Neutrino Oscillation Phenomenology

Boris Kayser
Pontecorvo School
September, 2019

NASA Hubble Photo

What Are Neutrinos Good For?

Energy generation in the sun starts with the reaction —

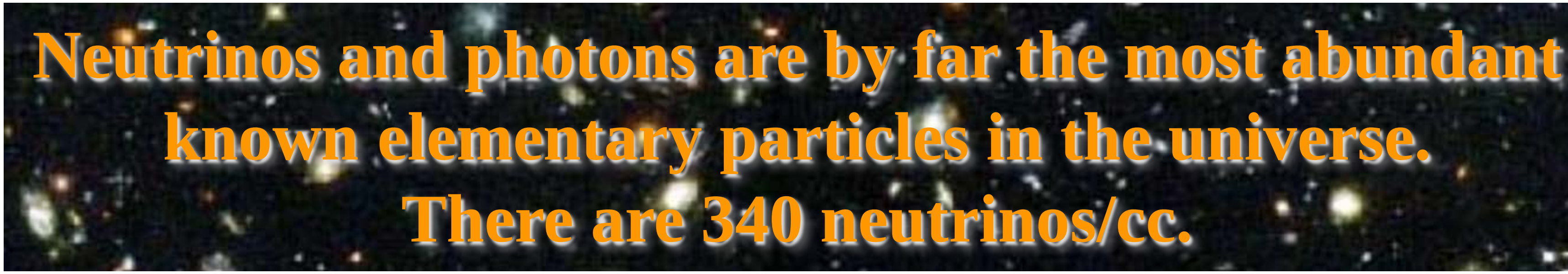
$$p + p \rightarrow d + e^{+} + \nu$$

Spin: $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$

Without the neutrino, angular momentum would not be conserved.

Uh, oh ……

The Neutrinos



The neutrinos are spin – 1/2, electrically neutral, leptons.

The only known forces they experience are the weak force and gravity.

This means that they do not interact with other matter very much at all.
Thus, neutrinos are difficult to detect and study.

Their weak interactions are successfully described by the Standard Model.

The Neutrino Revolution
(1998 – ...)

Neutrinos have nonzero masses!

Leptons mix!

The 2015 Nobel Prize in Physics went to **Takaaki Kajita** and **Art McDonald** for the experiments that proved this.

Super–Kamiokande, Japan

Sudbury Neutrino Observatory, Canada

The discovery of neutrino mass
and leptonic mixing
comes from the observation of
neutrino flavor change
(neutrino oscillation).

The Physics of Neutrino Oscillation

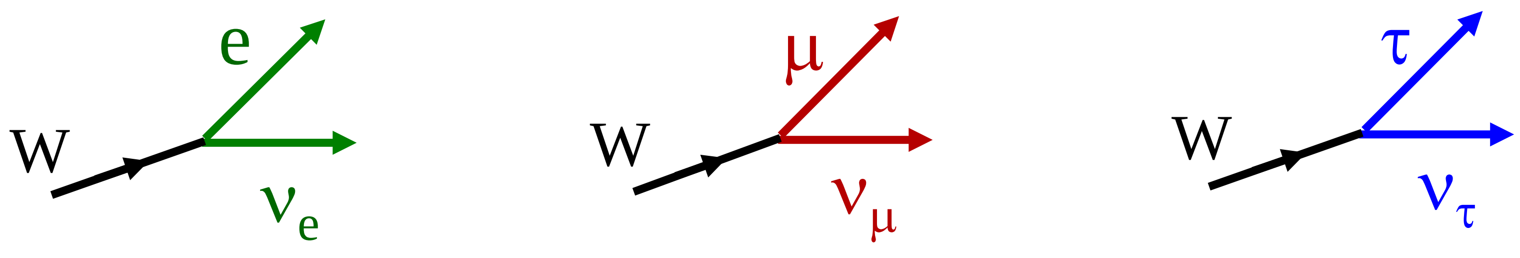
— Preliminaries

The Neutrino Flavors

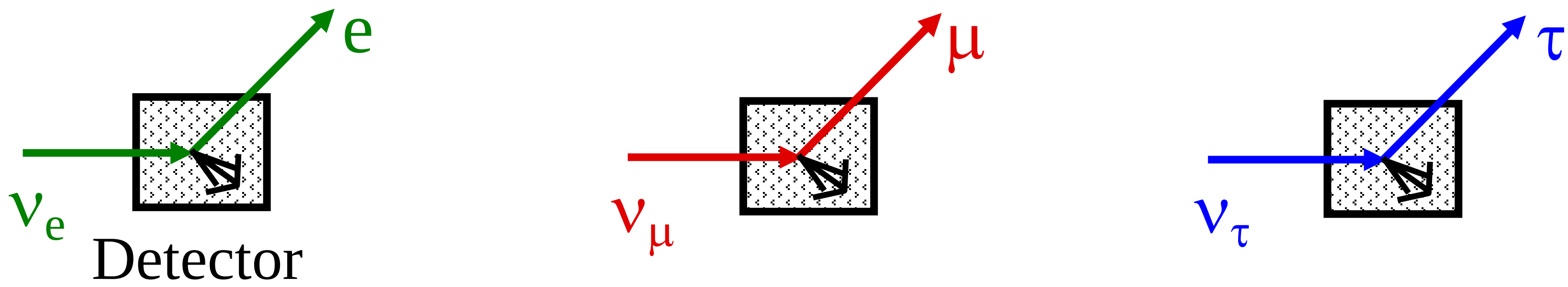
There are three flavors of charged leptons: e , μ , τ

There are three known flavors of neutrinos: ν_e, ν_μ, ν_τ

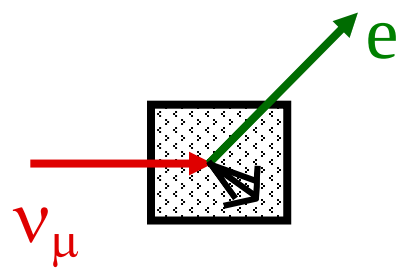
We ***define*** the neutrinos of specific flavor, ν_e, ν_μ, ν_τ, by W boson decays:



As far as we know, when a neutrino of given flavor interacts and turns into a charged lepton, that charged lepton will always be of the same flavor as the neutrino.



but not

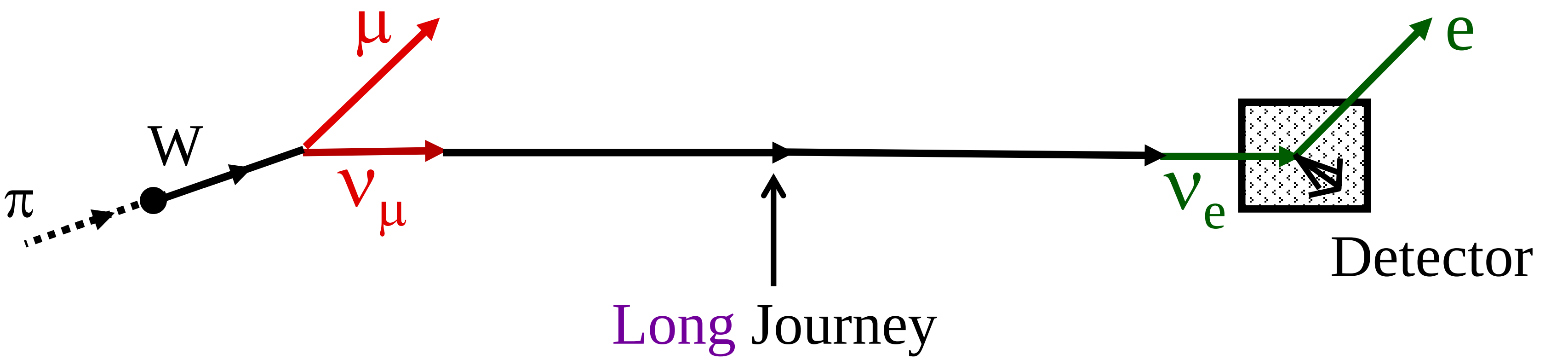


Lederman
Schwartz
Steinberger

The weak interaction couples the neutrino of a given flavor only to the charged lepton of the same flavor.

Neutrino Flavor Change ("Oscillation")

If neutrinos have masses, and leptons mix, we can have —



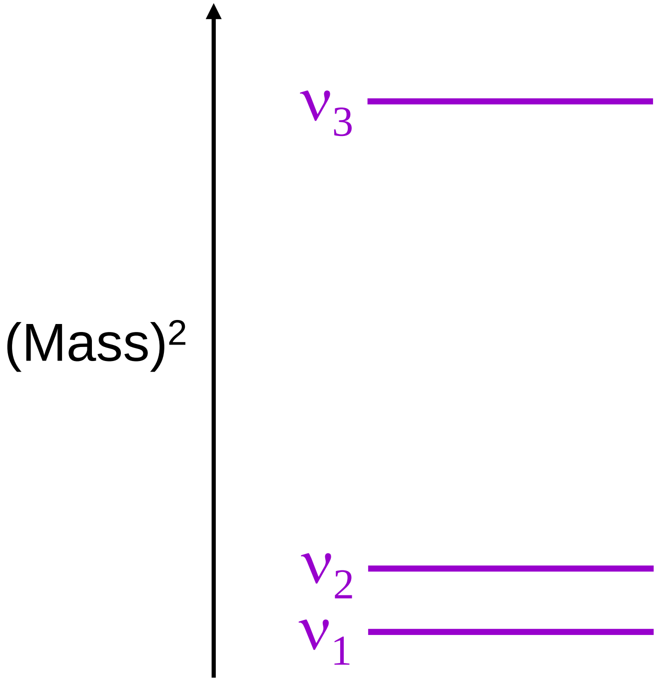
Give a ν time to change character, and you can have

for example: $\nu_\mu \longrightarrow \nu_e$

The last 21 years have brought us compelling evidence that such flavor changes actually occur.

Flavor Change Requires ***Neutrino Masses***

There must be some spectrum of neutrino mass eigenstates ν_i:



Mass $(\nu_i) \equiv m_i$

Flavor Change Requires ***Leptonic Mixing***

The neutrinos $\nu_{e,\mu,\tau}$ of definite flavor

$(W \rightarrow e\,\nu_e$ or $\mu\,\nu_\mu$ or $\tau\,\nu_\tau)$

must be superpositions of the mass eigenstates:

$$|\nu_\alpha> = \sum_i U^*_{\alpha i} \, |\nu_i> \quad .$$

Neutrino of flavor
α = e, μ, or τ

Neutrino of definite mass m_i

"PMNS" Leptonic Mixing Matrix

Pontecorvo

The leptonic mass eigenstates are e, μ, τ, and ν_1, ν_2, ν_3.

Notation: ℓ denotes a charged lepton. $\ell_e \equiv e$, $\ell_\mu \equiv \mu$, $\ell_\tau \equiv \tau$.

Since the only charged lepton ν_α couples to is ℓ_α,
the 3 ν_α must be orthogonal.

To make up 3 orthogonal ν_α, we must have at least 3 ν_i.
Unless some ν_i masses are degenerate,
all ν_i will be orthogonal.

Then —

$$\delta_{\alpha\beta} = \langle \nu_\alpha | \nu_\beta \rangle = \left\langle \sum_i U^*_{\alpha i} \nu_i \middle| \sum_j U^*_{\beta j} \nu_j \right\rangle$$

$$= \sum_{i,j} U_{\alpha i} U^*_{\beta j} \langle \nu_i | \nu_j \rangle = \sum_i U_{\alpha i} U^*_{\beta i}$$

This says that U is unitary, but note the unitary U may not be 3 x 3.

Leptonic mixing is easily incorporated into the Standard Model (SM) description of the $\ell\nu W$ interaction.

For this interaction, we then have —

Semi-weak coupling

Left-handed

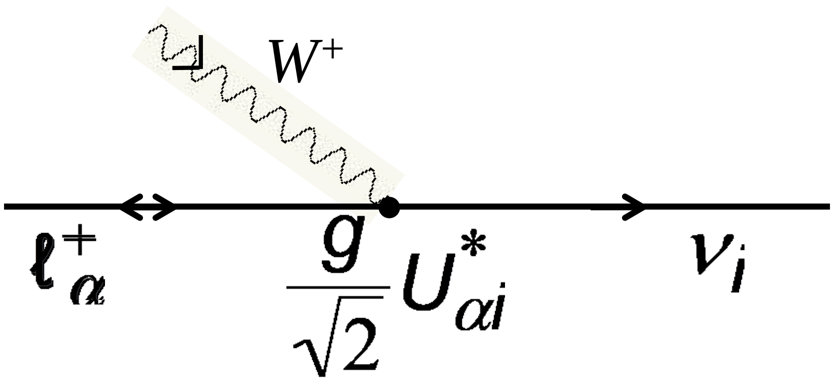
$$\mathcal{L}_{SM} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \left(\bar{\ell}_{L\alpha} \gamma^{\lambda} \nu_{L\alpha} W_{\lambda}^{-} + \bar{\nu}_{L\alpha} \gamma^{\lambda} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

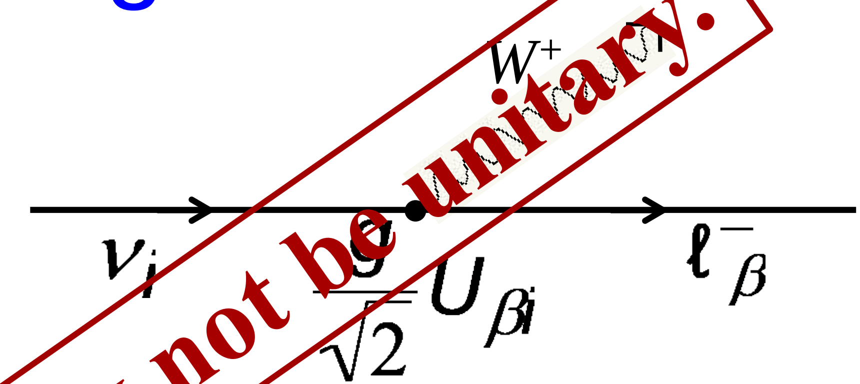
$$= -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} \left(\bar{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}^{-} + \bar{\nu}_{Li} \gamma^{\lambda} U_{\alpha i}^{*} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

Taking mixing into account

The SM interaction conserves the Lepton Number *L*, defined by $L(\nu) = L(\ell^{-}) = -L(\bar{\nu}) = -L(\ell^{+}) = 1.$

The Meaning of *U*





Caution: The 3 x 3 *U* may not be unitary.

$$U = \begin{array}{c} \\ e \\ \mu \\ \tau \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \end{array}$$

Please talk about ***leptonic*** mixing, not neutrino mixing.

The *e* row of *U*: The linear combination of neutrino mass eigenstates that couples to *e*.

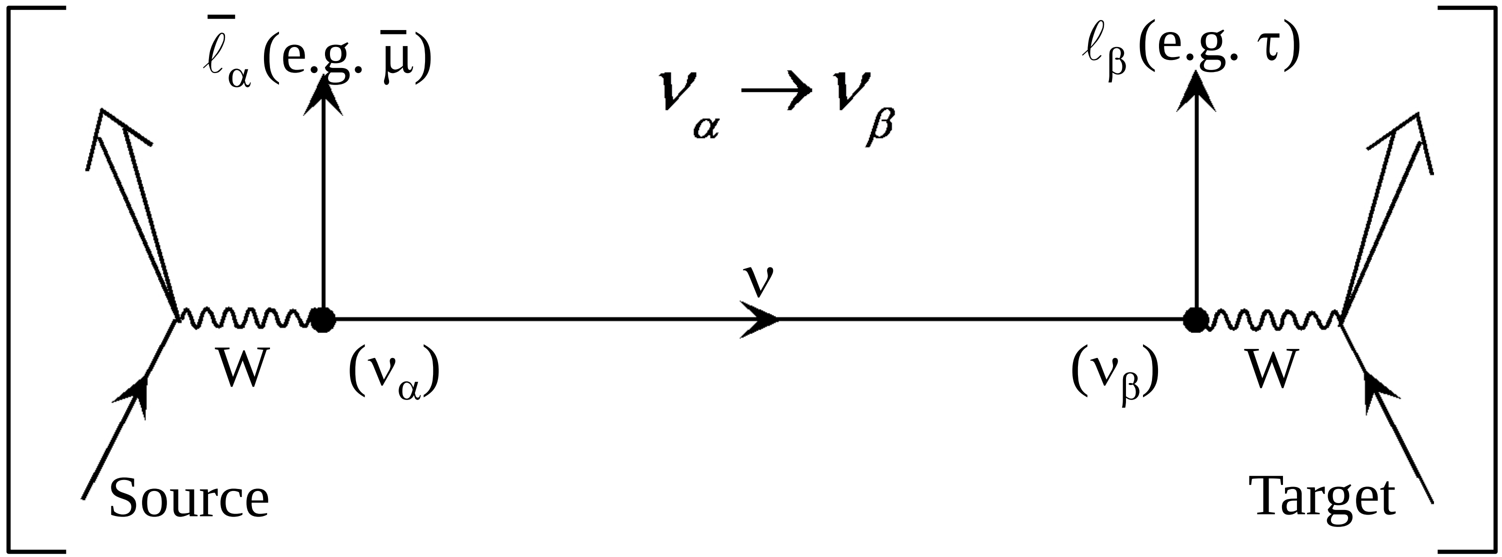
The ν_1 column of *U*: The linear combination of charged-lepton mass eigenstates that couples to ν_1 .

