# Introduction to Neutrino

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## Introduction

## Are neutrino special particles?

Yes, but we need to specify in what sense

$SU_L(2)$ doublet

$$\psi_{lL}(x) = \left( \begin{array}{c} \nu'_{lL}(x) \\ l'_L(x) \end{array} \right), \quad l = e, \mu, \tau.$$

Before spontaneous symmetry breaking $\nu'_{lL}$ and $l'_L$ are components of one field (like $p$ and $n$ are different projections of the isospin of nucleon)

However, neutrino are the only fundamental fermions with equal to zero electric charges

Two major consequences which make neutrinos special

- Neutrinos have no direct electromagnetic interaction. At relatively small energies neutrinos have only weak interaction, characterized by $G_F \simeq 10^{-5}\ \frac{1}{m_p^2}$. As a result, only in neutrino experiments central region of the sun, where solar energy is generated, mechanism of the SN explosion etc. can be probed
- Neutrinos are the only fundamental fermions which can be Dirac or Majorana particles. We have in mind neutrinos with definite masses (massless Majorana and massless Dirac neutrinos are equivalent). Small Majorana neutrino masses can be generated only by a beyond the Standard Model mechanism. Thus, possibility to be Majorana particles open for neutrinos the way to probe a new physics

I do not think that different speculations (like famous $v > c$ neutrinos etc) will work. I hope that indications in favor of sterile neutrinos will be not confirmed

Two-component neutrino theory (1957) is the first (basically correct) neutrino theory

Was confirmed by the experiment (1958)

This theory allowed to create current$\times$current $V-A$ theory and later the Standard Model

Was proposed by Weil in 1929 (before neutrino 1930)

Historic remark. In 1928 Dirac proposed the equation for a relativistic particle with spin 1/2 and mass $m$

$$i\gamma^{\alpha}\partial_{\alpha}\ \psi(x) - m\ \psi(x) = 0, \quad \alpha = 0, 1, 2, 3$$

$\psi_{\sigma}(x)$ is four-component spinor. Why for spin 1/2 particle $\sigma$ takes four values?

In order to ensure Lorenz invariance, space coordinates $x^1, x^2, x^3$ and time coordinate $x^0$ must enter symmetrically. Four matrices $\gamma^{\alpha}$ are needed. In order to ensure that for a particle with momentum $\vec{p}$ and energy $E$ the relation $E_p^2 = p^2 + m^2$ is satisfied

$$\gamma^{\alpha}\gamma^{\beta} + \gamma^{\beta}\gamma^{\alpha} = 2g^{\alpha\beta}$$

These relations can be fulfilled if $\gamma^{\alpha}$ are $4 \times 4$ matrices

Four solutions for a particle with momentum $\vec{p}$: solutions with positive energy $E_p = \sqrt{m^2 + p^2}$ and two projections of the spin, and negative energy $E_p = -\sqrt{m^2 + p^2}$ and two projections of the spin. The problem of solutions with negative energy was solved by Dirac by the redefinition of the vacuum state. The quantum field $\psi(x)$ is the field of particles and antiparticles with the same mass $m$. Positron was predicted (Dirac) and discovered in 1932 (Anderson)

In 1929 H. Weil proposed a two-component equation for a relativistic spin 1/2 particle He introduced left-handed (right-handed) two-component spinors

$$\psi_{L,R}(x) = \frac{1}{2}(1 \mp \gamma_5)\ \psi(x), \quad \psi(x) = \psi_L(x) + \psi_R(x)$$

From the Dirac equation we have coupled equations

$$i\gamma^{\alpha}\partial_{\alpha}\ \psi_L(x) - m\ \psi_R(x) = 0, \quad i\gamma^{\alpha}\partial_{\alpha}\ \psi_R(x) - m\ \psi_L(x) = 0.$$

For $m = 0$ two decoupled two-component Weil equations

$$i\gamma^{\alpha}\partial_{\alpha}\psi_{L,R}(x) = 0$$

## Physical meaning of $\psi_{L,R}(x)$ for $m = 0$

$$\gamma \cdot p \ u^r \ (p) = 0, \quad (\gamma_5 \gamma^0 \vec{\gamma} \vec{n}) \ u^r(p) = r \ u^r(p)$$

$r$ is the helicity; $\gamma_5 \ u^r(p) = r \ u^r(p)$

We have $\frac{1}{2}(1 \mp \gamma_5) \ u^r(p) = \frac{1}{2}(1 \mp r) \ u^r(p)$

Projection operators: $r = \mp 1$ for particle and $r = \pm 1$ for antiparticle

Notice, that the requirement $m = 0$ is connected with the fact that we assumed that $\psi_R$ and $\psi_L$ are independent

