}Xe $\rightarrow$ ^{136}Ba	$2.2 \cdot 10^{21}$	2.462	9	nEXO, KamLAND-Zen, NEXT, DARWIN
^{150}Nd $\rightarrow$ ^{150}Sm	$8.2 \cdot 10^{18}$	3.667	6	

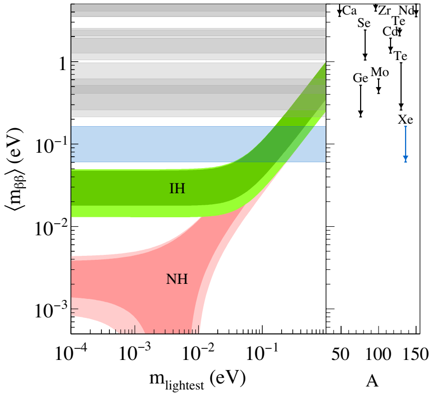
Worldwide running and planned experiments on different isotopes

Next generation $0\nu\beta\beta$: inverted hierarchy

Decay rate sensitive to neutrino masses, hierarchy
$m_{\beta\beta} = |\sum U_{ek}^2 m_k|$

$$T_{1/2}^{0\nu\beta\beta}\left(0^+ \to 0^+\right)^{-1} = G_{0\nu}\, g_A^4 \left|M^{0\nu\beta\beta}\right|^2 m_{\beta\beta}^2$$





KamLAND-Zen, PRL117 082503(2016)

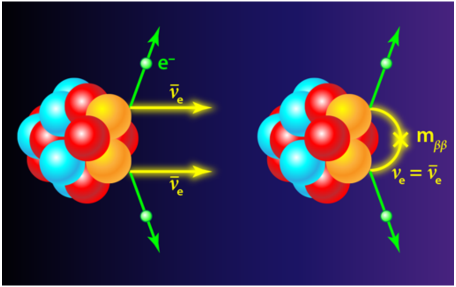
Matrix elements assess if next generation experiments fully explore "inverted hierarchy"

Calculating nuclear matrix elements

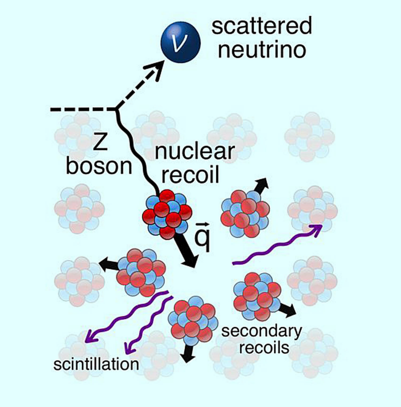
Nuclear matrix elements needed in low-energy new physics searches

$$\langle \text{Final} | \mathcal{L}_{\text{leptons}-\text{nucleons}} | \text{Initial} \rangle = \langle \text{Final} | \int dx\, j^{\mu}(x) J_{\mu}(x) | \text{Initial} \rangle$$

- Nuclear structure calculation of the initial and final states:
  Shell model, QRPA, IBM,
  Energy-density functional
  Ab initio many-body theory
  GFMC, Coupled-cluster, IM-SRG...
- Lepton-nucleus interaction:
  Hadronic current in nucleus:
  phenomenological,
  effective theory of QCD



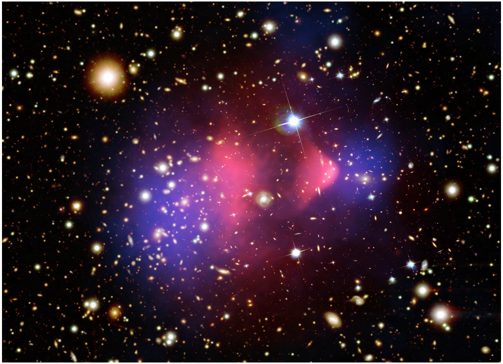
Coherent ν-nucleus scattering, dark matter detection



Coherent ν-nucleus scattering

Neutral current process, tiny cross-section

Neutrinos couple to neutrons, complements EM interactions



Dark matter scattering off nuclei

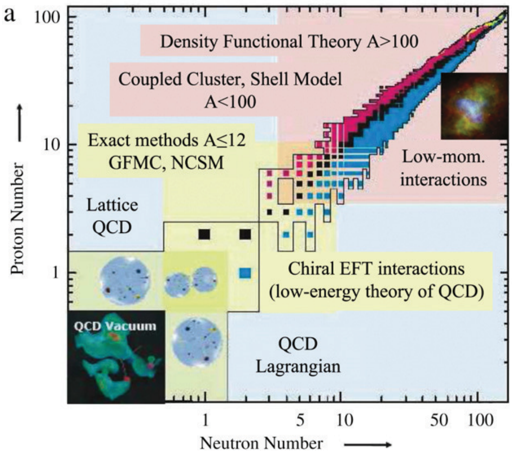
What is dark matter made of?

Outline

Outline

Nuclear Structure from First Principles

All nuclear structure calculations are, to some extent, phenomenological



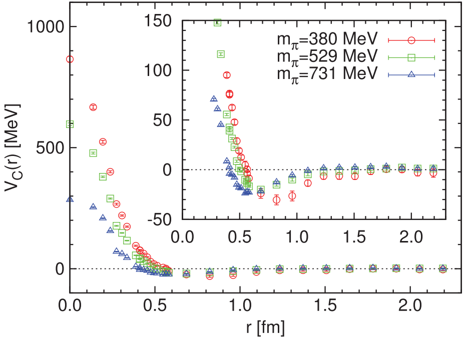
Relevant degrees of freedom: protons and neutrons
Many-body problem too hard in general, approximations are needed

Nuclear force at low (nuclear structure) energies: adjustments to reproduce finite nuclei needed

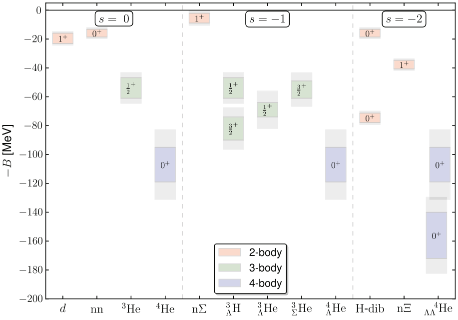
Can we connect nuclear structure calculations to quantum chromodynamics (QCD)?

Lattice QCD

QCD non-perturbative at low energies relevant for nuclear structure
Lattice QCD solves the QCD Lagrangian in discretized space-time Lattice



HALQCD Collaboration



NPLQCD Collaboration

Nuclear potentials, and lightest nuclei and hypernuclei solved
at non-physical pion mass $m_\pi \sim 400 - 800$ MeV, ongoing improvements

Theory for nuclear forces

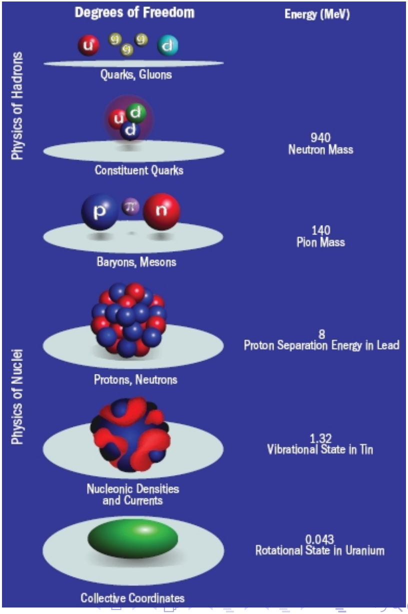
Difficult to find NN potential with consistent NNN forces and connected to QCD...

Use concept of separation of scales!

The energy scale relevant determines the degrees of freedom

For nuclear structure, typical energies of interest point to nucleons and pions (pions are particularly light mesons!)

Effective theory with nucleons and pions as degrees of freedom, with connection to QCD

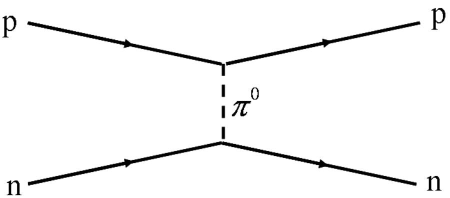


Nuclear interaction: Yukawa

The foundation for a theory of the nuclear interaction given by Yukawa (1935)

Predicted new particle (pion!) responsible for attractive nuclear force $r \sim 1$ fm $\Rightarrow m \sim 1/r$

Pion discovered in 1947!