How Neutrino Oscillation In Vacuum Works

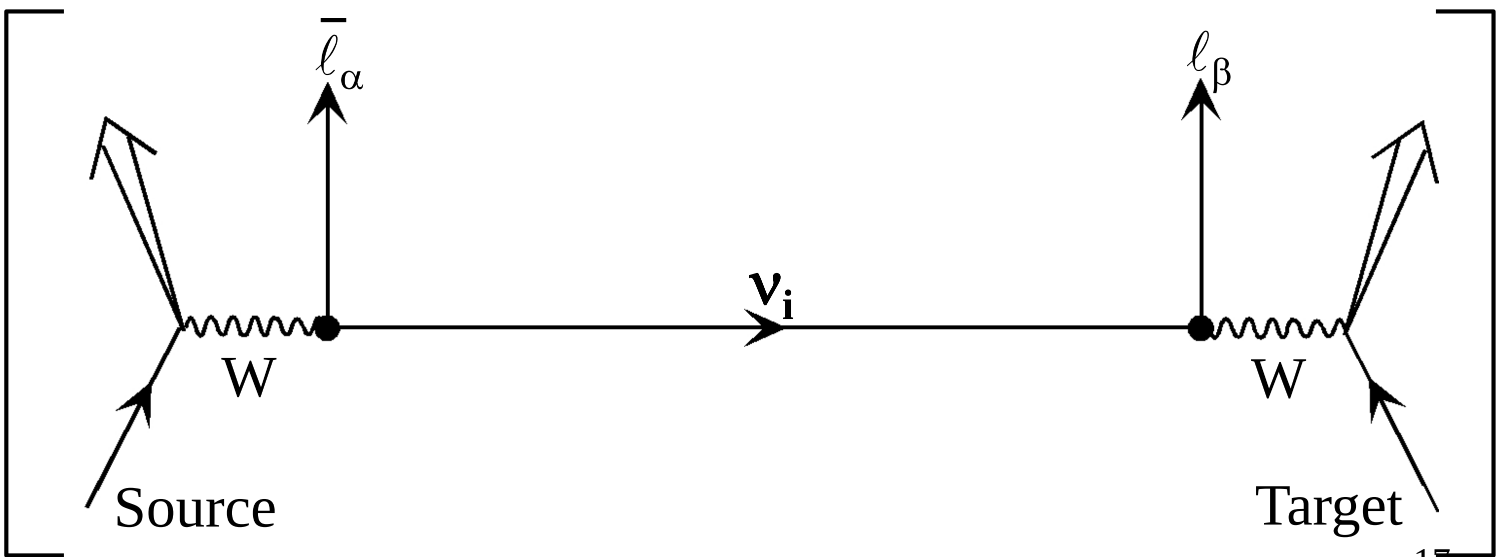
Neutrino Oscillation

(Approach of B.K. and Stodolsky)

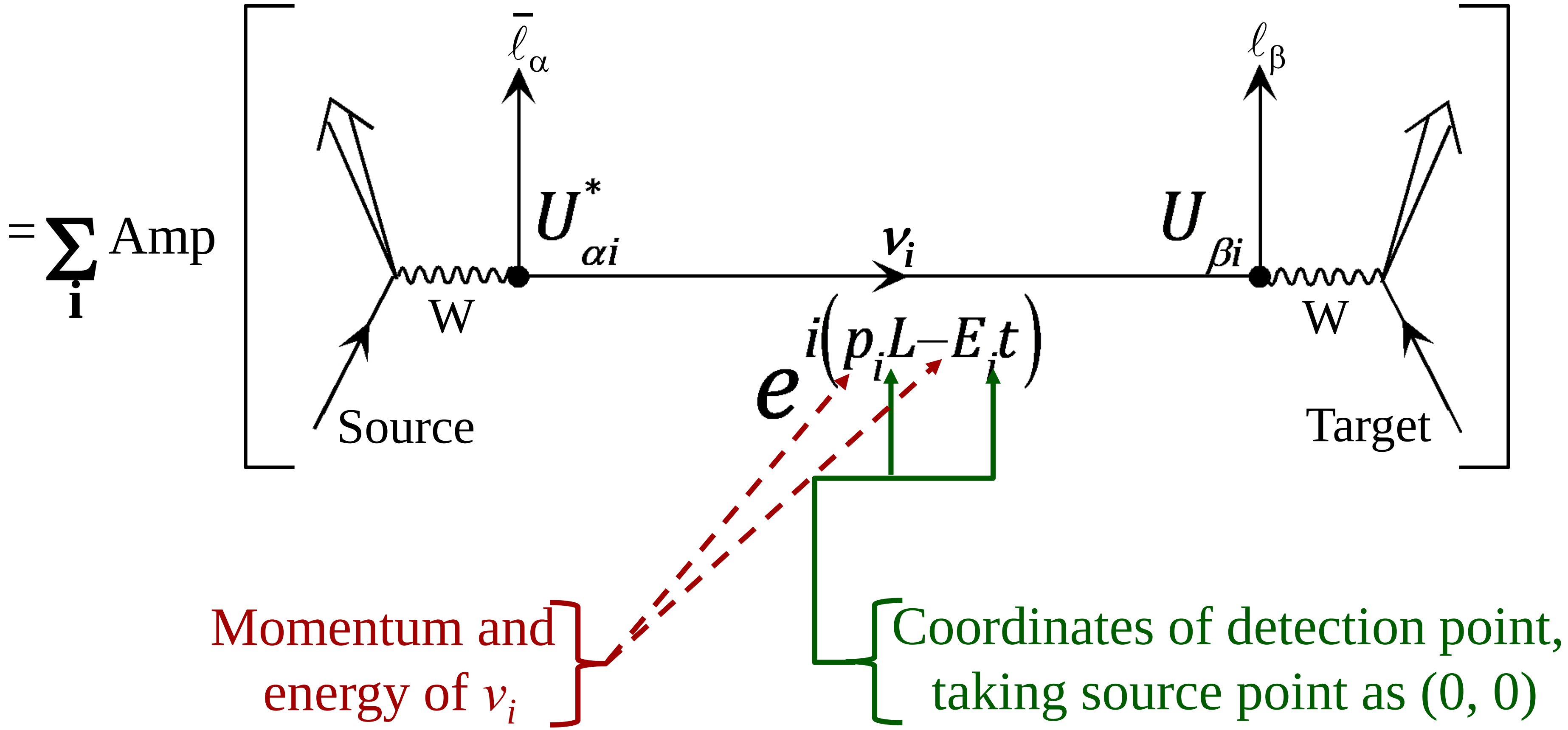
Amp



$= \sum_i$ Amp



$$\mathrm{Amp}\left(\nu_\alpha \rightarrow \nu_\beta\right)$$



Neutrino sources are ~ constant in time.

Averaged over time, the

$$e^{-iE_1 t} - e^{-iE_2 t}$$ interference

is —

$$\left\langle e^{-i(E_1 - E_2)t} \right\rangle_t = 0$$ *unless* $E_2 = E_1$.

Only neutrino mass eigenstates with a common energy E are coherent.

(Stodolsky)

For each mass eigenstate ν_i,

$$p_i = \sqrt{E^2 - m_i^2} \cong E - \frac{m_i^2}{2E} \,.$$

Then the plane-wave factor $e^{i(p_i L - E_i t)}$ is —

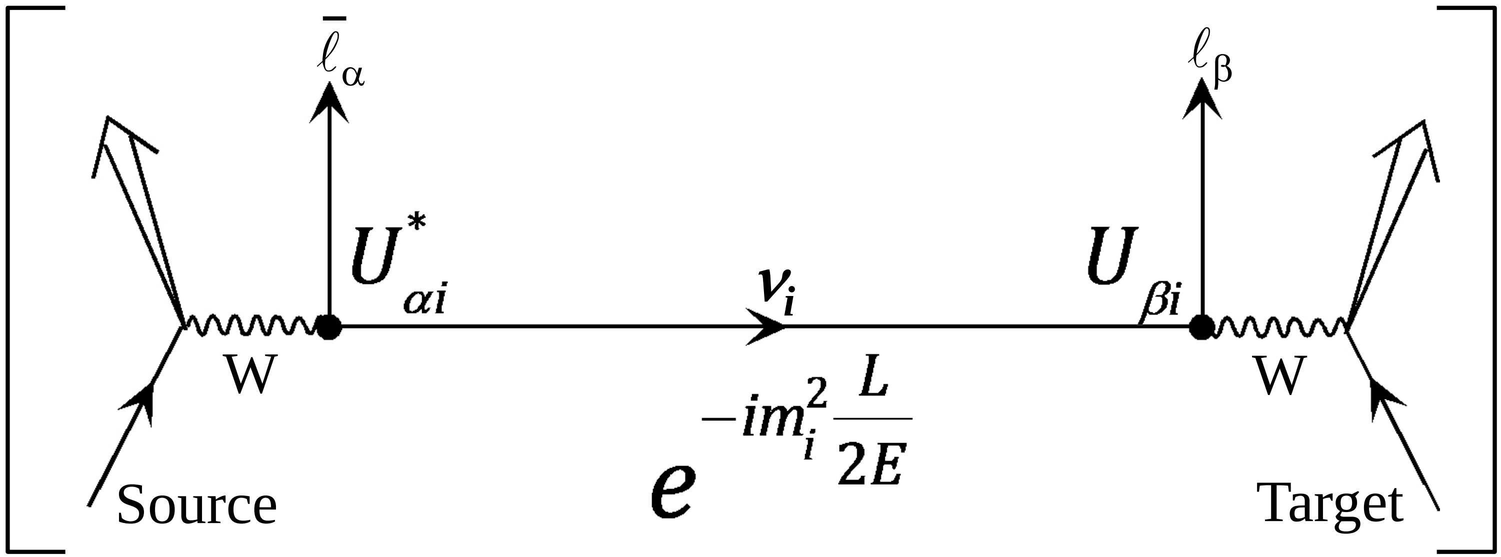
$$e^{i(p_i L - E_i t)} \cong e^{i\left\{\left(E - \frac{m_i^2}{2E}\right)L - Et\right\}} = e^{iE(L-t)} e^{-im_i^2 \frac{L}{2E}}$$

Irrelevant overall phase factor

Then —

$$\mathrm{Amp}\left(\nu_\alpha \rightarrow \nu_\beta\right)$$

$$= \sum_{\mathbf{i}} \mathrm{Amp}$$



$$= \sum_i U^*_{\alpha i} e^{-im_i^2 \frac{L}{2E}} U_{\beta i}$$

Probability of Neutrino Oscillation in Vacuum

$$P(\nu_\alpha \to \nu_\beta) = \left|\text{Amp}(\nu_\alpha \to \nu_\beta)\right|^2 =$$

$$= \delta_{\alpha\beta} - 4\sum_{i>j} \text{Re}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin^2\left(\Delta m^2_{ij} \frac{L}{4E}\right)$$

$$+ 2\sum_{i>j} \text{Im}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin\left(\Delta m^2_{ij} \frac{L}{2E}\right)$$

where $\Delta m^2_{ij} \equiv m^2_i - m^2_j$.

Neutrino flavor change implies neutrino mass!

Neutrinos vs. Antineutrinos

$$\left[\bar{\nu}_\alpha(\mathrm{RH}) \rightarrow \bar{\nu}_\beta(\mathrm{RH})\right] = \mathrm{CP}\left[\nu_\alpha(\mathrm{LH}) \rightarrow \nu_\beta(\mathrm{LH})\right]$$

A difference between the probabilities of these two oscillations in vacuum would be a leptonic violation of CP invariance.

Assuming CPT invariance —

$$P\left[\bar{\nu}_\alpha(\mathrm{RH}) \rightarrow \bar{\nu}_\beta(\mathrm{RH})\right] = P\left[\nu_\beta(\mathrm{LH}) \rightarrow \nu_\alpha(\mathrm{LH})\right]$$

Probability

$$P\left(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta\right) =$$

$$= \delta_{\alpha\beta} - 4 \sum_{i>j} \mathrm{Re}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin^2\left[\Delta m^2_{ij} \frac{L}{4E}\right]$$

$$\underset{(-)}{+} 2 \sum_{i>j} \mathrm{Im}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin\left[\Delta m^2_{ij} \frac{L}{2E}\right]$$

In neutrino oscillation, CP non-invariance comes from phases in the leptonic mixing matrix U.

The plane-wave treatment of neutrino oscillation is not completely correct,

since $\Delta x \Delta p \geq \frac{\hbar}{2}$.

The expression for the oscillation probability resulting from this treatment is wrong at very large L.

But a wave packet treatment shows that the plane-wave expression is correct under almost all circumstances.

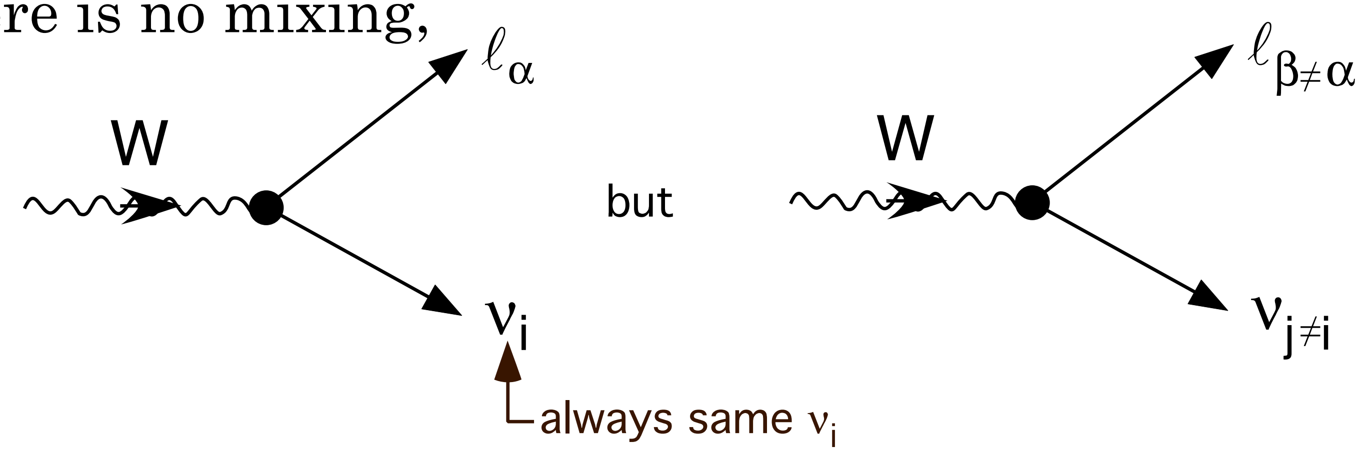
— Comments —

1. If all $m_i = 0$, so that all $\Delta m_{ij}^2 = 0$,

$$P\,(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) = \delta_{\alpha\beta}$$

Flavor *change* $\Rightarrow$ ν Mass

2. If there is no mixing,



$$\Rightarrow U_{\alpha i} U_{\beta\neq\alpha, i} = 0, \text{ so that } P\,(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) = \delta_{\alpha\beta}.$$

3. One can detect ($\nu_\alpha \rightarrow \nu_\beta$) in two ways:

See $\nu_{\beta\neq\alpha}$ in a ν_α beam (Appearance)

See some of known ν_α flux disappear (Disappearance)

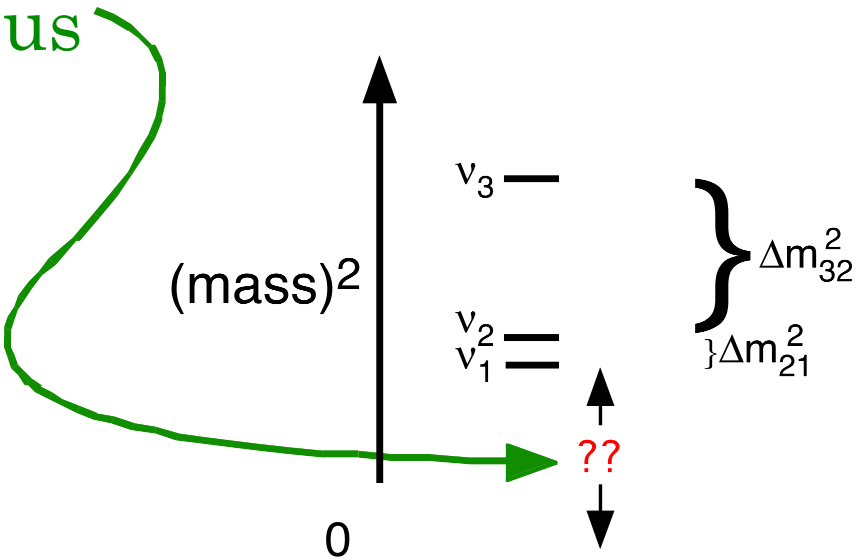
4. Including $\hbar$ and c

$\sin^2[1.27\Delta m^2(\mathrm{eV})^2 \frac{L(\mathrm{km})}{E(\mathrm{GeV})}]$ becomes appreciable when its argument reaches $\mathcal{O}(1)$.

An experiment with given L/E is sensitive to

5. Flavor change in vacuum oscillates with L/E. Hence the name "neutrino oscillation". {The L/E is from the proper time τ.}

6. P $(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta)$ depends only on squared-mass splittings. Oscillation experiments cannot tell us



7. Neutrino flavor change does not change the total flux in a beam.

It just redistributes it among the flavors.

$$\sum_{\text{All } \beta} P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) = 1$$

But some of the flavors $\beta \neq \alpha$ could be sterile.

Then some of the *active* flux disappears:

Important Special Cases

Three Flavors

For $\beta \neq \alpha$,

$$e^{-im_1^2\frac{L}{2E}}\mathrm{Amp}^*(\nu_\alpha \to \nu_\beta) = \sum_i U_{\alpha i}U^*_{\beta i}e^{im_i^2\frac{L}{2E}}e^{-im_1^2\frac{L}{2E}}$$

$$= U_{\alpha 3}U^*_{\beta 3}e^{2i\Delta_{31}} + U_{\alpha 2}U^*_{\beta 2}e^{2i\Delta_{21}} \underbrace{-(U_{\alpha 3}U^*_{\beta 3} + U_{\alpha 2}U^*_{\beta 2})}_{\text{Unitarity}}$$

$$= 2i[U_{\alpha 3}U^*_{\beta 3}e^{i\Delta_{31}}\sin\Delta_{31} + U_{\alpha 2}U^*_{\beta 2}e^{i\Delta_{21}}\sin\Delta_{21}]$$

$$\text{where } \Delta_{ij} \equiv \Delta m^2_{ij}\frac{L}{4E} \equiv (m_i^2 - m_j^2)\frac{L}{4E} \quad .$$

$$P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) = \left| e^{-im_1^2 \frac{L}{2E}} \, \text{Amp}^*(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) \right|^2$$

$$= 4[|U_{\alpha 3}U_{\beta 3}|^2 \sin^2\Delta_{31} + |U_{\alpha 2}U_{\beta 2}|^2 \sin^2\Delta_{21}$$

$$+2|U_{\alpha 3}U_{\beta 3}U_{\alpha 2}U_{\beta 2}| \sin\Delta_{31} \sin\Delta_{21} \cos(\Delta_{32} \underset{(-)}{+} \delta_{32})] \quad .$$

Here $\delta_{32} \equiv \arg(U_{\alpha 3}U^*_{\beta 3}U^*_{\alpha 2}U_{\beta 2})$, a CP – violating phase.

Two waves of different frequencies,

and their ~~CP~~ interference.

When the Spectrum Is—

Invisible if

For $\beta \neq \alpha$,

(—) (—)

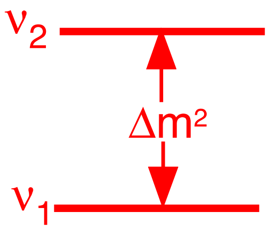
$|^2 \sin^2(\Delta m^2 \frac{L}{4E})$.

For no flavor change,

(—) (—)

Experiments with can determine the flavor content of ν_3.

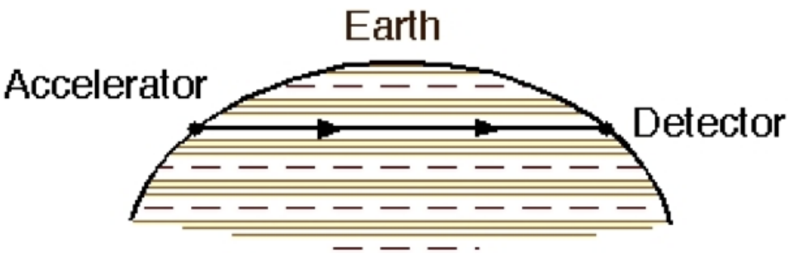
When There are Only Two Flavors and Two Mass Eigenstates



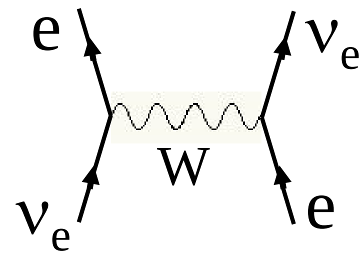
$$\begin{matrix} i\xi & 0 \\ 0 & 1 \end{matrix} \Big]$$

Mixing angle

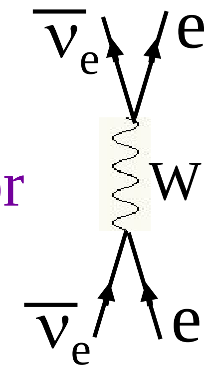
Neutrino Flavor Change In Matter



involves



or



Coherent forward scattering via this
W-exchange interaction leads to
an extra interaction potential energy —

$$V_W = \begin{cases} +\sqrt{2}G_F N_e, & \nu_e \\ -\sqrt{2}G_F N_e, & \overline{\nu_e} \end{cases}$$

Fermi constant (G_F)

Electron density (N_e)

This raises the effective mass of ν_e, and lowers that of $\overline{\nu_e}$.