If $\psi_L$ and $\psi_R$ are $L$ and $R$ components of Majorana fields

$$\chi_1 = \psi_L + (\psi_L)^c, \quad \chi_2 = \psi_R + (\psi_R)^c$$

$$\chi_k = \chi_k^c = C(\bar{\chi}_k)^T, \quad C\gamma_\alpha^T C^{-1} = -\gamma_\alpha \quad (k = 1, 2)$$

we obtained two-component equations for relativistic $m \neq 0$ Majorana particles

$$i\gamma^\alpha \partial_\alpha \ \chi_k(x) - m \ \chi_k(x) = 0$$

The Weyl equations are not invariant under the space inversion (do not conserve parity)

$$\psi'(x') = \eta\ \gamma^0 \psi(x), \quad \psi'_{R,L}(x') = \eta\ \gamma^0 \psi_{L,R}(x), \ \ x' = (x^0, -\vec{x})$$

The state with $r = -1$ ($r = 1$) in R-system, is the state with $r = 1$ ($r = -1$) in L-system

Pauli in the book "Quantum Mechanics" wrote "...because the equation for $\psi_L(x)$ ($\psi_R(x)$) is not invariant under space reflection it is not applicable to the physical reality".

When in 1957 it was discovered that in the weak interaction the parity is not conserved, Landau , Lee and Yang and Salam proposed the theory of the two-component Weyl neutrino

At that time from the measurement of tritium $\beta$-spectrum was found $m_\nu < 200\ \mathrm{eV} \simeq 4 \cdot 10^{-4}\ m_e$. It was natural to suggest that neutrino is a massless particle

Impressive confirmation of the two-component neutrino theory was obtained in the classical Goldhaber et al. experiment (1958) on the measurement of the neutrino helicity. Neutrino helicity was determined from the measurement of the circular polarization of $\gamma$'s produced in the chain of reactions

$$e^{-} +^{152} \mathrm{Eu} \to \nu +^{152} \mathrm{Sm}^{*}, \quad {}^{152}\mathrm{Sm}^{*} \to^{152} \mathrm{Sm} + \gamma$$

Goldhaber et al. concluded "... our result is compatible with 100% negative helicity of neutrino."(neutrino field is $\nu_L$)

The two-component neutrino theory played extremely important role in creation of the $V - A$ current$\times$current weak interaction theoryand Standard Model

Feynman and Gell-Mann, Marshak and Sudarshan $V - A$ theory was based on Fermi effective Hamiltonian approach (Hamiltonian is built from fields of particles which participate in a process)

$$n \to p + e^- + \bar{\nu}$$

Generalized effective Hamiltonian

$$\mathcal{H}_I^\beta = \sum_{i=S,V,\dots} G_i \bar{p} O^i n \; \bar{e} O_i \nu + \mathrm{h.c.}, O^i = 1, \gamma^\alpha, \sigma^{\alpha\beta}, \gamma^\alpha\gamma_5, \gamma_5$$

Fermi assumed vector interaction. ( $\bar{p}\gamma^\alpha n$ is current (analog to the electromagnetic current $\bar{p}\gamma^\alpha p$)

Two-component neutrino theory : $\nu \to \nu_L$

F-G, M-S: Into the Hamiltonian of the $\beta$-decay enter left-handed components of all fields: $p \to p_L, n \to n_L, e \to e_L$

Only one term survive ($S$, $T$, $P$ terms are equal to zero, $A = -V$)

$$\bar{p}_L O^i n_L \to \bar{p}_L \gamma^\alpha n_L, \quad \bar{e}_L O_i \nu_L \to \bar{e}_L \gamma_\alpha \nu_L$$

We come to unique Hamiltonian

$$\mathcal{H}_I^\beta = \frac{G_F}{\sqrt{2}}\, 4\, \bar{p}_L \gamma^\alpha n_L \;\; \bar{e}_L \gamma_\alpha \nu_L + \mathrm{h.c.}$$

which has a form of product of two currents

With an idea of left-handed components (started with neutrino) the notion of charged weak current appeared

$$j^{\alpha} = 2\left(\bar{p}_L \gamma^{\alpha} n_L + \bar{\nu}_{eL} \gamma^{\alpha} e_L + \bar{\nu}_{\mu L} \gamma^{\alpha} \mu_L\right)$$

The effective Hamiltonian of the weak interaction

$$\mathcal{H}_I = \frac{G_F}{\sqrt{2}}\, j^{\alpha}\, j^{+}_{\alpha}$$

was in a perfect agreement with existed data
Currents naturally appear in theories based on local gauge invariance. In order to come to the current $\bar{\nu}_{lL}\gamma^{\alpha} l_L$ we need to assume that

$$\psi_{lL} = \begin{pmatrix} \nu'_{lL} \\ l'_L \end{pmatrix}, \quad l = e, \mu, \tau$$

is $SU_L(2)$ left-handed doublet ($\nu'_{lL}$ and $l'_L$ are two-component left-handed Weyl fields)