Spin dependent

Isospin dependent

Non-central interaction,
Tensor component
$S_{12} = \{3\,(\sigma_1 \cdot \hat{r})\,(\sigma_2 \cdot \hat{r}) - \sigma_1 \cdot \sigma_2\}$

$$V(r) = -\frac{m_\pi^2\, g_A^2}{f_\pi^2}\,(\tau_1 \cdot \tau_2)\left[\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + S_{12}\left(1 + \frac{3}{mr} + \frac{3}{(mr)^2}\right)\right]\frac{e^{-mr}}{mr}$$

Effective theories

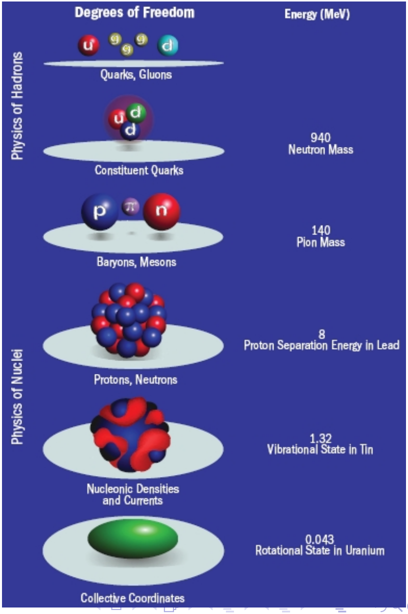
Effective theory:
approximation of the full theory
valid at relevant scales

Expansion in terms of small parameter:
typical scale / breakdown scale

In an effective theory
the physics resolved
at relevant energies is explicit

Terms at different orders given by
symmetries of the full theory

Unresolved physics
encoded in Low Energy Couplings

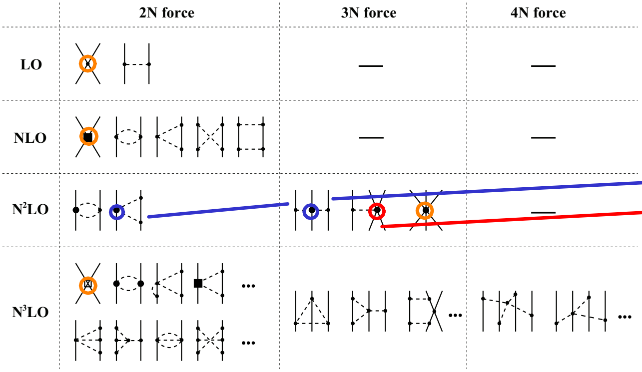


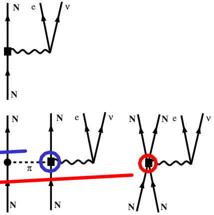
Chiral Effective Field Theory (EFT)

Chiral EFT: low energy approach to QCD, nuclear structure energies

Approximate chiral symmetry: pion exchanges, contact interactions

Systematic expansion: nuclear forces and electroweak currents





Park, Krebs, Baroni...

Short-range couplings
fitted to experiment once

Weinberg, van Kolck, Kaplan, Savage, Wise, Meißner, Epelbaum...

How does chiral EFT work?

The chiral EFT Lagrangian is an expansion, in different orders of pion-pion, pion-nucleon and nucleon-nucleon parts

$$\begin{aligned}\mathcal{L}_{\chi EFT} =& \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \dots \\ =& \mathcal{L}^{(0)}_{\pi\pi} + \mathcal{L}^{(0)}_{\pi N} + \mathcal{L}^{(0)}_{NN} + \mathcal{L}^{(1)}_{\pi\pi} + \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(1)}_{NN} + \dots\end{aligned}$$

For example:

$$\mathcal{L}^{(0)}_{\pi\pi} = \frac{f_\pi^2}{4} Tr\left[\partial^\mu U \partial_\mu U^\dagger + m_\pi^2 \left(U + U^\dagger\right)\right], \quad U = exp\left[i\frac{\boldsymbol{\pi}\cdot\boldsymbol{\tau}}{f_\pi}\right]$$

$$\mathcal{L}^{(0)}_{\pi N} = \bar{N}\left(i\gamma_\mu \mathcal{D}^\mu + \frac{g_A}{2}\gamma^\mu\gamma_5 u(\pi)_\mu - M\right) N$$

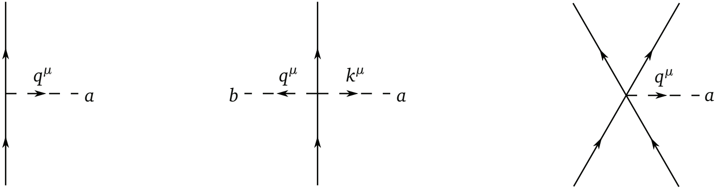
$$\dots$$

Evaluate these expressions to lowest orders in pion fields obtain Feynman diagrams for each vertex λ_i

The chiral order of a diagram is $\nu = 2(N - C) + 2L + \sum_i \lambda_i$ with N nucleons, C disconnected parts and L loops

Examples of chiral EFT diagrams

Feynman diagrams are read off from the Lagrangian:



(a) (b) (c)

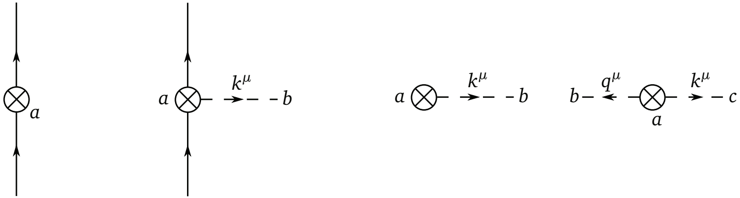
	(a)	(b)	(c)
LO	$-\frac{g_A}{2F_\pi}\gamma_5 \not{q}\tau^a$	$\frac{1}{4F_\pi^2}\epsilon^{abc}\not{q}\tau^c$	—
NLO	—	$\frac{2i}{F_\pi^2}\left(c_4\epsilon^{abc}\frac{\tau^c}{2}k_\mu q_\nu\sigma^{\mu\nu} - c_3 k_\mu q^\mu\delta^{ab} - 2c_1 m_\pi^2\delta_{ab}\right)$	$\frac{d_1}{F_\pi}\tau^a\boldsymbol{\sigma}_1\cdot\mathbf{q} + (1\leftrightarrow 2)$ $+\frac{d_2}{F_\pi}(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)^a\mathbf{q}\cdot(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)$

for the lowest order pion-nucleon diagrams from $\mathcal{L}_{\pi\pi}^{(0)} + \mathcal{L}_{\pi N}^{(0)} + \mathcal{L}_{NN}^{(1)}$

Chiral EFT currents

In addition, from the Chiral EFT Lagrangian we obtain the currents on how nucleons (and pions) couple to different probes of scalar, vector, axial... character

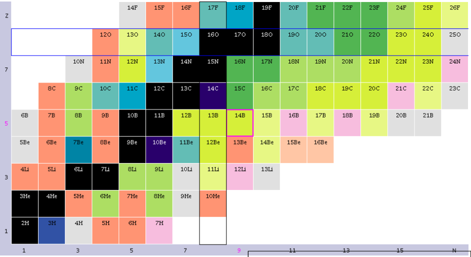
This is consistent with nuclear forces (same couplings)!



(a) (b) (c) (d)

	(a)	(b)	(c)	(d)
LO axial	$ig_A\gamma^\mu\gamma_5\frac{\tau^a}{2}$	$-\frac{i}{F_\pi}\epsilon^{abc}\gamma^\mu\frac{\tau^c}{2}$	$F_\pi k^\mu\delta^{ab}$	—
LO vector	$i\gamma^\mu\frac{\tau^a}{2}$	$-i\frac{g_A}{F_\pi}\epsilon^{abc}\gamma^\mu\gamma_5\frac{\tau^c}{2}$	—	$-\epsilon^{abc}k^\mu$
NLO axial	—	$\frac{2}{F_\pi}\left(-c_4\epsilon^{abc}\frac{\tau^c}{2}k_\nu\sigma^{\mu\nu}+c_3k^\mu\delta^{ab}\right)$	—	—
NLO vector	—	—	—	—

Calculations with NN, NN+3N forces

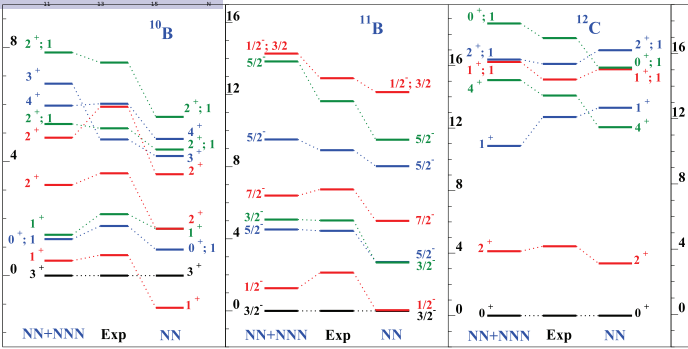


Ab initio many-body calculations feasible in light nuclei

No Core Shell Model
Green's Function Monte Carlo

NN forces
do not reproduce
binding energies
and spectra:
need 3N forces

Agrees with
experience
from Shell Model
in heavier systems

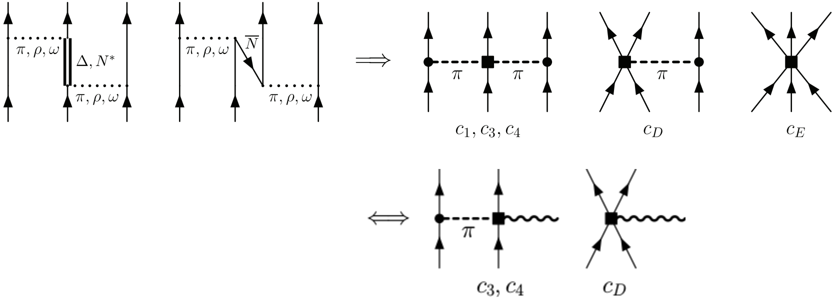


Three-nucleon forces, meson-exchange currents

Forces between 3 nucleons, external probe couplings to 2 nucleons known in nuclear theory for a long time

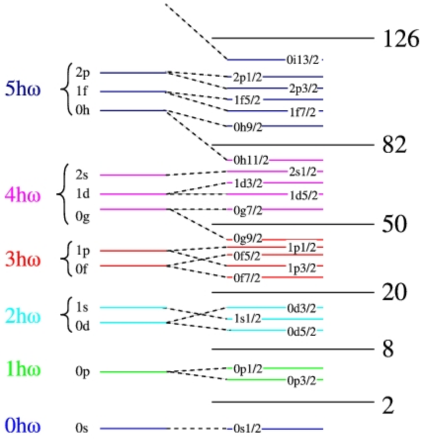
Fujita and Miyazawa PTP17 (1957), Towner Phys. Rep. 155 (1987)...