The fractional importance of matter effects on an oscillation involving a vacuum splitting Δm^2 is —

$$\underbrace{[\sqrt{2}G_F N_e]}_{\text{Interaction energy}} / \underbrace{[\Delta m^2/2E]}_{\text{Vacuum energy}} .$$

The matter effect —

— Grows with neutrino energy E

— Is sensitive to Sign(Δm^2)

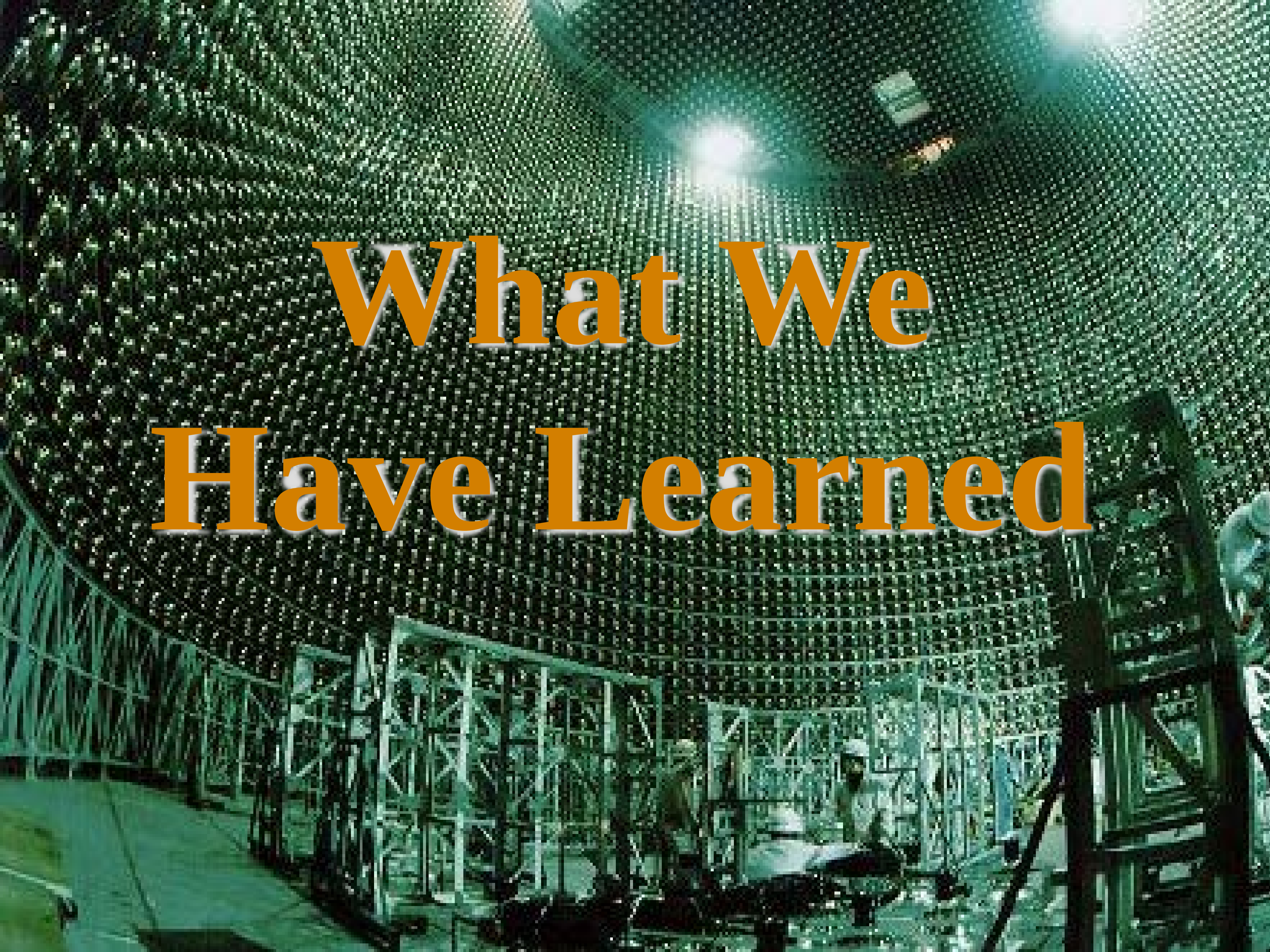
— Reverses when ν is replaced by $\bar{\nu}$

This last is a "fake CP violation" that has to be taken into account in searches for genuine CP violation.

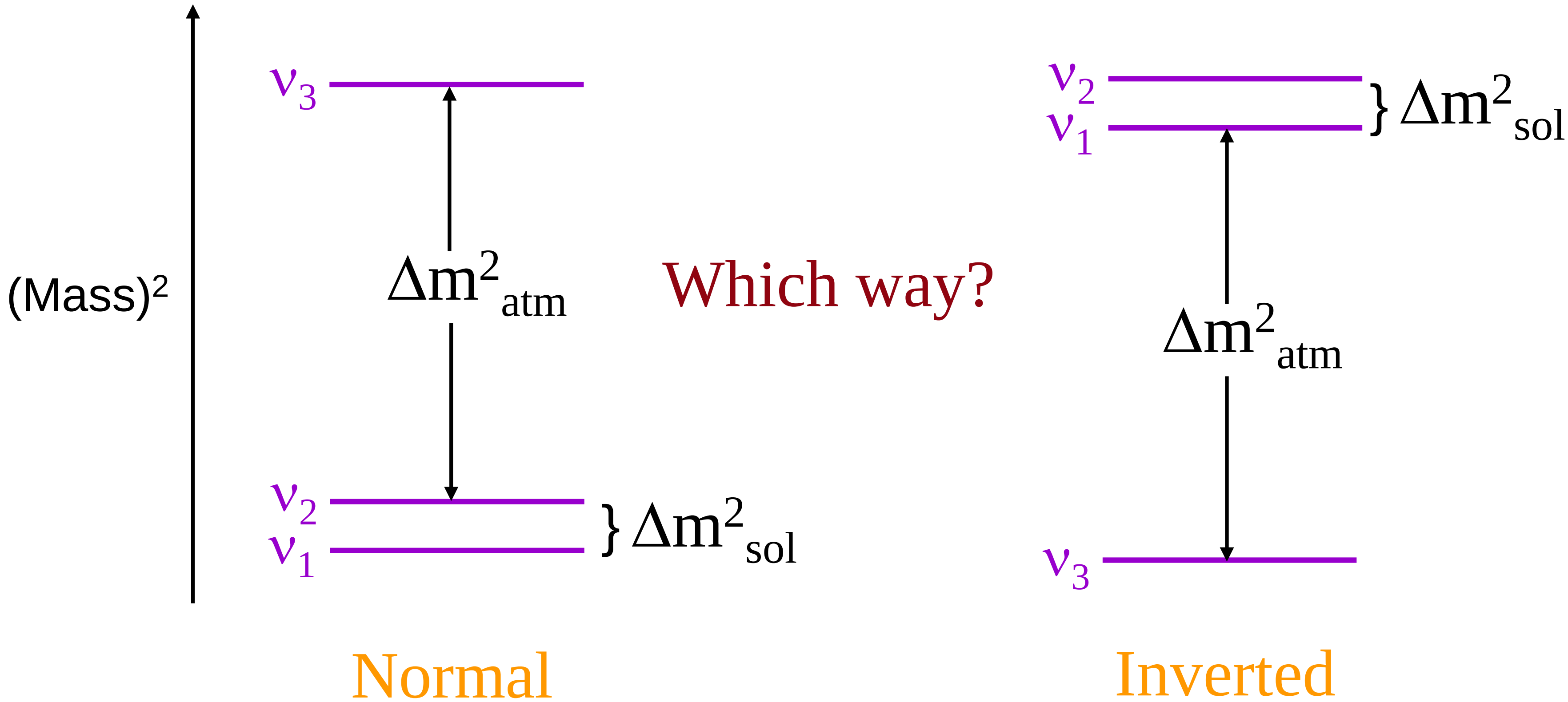
Evidence For Flavor Change

Neutrinos	Evidence of Flavor Change
Solar	Compelling
Reactor (Long-Baseline)	Compelling
Atmospheric	Compelling
Accelerator (Long-Baseline)	Compelling
Accelerator, Reactor, and Radioactive Sources (Short-Baseline)	"Interesting"

What We Have Learned



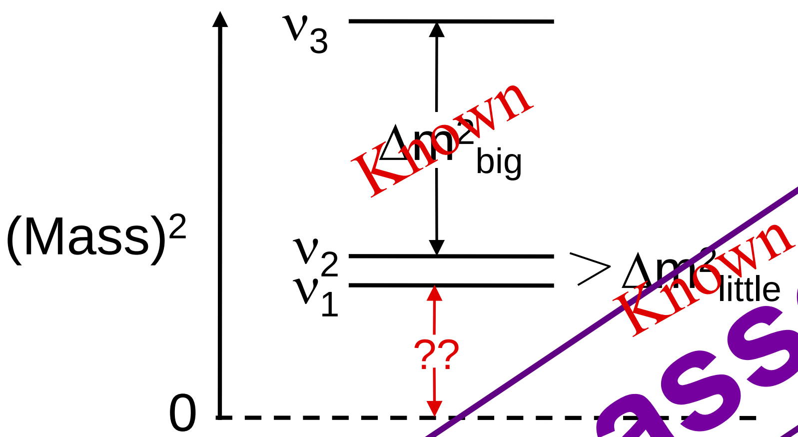
The (Mass)^2 Spectrum



$\Delta m^2_{sol} \cong 7.6 \times 10^{-5}\ eV^2$, $\Delta m^2_{atm} \cong 2.5 \times 10^{-3}\ eV^2$

Are there ***more*** mass eigenstates?

Constraints On the Absolute Scale of Neutrino Mass



How far above zero is the whole pattern?

Neutrino masses are tiny

Cosmology, *under certain assumptions* ⟹ $\sum m(\nu_i) < 0.17$ eV

All *i*

Tritium beta decay ⟹ $\sqrt{0.69m^2(\nu_1)+0.29m^2(\nu_2)+0.02m^2(\nu_3)} < 2\text{ eV}$

Oscillation ⟹ Mass[Heaviest ν_i] > $\sqrt{\Delta m^2_{big}}$ > 0.05 eV

Measurements of the tritium β energy spectrum bound the average neutrino mass —

$$\langle m_\beta \rangle = \sqrt{\sum_i |U_{ei}|^2 m_i^2}$$ (Farzan & Smirnov)

Presently: $\langle m_\beta \rangle < 2$ eV (Mainz & Troitzk)

(Lecture by Kathrin Valerius)

Leptonic Mixing

Mixing means that —

$$|\nu_\alpha> = \sum_i U^*_{\alpha i} \, |\nu_i> \; .$$

Neutrino of flavor α = e, μ, or τ

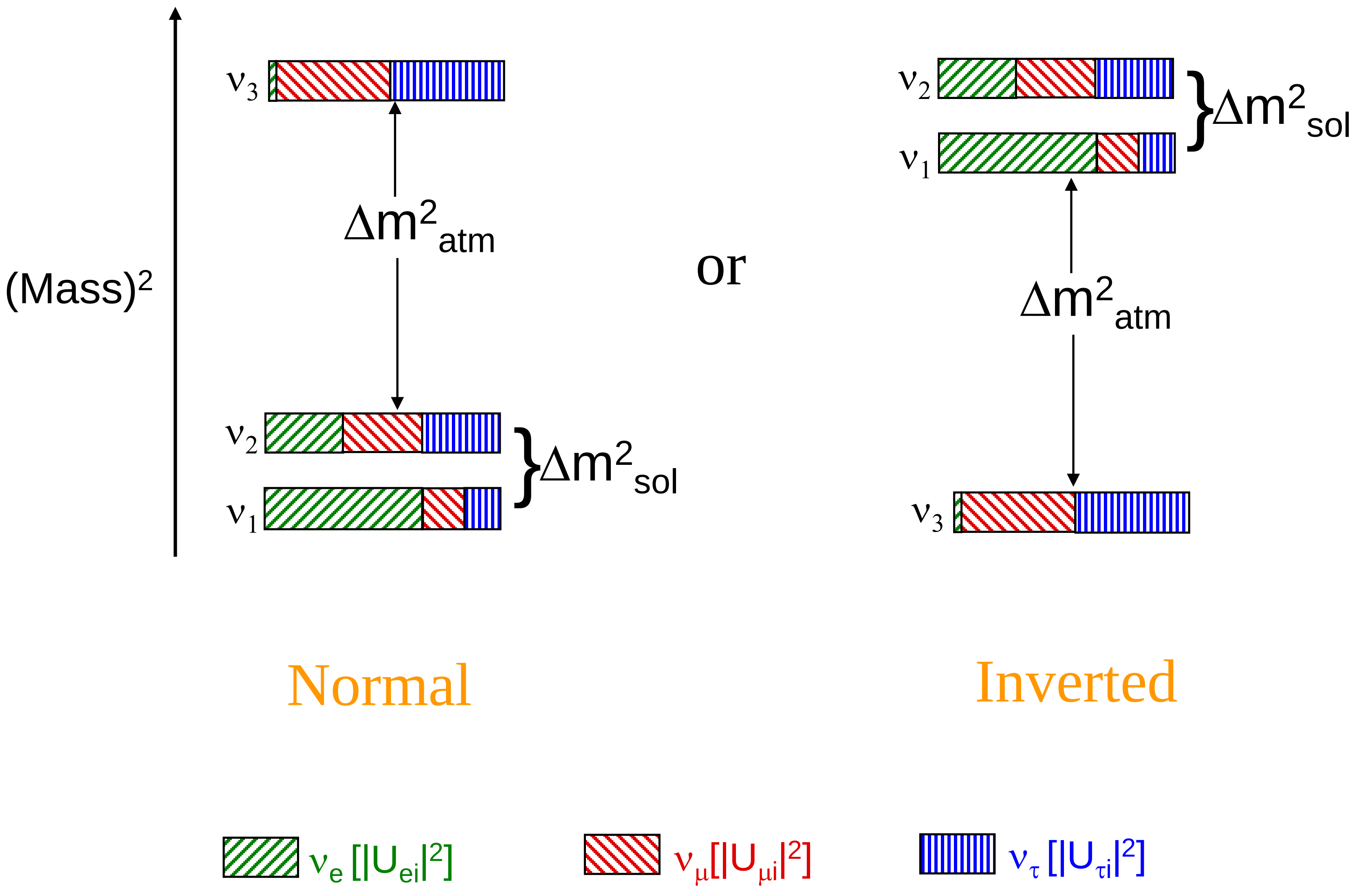
Neutrino of definite mass m_i

Inversely, $|\nu_i> = \sum_\alpha U_{\alpha i} \, |\nu_\alpha> \; .$ (***if*** U is unitary)

Flavor-α fraction of $\nu_i = |U_{\alpha i}|^2$.

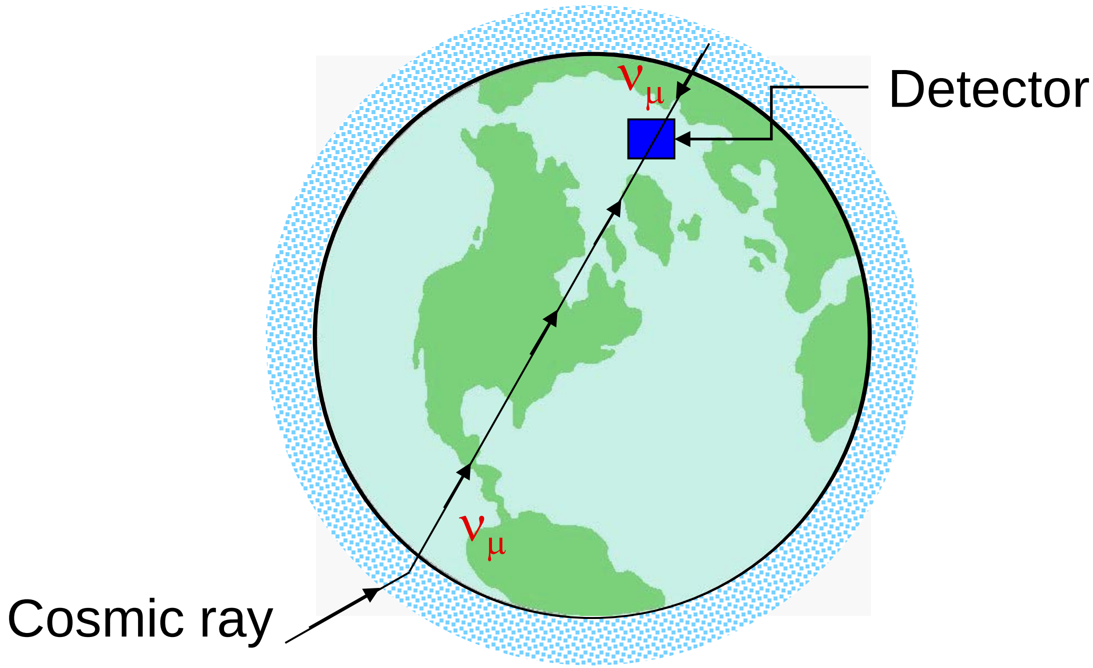
When a ν_i interacts and produces a charged lepton, the probability that this charged lepton will be of flavor α is $|U_{\alpha i}|^2$.