From the local $SU_L(2)$ invariance it follows that vector bosons must exist and that the interaction Lagrangian has the form

$$\mathcal{L}_I = (-\frac{g}{2\sqrt{2}} j_\alpha^{CC} W^\alpha + \mathrm{h.c.}) - g j_\alpha^3 A^{3\alpha}.$$

$$j_\alpha^{CC} = 2 \sum_{l=e,\mu,\tau} \bar{\nu}'_{lL} \gamma_\alpha l'_L$$

is the charged current. (We consider leptons; similar expressions for quarks). $W^\alpha$ is the field of $W^\pm$ bosons and $A^{3\alpha}$ is the field of neutral bosons

The Standard Model is unified theory of weak and electromagnetic interactions. It is based on the local $SU_L(2) \times U_Y(1)$ group, $U_Y(1)$ is the group of the hypercharge $Y$ determined by the Gell-Mann-Nishijima relation

$$Q = I_3 + \frac{1}{2} Y$$

From unification requirement it follows that Neutral Current and neutral $Z^0$ boson must exist. The interaction Lagrangian induced by $SU_L(2)\times U_Y(1)$ local gauge invariance consist of CC, NC and EM parts

$$\mathcal{L}_I = (-\frac{g}{2\sqrt{2}} j^{CC}_{\alpha} W^{\alpha} + \mathrm{h.c.}) - \frac{g}{2\cos\theta_W} j^{NC}_{\alpha} Z^{\alpha} - e j^{EM}_{\alpha} A^{\alpha}$$

$$j^{NC}_{\alpha} = \sum_{l=e,\mu,\tau} (\bar{\nu}'_L \nu'_L - \bar{l}'_L l'_L) - 2\sin^2\theta_W j^{EM}_{\alpha}, \quad j^{EM}_{\alpha} = (-) \sum_{l=e,\mu,\tau} \bar{l}'_L l'_L$$

and $\sin\theta_W = \frac{e}{g}$. NC was predicted by SM

In the $SU_L(2)\times U_Y(1)$ invariant Lagrangian there no mass terms. SM is based on the spontaneous breaking of the symmetry. In the SM Lagrangian the scalar Higgs field, $SU_L(2)$ doublet,

$$\phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix}$$

enter. Higgs field is a special. Due to Higgs potential

$$V(\phi^{\dagger}\phi) = -\mu^2 \phi^{\dagger}\phi + \lambda(\phi^{\dagger}\phi)^2, \quad \mu^2 > 0, \quad \lambda > 0$$

Vacuum values of the Higgs field are different from zero and degenerate

If we choose any definite vacuum value of the Higgs field, the symmetry will be broken

$$\phi(x) = \left( \begin{array}{c} 0 \\ \frac{v+H(x)}{\sqrt{2}} \end{array} \right)$$

All particles, except photon and neutrinos (?) acquire masses

$v$ is VEV of the Higgs field

All SM masses are proportional to $v$, the only parameter which has dimension of $M$

$$m_W^2 = \frac{1}{4} g^2 \ v^2, \quad m_Z^2 = \frac{1}{4} \frac{g^2}{\cos^2\theta_W} \ v^2$$

Taking into account that $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$ we have

$$v = (\sqrt{2} G_F)^{-1/2} \simeq 246 \ \mathrm{GeV}$$

Fermion masses (and fermion-Higgs interactions) are generated by $SU_L(2) \times U_Y(1)$ invariant Yukawa interactions. For leptons

$$\mathcal{L}_I^Y = -\sqrt{2} \sum_{l_1, l_2} \bar{\psi}_{l_1 L} Y_{l_1 l_2} l'_{2R} \phi + \mathrm{h.c.}$$

Right-handed fields $l'_R$ are $SU_L(2)$ singlets, $Y$ is $3 \times 3$ complex dimensionless matrix (not constrained by the symmetry) After the spontaneous symmetry breaking for the mass term we have

$$\mathcal{L}_I^Y = -\sum_{l_1, l_2} \bar{l}'_{1L} Y_{l_1 l_2} l'_{2R} v + \mathrm{h.c.} = -\sum_{l=e,\mu,\tau} m_l \ \bar{l} l$$