3N forces, 2b currents needed because of missing degrees of freedom (N-body forces appear in any effective theory)



The Δ isobar, with $M_\Delta = 1232$ MeV
relatively low excitation of the nucleon, $M_N = 939$ MeV

The (no core) shell model



The (no core) Shell Model

Many-body wave function
linear combination of
Slater Determinants
from single particle states in the basis
(3D harmonic oscillator)

$$|i\rangle = |n_i l_i j_i m_{j_i} m_{t_i}\rangle$$

$$|\phi_\alpha\rangle = a^+_{i1} a^+_{j2} \dots a^+_{kA} |0\rangle$$

$$|\Psi\rangle = \sum_\alpha c_\alpha |\phi_\alpha\rangle$$

$$H|\Psi\rangle = E|\Psi\rangle$$

$$\text{Dim} \sim \begin{pmatrix} (p+1)(p+2)_\nu \\ N \end{pmatrix} \begin{pmatrix} (p+1)(p+2)_\pi \\ Z \end{pmatrix}$$

Dimensions increase
combinatorially...

Ab initio calculations for oxygen

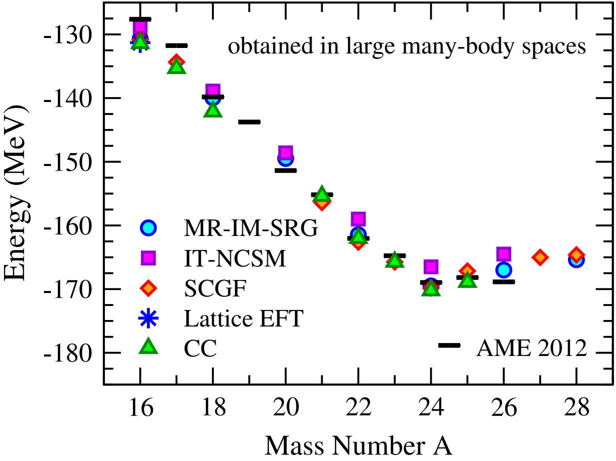
Ab initio calculations by different approaches,
treating explicitly all nucleons as degrees of freedom

No-core shell model
(Importance-truncated)

In-medium SRG

Self-consistent
Green's function

Coupled-cluster



Benchmark with the same initial Hamiltonian

The Coupled Cluster method is based on a reference state
and acting particle-hole excitation operators
impose no particle-hole excitations in the reference state

$$|\Psi\rangle = e^{-(T_1+T_2+T_3\cdots)}\,|\Phi\rangle$$

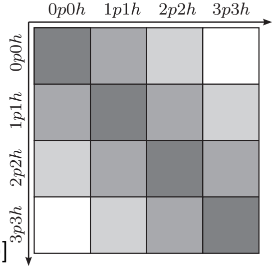
with $T_1 = \sum_{\alpha,\bar\alpha} t_\alpha^{\bar\alpha}\left\{a^\dagger_{\bar\alpha}, a_\alpha\right\}$, $T_2 = \sum_{\alpha\beta,\bar\alpha\bar\beta} t_{\alpha\beta}^{\bar\alpha\bar\beta}\left\{a^\dagger_{\bar\alpha} a^\dagger_{\bar\beta}, a_\alpha a_\beta\right\}$, $\cdots$

solve $\left\langle \Phi_\alpha^{\bar\alpha}\right| e^{\sum T_i} H e^{-\sum T_i}\,|\Phi\rangle = 0$, $\left\langle \Phi_{\alpha\beta}^{\bar\alpha\bar\beta}\right| e^{\sum T_i} H e^{-\sum T_i}\,|\Phi\rangle = 0$

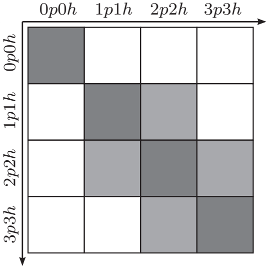
The In-medium similarity renormalization group method uses a similarity (unitary) transformation
to decouple reference state from particle-hole excitations

$$H = T + V \rightarrow H(s) = U(s) H U^\dagger(s)$$

$$\frac{dH}{ds} = [\eta(s), H(s)] \quad \text{with } \eta(s) = [G(s), H(s)]$$



$\langle i|\, H(0)\, |j\rangle$



$\langle i|\, H(\infty)\, |j\rangle$

Effective Shell Model interactions

Coupled Cluster:

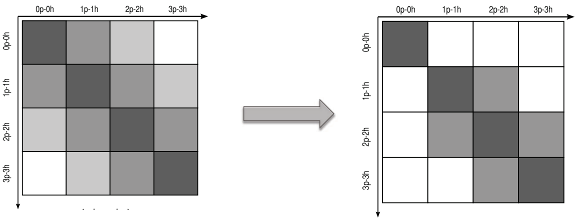
Solve coupled-cluster equations for core (reference state $|\Phi\rangle$), $A+1$ and $A+2$ systems

Project the coupled-cluster solution into valence space (Okubo-Lee-Suzuki transformation)

In-medium similarity renormalization group

decouple core from excitations

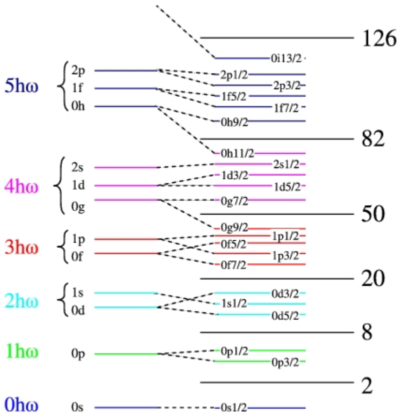
decouple A particles in valence space from rest



$\langle i|H|j\rangle$ $\qquad$ $\langle \mathrm{npnh}|H(\infty)|\Phi_{\mathrm{core}}\rangle = 0$

In addition to H_{eff}, these non-perturbative methods provide the core energy

Nuclear shell model (with core)



The Shell Model solves the many-body problem by direct diagonalization in a relatively small configuration space

The total space is separated into

- Outer orbits:
  orbits that are always empty
- Valence space: the space in which we explicitly solve the problem
- Inner core:
  orbits that are always filled

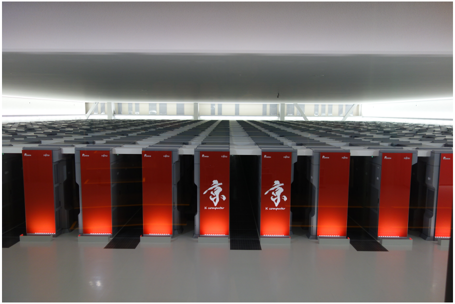
Diagonalization in valence space: $H\left|\Psi\right\rangle = E\left|\Psi\right\rangle \rightarrow H_{eff}\left|\Psi\right\rangle_{eff} = E\left|\Psi\right\rangle_{eff}$
where H_{eff} includes the effect of inner core and outer orbits

Nuclear shell model: computational power

Computational power critical for
size of nuclear shell model configuration space to be considered

1 major oscillator shell
$\sim 10^9$ Slater dets.

Caurier et al. RMP77 (2005)



$>$ 1 major oscillator shells
$\sim 10^{11}$ Slater dets.

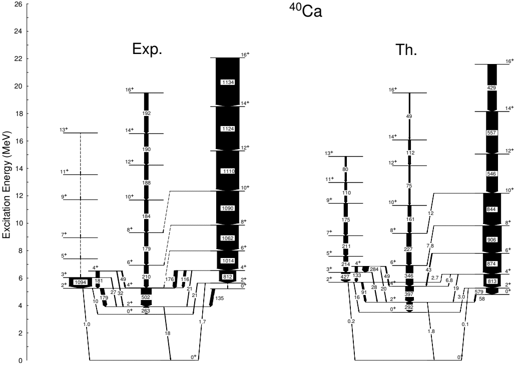
Caurier et al. RMP77 (2005)

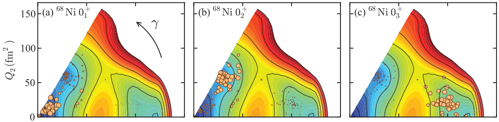
$\gtrsim 10^{24}$ Slater dets. with Monte Carlo SM

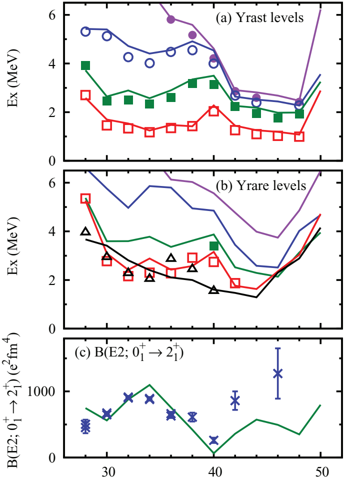
Otsuka, Shimizu, Tsunoda

Nuclear shell model: examples

The Shell Model is the method of choice for shell model nuclei: energies, deformation, electromagnetic and beta transition rates...