Experimentally, the flavor fractions are —



Observations
We Can Use
To Understand
The Flavor Fractions

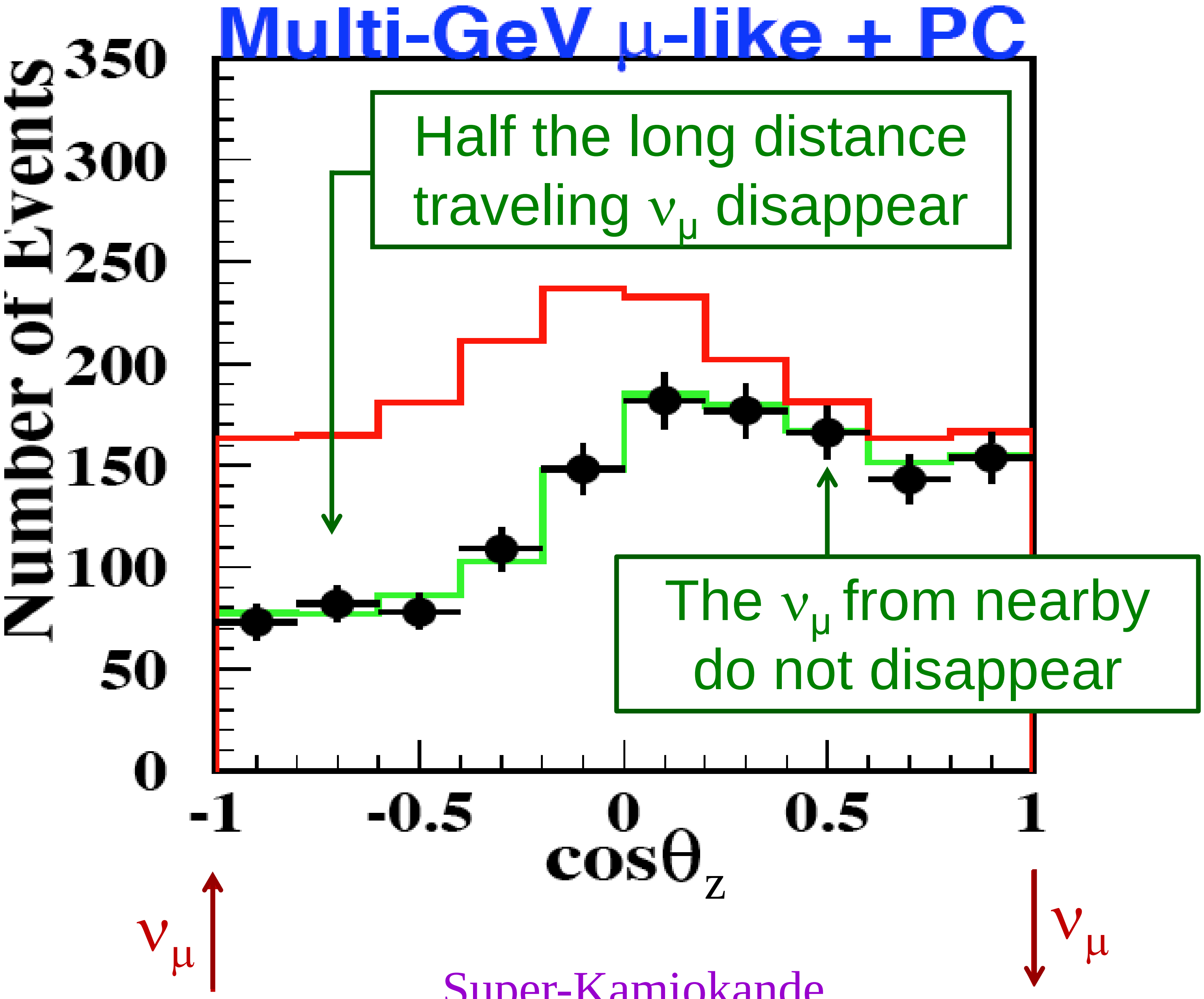
The Disappearance of Atmospheric ν_μ



Isotropy of the $\gtrsim$ 2 GeV cosmic rays + Gauss' Law + No ν_μ disappearance

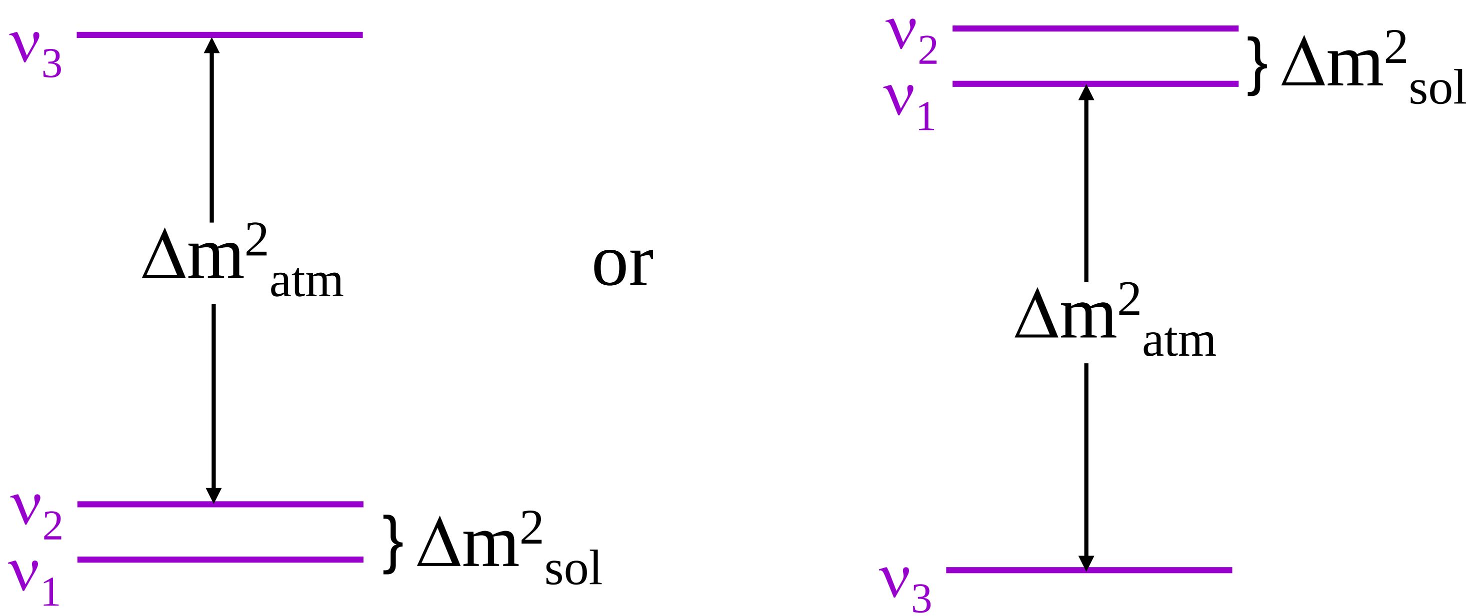
$$\Rightarrow \frac{\phi_{\nu_\mu}(\text{Up})}{\phi_{\nu_\mu}(\text{Down})} = 1 \,.$$

But Super-Kamiokande finds for $E_\nu > 1.3$ GeV —



Super-Kamiokande

At $E_\nu > 1.3$ GeV, in —



the solar splitting is largely invisible. Then—

$$\underbrace{P(\nu_\mu \to \nu_\mu)}_{\frac{1}{2}} \cong 1 - \underbrace{4|U_{\mu 3}|^2\left(1-|U_{\mu 3}|^2\right)}_{1} \underbrace{\sin^2\left[1.27\Delta m^2_{atm}\frac{L(\text{km})}{E(\text{GeV})}\right]}_{\frac{1}{2}}$$

$$1 \Rightarrow |U_{\mu 3}|^2 = \frac{1}{2}$$

At large L/E

Reactor – Neutrino Experiments and $|U_{e3}|^2 = \sin^2\theta_{13}$

Reactor $\bar{\nu}_e$ have $E \sim 3$ MeV, so if $L \sim 1.5$ km,

$$\sin^2\left[1.27\Delta m^2(\mathrm{eV}^2)\frac{L(\mathrm{km})}{E(\mathrm{GeV})}\right]$$ will be sensitive to —

$$\Delta m^2 = \Delta m^2_{\mathrm{atm}} = 2.5\,\mathrm{x}\,10^{-3}\,\mathrm{eV}^2 = \frac{1}{400}\mathrm{eV}^2$$

but not to —

$$\Delta m^2 = \Delta m^2_{\mathrm{sol}} = 7.6\,\mathrm{x}\,10^{-5}\,\mathrm{eV}^2 \approx \frac{1}{13{,}000}\mathrm{eV}^2$$

Then —

$$P(\bar{\nu}_e \to \bar{\nu}_e) \cong 1 - 4|U_{e3}|^2\left(1 - |U_{e3}|^2\right)\sin^2\left[1.27\Delta m^2_{\text{atm}}\,\frac{L(\text{km})}{E(\text{GeV})}\right]$$

Measurements by the Daya Bay, RENO,
and Double CHOOZ reactor neutrino experiments,
(and by the T2K accelerator neutrino experiment)

➡ $|U_{e3}|^2 \cong 0.02$

(Lecture by Yifang Wang)

The Change of Flavor of Solar ν_e

Nuclear reactions in the core of the sun produce ν_e. Only ν_e.

The Sudbury Neutrino Observatory (SNO) measured, for the high-energy part of the solar neutrino flux:

$$\nu_{sol}\, d \rightarrow e\, p\, p \;\Rightarrow\; \phi_{\nu_e}$$

$$\nu_{sol}\, d \rightarrow \nu\, n\, p \;\Rightarrow\; \phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau} \qquad (\nu \text{ remains a } \nu)$$

From the two reactions,

$$\frac{\phi_{\nu_e}}{\phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau}} = 0.301 \pm 0.033$$

For solar neutrinos, $P(\nu_e \rightarrow \nu_e) = 0.3$

The Significance of $P(\nu_e \rightarrow \nu_e)$

For SNO-energy-range solar neutrinos,
there is a very pronounced solar matter effect.

(Mikheyev, Smirnov, Wolfenstein)

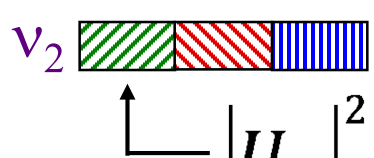
At these energies —

A solar neutrino is born in the core of the sun as a ν_e.

But by the time it emerges from the outer edge of the sun, with 91% probability it is a ν_2.

(Nunokawa, Parke, Zukanovich-Funchal)

$$\text{Then } P(\nu_e \rightarrow \nu_e) \text{ at earth } = \left|\langle \nu_e | \nu_2 \rangle\right|^2 = \left|U_{e2}\right|^2.$$



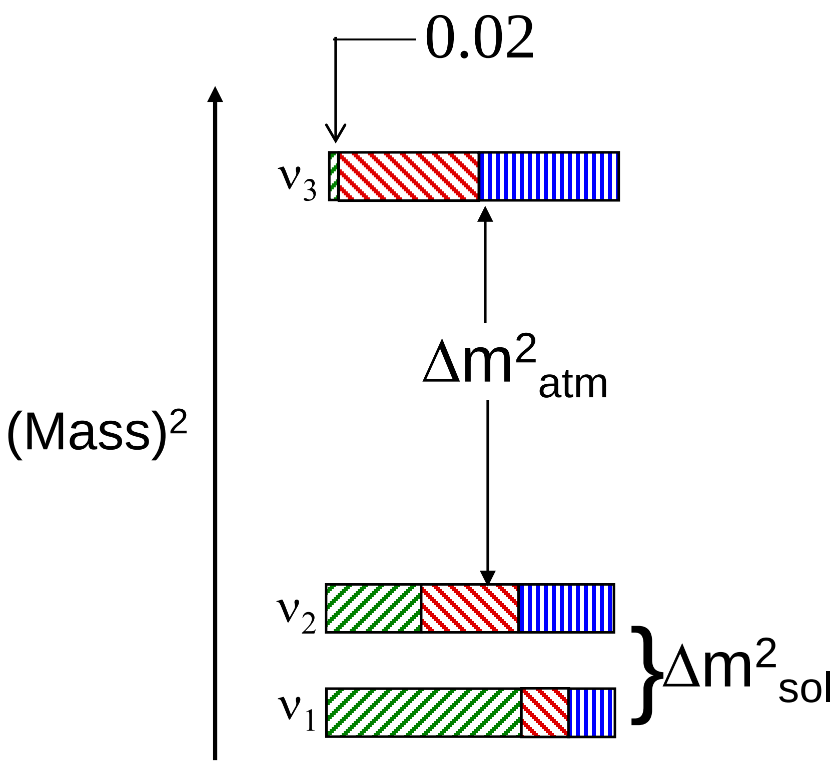
$$\left|U_{e2}\right|^2 = 0.3.$$

Constructing the Approximate Mixing Matrix

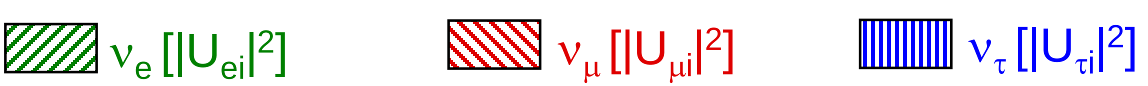
(A Blackboard Exercise)

The result —

$$
\begin{array}{cccc}
 & \nu_1 & \nu_2 & \nu_3 \\
U \approx & \left[\begin{array}{ccc}
\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\
-\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\
\sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}}
\end{array} \right] & \begin{array}{c} \nu_e \\ \nu_\mu \\ \nu_\tau \end{array}
\end{array}
$$



$$
U \approx
\begin{array}{c}
\begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\
\left[\begin{array}{ccc}
\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\
-\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\
\sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}}
\end{array}\right]
\end{array}
\begin{array}{c} \nu_e \\ \nu_\mu \\ \nu_\tau \end{array}
$$



The Lepton Mixing Matrix U

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \times \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \times \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$c_{ij} \equiv \cos\theta_{ij}$$
$$s_{ij} \equiv \sin\theta_{ij}$$

$$\times \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Note big mixing!

Majorana phases

$\theta_{12} \approx 35^\circ$, $\theta_{23} \approx 42\text{-}51^\circ$, $\theta_{13} \approx 8.5^\circ$ ← *Not very small!*

The phases violate CP. δ would lead to $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$.

↑ ~~CP~~

But note the crucial role of $s_{13} \equiv \sin\theta_{13}$.

There is already a 2σ hint of ~~CP~~ ($\sin\delta \neq 0$) from T2K.

The Majorana ~~CP~~ Phases

The phase α_i is associated with neutrino mass eigenstate ν_i:

$$U_{\alpha i} = U^0{}_{\alpha i} \exp(i\alpha_i/2) \text{ for all flavors } \alpha.$$

$$\text{Amp}(\nu_\alpha \rightarrow \nu_\beta) = \sum_i U_{\alpha i}{}^* \exp(-\, i m_i{}^2 L/2E)\, U_{\beta i}$$

is insensitive to the Majorana phases α_i .

Only the phase δ can cause CP violation in neutrino oscillation.

There Is Nothing Special About θ_{13}

All mixing angles must be nonzero for ~~*CP*~~ in oscillation.

For example —

$$P(\bar{\nu}_\mu \to \bar{\nu}_e) - P(\nu_\mu \to \nu_e) = 2\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin\delta$$
$$\times \sin\left(\Delta m^2{}_{31}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{32}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{21}\frac{L}{4E}\right)$$

In the factored form of *U*, one can put δ next to θ_{12} instead of θ_{13}.

Looking to the Future

Open Questions

•Are neutrinos their own antiparticles?

•Is the physics behind the masses of neutrinos different from that behind the masses of all other known particles?

•What is the absolute scale of neutrino mass?

•Is the spectrum like  or  ?

- Is θ_{23} maximal?

- Do neutrino interactions violate CP?

Is $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$?

- Is CP violation involving neutrinos the key to understanding the matter – antimatter asymmetry of the universe?

- What can neutrinos and the universe tell us about one another?

- Are there ***more*** than 3 mass eigenstates?
  - Are there "sterile" neutrinos that don't couple to the W or Z?

- Do neutrinos have Non-Standard-Model interactions?

- Do neutrinos break the rules?
  - Violation of Lorentz invariance?
  - Violation of CPT invariance?
  - Departures from quantum mechanics?

Are Neutrinos Their Own Antiparticles?
(The Majorana vs. Dirac Question)

Is each neutrino mass eigenstate, such as ν_1,

a Majorana fermion $\overline{\nu_1}(h) = \nu_1(h)$

↑ Helicity

or

a Dirac fermion $\overline{\nu_1}(h) \neq \nu_1(h)$

This question is particularly interesting because of its relation to another question:

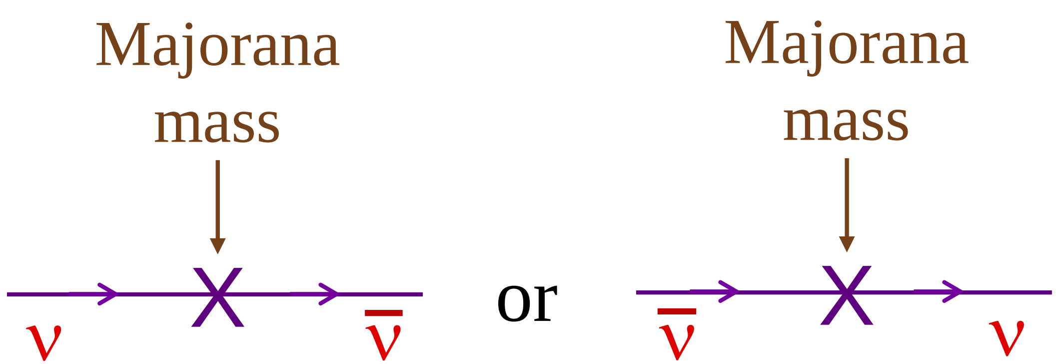
What is the origin of the neutrino masses?

(Lectures by Alexei Smirnov)

In particular, are there neutrino *Majorana mass terms*?

Imagine a world with just one flavor,
and correspondingly, just one neutrino mass eigenstate.

Acting on underlying, distinct, neutrino states ν and $\bar{\nu}$
out of which the mass eigenstate "ν_1" is composed,
a Majorana mass has the effect —



A Majorana mass for any fermion f causes $f \leftrightarrow \bar{f}$.

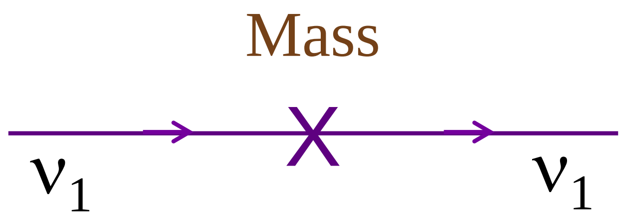
Quark and ***charged-lepton*** Majorana masses
are **forbidden** by electric charge conservation.

Among the fundamental fermions,
only ***neutrinos*** may have Majorana masses.

Having them would make the neutrinos special.

The Mass Eigenstates When There Are Majorana Masses

For any fermion mass eigenstate, e.g. ν_1, the action of its mass must be —

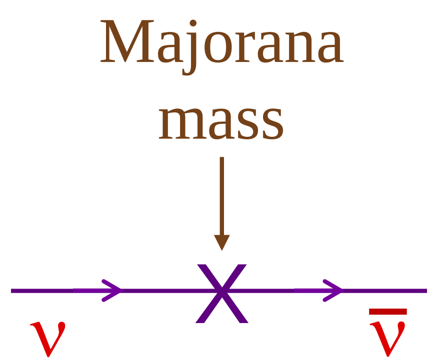


The mass eigenstate must be sent back into itself:

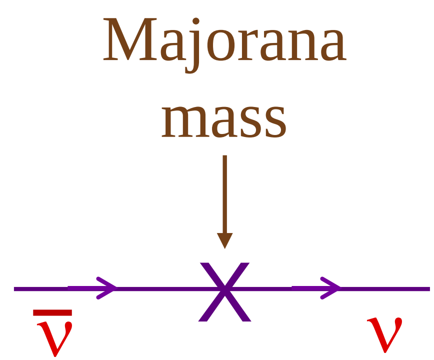
$$H|\nu_1\rangle = m_1|\nu_1\rangle$$

Recall that —

A ***Majorana*** mass has the effect:



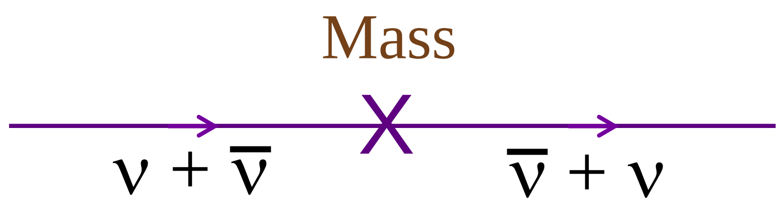
or



Then the mass eigenstate neutrino ν_1 must be —

$$\nu_1 = \nu + \bar{\nu} \,,$$

since this is the neutrino that the Majorana mass term sends back into itself, as required for any mass eigenstate particle:



Consequence: The neutrino mass eigenstates ν_1, ν_2, ν_3 are their own antiparticles.

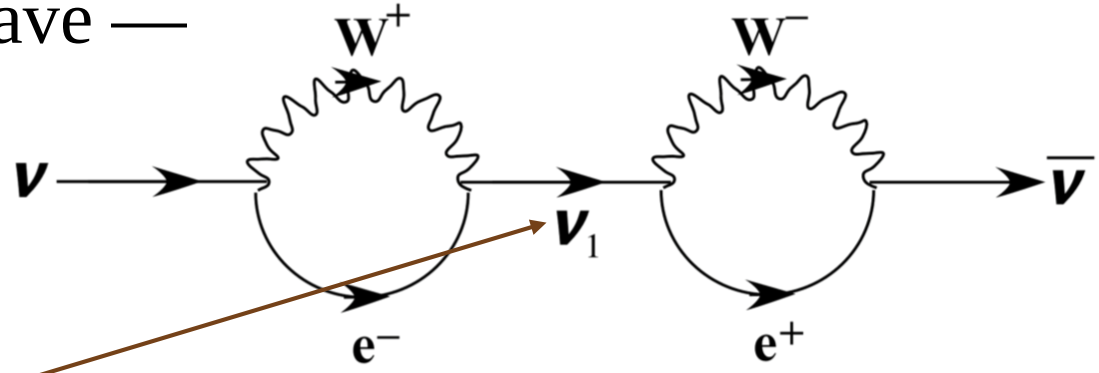
$$\bar{\nu}_i = \nu_i$$ For given helicity

If the masses of the neutrino mass eigenstates come from ***both*** Majorana mass terms and Standard Model-style "Dirac" mass terms, the neutrino mass eigenstates will still be Majorana particles.