$m_l$ is the mass of the lepton $l$ $(l = e, \mu, \tau)$

$$m_l = y_l \ v, \quad l'_{1L} = \sum_{l_2} (V_L)_{l_1 l_2} l_{2L}$$

$y_l$ is eigenvalue of the matrix $Y$ (Yukawa coupling), $V_L^\dagger V_L = 1$

Assume that in the Lagrangian enter the Yukawa interaction

$$\mathcal{L}_I^{Y\nu} = -\sqrt{2}\sum_{l_1,l_2} \bar{\psi}_{l_1 L} Y^{\nu}_{l_1 l_2} \nu'_{l_2 R} \tilde{\phi} + \mathrm{h.c.}$$

which generates neutrino mass term

We assumed that in the Lagrangian enter left-handed fields $\nu_{lL}$ and right-handed fields $\nu_{lR}$, $SU_L(2)$ singlets

After spontaneous symmetry breaking the Dirac mass term is generated

$$\mathcal{L}_I^D = -\sum_{i=1}^{3} m_i \bar{\nu}_i \nu_i, \quad m_i = y_i v$$

$\nu_i$ is the Dirac field with mass $m_i$

Because neutrino masses are much smaller than masses of leptons and quarks neutrino Yukawa couplings $y_i$ are much smaller than Yukawa couplings of leptons and quarks

$$y_t \simeq 7 \cdot 10^{-1}, \quad y_b \simeq 2 \cdot 10^{-2}, \quad y_\tau \simeq 7 \cdot 10^{-3} \quad y_3 \leq 10^{-12}$$

it is very unlikely that neutrino masses are of the same Standard Model origin as masses of leptons and quarks

Notice that in the EM current enter left-handed and right-handed components of charged fields. Yukawa interactions do not increase the number of degrees of freedom (dof). Neutrinos have no EM interaction. The neutrino Yukawa interaction, doubles the number of neutrino dof.The most economical possibility is the Standard Model with massless neutrinos

The most plausible possibility: NEUTRINO MASSES AND MIXING ARE OF BEYOND THE STANDARD MODEL ORIGIN

For the first time neutrino masses and mixing were introduced via neutrino mass term by Gribov and Pontecorvo in 1969

They consider two flavor neutrinos. General case was discussed later by Petcov and SB

The SM charged current

$$j^{CC}_{\alpha}(x) = 2 \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}(x)\gamma_{\alpha} l_L(x)$$

$l_L(x)$ is the field of lepton $l$ with mass $m_l$ and $\nu_{lL}(x)$ is flavor neutrino field ($\nu_{lL}(x) = \sum_{l_1}(V_L)_{ll_1} l'_1(x)$)

A mass term is a Lorenz-invariant product of $L$ and $R$ components

Can we built a neutrino mass term in which only left-handed flavor fields $\nu_{lL}(x)$ enter?

Gribov-Pontecorvo answer: yes.

If we assume that the mass term does not conserve the total lepton number $L$ and take into account that the conjugated field $\nu^c_{lL} \equiv (\nu_{lL})^c = C\bar{\nu}^T_{lL}$ is right-handed field

$C$ is the matrix of the charge conjugation ($C\gamma^T_{\alpha}C^{-1} = -\gamma_{\alpha}$ )

## Majorana mass term

$$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2}\sum_{l',l} \bar{\nu}_{l'L} M^{M}_{l'l} \nu^{c}_{lL} + \mathrm{h.c.}$$

$$M^M = (M^M)^T$$

After the standard diagonalization ($M^M = U\ m\ U^T$)

$$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2}\sum_{i=1}^{3} m_i\ \bar{\nu}_i \nu_i$$

$$\nu_i(x) = \nu_i^c(x) = C\bar{\nu}_i^T(x)$$

is the field of the Majorana neutrino with the mass $m_i$

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li}\ \nu_{iL}(x), \quad U^\dagger U = 1$$

$U$ is Pontecorvo-MNS unitary mixing matrix

- The Majorana mass term is the most economical mass term: no $\nu_{lR}$, minimal number of dof.
- Neutrino masses $m_i$ are parameters. No any clues, why neutrino masses are so small.

Beyond SM neutrino masses

The method of the effective Lagrangian is a general method which allows to describe effects of a new physics. The effective Lagrangian is a dimension five or more non renormalizable Lagrangian, invariant under $SU(2)_L \times U(1)_Y$ transformations and built from the Standard Model fields

Let us consider dimension $M^{5/2}$, $SU(2)_L \times U(1)_Y$ invariant $\tilde{\phi}^\dagger \; \psi_{lL} \quad (l = e, \mu, \tau)$ $\tilde{\phi} = i\tau_2\phi^*$ is conjugated Higgs doublet