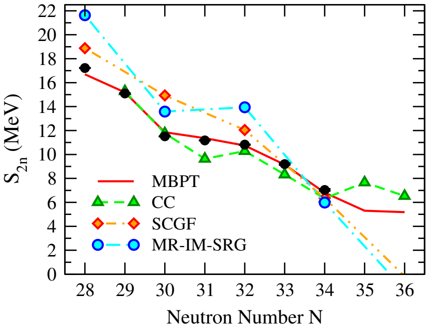


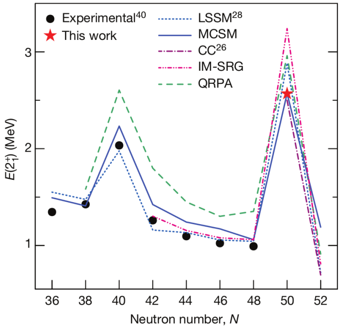




Shell evolution in medium-mass nuclei

Calculations with NN+3N forces predict doubly-magic nuclei ^{52}Ca, ^{54}Ca, ^{78}Ni
groundbreaking mass / 2^+ measurements at ISOLDE / RIBF





LETTER

doi:10.1038/nature12226

Masses of exotic calcium isotopes pin down nuclear forces

F. Wienholtz[1], D. Beck[2], K. Blaum[3], Ch. Borgmann[3], M. Breitenfeldt[4], R. B. Cakirli[3,5], S. George[1], F. Herfurth[2], J. D. Holt[6,7], M. Kowalska[8], S. Kreim[3,8], D. Lunney[9], V. Manea[9], J. Menéndez[6,7], D. Neidherr[2], M. Rosenbusch[1], L. Schweikhard[1], A. Schwenk[7,6], J. Simonis[6,7], J. Stanja[10], R. N. Wolf[1] & K. Zuber[10]

ARTICLE

https://doi.org/10.1038/s41586-019-1155-x

^{78}Ni revealed as a doubly magic stronghold against nuclear deformation

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Many-body methods for $0\nu\beta\beta$ decay

Different many-body methods are used in $0\nu\beta\beta$ decay

- Nuclear shell Model
  Madrid-Strasbourg, Michigan, Bucharest, Tokyo
  Relatively small valence spaces (one shell), all correlations included
- Quasiparticle random-phase approximation (QRPA) method
  Tübingen, Bratislava, Jyvaskyla, Chapel Hill, Prague...
  Several shells, only simple correlations included
- Interacting Boson Model
  Yale-Concepción
  Small space, important proton-neutron pairing correlations missing
- Energy Density Functional theory
  Madrid, Beijing
  >10 shells, important proton-neutron pairing correlations missing

Ab initio many-body methods:
No Core Shell Model, Green's Function Monte Carlo, Coupled Cluster...

Outline

Weak interactions in nuclei

β and $\beta\beta$ decay processes are driven by the Weak interaction

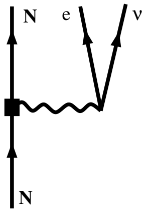
$$H_W = \frac{G_F}{\sqrt{2}} \left(j_{L\mu} J_L^{\mu\dagger} \right) + H.c.$$

$j_{L\mu}$ is the leptonic current (electron, neutrino): $j_{L\mu} = \overline{e}\gamma_\mu \left(1 - \gamma_5\right) \nu_{eL}$

The Lorentz structure is Vector $-$ Axial-Vector ($V - A$) current,
as indicated by the Standard Model of Particle Physics

For neutrinos,
interaction eigenstates are not mass eigenstates:
$\nu_{eL} = \sum_i U_{ei} \nu_{iL}$,
with U the **Pontecorvo**-MNS neutrino-mixing matrix

The treatment of electrons and neutrinos
is relatively easy because
they are elementary particles



Weak interactions: hadronic current

β and $\beta\beta$ decay processes are driven by the Weak interaction

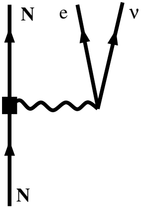
$$H_W = \frac{G_F}{\sqrt{2}} \left(j_{L\mu} J_L^{\mu\dagger} \right) + H.c.$$

$J_L^{\mu\dagger}$ is the hadronic current:
it is not so straightforward because the Standard Model predicts $J_L^{\mu\dagger}$ at the level of quarks and we need $J_L^{\mu\dagger}$ at the level of nucleons:

- Obtain $J_L^{\mu\dagger}$ phenomenologically
- Obtain $J_L^{\mu\dagger}$ using an effective theory: Chiral EFT!

In nuclei (non-relativistic), β decay is simply

$$\langle F | \sum_i g_V \, \tau_i^- + g_A \, \boldsymbol{\sigma}_i \tau_i^- \, | I \rangle$$



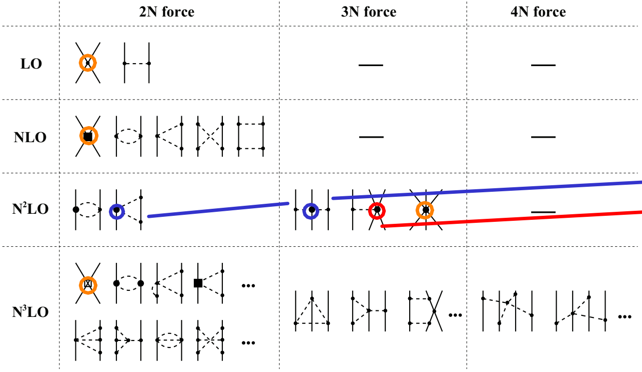
corresponding to Fermi and Gamow-Teller transitions,
corrections (forbidden transitions)
involve an expansion of the lepton wavefunctions

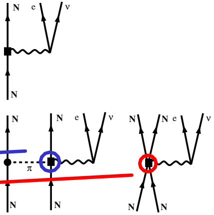
Chiral Effective Field Theory

Chiral EFT: low energy approach to QCD, nuclear structure energies

Approximate chiral symmetry: pion exchanges, contact interactions

Systematic expansion: nuclear forces and electroweak currents





Park, Krebs, Baroni...

Short-range couplings fitted to experiment once

Weinberg, van Kolck, Kaplan, Savage, Wise, Meißner, Epelbaum...

Remember, the weak interaction is V-A: vector-axial
Chiral EFT currents: calculate systematically at Q^0, Q^2 ... order

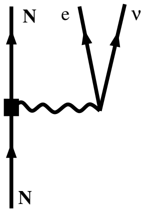
At order Q^0 standard Fermi, Gamow-Teller operators

$$J_i^0(p) = g_V \tau^-, \qquad \boldsymbol{J}_i(p) = g_A \boldsymbol{\sigma}$$

At oder Q^2 loop and pion-pole corrections

$$J_i^0(p) = g_V(p^2)\tau^-,$$
$$\boldsymbol{J}_i(p) = \left[g_A(p^2)\boldsymbol{\sigma} - g_P(p^2)\frac{(\boldsymbol{p}\cdot\boldsymbol{\sigma}_i)\boldsymbol{p}}{2m} + i\,(g_M + g_V)\,\frac{\boldsymbol{\sigma}_i \times \boldsymbol{p}}{2m} \right]\tau^-,$$

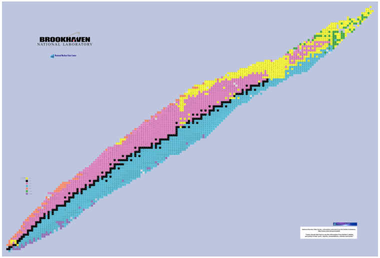
$$g_V(p^2) = g_V\,(1 - 2\frac{p^2}{\Lambda_V^2}), \qquad g_A(p^2) = g_A\,(1 - 2\frac{p^2}{\Lambda_A^2}),$$
$$g_P(p^2) = \frac{2g_{\pi pn}F_\pi}{m_\pi^2 + p^2} - 4\,g_A(p^2)\frac{m}{\Lambda_A^2}, \qquad g_M = \kappa_p - \kappa_n = 3.70,$$

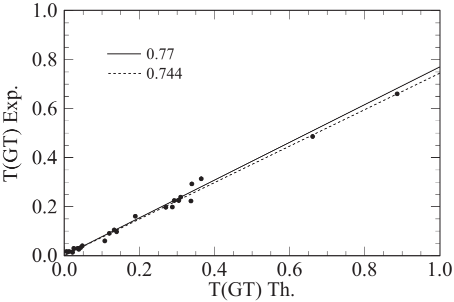


Order Q^2 corrections are not relevant for single-β decay, $2\nu\beta\beta$ decay, because in these processes $\mathbf{p} \sim 0$

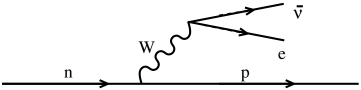
β decay: theory vs experiment

β decays (e^- capture) main decay model along nuclear chart
In general well described by nuclear structure theory: shell model...