(Bilenky and Petcov)

And going the other way —

If the neutrino mass eigenstates are Majorana particles, we can have —



Mass eigenstate

This is a Majorana mass.

Majorana masses ⟺ ***Majorana neutrinos***

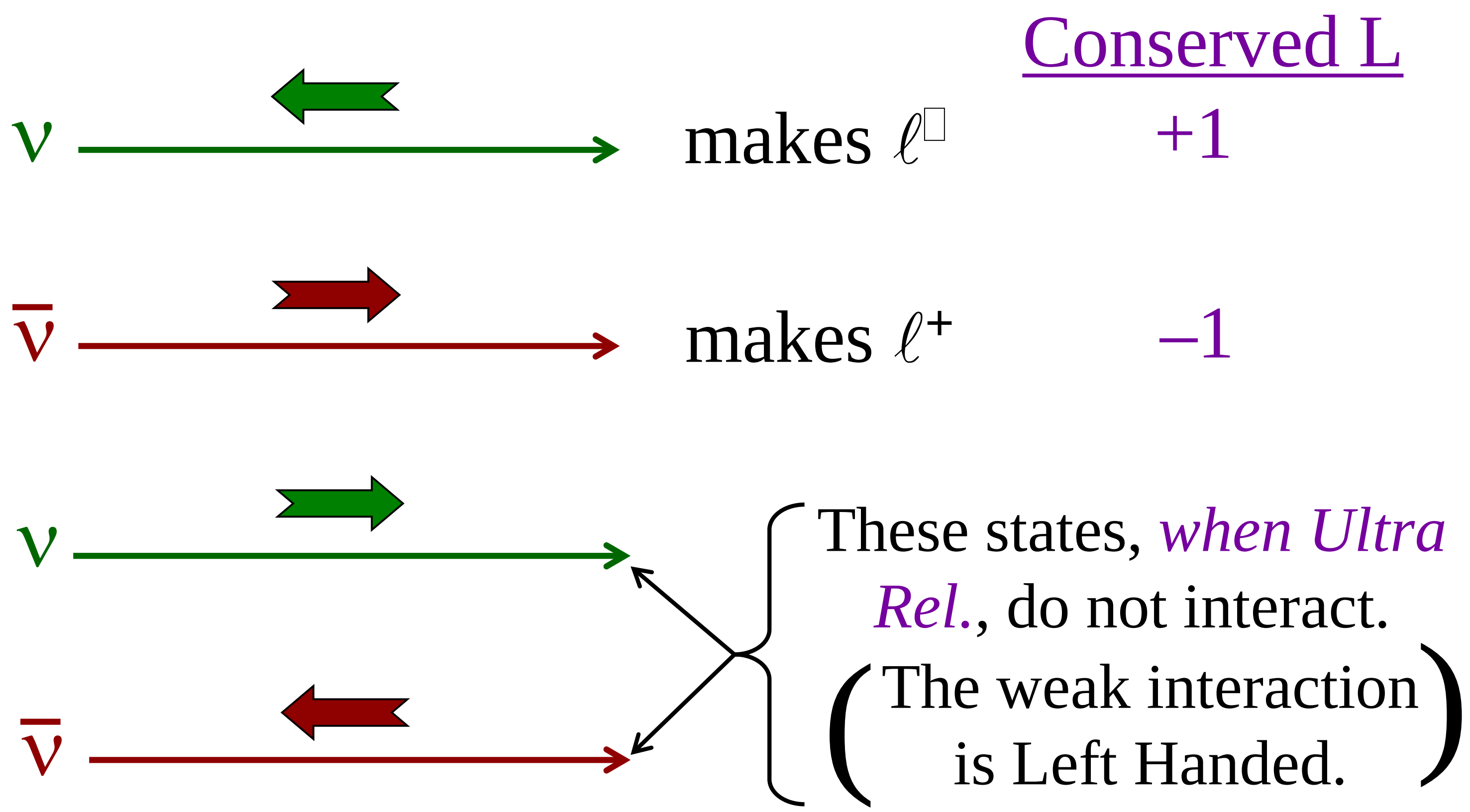
⇓

L nonconservation

The Interactions of Dirac, and Especially Majorana, Neutrinos

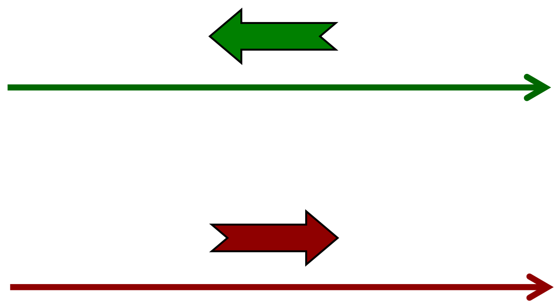
SM Interactions Of A Dirac Neutrino

We have 4 mass-degenerate states:



SM Interactions Of A Majorana Neutrino

We have only 2 mass-degenerate states:

ν  makes ℓ^-

$\bar{\nu}$ makes ℓ^+

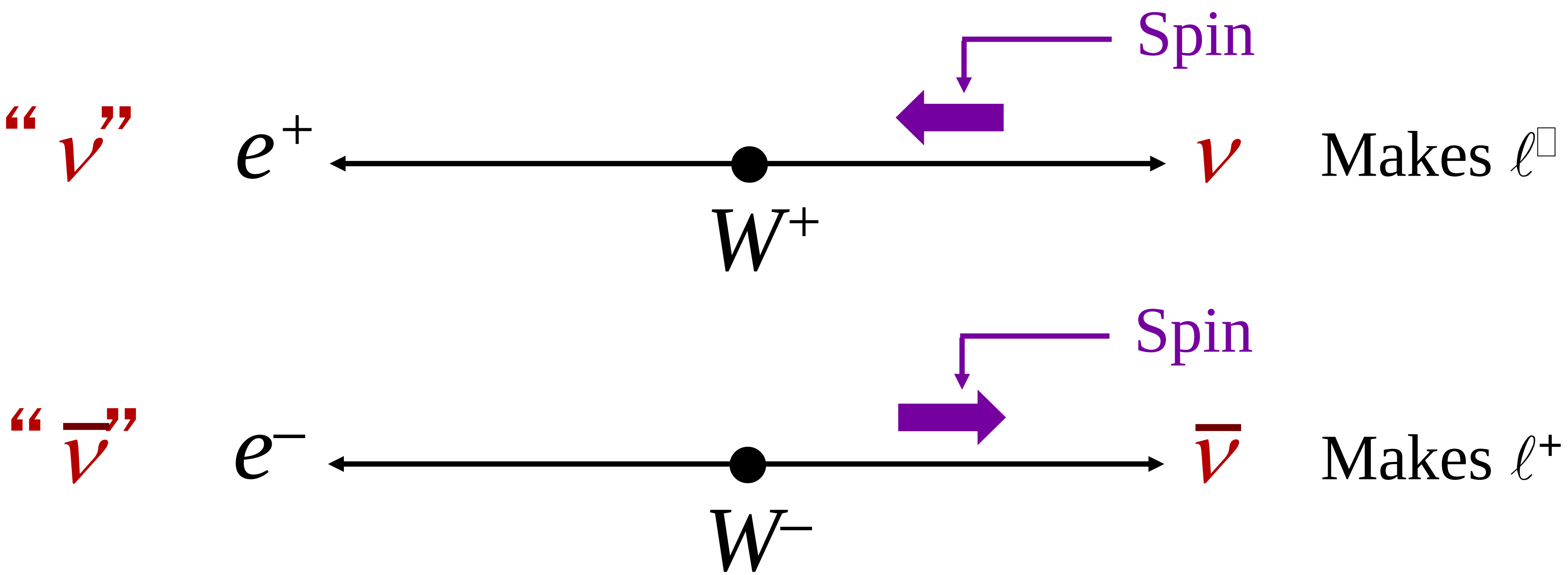
The SM weak interactions violate *parity*.

(They can tell *Left* from *Right*.)

An incoming left-handed neutral lepton makes ℓ^-.

An incoming right-handed neutral lepton makes ℓ^+.

Note: “ν” and “$\bar{\nu}$” are ***produced*** with opposite helicity.



The weak interactions violate ***Parity***. ***Particles with left-handed and right-handed helicity can behave differently.***

For ***ultra-relativistic Majorana*** neutrinos,
helicity is a "substitute" for lepton number.

Majorana neutrinos behave
indistiguishably from Dirac neutrinos.

However, for ***non-relativistic*** neutrinos,
there can be a big difference between the behavior
of Majorana neutrinos and Dirac neutrinos.

To Determine Whether

$$\bar{\nu} = \nu$$

The Major Approach — Seek

Neutrinoless Double Beta Decay [0νββ]

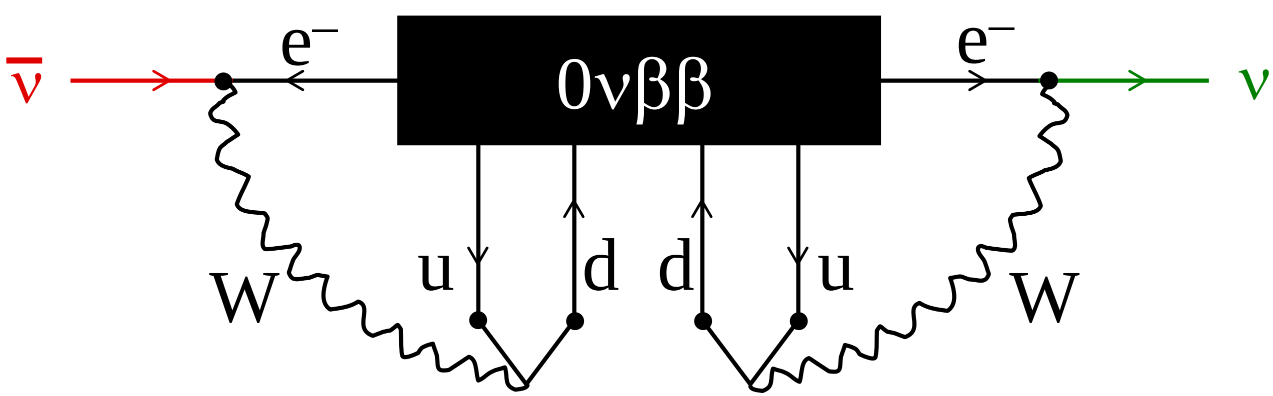
(Lectures by Andrea Giuliani and Javier Menendez)

Observation at any non-zero level would imply —

- Lepton number L is not conserved ($\Delta L = 2$)
- Neutrinos have Majorana masses
- Neutrinos are Majorana particles (self-conjugate)

Whatever diagrams cause 0νββ, its observation would imply the existence of a Majorana mass term:

(Schechter and Valle)



$\bar{\nu} \rightarrow \nu$: A (tiny) Majorana mass term

$\therefore$ 0νββ ➡ $\bar{\nu}_i = \nu_i$

An Alternative Approach:
Study the Decays of a Heavy Neutral Lepton

Of course, this requires the *existence* of a heavy neutral lepton.

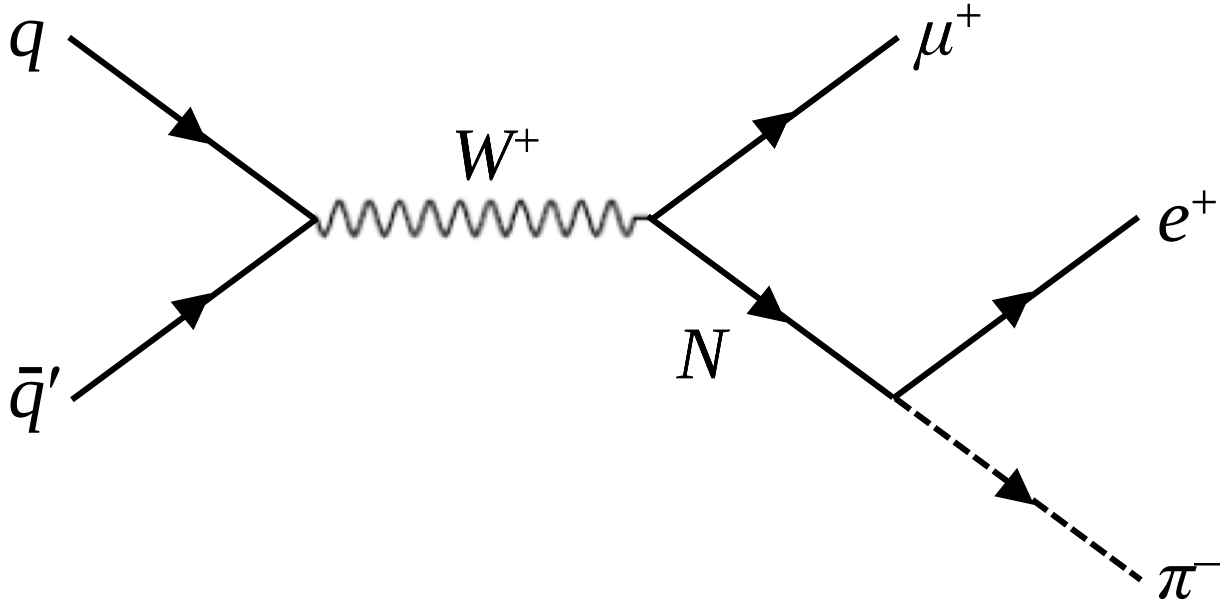
(Lecture by Dmitry Gorbunov)

A heavy neutral lepton is being sought at CERN, J-PARC, Fermilab, and perhaps elsewhere.

How Decays Can Be Revealing

Suppose there is a heavy neutral lepton ***N*.**

A chain (say at the LHC) like —



would violate lepton number conservation, and signal that N is a Majorana neutrino.

To look for this, a detector must have charge discrimination.

Working Without Charge Discrimination

The angular distributions in the decays —

$$N \rightarrow \nu + X$$

$\nu_1, \nu_2, \text{or } \nu_3$; $X = \bar{X}$

could also reveal whether *N* is a Dirac or a Majorana particle.

(Balantekin, de Gouvêa, B. K.)

Depending on the mass of *N*, we could have —

$$X = \gamma, \pi^0, \rho^0, Z^0, \text{or } H^0$$

.

The leptons N are expected to be highly polarized by their production mechanism. Nevertheless —

From nothing but rotational and CPT invariance, if N is a Majorana particle, the angular distribution of daughter X in the N rest frame will be ***isotropic***.

But if N is a Dirac particle, then —

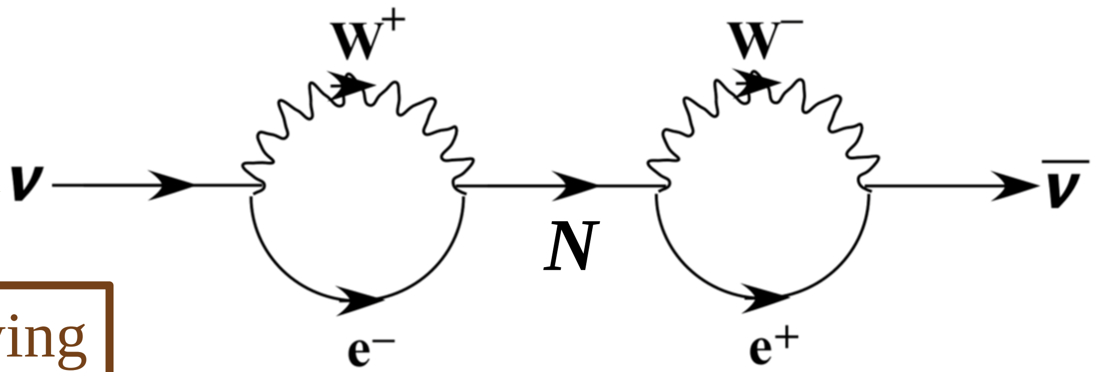
$$\frac{d\Gamma(N \to \nu + X)}{d(\cos\theta_X)} = \Gamma_0 (1 + \alpha \cos\theta_X)$$

(θ_X w.r.t. $\vec{s}_N$)

with —

X	γ	π^0	ρ^0	Z^0	H^0
Quite non-isotropic					

If *N* is a Majorana particle, then ***all*** the neutrinos are Majorana particles.



Some underlying neutrino

This is a Majorana mass.

Majorana masses ⇒ ***Majorana neutrinos***

The Search for CP Violation When It Might Be That

$$\bar{\nu} = \nu$$

Whether neutrino interactions violate CP invariance is a major open question.

The experimental approach to testing for this violation is almost always described as the attempt to see whether —

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \neq P(\nu_\mu \rightarrow \nu_e).$$

This description is valid if $\bar{\nu} \neq \nu$, but not if $\bar{\nu} = \nu$.

However, the present and future experimental probes of leptonic CP-invariance violation are valid probes of this violation whether $\bar{\nu} \neq \nu$ *or* $\bar{\nu} = \nu$.

These experiments are ***completely insensitive*** to whether $\bar{\nu} \neq \nu$ or $\bar{\nu} = \nu$.

For any process $i \to f$, and its CP-mirror image $\mathrm{CP}(i) \to \mathrm{CP}(f)$, CP invariance means that —

$$\left|\langle f|T|i\rangle\right|^2 = \left|\langle \mathrm{CP}(f)|T|\mathrm{CP}(i)\rangle\right|^2.$$

So, compare two CP-mirror-image processes.

If they have different rates, CP invariance is violated.

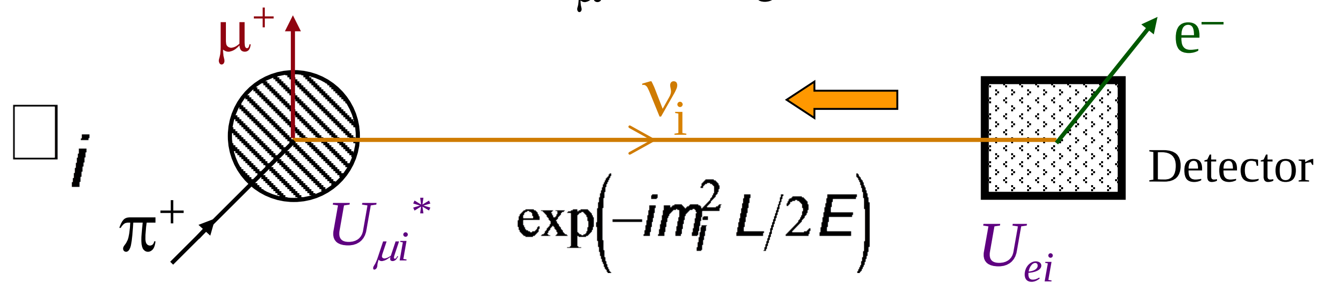
Acting on a particle ψ with momentum $\vec{p}$ and helicity h,

$$\mathrm{Rotation}(\pi)\mathrm{CP}\left|\psi(\vec{p},h)\right\rangle = \eta\left|\bar{\psi}(\vec{p},-h)\right\rangle$$

η ← Irrelevant phase factor

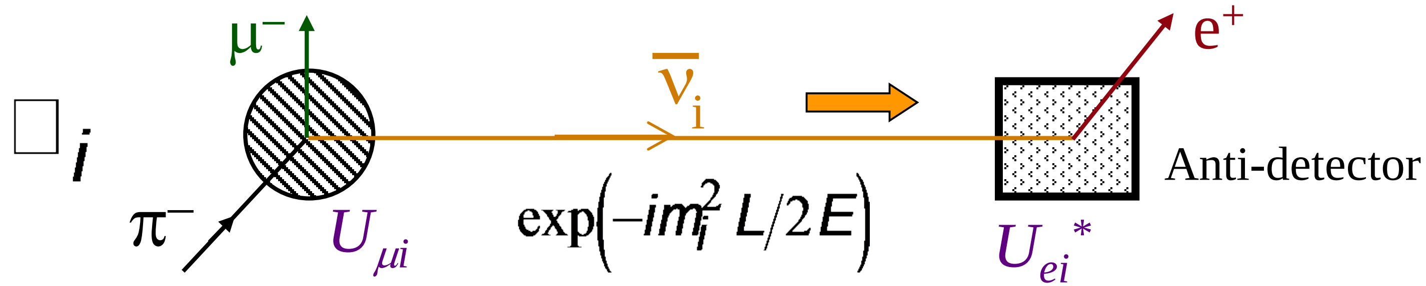
Compare

$$\nu_\mu \rightarrow \nu_e$$



with

$$\text{“}\ \bar{\nu}_\mu \rightarrow \bar{\nu}_e\ \text{”}$$



If these two CP-mirror-image processes have different rates, CP invariance is violated.