After spontaneous symmetry breaking

$$\tilde{\phi} = \begin{pmatrix} \frac{v+H}{\sqrt{2}} \\ 0 \end{pmatrix}$$

$$\tilde{\phi}^\dagger \; \psi_{lL} \rightarrow \frac{v}{\sqrt{2}} \; \nu'_{lL}$$

The only effective Lagrangian which generates the neutrino mass term (Weinberg)

$$\mathcal{L}_I^{\mathrm{eff}} = -\frac{1}{\Lambda} \sum_{l_1,l_2} (\bar{\psi}_{l_1 L}\tilde{\phi})\, X_{l_1 l_2}\, (\tilde{\phi}^T \psi^c_{l_2 L}) + \mathrm{h.c.}$$

$X$ is a $3 \times 3$ dimensionless, symmetrical matrix; the operator has minimal dimension $M^5$

The constant $\Lambda$ (dimension $M$) characterizes a scale of a beyond the SM physics

General remark. $\mathcal{L}_I^{\mathrm{eff}}$ does not conserve $L$. Global invariance and conservation of $L$ is not a fundamental symmetry of a Quantum Field Theory. In the SM local gauge symmetry ensure conservation of $L$. Non conservation of $L$ is, apparently, natural in a theory beyond the SM

After the spontaneous symmetry breaking we come to the Gribov-Pontecorvo Majorana mass term

Major difference

Neutrino masses

$$m_i = \frac{v^2}{\Lambda}\, x_i$$

$x_i$ is the eigenvalue of the matrix X

Masses of all particles, generated by the SM Higgs mechanism, are proportional to $v \simeq 246$ GeV

Neutrino masses, generated by the beyond the SM effective Lagrangian, contain additional factor

$$\frac{v}{\Lambda} = \frac{\text{electroweak scale}}{\text{scale of a new physics}}$$

If $\Lambda \gg v$ the puzzle of neutrino masses (smallness with respect to SM masses of lepton and quarks) will be resolved

The absolute values of neutrino masses at present are unknown
From existing oscillation data and cosmology for the largest neutrino mass it can be found the bounds

$$\sqrt{\Delta m_A^2} \simeq 5 \cdot 10^{-2} \leq m_3 \leq \frac{\sum_{i=1}^{3} m_i}{3} \simeq 3 \cdot 10^{-1}\ \mathrm{eV}$$

From the range above

$$\Lambda \simeq (2 \cdot 10^{14} - 10^{15})\ x_3\ \mathrm{GeV}$$

$x_3$ is unknown parameter. If we assume $x_3 \simeq 1$ (like square of the Yukawa coupling of the t-quark) we come to a GUT scale

$$\Lambda \simeq (2 \cdot 10^{14} - 10^{15})\ \mathrm{GeV}$$

Such large scale could be probed only through cosmology

How the Weinberg effective Lagrangian can be generated?
Estimates of a scale of a new physics depend on a mechanism of the generation of the effective Lagrangian
The most economical (and natural) possibility: exchange of heavy virtual Majorana leptons , $SU_L(2) \times U_Y(1)$ singlets, between lepton-Higgs pairs
Assume $L$ is not conserved and exist heavy Majorana leptons $N_i$

$$N_i = N_i^c = C(\bar{N}_i)^T, \quad i = 1, 2, ...$$

which have the following Yukawa interaction with SM leptons and Higgs bosons

$$\mathcal{L}_I = -\sqrt{2} \sum_{l,i} (\bar{\psi}_{lL} \tilde{\phi}) y_{li} \; N_{iR} + \mathrm{h.c.}$$

In the three-approximation with virtual heavy leptons we come to the Weinberg effective Lagrangian in which

$$\frac{1}{\Lambda}X_{l'l} = \sum_i y_{l'i}\frac{1}{M_i}y_{l'i}, \quad M_i \text{ is } N_i \text{ mass}$$

Majorana mass matrix is given by

$$M^M = y\ \frac{v^2}{M}\ y^T$$

The scale $\Lambda$ is determined by masses of heavy Majorana leptons ( type I seesaw mechanism).

This is not the only three-approximation possibility to generate the Weinberg effective Lagrangian

Other possibilities: exchange of heavy triplet scalar bosons between lepton and Higgs pairs (type II seesaw) and exchange of heavy Majorana triplet leptons between Higgs-lepton pairs (type III seesaw)

The mechanism we considered is a very attractive mechanism of neutrino mass generation

- It gives a possibility to explain not only smallness of neutrino masses but also the barion asymmetry of the Universe. In fact, $CP$-violating decays of heavy Majorana leptons, created in the early Universe, could produce a lepton asymmetry which would generate the barion asymmetry (leptogenesis)
- The Weinberg effective Lagrangian has a minimal dimension (five). Investigation of effects of small neutrino masses allow us to probe a beyond the SM , $L$-violating physics at a very large scales, unreachable in the laboratory