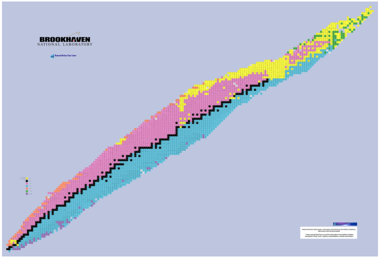


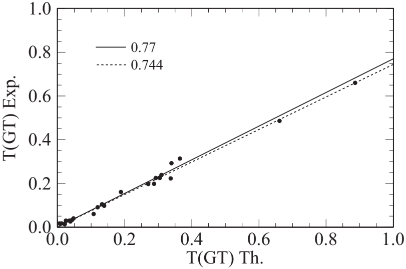
Martinez-Pinedo et al. PRC53 2602(1996)



$$\langle F | \sum_i [g_A \, \boldsymbol{\sigma}_i \tau_i^-] \, | I \rangle$$

β decays (e^- capture) main decay model along nuclear chart
In general well described by nuclear structure theory: shell model...





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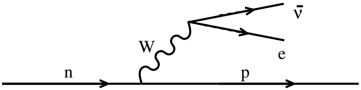


Gamow-Teller transitions:
theory needs $\sigma_i\tau$ "quenching"

$$\langle F | \sum_i [g_A\, \boldsymbol{\sigma}_i \tau_i^-]^{\text{eff}} | I \rangle\,, \quad [\boldsymbol{\sigma}_i\tau]^{\text{eff}} \approx 0.7\boldsymbol{\sigma}_i\tau$$

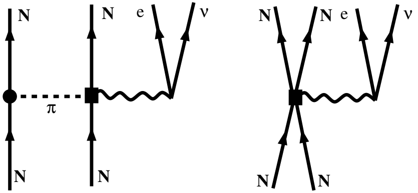
Deficient many-body approach,
or transition operator?

Two-body currents currents

At order Q^3 chiral EFT predicts contributions from two-body (2b) currents

Reflect interactions between nucleons

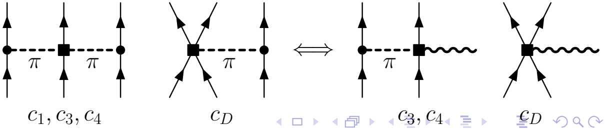
Long-range currents dominate



The expression for the leading Q^3 2b currents is

$$\mathbf{J}_{12}^3 = -\frac{g_A}{4F_\pi^2}\frac{1}{m_\pi^2+k^2}\left[2\left(c_4+\frac{1}{4m}\right)\mathbf{k}\times(\sigma_\times\times\mathbf{k})\tau_\times^3 + 4c_3\mathbf{k}\cdot\left(\sigma_1\tau_1^3+\sigma_2\tau_2^3\right)\mathbf{k} - \frac{i}{m}\mathbf{k}\cdot(\sigma_1-\sigma_2)\mathbf{q}\tau_\times^3\right]$$

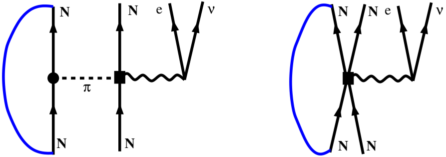
Long-range currents depend on c_3, c_4 couplings of nuclear forces



2b currents: normal-ordering

Approximate in medium-mass nuclei:
2b currents imply that the $\beta\beta$ decay operator is 4-body...
normal-ordered 1b part with respect to spin/isospin symmetric Fermi gas

Sum over one nucleon, direct and the exchange terms



$\Rightarrow \mathbf{J}^{\text{eff}}_{n,2b}$ normal-ordered 1b current

Corrections $\sim (n_{\text{valence}}/n_{\text{core}})$
in Fermi systems

The normal-ordered two-body currents modify GT operator

$$\mathbf{J}^{\text{eff}}_{n,2b} = \sum_{\boldsymbol{\sigma}_m}^{FG} \sum_{\tau_m}^{FG} \int \frac{p_m^2 dp_m}{(2\pi)^3} \, \mathbf{J}_{m,n,2b} \, (1 - P_{mn})$$

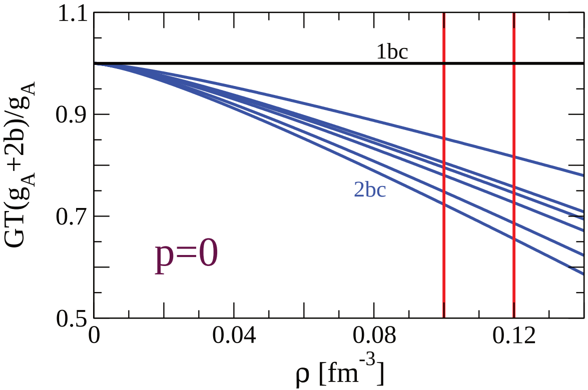
$$= -\frac{g_A \rho}{f_\pi^2} \tau_n^- \sigma_n \left[\frac{2}{3} c_3 \frac{\mathbf{p}^2}{4m_\pi^2 + \mathbf{p}^2} + I(\rho, P) \left(\frac{1}{3} (2c_4 - c_3) + \frac{1}{6m_N} \right) \right],$$

long-range p dependent   long-range p independent

2b currents at zero momentum-transfer

2b currents at $p = 0$: relevant for Gamow-Teller decays, $2\nu\beta\beta$ decay

$$\mathbf{J}^{\rm eff}_{n,2b} = -\frac{g_A \rho}{f_\pi^2} \tau_n^- \sigma_n \left[I(\rho, P) \left(\frac{1}{3}(2c_4 - c_3) + \frac{1}{6m_N} \right) \right],$$



Black horizontal line represents 1b current

Difference horizontal line and blue lines 2b effect

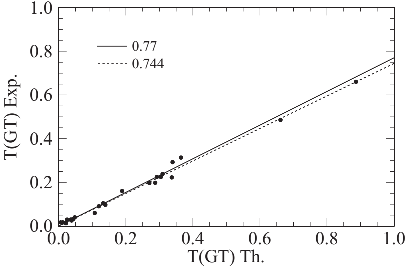
Nuclear density range $\rho = 0.10 \ldots 0.12$ fm^{-3}

Couplings c_3, c_4 taken from NN potentials

2b currents, in normal-ordered approximation predict g_A quenching

Gamow-Teller β decay with IM-SRG

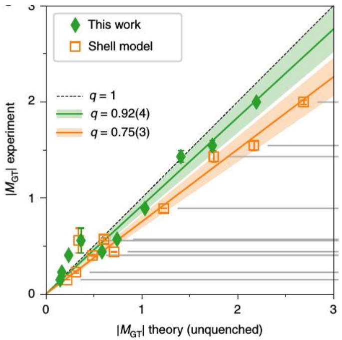
β decays (e^- capture) challenge for nuclear theory



Martinez-Pinedo et al. PRC53 2602(1996)

$$\langle F | \sum_i [g_A \, \boldsymbol{\sigma}_i \tau_i^-]^{\text{eff}} | I \rangle \,, \quad [\boldsymbol{\sigma}_i \tau]^{\text{eff}} \approx 0.7 \boldsymbol{\sigma}_i \tau$$

Phenomenological models need $\boldsymbol{\sigma}_i \tau$ “quenching”

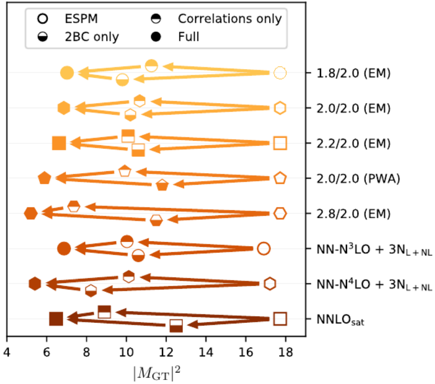


Gysbers et al. Nature Phys. 15 428 (2019)

Ab initio calculations including meson-exchange currents do not need any “quenching”

Origin of β-decay "quenching"

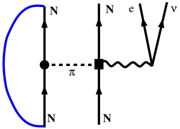
Complementary, similar impact of nuclear correlations and meson-exchange currents



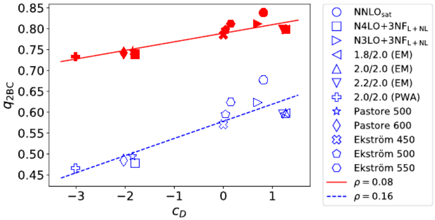
Gysbers et al. Nature Phys. 15 428 (2019)

2b currents modify GT operator

JM, Gazit, Schwenk PRL107 062501 (2011)



$$\boldsymbol{J}^{\rm eff}_{n,2b} \simeq -\frac{g_A \rho}{f_\pi^2} \tau_n^- \boldsymbol{\sigma}_n \times \left[I(\rho) \frac{2c_4 - c_3}{3} - \frac{c_D}{4 g_A \Lambda_\chi} - \frac{2 g_A \rho}{3 f_\pi^2} c_3 \frac{p^2}{m_\pi^2 + p^2} \right]$$