Important Notice

To correct for our not using an anti-detector, we must know how the cross sections for left-handed and right-handed neutrinos to interact in a detector compare.

Experiments to determine these cross sections are very important.

(Lecture by Jan Sobczyk)

Neutrino Physics References

Books

The Physics of Massive Neutrinos, B. K., with F. Gibrat-Debu and F. Perrier (World Scientific, Singapore, 1989).

Physics of Neutrinos, M. Fukugita and T. Yanagida (Springer, Berlin/Heidelberg, 2003).

Fundamentals of Neutrino Physics and Astrophysics, C. Giunti and C. Kim (Oxford University Press, Oxford, 2007).

The Physics of Neutrinos, V. Barger, D. Marfatia, and K. Whisnant (Princeton University Press, Princeton, 2012).

Papers

"Neutrino Masses, Mixing, and Oscillations,"
K. Nakamura and S. Petcov, in M. Tanabashi et al. (PDG), Phys. Phys. D 98, 030001 (2018). Also at http://pdg.lbl.gov/2019/reviews/rpp2018-rev-neutrino-mixing.pdf

"Neutrino Mass, Mixing, and Flavor Change," B. K., in **Neutrino Mass**, eds. G. Altarelli and K. Winter (Springer, Berlin/Heidelberg, 2003). Also eprint hep-ph/0211134. This paper discusses quite a few of the topics covered in the lectures.

“Neutrino Oscillation Physics,” B. K., in **Proceedings of the International School on AstroParticle Physics**, eds. G. Bellini and L. Ludhova (IOS Press, Amsterdam, 2012), and in **Proceedings of the 2011 European School of High-Energy Physics**, eds. C. Grojean and M. Mulders (CERN, Geneva, 2014). Also arXiv:1206.4325. This paper derives the probability for neutrino oscillation without assuming that all neutrino mass eigenstates in a beam have the same energy, or else the same momentum.

“Light Sterile Neutrinos: A White Paper,” K. Abazajian et al., arXiv:1204.5379.

Good luck!

Back Up

The Origin of Neutrino Mass

The fundamental constituents of matter
are the ***quarks***, the ***charged leptons***,
and the ***neutrinos***.

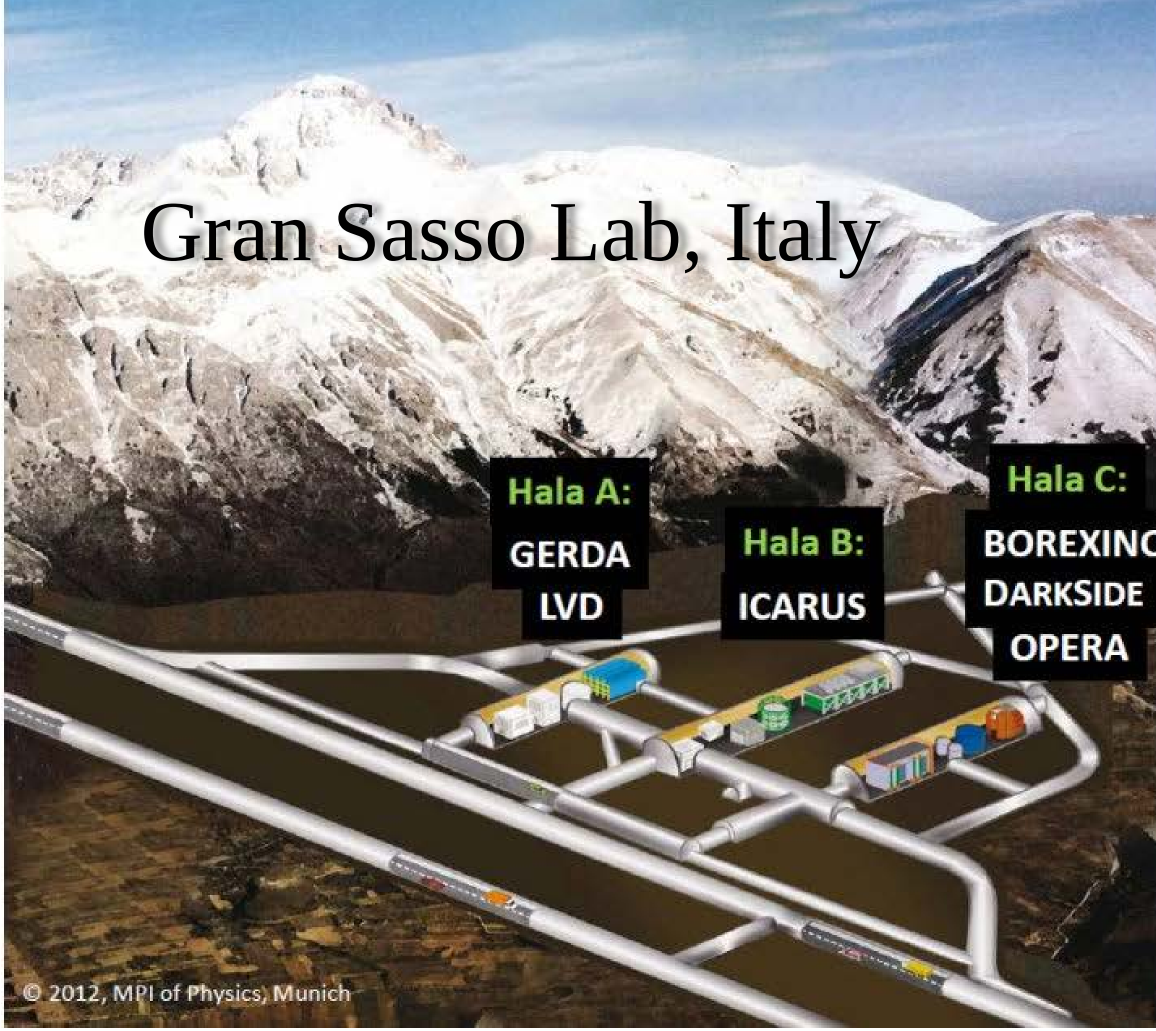
The discovery and study of the ***Higgs boson*** at CERN's Large Hadron Collider has provided strong evidence that the ***quarks*** and ***charged leptons*** derive their masses from an interaction with the ***Higgs field***.

Most theorists strongly suspect that the origin of the neutrino masses is different from the origin of the quark and charged lepton masses.

The Standard-Model ***Higgs field*** is probably still involved, but there is probably something more — something way outside the Standard Model —

Majorana masses.

More later ……….



Is the Origin of Neutrino Mass Different?

Neutrino Masses Without Field Theory

We will describe what the quantum field theory does, but without equations.

For simplicity, let us treat a world with just one flavor, and correspondingly, just one neutrino mass eigenstate.

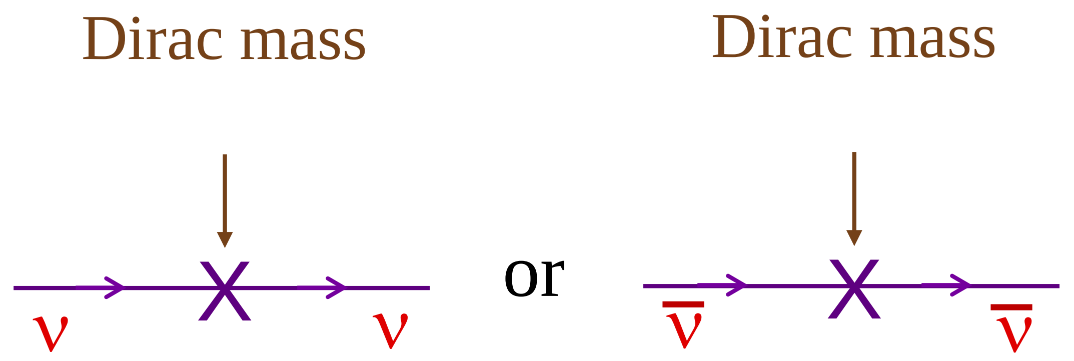
We start with underlying neutrino states ν and $\bar{\nu}$ that are distinct from each other, like other familiar fermions, and are not the mass eigenstates.

We will have to see what the mass eigenstates are later.

We can have two types of masses:

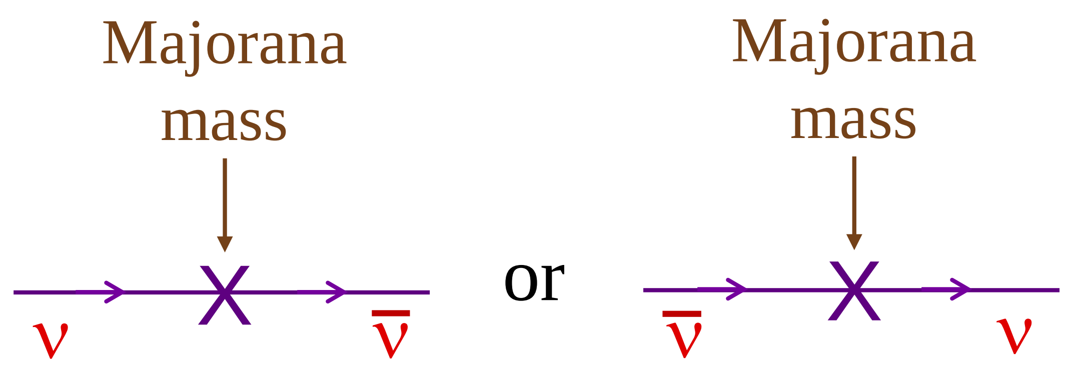
Dirac Mass

A Dirac mass has the effect:



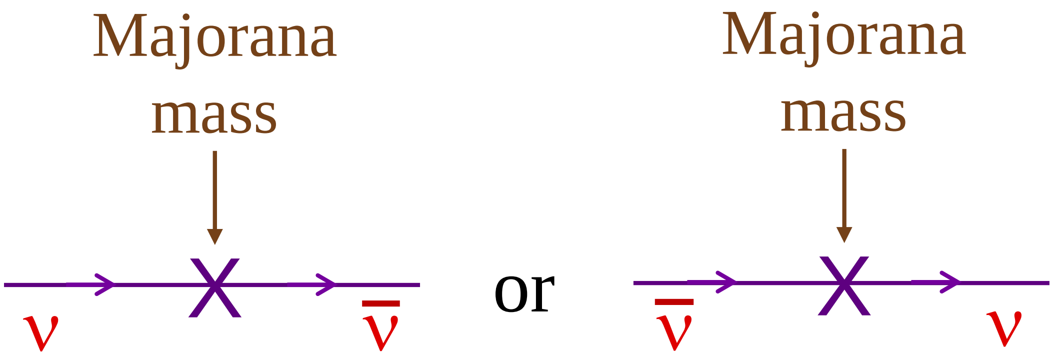
Majorana Mass

A Majorana mass has the effect:



Majorana Mass

A Majorana mass has the effect:



Majorana masses mix ν and ν̄, so they do not conserve the Lepton Number L that distinguishes leptons from antileptons:

$$L(\nu) = L(\ell^-) = -L(\bar{\nu}) = -L(\ell^+) = 1$$

If there are no visibly large non-SM interactions that violate lepton number L, any violation of L that we might discover would have to come from Majorana neutrino masses.

A Majorana mass for any fermion f causes $f \longleftrightarrow \bar{f}$.

Quark and ***charged-lepton*** Majorana masses are **forbidden** by electric charge conservation.

But ***neutrinos*** are electrically neutral, so they **can** have Majorana masses.

Neutrino Majorana masses would make the neutrinos *very* distinctive, because —

Majorana neutrino masses have a different origin than the quark and charged-lepton masses.

(Lectures by Alexei Smirnov)

The Terminology

Suppose ν_i is a ***mass eigenstate***,

with given helicty h.

- $\overline{\nu_i}(h) = \nu_i(h)$ *Majorana neutrino*

or

- $\overline{\nu_i}(h) \neq \nu_i(h)$ *Dirac neutrino*

We have just shown that if the underlying neutrino masses are *Majorana masses*, then the mass eigenstates are *Majorana neutrinos*.

What Tritium β Decay Measures

Tritium decay: $\quad {}^{3}H \rightarrow {}^{3}He + e^{-} + \bar{\nu}_i \;;\; i = 1, 2, \text{ or } 3$

There are 3 distinct final states.

The amplitudes for the production of these 3 distinct final states contribute *incoherently*.

$$BR\left({}^{3}H \rightarrow {}^{3}He + e^{-} + \bar{\nu}_i\right) \propto \left|U_{ei}\right|^2$$

In ${}^{3}H \rightarrow {}^{3}He + e^{-} + \bar{\nu}_i$, the bigger m_i is, the smaller the maximum electron energy is.

There are 3 separate thresholds in the β energy spectrum.

The β energy spectrum is modified according to —

$$(E_0 - E)^2 \Theta[E_0 - E] \to \sum_i |U_{ei}|^2 (E_0 - E)\sqrt{(E_0 - E)^2 - m_i^2}\, \Theta[(E_0 - m_i) - E]$$

Maximum β energy when there is no neutrino mass

β energy

Present experimental energy resolution is insufficient to separate the thresholds.

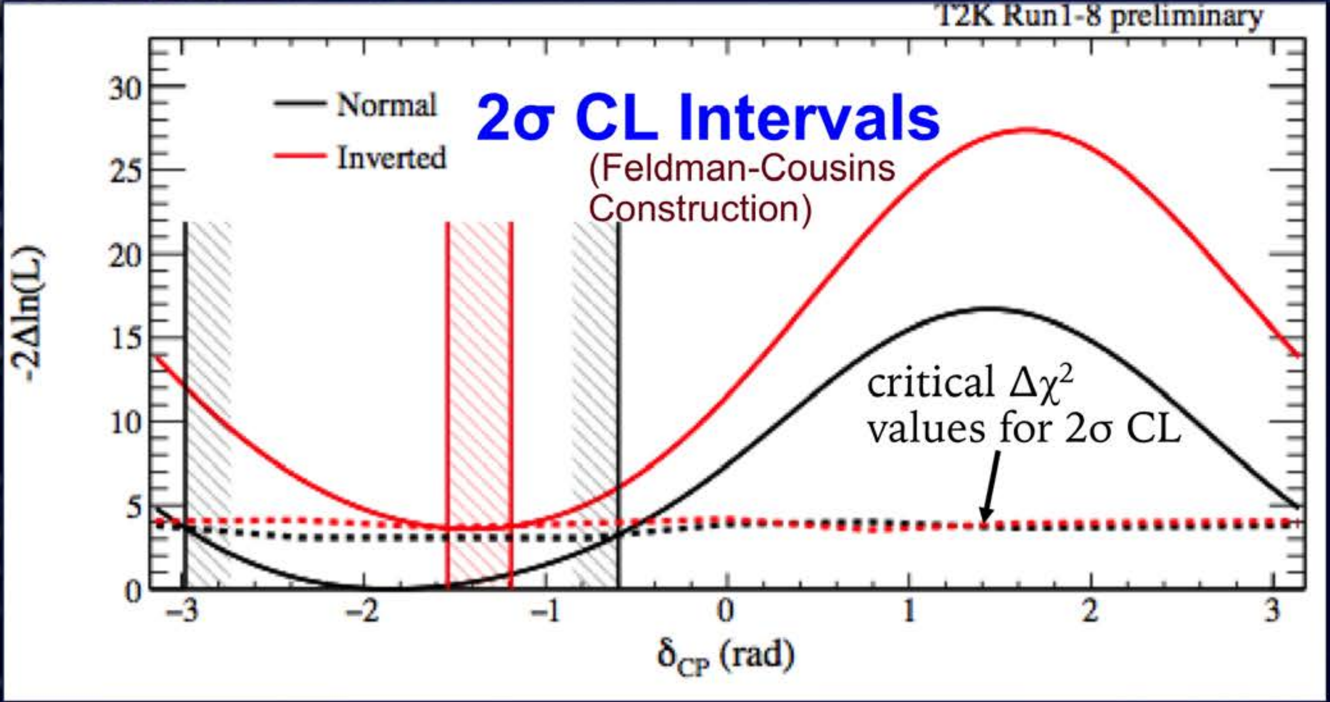
Measurements of the tritium β energy spectrum bound the average neutrino mass —

$$\langle m_\beta \rangle = \sqrt{\sum_i |U_{ei}|^2 m_i^2}$$ (Farzan & Smirnov)

Presently: $\langle m_\beta \rangle < 2\ \text{eV}$ (Mainz & Troitzk)

(Lectures by Igor Tkachev & Loredana Gastaldo)

"Full" Joint Analysis Results on δ_{CP} (with Reactors' Measurement of $\sin^2\theta_{13}$ as a Constraint)



From T2K

- Best fit: $\delta_{CP} = -1.83$ rads in NH
- 1σ CL interval:
  - NH: [−2.49, −1.23] rads
- 2σ CL intervals:
  - NH: [−2.98, −0.60] rads
  - IH: [−1.54, −1.19] rads

→ CP conserving values (0, ±π) fall outside of the 2σ intervals

Parametrizing the 3 X 3 Unitary Leptonic Mixing Matrix

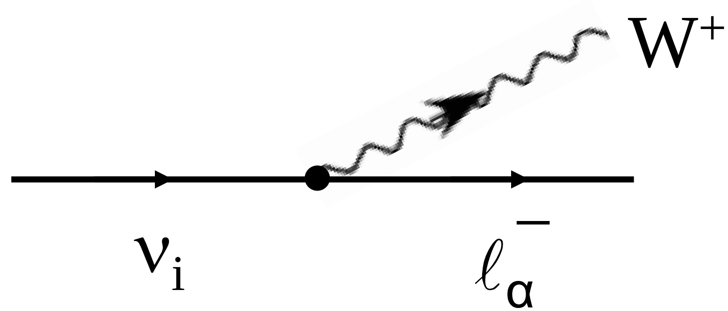
Caution: We are *assuming* the mixing matrix *U* to be 3 x 3 and unitary.

$$\mathcal{L}_{SM} = -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} \left(\overline{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}^{-} + \overline{\nu}_{Li} \gamma^{\lambda} U_{\alpha i}^{*} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

$$(CP)\left(\overline{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}^{-} \right)(CP)^{-1} = \overline{\nu}_{Li} \gamma^{\lambda} U_{\alpha i} \ell_{L\alpha} W_{\lambda}^{+}$$

Phases in *U* will lead to CP violation, unless they are removable by redefining the leptons.