However, the validity of this mechanism can not be directly tested

There exist numerous neutrino mass models based on the assumption that SM neutrinos are massless which generate Weinberg effective Lagrangian. In these models neutrino masses are suppressed by different loop mechanisms which require existence of beyond the Standard Model particles with masses which are smaller than $10^{14} - 10^{15}$ GeV (including masses which can be probed at LHC)

A wide class of neutrino mass models, minimal in a sense that there are no $\nu_R$ in the SM, through Weinberg effective Lagrangian lead to the Majorana mass term

We can conclude

- The smallness of neutrino masses is a signature of a beyond the Standard Model physics
- There are many possibilities to explain the smallness of neutrino masses by different beyond the SM mechanisms which lead to Majorana mass term

From the Majorana mass term we can come to the following general conclusions which can be probed by present-day and future experiments

1. Neutrino with definite masses $\nu_i$ are Majorana particles. Neutrinoless double $\beta$-decay is allowed process.
2. The number of massive neutrinos is equal to the number of flavor neutrinos (three). There are no transitions of flavor neutrinos into sterile states

## Neutrino Oscillations

If a neutrino mass term contain flavor fields $\nu_{lL}(x)$ ($l = e, \mu, \tau$) and some fields $\nu_{sL}(x)$ ($s = s_1, ...s_{n_{st}}$) which do enter into weak interaction (called sterile) for the mixing we have

$$\nu_{lL}(x) = \sum_{i=1}^{3+n_{st}} U_{li}\ \nu_{iL}(x), \quad \nu_{sL}(x) = \sum_{i=1}^{3+n_{st}} U^*_{si}\ \nu_{iL}(x)$$

Here $U$ is unitary $(3 + n_{st}) \times (3 + n_{st})$ mixing matrix, and $\nu_{iL}(x)$ is the field of neutrino with mass $m_i$

If all neutrino masses are small the states of the flavor neutrinos

$$|\nu_l\rangle = \sum_{i=1}^{3+n_{st}} U^*_{li}\ |\nu_{iL}\rangle \quad (l = e, \mu, \tau)$$

$|\nu_i\rangle$ is the state with momentum $\vec{p}$ and energy $E_i \simeq p + \frac{m_i^2}{2E}$

## Sterile states

$$|\nu_s\rangle = \sum_{i=1}^{3+n_{st}} U_{si}^* \, |\nu_{iL}\rangle, \quad (s = s_1, ...s_{n_{st}})$$

From unitarity of $U$

$$\langle \nu_{l'} | \nu_l \rangle = \delta_{l'l}, \;\; \langle \nu_{s'} | \nu_s \rangle = \delta_{s's}, \;\; \langle \nu_s | \nu_l \rangle = 0$$

## Neutrino transitions in vacuum

$$\mathrm{P}(\nu_l \to \nu_{l'}) = |\sum_{i=1}^{3+n_{st}} U_{l'i} e^{-iE_i t} U_{li}^*|^2 = |\delta_{l'l} - 2i \sum_{i \neq r} e^{-i\Delta_{ri}} U_{l'i} U_{li}^* \sin \Delta_{ri}|^2$$

$$\Delta_{ri} = \frac{\Delta m_{ri}^2 L}{4E}, \;\; \Delta m_{ik}^2 = m_k^2 - m_i^2$$

$L \simeq t$ is the distance between neutrino source and detector and $r$ is an arbitrary, fixed index

## Transition probability

$$\mathrm{P}(\overset{(-)}{\nu}_l \to \overset{(-)}{\nu}_{l'}) = \delta_{l'l} - 4\sum_{i\neq r}|U_{li}|^2(\delta_{l'l} - |U_{l'i}|^2)\sin^2\Delta_{ri}$$

$$+8\sum_{i>k;i,k\neq r}\mathrm{Re}\,(U_{l'i}U^*_{li}U^*_{l'k}U_{lk})\cos(\Delta_{ri}-\Delta_{rk})\sin\Delta_{ri}\sin\Delta_{rk}$$

$$\pm 8\sum_{i>k;i,k\neq r}\mathrm{Im}\,(U_{l'i}U^*_{li}U^*_{l'k}U_{lk})\sin(\Delta_{ri}-\Delta_{rk})\sin\Delta_{ri}\sin\Delta_{rk}$$

All LBL accelerator, LBL reactor, atmospheric and solar neutrino oscillation data are described by the three-neutrino mixing

Six parameters: $\Delta m^2_S$, $\Delta m^2_A$, mixing angles $\theta_{12}$, $\theta_{23}$, $\theta_{13}$, one CP phase $\delta$

Possible neutrino mass spectra. $\Delta m^2_S = \Delta m^2_{12} = (m^2_2 - m^2_1) > 0$.