Outline

Two-neutrino $\beta\beta$ decay matrix elements

Two-neutrino double-beta decay matrix element, second order process

$$M^{2\nu\beta\beta} = \sum_k \frac{\langle 0_f^+ | \sum_n \tau_n^- + \sigma_n\tau_n^- | J_k^+ \rangle \langle J_k^+ | \sum_m \tau_m^- + \sigma_m\tau_m^- | 0_i^+ \rangle}{E_k - (M_i + M_f)/2}$$
$$= \sum_k \frac{\langle 0_f^+ | \sum_n \sigma_n\tau_n^- | J_k^+ \rangle \langle J_k^+ | \sum_m \sigma_m\tau_m^- | 0_i^+ \rangle}{E_k - (M_i + M_f)/2}$$
$$= \sum_k \frac{\langle 0_f^+ | \sum_n \sigma_n\tau_n^- | 1_k^+ \rangle \langle 1_k^+ | \sum_m \sigma_m\tau_m^- | 0_i^+ \rangle}{E_k - (M_i + M_f)/2}$$

- $\tau_n^- \tau_m^-$ transform two neutrons into two protons
- Only Gamow-Teller spin operator contributes:
  Fermi contribution vanishes due to isospin conservation:

$$\langle 0_f^+ | \sum_m \tau_m^- | J_k^+ \rangle = \langle 0_f^+ | T^- | J_k^+ \rangle \sim 0$$

- Neutrinos are emitted, do not appear in the transition operator

$\Rightarrow$ Only intermediate nucleus $|1_k^+\rangle$ states contribute

Two-neutrino $\beta\beta$ decay calculations

$$M^{2\nu\beta\beta} = \sum_k \frac{\langle 0_f^+ | \sum_n \sigma_n \tau_n^- | 1_k^+ \rangle \langle 1_k^+ | \sum_m \sigma_m \tau_m^- | 0_i^+ \rangle}{E_k - (M_i + M_f)/2}$$

Shell Model $2\nu\beta\beta$ decay calculations
in good agreement to experiment

GT quenching is needed

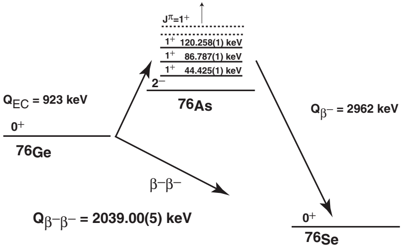
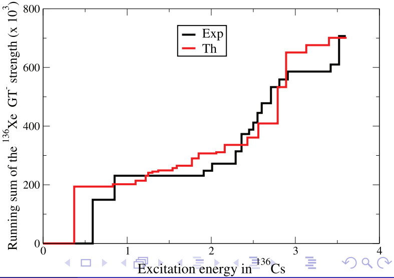


Table 2
The ISM predictions for the matrix element of several 2ν double beta decays (in MeV^{-1}). See text for the definitions of the valence spaces and interactions.

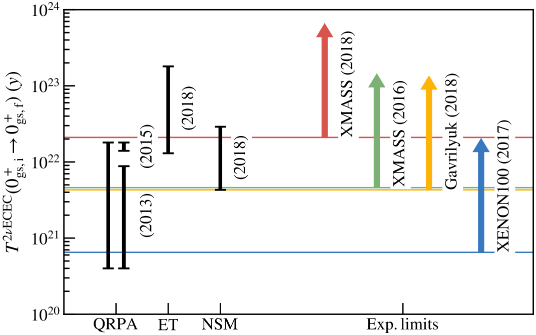
	$M^{2\nu}$(exp)	q	$M^{2\nu}$(th)	INT
$^{48}Ca \to {}^{48}Ti$	0.047 ± 0.003	0.74	0.047	kb3
$^{48}Ca \to {}^{48}Ti$	0.047 ± 0.003	0.74	0.048	kb3g
$^{48}Ca \to {}^{48}Ti$	0.047 ± 0.003	0.74	0.065	gxpf1
$^{76}Ge \to {}^{76}Se$	0.140 ± 0.005	0.60	0.116	gcn28:50
$^{76}Ge \to {}^{76}Se$	0.140 ± 0.005	0.60	0.120	jun45
$^{82}Se \to {}^{82}Kr$	0.098 ± 0.004	0.60	0.126	gcn28:50
$^{82}Se \to {}^{82}Kr$	0.098 ± 0.004	0.60	0.124	jun45
$^{128}Te \to {}^{128}Xe$	0.049 ± 0.006	0.57	0.059	gcn50:82
$^{130}Te \to {}^{130}Xe$	0.034 ± 0.003	0.57	0.043	gcn50:82
$^{136}Xe \to {}^{136}Ba$	0.019 ± 0.002	0.45	0.025	gcn50:82



Gamow-Teller Strengths
(each leg of the $\beta\beta$ decay) are well reproduced

Two-neutrino ECEC of ^{124}Xe

Predicted 2νECEC half-life:
shell model error bar largely dominated by "quenching" uncertainty



Suhonen
JPG 40 075102 (2013)

Pirinen, Suhonen
PRC 91, 054309 (2015)

Coello Pérez, JM, Schwenk
PLB 797 134885 (2019)

Shell model, QRPA and Effective theory (ET) predictions
suggest experimental detection close to XMASS 2018 limit

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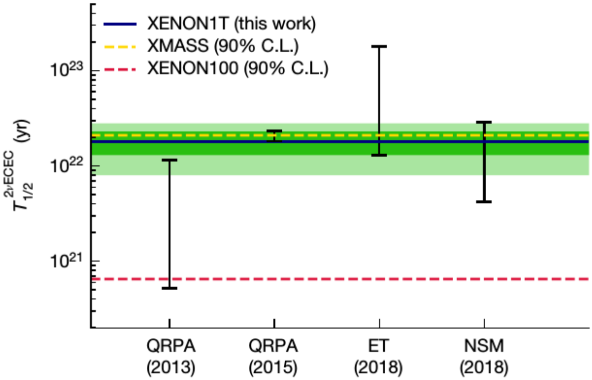

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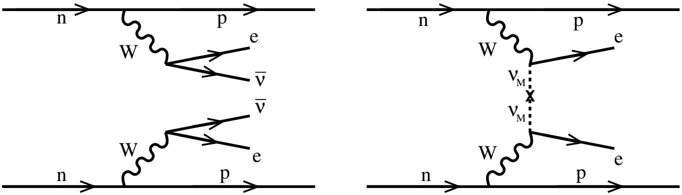
XENON1T
Nature 568 532 (2019)

Shell model, QRPA and Effective theory (ET) predictions
suggest experimental detection close to XMASS 2018 limit

$0\nu\beta\beta$ decay vs $2\nu\beta\beta$ decays

From the theoretical point of view, $0\nu\beta\beta$ and $2\nu\beta\beta$ decays are also different



- In $2\nu\beta\beta$ decay, the momentum transfer to the leptons is limited by $Q_{\beta\beta}$, while for $0\nu\beta\beta$ decay larger momentum transfers are permitted
- In $0\nu\beta\beta$ decay the Majorana neutrinos annihilate each other which is only possible if neutrinos have mass
- In $0\nu\beta\beta$ decay the Majorana neutrinos are part of the transition operator, via the so-called neutrino potential

$0\nu\beta\beta$: closure approximation

The neutrinos can carry large momentum, $p \sim 100$ MeV,
much larger than the excitation energies in the intermediate states $|N_a\rangle$

The closure approximation can be used (good to 90%)

$$\sum_a \frac{\langle N_f | J_L^{\mu\dagger}(\mathbf{x}) | N_a\rangle \langle N_a | J_L^{\rho\dagger}(\mathbf{y}) | N_i\rangle}{p + E_a - \frac{1}{2}(E_i + E_f)}$$

$$\simeq \frac{1}{p + \langle E\rangle - \frac{1}{2}(E_i + E_f))} \sum_a \langle N_f | J_L^{\mu\dagger}(\mathbf{x}) | N_a\rangle \langle N_a | J_L^{\rho\dagger}(\mathbf{y}) | N_i\rangle$$

$$= \frac{1}{p + \langle E\rangle - \frac{1}{2}(E_i + E_f))} \langle N_f | J_L^{\mu\dagger}(\mathbf{x}) J_L^{\rho\dagger}(\mathbf{y}) | N_i\rangle \,.$$

This simplifies the calculation, only initial and final states are needed

We still need the transition operator!

From currents to transition operator

The transition operator originates from the nuclear currents and the neutrinos

$$M^{0\nu\beta\beta}(m_j) = \langle 0_f^+ | \sum_{n,m} \tau_n^- \tau_m^- \frac{R}{g_A^2} \int \frac{d\mathbf{p}}{2\pi^2} e^{i\mathbf{p}\cdot(\mathbf{r}_n - \mathbf{r}_m)} \frac{\mathbf{J}_n \mathbf{J}_m(p^2)}{p\,(p+\mu)} |0_i^+\rangle$$

$$= \langle 0_f^+ | \sum_{n,m} \tau_n^- \tau_m^- \frac{R}{g_A^2} \int \frac{d\mathbf{p}}{2\pi^2} e^{i\mathbf{p}\cdot(\mathbf{r}_n - \mathbf{r}_m)} \frac{\Omega_{nm}(p^2)}{p\,(p+\mu)} |0_i^+\rangle$$

$$= \langle 0_f^+ | \sum_{n,m} \tau_n^- \tau_m^- \left(-H^F(r) + H^{GT}(r) \boldsymbol{\sigma}_n \boldsymbol{\sigma}_m - H^T(r) \mathbf{S}_{nm}^r \right) |0_i^+\rangle$$

The integral over **p** is performed and
$r = |\mathbf{r}_n - \mathbf{r}_m|$ distance between decaying neutrons, $H(r)$ neutrino potentials.