$U_{\alpha i}$ describes —



$$U_{\alpha i} \sim \langle \ell_\alpha^- W^+ | H | \nu_i \rangle$$

When $| \nu_i \rangle \rightarrow | e^{i\varphi} \nu_i \rangle$, $U_{\alpha i} \rightarrow e^{i\varphi} U_{\alpha i}$, all α

When $| \ell_\alpha^- \rangle \rightarrow | e^{i\varphi} \ell_\alpha^- \rangle$, $U_{\alpha i} \rightarrow e^{-i\varphi} U_{\alpha i}$, all i

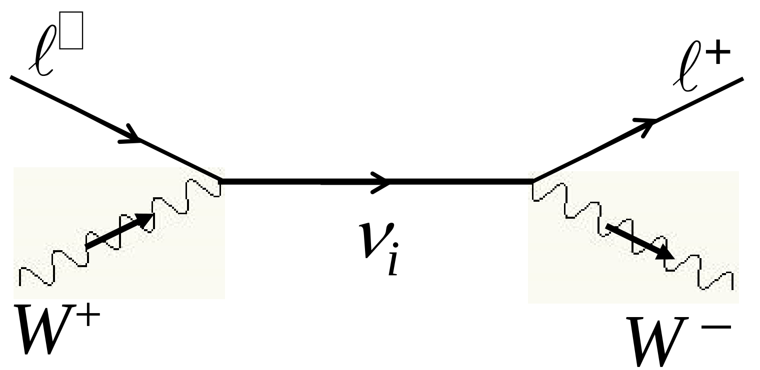
Thus, one may multiply any column, or any row, of U by a complex phase factor without changing the physics.

Some phases may be removed from U in this way.

When the Neutrino Mass Eigenstates Are Their Own Antiparticles

When this is the case, processes that do not conserve the lepton number $L \equiv$ #(Leptons) – #(Antileptons) can occur.

Example:



The amplitude for any such L-violating process contains an extra factor.

When we phase-redefine ν_i to remove a phase from U, that phase just moves to the extra factor.

It does not disappear from the physics.

Hence, when $\bar{\nu}_i = \nu_i$, U can contain extra physically-significant phases.

These are called Majorana phases.

How Many Mixing Angles and ~~CP~~ Phases Does U Contain?

Real parameters before constraints:	18
Unitarity constraints — $\sum_i U_{\alpha i}^* U_{\beta i} = \delta_{\alpha\beta}$	
Each row is a vector of length unity:	– 3
Each two rows are orthogonal vectors:	– 6
Rephase the three ℓ_α:	– 3
Rephase two ν_i, if $\overline{\nu}_i \neq \nu_i$:	– 2
Total physically-significant parameters:	4
Additional (Majorana) ~~CP~~ phases if $\overline{\nu}_i = \nu_i$:	2

How Many Of The Parameters Are Mixing Angles?

The *mixing angles* are the parameters in U when it is *real*.

U is then a three-dimensional rotation matrix.

Everyone knows such a matrix is described in terms of 3 angles.

Thus, U contains 3 mixing angles.

Summary

Mixing angles	$\not{CP}$ phases if $\bar{\nu}_i \neq \nu_i$	$\not{CP}$ phases if $\bar{\nu}_i = \nu_i$
3	1	3

Multiplied out, the leptonic mixing matrix U is —

$c_{ij} \equiv \cos\theta_{ij}$
$s_{ij} \equiv \sin\theta_{ij}$

$$U = \begin{array}{c} \\ e \\ \mu \\ \tau \end{array}\!\!\!\!\!\!\begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array}$$

$$U = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \begin{array}{c} e \\ \mu \\ \tau \end{array}$$

$$\times \mathrm{diag}\left(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1\right)$$

Probability of Flavor Change in Matter of Constant Density

(Freund; Nakamura & Petcov PDG review)

We quote an approximate expression for neutrinos from an accelerator traveling through earth matter to a detector hundreds of kilometers away.

$$\text{Let } A \square \sqrt{2} G_F N_e \frac{2E}{\Delta m_{31}^2}, \; \Delta \square \frac{\Delta m_{31}^2 L}{4E}, \; \text{and } \alpha \square \frac{\Delta m_{21}^2}{\Delta m_{31}^2}.$$

Also, let —

$$J_{CP} \square -\text{Im}\left(U_{\mu 3}^* U_{e3} U_{\mu 2} U_{e2}^*\right) = \frac{1}{8} \cos\theta_{13} \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin\delta$$

This quantity, the **leptonic Jarlskog invariant**, is one measure of how big leptonic CP violation is.

In terms of these quantities —

$$P\left(\nu_\mu \to \nu_e\right) \cong P_0 + P_{\sin\delta} + P_{\cos\delta} + P_3$$

where $$P_0 = \sin^2\theta_{23}\sin^2 2\theta_{13}\frac{\sin^2\left[\left(1-A\right)\Delta\right]}{\left(1-A\right)^2}$$

$$P_{\sin\delta} = -\alpha\left(8J_{CP}\right)\sin\Delta\frac{\left(\sin A\Delta\right)\left(\sin\left[\left(1-A\right)\Delta\right]\right)}{A\left(1-A\right)}$$

$$P_{\cos\delta} = \alpha\left(8J_{CP}\cot\delta\right)\cos\Delta\frac{\left(\sin A\Delta\right)\left(\sin\left[\left(1-A\right)\Delta\right]\right)}{A\left(1-A\right)}$$

and $$P_3 = \alpha^2\cos^2\theta_{23}\sin^2 2\theta_{12}\frac{\sin^2 A\Delta}{A^2}$$

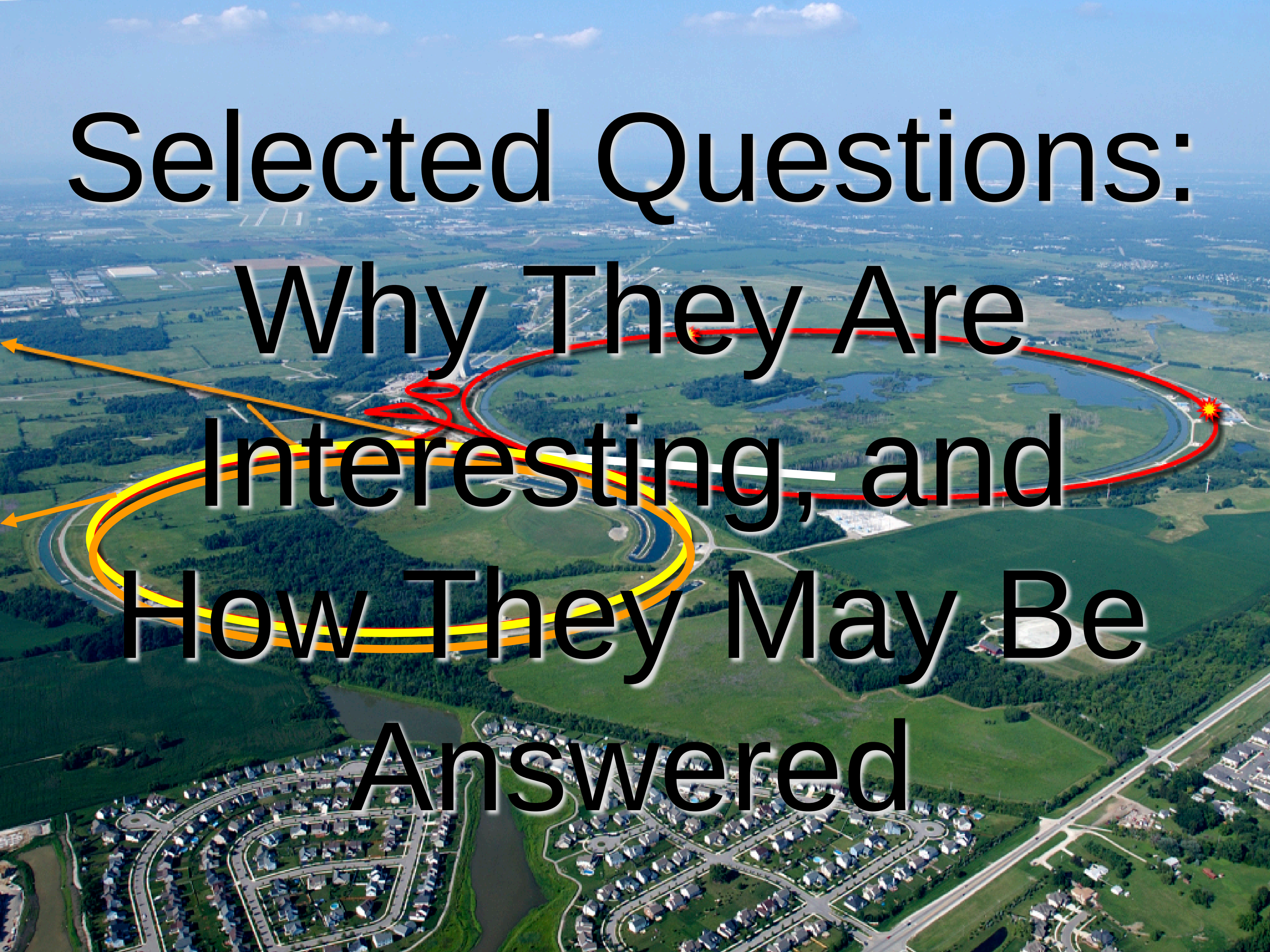
This approximation keeps terms up to 2nd order in $|\alpha| \simeq 0.03$ and $\sin^2\theta_{13} \simeq 0.02$.

$$P\left(\bar{\nu}_\mu \to \bar{\nu}_e\right) = P\left(\nu_\mu \to \nu_e\right) \text{ with } \delta \to -\delta \text{ and } A \to -A$$

The $P_{\sin\delta}$ term is the only one that changes due to *intrinsic CP violation* when we go from neutrino oscillation to antineutrino oscillation.

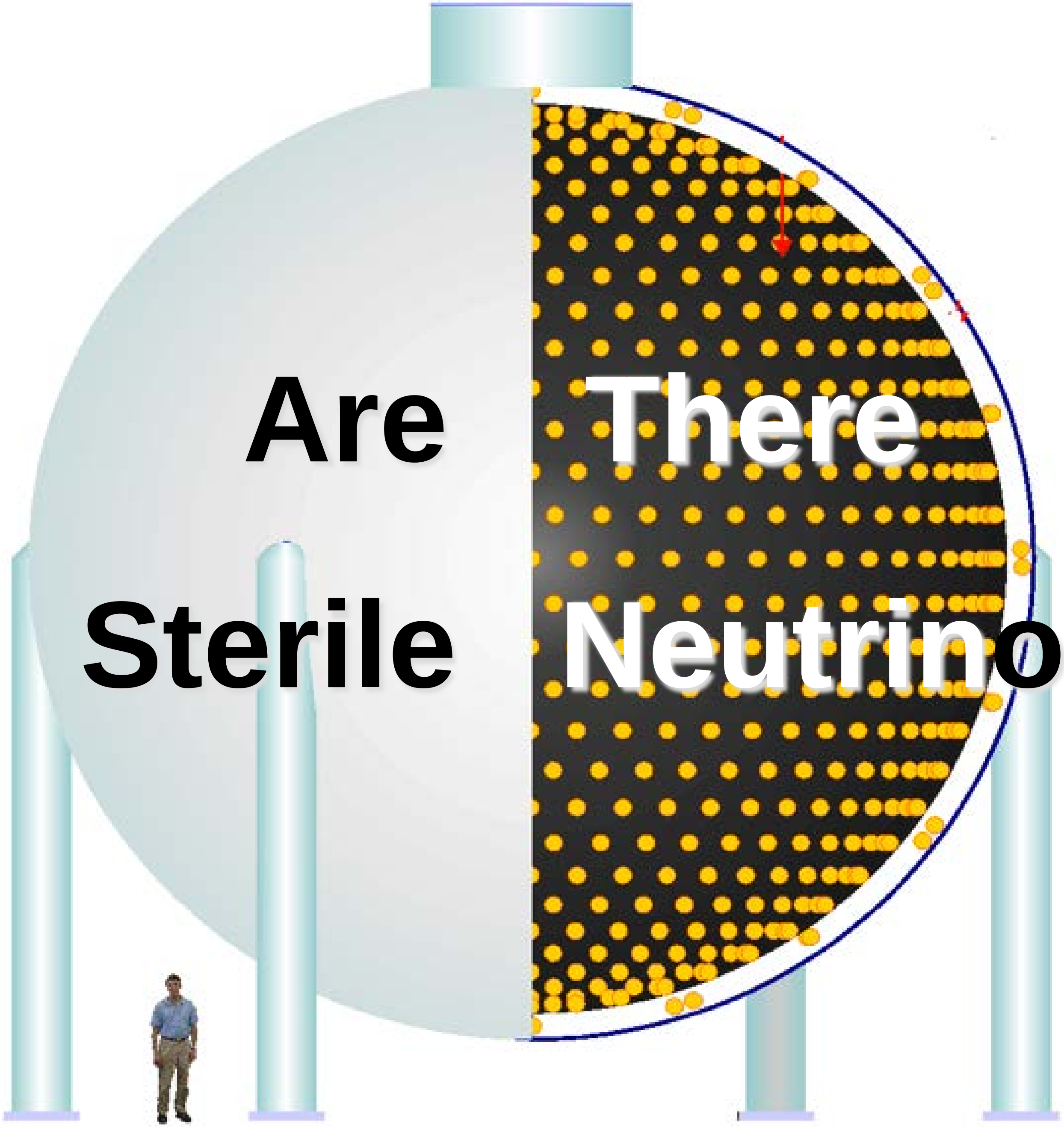
But even if there is no intrinsic CP violation ($\sin\delta = 0$), the neutrino and antineutrino oscillation probabilities will differ due to the matter effect (A □ 0).

How they differ → Sign (A) → $\text{Sign}\left(\Delta m^2_{31}\right)$



Selected Questions: Why They Are Interesting, and How They May Be Answered

Are There Sterile Neutrinos?



(Lectures by Carlo Giunti & Jonathan Link)

Sterile Neutrino

One that does not couple
to the SM *W* or *Z* boson

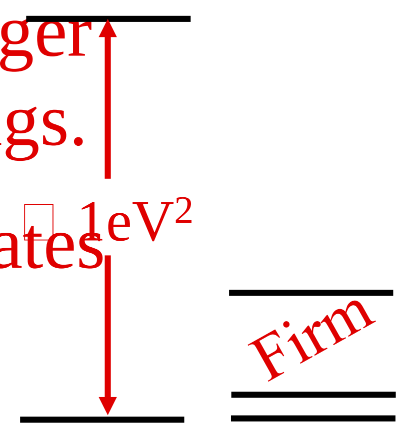
A “sterile” neutrino may well couple to some non-SM particles. These particles could perhaps be found at LHC or elsewhere.

The Hints of eV-Mass Sterile Neutrinos

$$\text{Probability(Oscillation)} \propto \sin^2\left[1.27\Delta m^2(\text{eV}^2)\frac{L(\text{m})}{E(\text{MeV})}\right]$$

There are several hints of oscillation with L(m)/E(MeV) ~ 1:

These ⇒ a $\Delta m^2 \sim 1\ \text{eV}^2$, bigger than the two established splittings.



⇒ At least **4** mass eigenstates

⇒ At least **4** flavors

Then $$\frac{\Gamma(Z \to \nu\bar{\nu})\big|_{\text{Exp}}}{\Gamma(Z \to \text{One } \nu\bar{\nu} \text{ Flavor})\big|_{\text{SM}}} = 2.984 \pm 0.009$$

⇒ At least **1** sterile neutrino

Resource Slides

KamLAND Evidence for Oscillatory Behavior

The KamLAND detector studied $\bar{\nu}_e$ produced by Japanese nuclear power reactors ~ 180 km away.

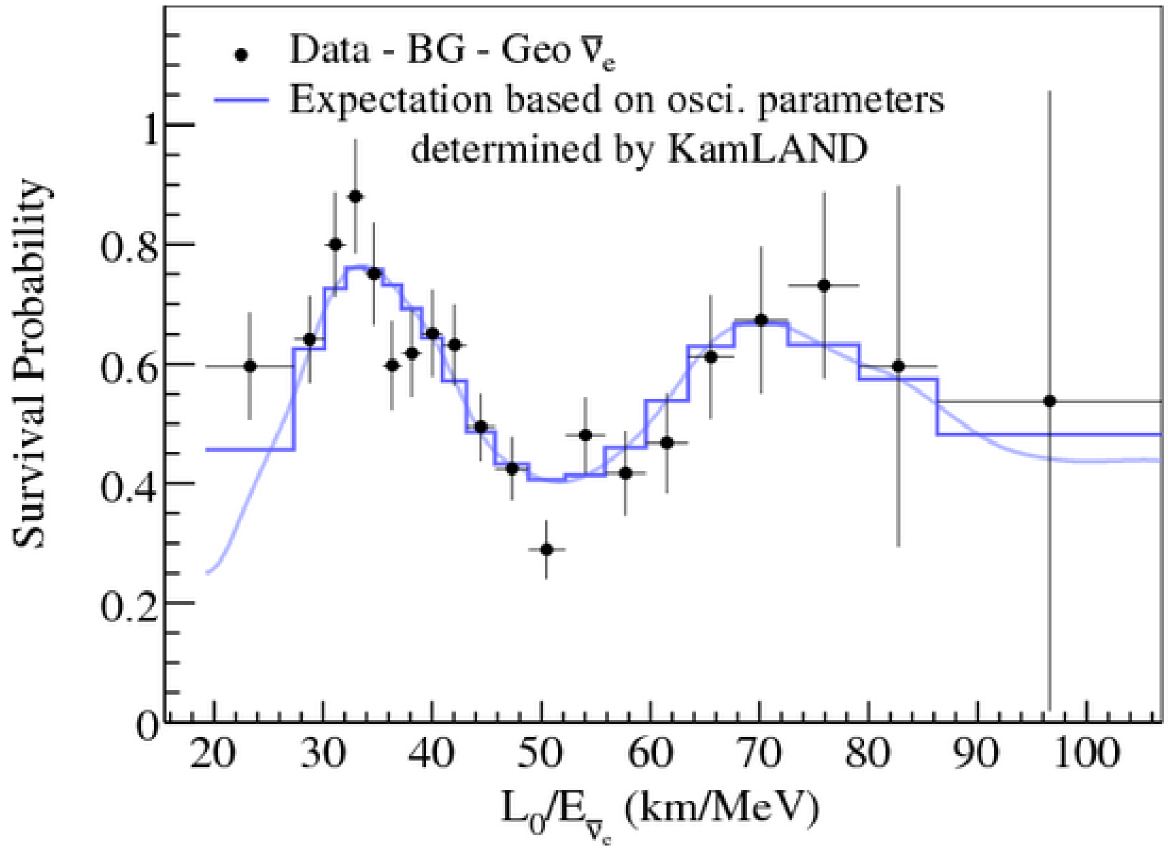
For KamLAND, $x_{Matter} < 10^{-2}$. Matter effects are negligible.

The $\bar{\nu}_e$ survival probability, $P(\bar{\nu}_e \to \bar{\nu}_e)$, should oscillate as a function of L/E following the vacuum oscillation formula.