For $m_3$ two possibilities

$$NO. \quad m_3 > m_2 > m_1, \quad \Delta m^2_A = \Delta m^2_{23}$$

$$IO. \quad m_2 > m_1 > m_3, \quad \Delta m^2_A = |\Delta m^2_{13}|$$

Table: Values of neutrino oscillation parameters obtained from the global fit of existing data

| Parameter | Normal Ordering | Inverted Ordering |
|---|---|---|
| $\sin^2\theta_{12}$ | $0.310^{+0.013}_{-0.012}$ | $0.310^{+0.013}_{-0.012}$ |
| $\sin^2\theta_{23}$ | $0.580^{+0.017}_{-0.021}$ | $0.584^{+0.016}_{-0.020}$ |
| $\sin^2\theta_{13}$ | $0.02241^{+0.00065}_{-0.00065}$ | $0.02264^{+0.00066}_{-0.00066}$ |
| $\delta$ (in $^\circ$) | $(215^{+40}_{-29})$ | $(284^{+27}_{-29})$ |
| $\Delta m^2_S$ | $(7.39^{+0.21}_{-0.20}) \cdot 10^{-5}\ \mathrm{eV}^2$ | $(7.39^{+0.21}_{-0.20}) \cdot 10^{-5}\ \mathrm{eV}^2$ |
| $\Delta m^2_A$ | $(2.525^{+0.033}_{-0.032}) \cdot 10^{-3}\ \mathrm{eV}^2$ | $(2.512^{+0.034}_{-0.032}) \cdot 10^{-3}\ \mathrm{eV}^2$ |

$\frac{\Delta m^2 L}{4E} \gtrsim 1$ is a condition to observe neutrino oscillations

For accelerator neutrinos ($E \simeq 1$ GeV) distances of hundreds km are required to be sensitive to $\Delta m^2_A \simeq 2.5 \cdot 10^{-3}\ \mathrm{eV}^2$, for reactor $\bar{\nu}_e$'s ($E \simeq 4$ MeV) distances of about two hundreds km are required to be sensitive to $\Delta m^2_S \simeq 8 \cdot 10^{-5}\ \mathrm{eV}^2$ etc

## Sterile neutrino "classical" anomalies

About 25 years exist LSND anomaly. In the LSND accelerator SBL experiment (source-detector distance 30 m) $\bar{\nu}_{\mu} \to \bar{\nu}_{e}$ transitions was observed. The best-fit values of the neutrino oscillation parameters are $\sin^2 2\theta = 0.003, \quad \Delta m^2 = 1.2 \ \mathrm{eV}^2$.

In 2011 reactor $\bar{\nu}_e$'s spectrum was recalculated. The mean flux of $\bar{\nu}_e$'s increased by 3.5% and ratio of measured and expected events in all old short-baseline reactor experiments became equal to $0.943 \pm 0.023$. (Reactor neutrino anomaly). Can be explained by neutrino oscillations with $\Delta m^2 > 1.5 \ \mathrm{eV}^2$, $\sin^2 2\theta = 0.14 \pm 0.08$

In short baseline calibration neutrino experiments performed by the GALLEX and SAGE collaborations the ratio $R$ of detected and expected neutrino events was found $R = 0.812^{+0.10}_{-0.11}$ (GALLEX) and $R = 0.791^{+0.084}_{-0.078}$ (SAGE) (Galium neutrino anomaly). Can be explained by neutrino oscillations ($\Delta m^2 > 0.35 \ \mathrm{eV}^2$, $\sin^2 2\theta > 0.07$)

These and other short-baseline data can be explained if we assume that in addition to three light neutrinos exist a fourth neutrino $\nu_4$ with mass about one eV (3+1 scheme). Taking into account that $\Delta m^2_{14} \gg \Delta m^2_A \gg \Delta m^2_S$ we have

$$P^{\mathrm{SBL}}(\overset{(-)}{\nu}_l \to \overset{(-)}{\nu}_l) = 1 - 4|U_{l4}|^2(1 - |U_{l4}|^2)\ \sin^2 \Delta_{14}, \quad l = e, \mu$$

$$P^{\mathrm{SBL}}(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e) = \sin^2 2\theta_{e\mu} \sin^2 \Delta_{14}$$

$$\sin^2 2\theta_{e\mu} = 4|U_{e4}|^2|U_{\mu4}|^2.$$

Important relation between SBL transition amplitudes

From results of LSND and reactor experiments experiments $|U_{\mu4}|^2 \simeq 10^{-1}$. Effects of neutrino oscillations was not observed in SBL $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu$ disappearance experiments: $|U_{\mu4}|^2 < 10^{-2}$ in the region $2 \cdot 10^{-1} < \Delta m^2_{14} < 10\ \mathrm{eV}^2$