There is three spin structures contributing to $0\nu\beta\beta$ decay:
Fermi ($\mathbb{1}$), Gamow-Teller ($\sigma_1 \sigma_2$), Tensor (S_{12})

The Gamow-Teller term is dominant ($\sim 85\%$)

$0\nu\beta\beta$ decay nuclear matrix elements

$0\nu\beta\beta$ process needs massive Majorana neutrinos ($\nu = \bar{\nu}$)
$\Rightarrow$ detection would proof Majorana nature of neutrinos

$$\left(T_{1/2}^{0\nu\beta\beta}\left(0^{+} \rightarrow 0^{+}\right)\right)^{-1} = G_{01}\, g_A^4 \left|M^{0\nu\beta\beta}\right|^2 m_{\beta\beta}^2$$

G_{01} is the phase space factor
includes information of $Q_{\beta\beta}$, electrons...

g_A is the axial coupling (hadronic matrix element)

$M^{0\nu\beta\beta}$ is the nuclear matrix element

$m_{\beta\beta} = |\sum U_{ek}^2 m_k|$, represents physics beyond the Standard Model

Sensitive to absolute neutrino masses, compete with other determinations:
single-β decay ($\sqrt{\sum |U_{ek}|^2 m_k^2}$) and cosmology ($\sum m_k$)

Outline

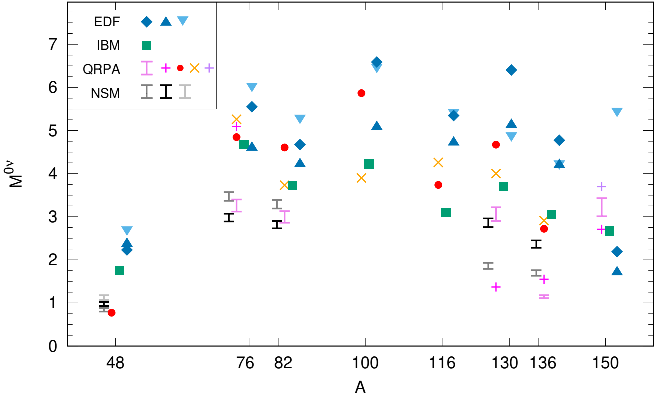
$0\nu\beta\beta$ decay nuclear matrix elements

Large difference in nuclear matrix element calculations: factor $\sim 2-3$

$$\langle 0_f^+ | \sum_{n,m} \tau_n^- \tau_m^- \sum_X H^X(r)\,\Omega^X\, | 0_i^+ \rangle$$

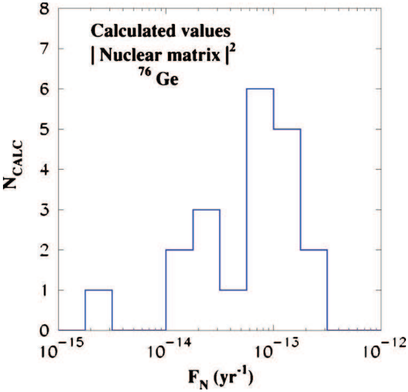
Ω^X = Fermi ($\mathbb{1}$), GT ($\sigma_n \sigma_m$), Tensor

$H(r)$ = neutrino potential



Engel, JM, Rep. Prog. Phys. 80 046301 (2017), updated

Spread about factor two – three in nuclear matrix element calculations



But this means a big improvement!

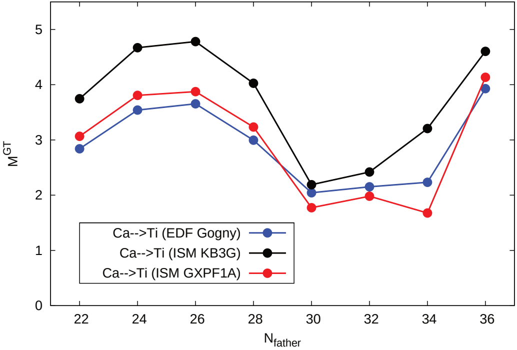
The uncertainty in the calculated nuclear matrix elements for neutrinoless double beta decay will constitute the principal obstacle to answering some basic questions about neutrinos. The essential problem is that the correct theory of nuclei

Bahcall, Murayama, Peña-Garay
PRD70 033012 (2004)

$0\nu\beta\beta$ decay without correlations

Non-realistic spherical (uncorrelated) mother and daughter nuclei:

- Shell model (SM): zero seniority, neutron and proton $J = 0$ pairs
- Energy density functional (EDF): only spherical contributions



In contrast to full (correlated) calculation SM and EDF NMEs agree!

NME scale set by pairing interaction

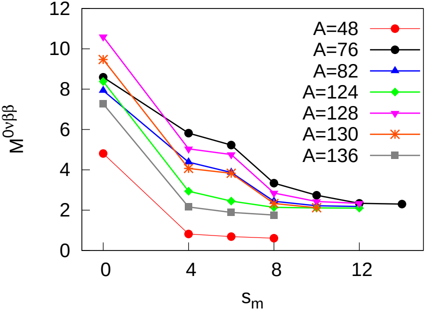
JM, Rodríguez, Martínez-Pinedo, Poves PRC90 024311(2014)

NME follows generalized seniority model:

$$M_{GT}^{0\nu\beta\beta} \simeq \alpha_\pi \alpha_\nu \sqrt{N_\pi + 1}\sqrt{\Omega_\pi - N_\pi}\sqrt{N_\nu}\sqrt{\Omega_\nu - N_\nu + 1},$$ Barea, Iachello PRC79 044301(2009)

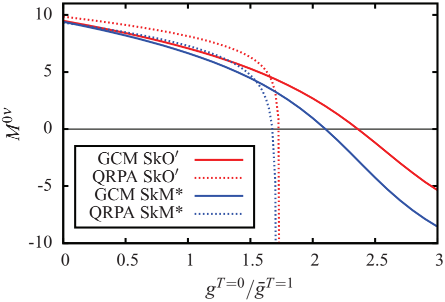
$0\nu\beta\beta$ decay favoured by proton-proton, neutron-neutron pairing, but it is disfavored by proton-neutron pairing

Ideal case: superfluid nuclei reduced with high-seniorities



Caurier et al. PRL100 052503 (2008)

Addition of isoscalar pairing reduces matrix element value



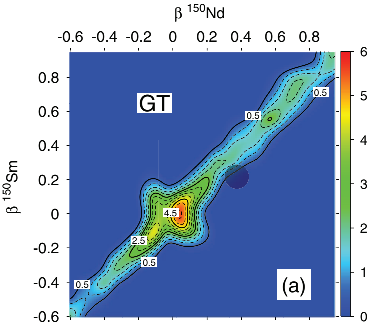
Hinohara, Engel PRC90 031301 (2014)

Related to approximate $SU(4)$ symmetry of the $\sum H(r)\boldsymbol{\sigma}_i\boldsymbol{\sigma}_j\tau_i\tau_j$ operator

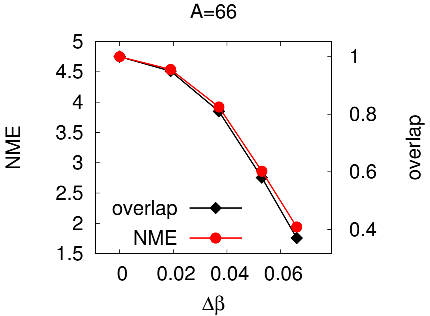
Deformation and $0\nu\beta\beta$ decay

$0\nu\beta\beta$ decay is disfavoured by quadrupole correlations

$0\nu\beta\beta$ decay very suppressed when nuclei have different structure



Rodríguez, Martínez-Pinedo
PRL105 252503 (2010)



JM, Caurier, Nowacki, Poves
JPCS267 012058 (2011)

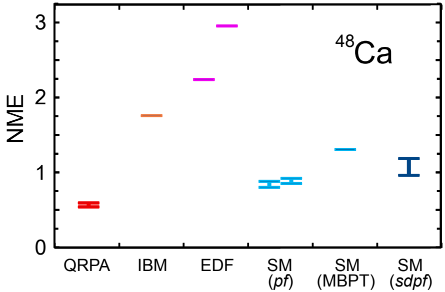
Suppression also observed with QRPA Fang et al. PRC83 034320 (2011)

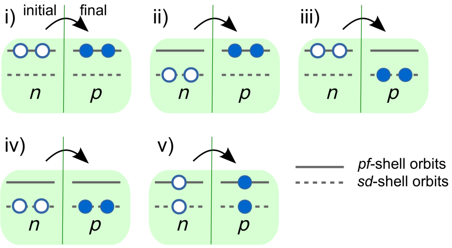
Shell model configuration space: two shells

^{48}Ca extended configuration space from *pf* to *sdpf*, 4 to 7 orbitals dimension 10^5 to 10^9