In the two-neutrino approximation, we expect —

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta \sin^2\left[1.27\Delta m^2\left(eV^2\right)\frac{L(km)}{E(GeV)}\right].$$

Survival probability $P(\overline{\nu}_e \rightarrow \overline{\nu}_e)$ of reactor $\overline{\nu}_e$

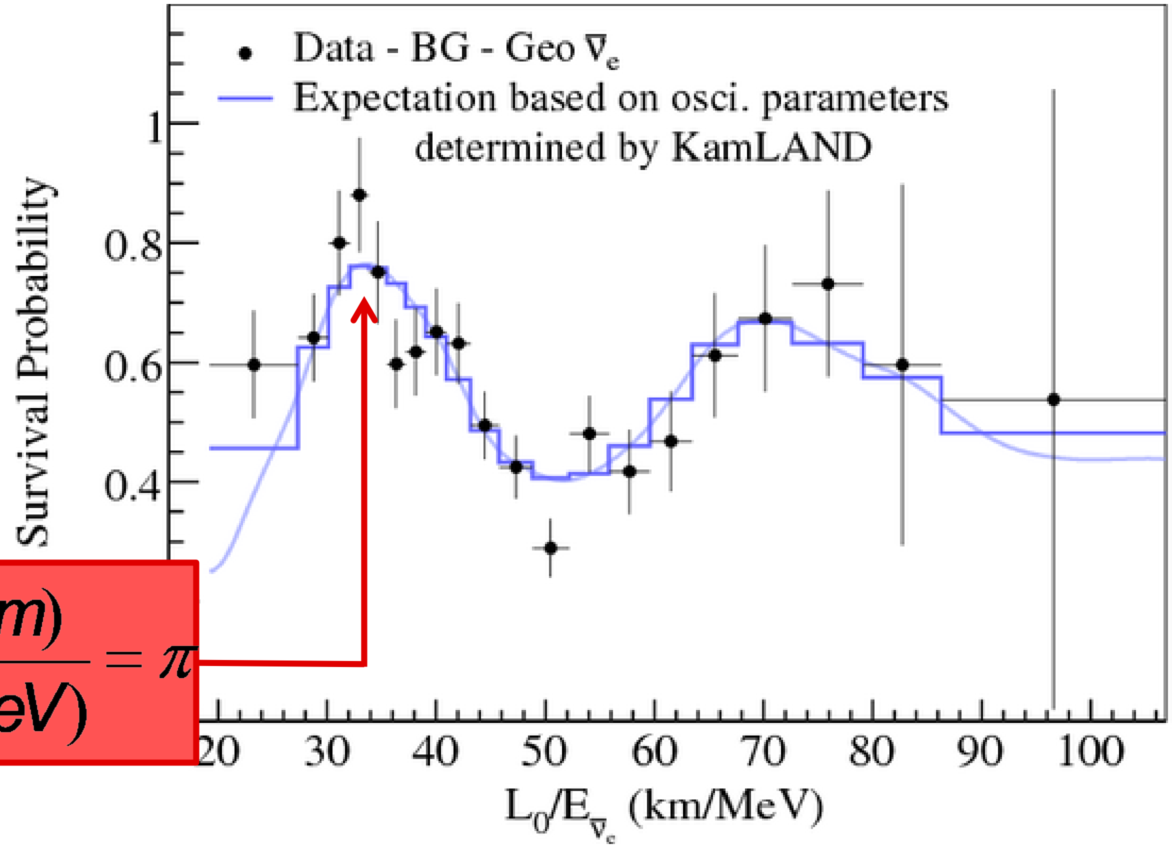


L_0 = 180 km is a flux-weighted average travel distance.

$P(\overline{\nu}_e \rightarrow \overline{\nu}_e)$ actually oscillates!

The Behavior of Reactor ν_e In KamLAND

Survival probability $P(\bar{\nu}_e \to \bar{\nu}_e)$ of reactor $\bar{\nu}_e$



$$1.27\Delta m^2_{\text{sol}}\left(eV^2\right)\frac{L(km)}{E(GeV)} = \pi$$

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta \sin^2\left[1.27\Delta m^2_{\text{sol}}\left(eV^2\right)\frac{L(km)}{E(GeV)}\right]$$

The leptonic mixing matrix U is —

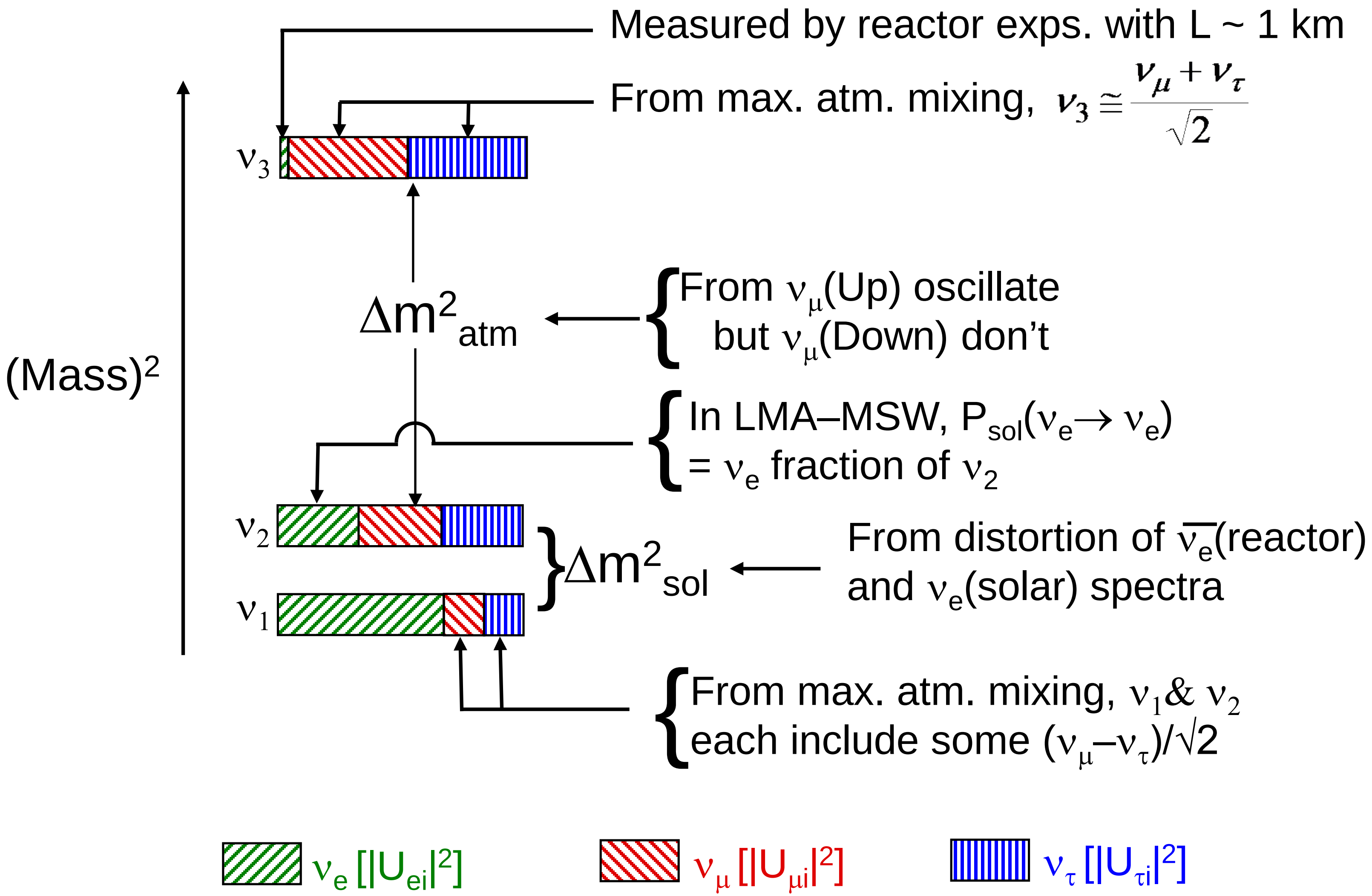
$c_{ij} \equiv \cos\theta_{ij}$
$s_{ij} \equiv \sin\theta_{ij}$

$$U = \begin{array}{c} \\ \\ \\ \end{array}\begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

(columns: ν_1, ν_2, ν_3)

$$\times \mathrm{diag}\left(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1\right)$$

Majorana phases

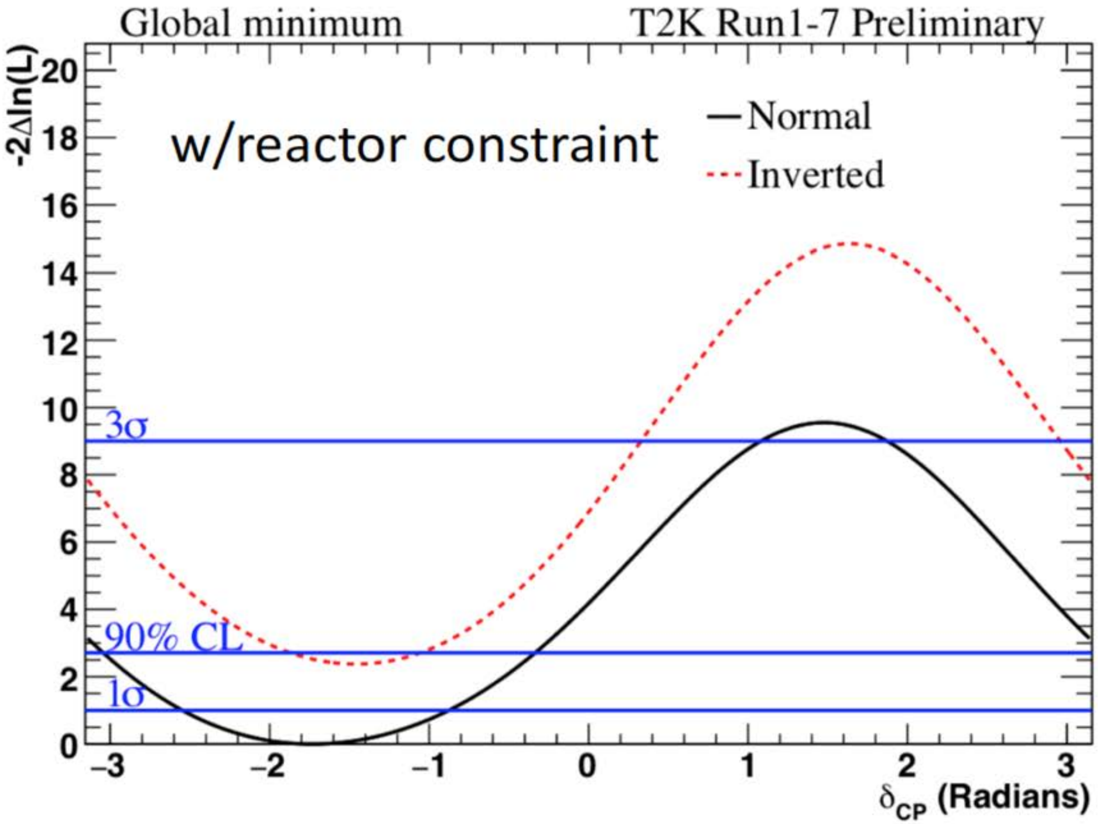


Measured by reactor exps. with L ~ 1 km
From max. atm. mixing, $\nu_3 \cong \frac{\nu_\mu + \nu_\tau}{\sqrt{2}}$
ν_3
Δm^2_{atm}
From ν_μ(Up) oscillate but ν_μ(Down) don't
(Mass)^2
In LMA–MSW, $P_{sol}(\nu_e \to \nu_e)$ = ν_e fraction of ν_2
ν_2
Δm^2_{sol}
From distortion of $\overline{\nu_e}$(reactor) and ν_e(solar) spectra
ν_1
From max. atm. mixing, ν_1& ν_2 each include some $(\nu_\mu - \nu_\tau)/\sqrt{2}$
ν_e $[|U_{ei}|^2]$
ν_μ $[|U_{\mu i}|^2]$
ν_τ $[|U_{\tau i}|^2]$

Has leptonic CP violation already been seen?

$$\text{CP violation} \propto \sin\delta_{CP}$$

From T2K

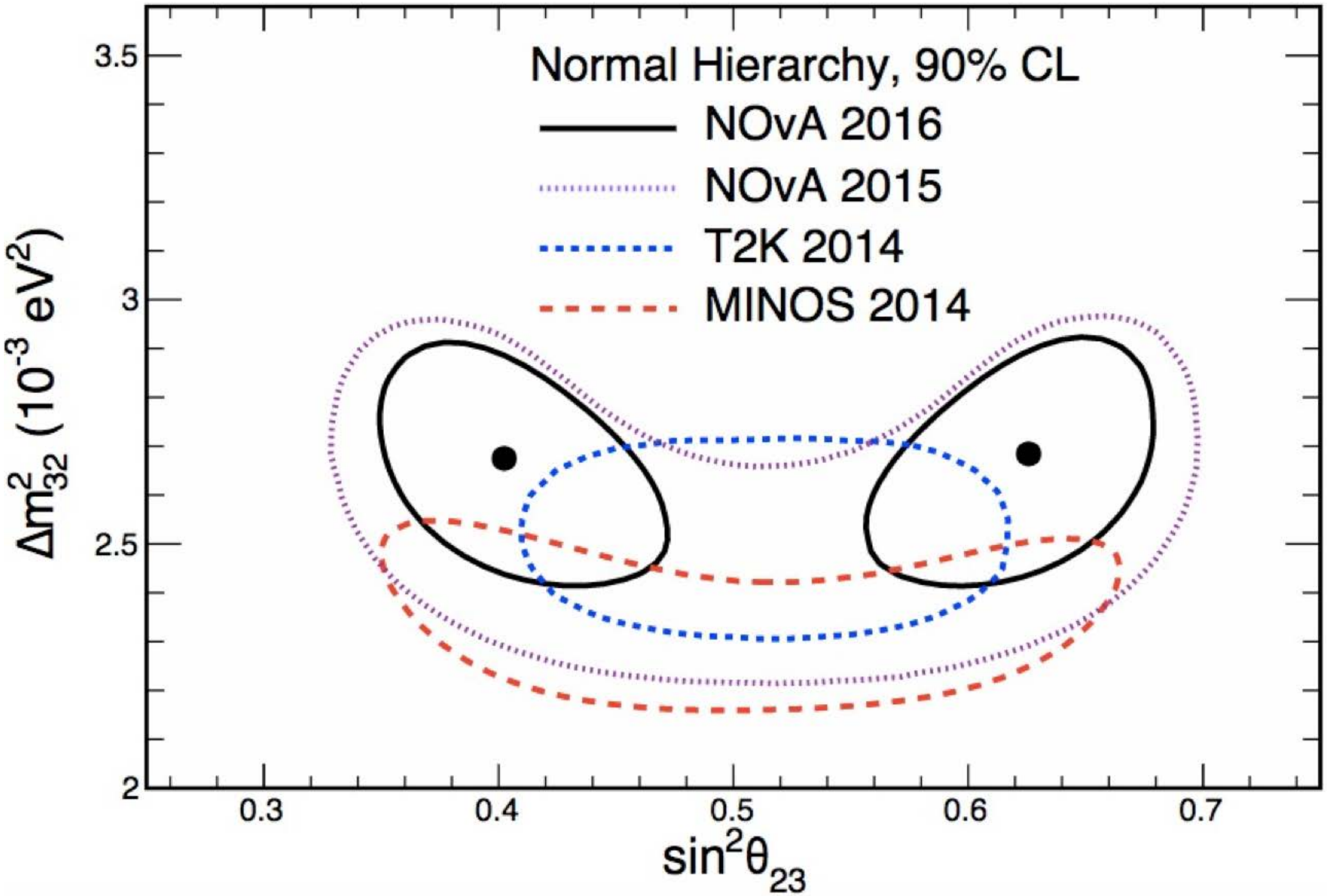


- Prefers large CP violation, $\delta_{CP}=0$ disfavored at 90% C.L.
- In combination with reactor results, disfavors $\delta_{CP}=0$ at 2σ

Current Issues

Is θ_{23} maximal (45°)?

If so, there may well be a symmetry behind that.



Has leptonic CP violation already been seen?

$$\text{CP violation} \propto \sin\delta_{CP}$$

When combined with reactor measurements, the hypothesis of CP conservation ($\delta_{CP} = 0$ or π) is excluded at 90% confidence level.

T2K, July 5, 2017

But —

	3σ range
$\sin^2\theta_{12}$	$0.271 \to 0.345$
$\theta_{12}/^\circ$	$31.38 \to 35.99$
$\sin^2\theta_{23}$	$0.385 \to 0.638$
$\theta_{23}/^\circ$	$38.4 \to 53.0$
$\sin^2\theta_{13}$	$0.01934 \to 0.02397$
$\theta_{13}/^\circ$	$7.99 \to 8.91$
$\delta_{\rm CP}/^\circ$	$0 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ {\rm eV}^2}$	$7.03 \to 8.09$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ {\rm eV}^2}$	$\begin{bmatrix} +2.407 \to +2.643 \\ -2.629 \to -2.405 \end{bmatrix}$

Global fit of Esteban, et al.

Most theorists strongly suspect that, unlike the quarks and the charged leptons, neutrinos have Majorana masses.

If true, this would make the neutrinos quite special.

So, what are Majorana masses?

Majorana Masses — In Pictures, Without Quantum Field Theory

The Possible Origins of Majorana Masses

According to the Standard Model —

Quark and charged lepton masses arise
from an interaction with the Higgs field.

Dirac neutrino masses would arise in the same way.

But ***Majorana*** neutrino masses cannot arise as
the quark and charged lepton masses do.

Majorana neutrino masses are from
physics way outside the Standard Model.

A ***Majorana*** neutrino mass can arise
without interaction with any Higgs field,

— or through interaction with a Higgs-like field
which is not in the Standard Model,
and carries a different value of the "weak isospin"
quantum number than the Standard Model Higgs,

— or through interaction with the Standard Model Higgs,
but not the same kind of interaction
as would generate the quark masses.

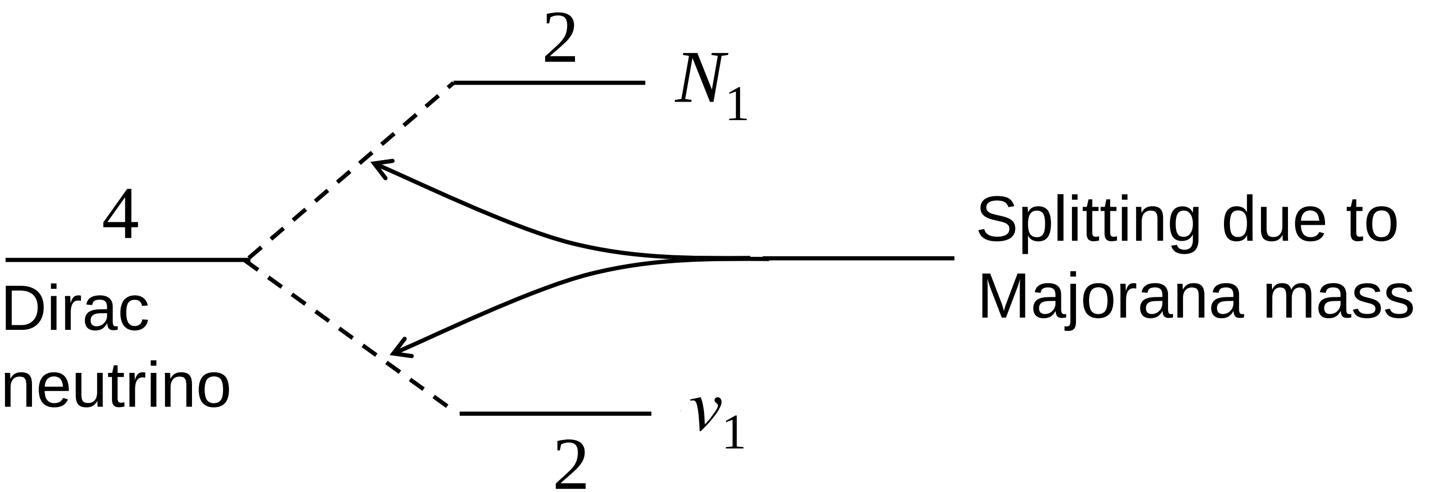
When an underlying ν has only a ***Dirac*** mass,
the resulting mass eigenstate is
one Dirac neutrino.

We have seen that when an underlying ν has only a ***Majorana*** mass, the resulting mass eigenstate is ***one Majorana*** neutrino.

When there are ***both*** Dirac and Majorana masses, ***two Majorana*** mass eigenstates result.

What Happens?

The Majorana mass term splits a Dirac neutrino into two Majorana neutrinos.



$4 = 2 + 2$