Clear contradiction to an interpretation of LSND and other results as neutrino oscillations. New more precise experiments are needed
Many new SBL neutrino experiments on the search for sterile neutrinos with masses of the order of eV are going on.
Large parts of allowed regions of oscillation parameters are excluded, however, no definite conclusions can be done
Crucial test of LSND and other results, apparently, will be performed in the accelerator SBL experiment at the Fermilab with three Liquid Argon Time Projection Detectors. Important feature of this experiment will be measurement in one experiment of $\nu_\mu \to \nu_e$ appearance and $\nu_\mu \to \nu_\mu$ disappearance

Neutrinoless double $\beta$-decay ($(A, Z) \to (A, Z+2) + e^- + e^-$)

If neutrinos with definite masses are Majorana particles neutrinoless double $\beta$-decay of $^{76}\mathrm{Ge}$, $^{130}\mathrm{Te}$, $^{136}\mathrm{Xe}$ and other even-even nuclei will be allowed

The half-life of the $0\nu\beta\beta$-decay

$$\frac{1}{T^{0\nu}_{1/2}} = |m_{\beta\beta}|^2 \, |M^{0\nu}|^2 \, G^{0\nu}(Q, Z).$$

$M^{0\nu}$ is the nuclear matrix element, $G^{0\nu}(Q, Z)$ is known phase-space factor and

$$|m_{\beta\beta}| = |\sum_{k=1}^{3} U^2_{ek} \, m_k|$$

is the effective Majorana mass

For the normal hierarchy

$$m_1 \ll m_2 \ll m_3$$

$$|m_{\beta\beta}| \leq 4 \cdot 10^{-3}\mathrm{eV}.$$

In the case of inverted hierarchy of the neutrino masses

$$m_3 \ll m_1 < m_2$$

$$2 \cdot 10^{-2}\ \mathrm{eV} \leq |m_{\beta\beta}| \leq 5 \cdot 10^{-2}\ \mathrm{eV}$$

Table: Lower limits of half-lives $T^{0\nu}_{1/2}$ and upper limits of the effective Majorana mass $|m_{\beta\beta}|$ obtained in recent experiments on the search for the $0\nu\beta\beta$-decay

| experiment | nucleus | NME | $T^{0\nu}_{1/2}(10^{25}\mathrm{yr})$ | $\lvert m_{\beta\beta}\rvert$ (eV) |
|---|---|---|---|---|
| Gerda | $^{76}$ Ge | 2.8-6.1 | 8.0 | (0.12-0.26) |
| Majorana | $^{76}$ Ge | 2.8-6.1 | 2.7 | (0.20-0.43) |
| KamLAND-Zen | $^{136}$ Xe | 1.6-4.8 | 10.7 | (0.05-0.16) |
| EXO | $^{136}$ Xe | 1.6-4.8 | 1.8 | (0.15-0.40) |
| CUORE | $^{130}$ Te | 1.4-6.4 | 1.5 | (0.11-0.50) |

Many new experiments on the search for the $0\nu\beta\beta$-decay are in preparation at present. In these future experiments the inverted hierarchy region and, possibly, part of the normal hierarchy region will be probed

Conclusion

It is very likely that SM neutrinos are massless (there are no right-handed neutrino fields $\nu_{lR}$ in the SM)

In this case neutrino masses are of a beyond the Standard Model origin

Dimension five Weinberg effective Lagrangian is the most simple and economical way of the explanation of the smallness of neutrino masses

The Weinberg Lagrangian can be generated in the tree approximation by interaction of Higgs-lepton pairs with heavy Majorana leptons. Smallness of neutrino masses, apparently, requires a new physics at a large scale $(10^{14} - 10^{15})$ GeV

There are many other models in which the Weinberg effective Lagrangian can be generated at a loop level with smaller scales of a beyond the SM physics

The Weinberg Lagrangian generates Majorana neutrino mass term. Existing "economical" models provide mechanisms of the suppression of neutrino masses but can not predict its values. However, there are two general consequences of these models which can be probed in the present-day experiments

- Neutrinos with definite masses are Majorana particles. $0\nu\beta\beta$ decay is allowed process.
- The number of neutrino with definite masses is equal to the number of flavor neutrinos. There are no transitions of flavor neutrinos into sterile states

Future experiments will show whether the nature choose economy and simplicity in the case of neutrino masses

Simplicity is a guide to the theory choice. A. Einstein

I always looked for simplicity and never stop to look for that. B. Pasternak