^{48}Ca 0_2^+ state lowered by 1.3 MeV nuclear matrix elements enhanced only moderately 30%

Iwata et al. PRL116 112502 (2016)





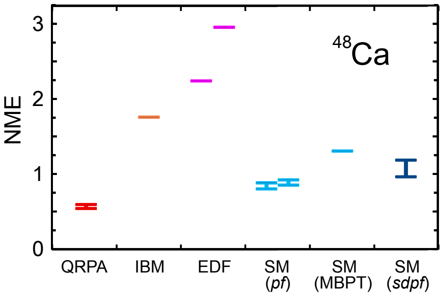
Terms dominated by pairing 2 particle – 2 hole excitations enhance the $\beta\beta$ matrix element

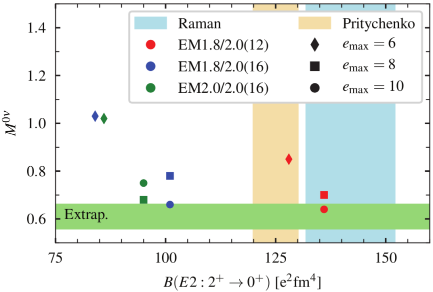
Terms dominated by 1 particle – 1 hole excitations suppress the $\beta\beta$ matrix element

Ab initio IMSRG nuclear matrix element for ^{48}Ca

^{48}Ca extended configuration space
from *pf* to *sdpf*, 4 to 7 orbitals
dimension 10^5 to 10^9
^{48}Ca 0_2^+ state lowered by 1.3 MeV
nuclear matrix elements
enhanced only moderately 30%

Iwata et al. PRL116 112502 (2016)





IM-SRG ^{48}Ca NME

Multi-reference calculation:
collective reference state

Generator coordinate method:
deformation, isoscalar pairing

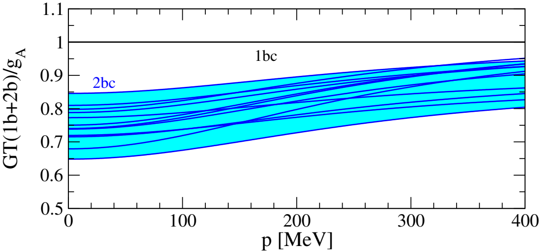
Yao et al. arXiv:1908.05424

2b currents: transferred-momentum dependence

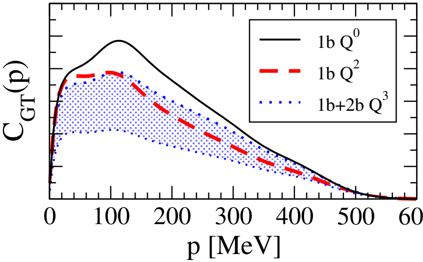
The $\sigma\tau^-$ term depends on transferred momentum *p*:

$$-\frac{g_A \rho}{f_\pi^2} \tau_n^- \sigma_n \frac{2}{3} c_3 \frac{\mathbf{p}^2}{4m_\pi^2 + \mathbf{p}^2}$$

Quenching reduced at $p > 0$
because chiral coupling $c_3 < 0$



Momentum transferred
dependence relevant for $0\nu\beta\beta$
decay where $p \sim m_\pi$



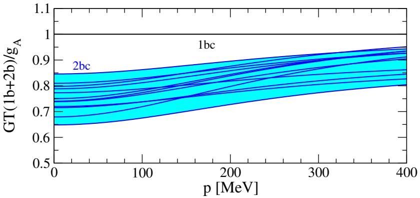
Typical momentum transfers set by
typical distance between decaying nucleons

$$M^{0\nu\beta\beta} = \int_0^\infty C(p)dp$$

2b currents in $\beta\beta$ decay

In $0\nu\beta\beta$ decay, two weak currents lead to four-body operator
when including the product of two 2b currents: computational challenge

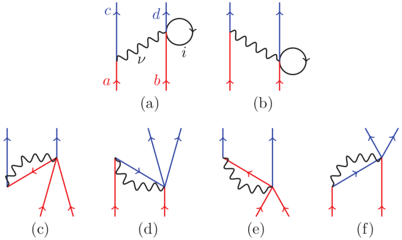
Approximate 2b current
as effective 1b current



Quenching reduced to $\sim 20\%$
at $p \sim m_\pi$ for $0\nu\beta\beta$ decay

JM et al. PRL107 062501(2011)

Approximate 4b operator
as effective 3b operator



Estimated effect $\sim 10\%$

Wang et al. PRC98 031301 (2018)

$0\nu\beta\beta$ decay light- and heavy-particle exchange

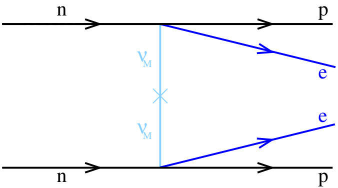
Neutrinoless $\beta\beta$ decay mediated by light or heavy particles

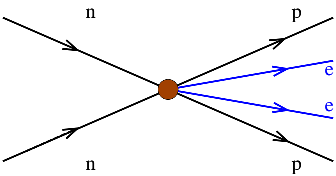
Barea, Horoi, JM, Šimkovic, Suhonen...

$$M^{0\nu\beta\beta} = \langle 0_f^+ | \sum_{n,m} \tau_n^- \tau_m^- \sum_X H^X(r)\,\Omega^X\, |0_i^+\rangle$$

$$H^X(r) = \frac{2}{\pi}\frac{R}{g_A^2}\int_0^\infty f^X(pr)\frac{h^X(p^2)}{\left(\sqrt{p^2+m_\nu^2}\right)\left(\sqrt{p^2+m_\nu^2}+\langle E^m\rangle-\frac{1}{2}(E_i-E_f)\right)}p^2\,dp$$

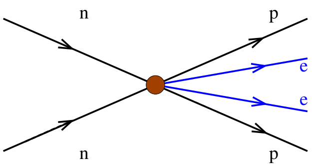
Same contributions in both channels
but in heavy-neutrino exchange the standard term becomes shorter range
$p \sim 100-200$ MeV, set by typical distance between decaying nucleons

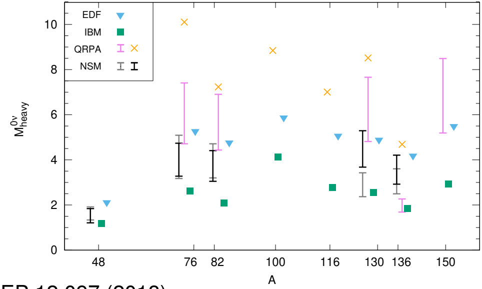




$0\nu\beta\beta$ mediated by BSM heavy particles

Extensions of the Stantard Model can also trigger $0\nu\beta\beta$ decay
typically mediated by exchange of heavy particle (heavy ν, M_R...)





Effective field theory Cirigliano et al JHEP 12 097 (2018)
dimension-7 ($\sim 1/\Lambda^3$), dimension-9 ($\sim 1/\Lambda^5$) operators can lead to $0\nu\beta\beta$

$$T_{1/2}^{-1} = G_{01} \left(g_A^2 \, M^{0\nu} + g_\nu^{\mathrm{NN}} \, m_\pi^2 \, M^{0\nu}_{\mathrm{cont}} \right)^2 \frac{m_{\beta\beta}^2}{m_e^2} + \frac{m_N^2}{m_e^2} \tilde{G} \, \tilde{g}^4 \, \tilde{M}^2 \left(\frac{v}{\Lambda} \right)^6 + \frac{m_N^4}{m_e^2 \, v^2} \tilde{G}' \, \tilde{g}'^4 \, \tilde{M}'^2 \left(\frac{v}{\Lambda'} \right)^{10} + \cdots ,$$

Phase-space, hadronic/nuclear matrix elements, known or calculated
Present experiments constrain dim-7 / dim-9 operators $\Lambda \gtrsim 250$ / 5 TeV

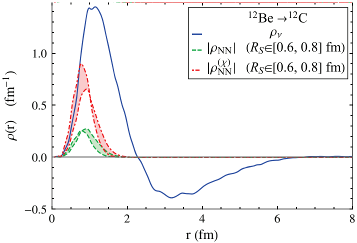
Light neutrino exchange: new contact operator

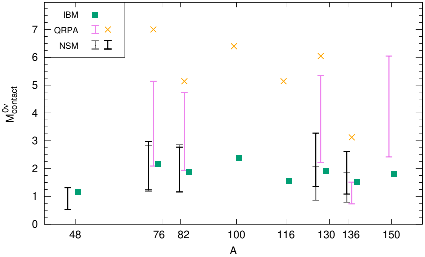
Contact operator suggested to contribute to light-neutrino exchange

Cirigliano et al. PRL120 202001(2018)

$$T_{1/2}^{-1} = G_{01} \left(g_A^2 \, M^{0\nu} + g_\nu^{\rm NN} \, m_\pi^2 \, M_{\rm cont}^{0\nu} \right)^2 \frac{m_{\beta\beta}^2}{m_e^2}$$

Unknown value of the hadronic coupling $g_\nu^{\rm NN}$!
to be determined experimentally or Lattice QCD calculations ($\sim g_A$)





Short-range operator similar disagreement than standard NME
larger error bars because uncertainty in short-range dynamics

Thank you very much for your attention!

Feel free to ask any questions!
I will be around until the end of the school