

# Theory of $\nu$ -masses and mixing

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Romania, September 2, 3, 2019*



# Lepton mixing: What is behind?

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R. Davis in 80ies: "Models of neutrino masses are so numerous that they can compose the Dark Matter in the Universe"

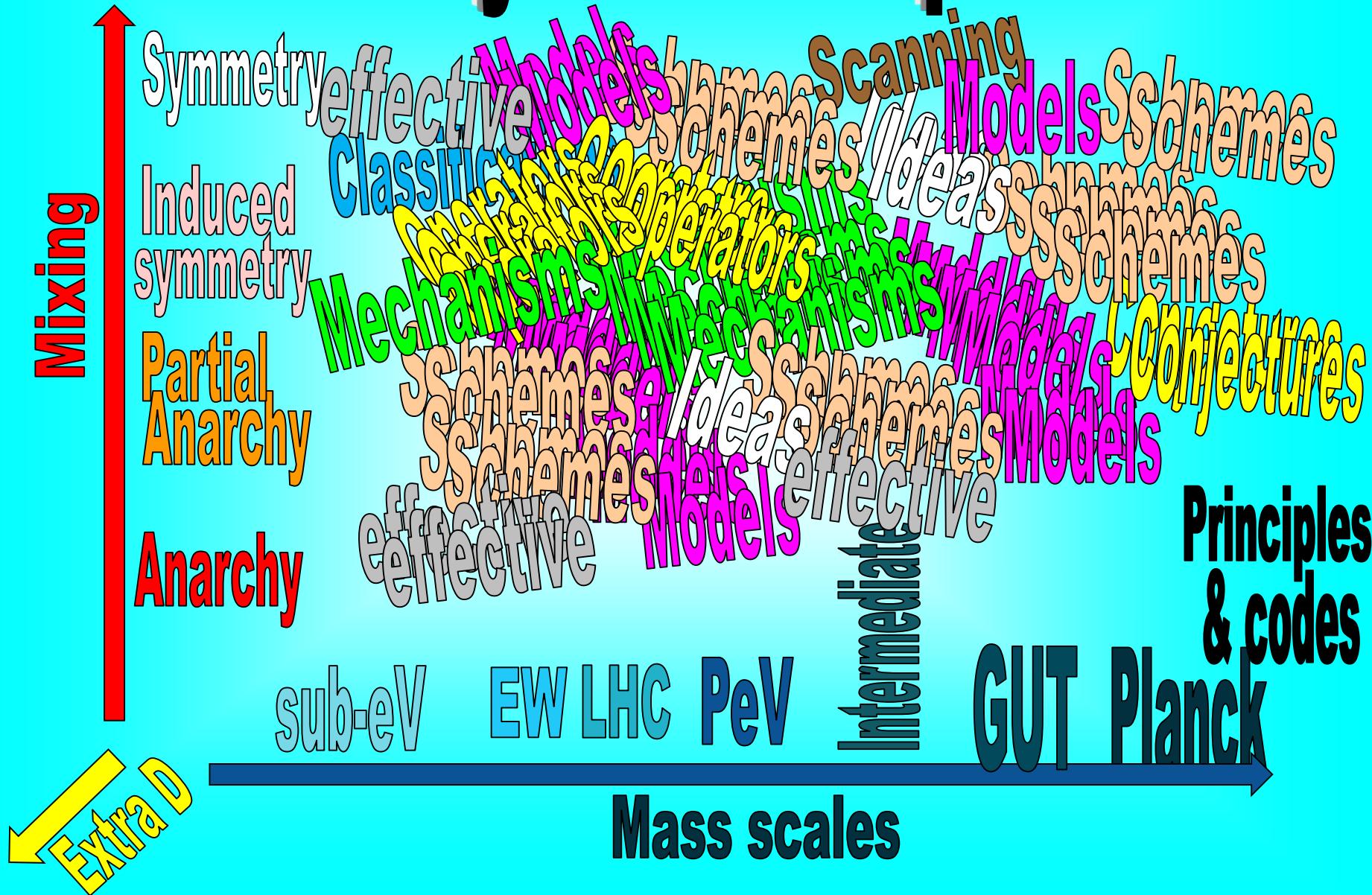
This can not be true: with present number of models the Universe would be overclosed many times contradicting observations

Davis's remark on  $\nu$  models- DM connection has new turn now:

- joint models of neutrino masses and dark matter
- understanding neutrino properties can stem from the Dark sector of theory or
- neutrino mass can be sourced by DM

Multi  
dimensional

# "Theory" Landscape



# What is the problem?

No theory  
of flavor,  
in general

No theory of quark  
masses and mixing

No physics BSM has been  
discovered yet at LHC ...  
Especially no SUSY →  
UV completion issue.

What is the hope?

Neutrinos are the key

Less ambitious:

It is the neutrino that sheds  
the light on all these problems

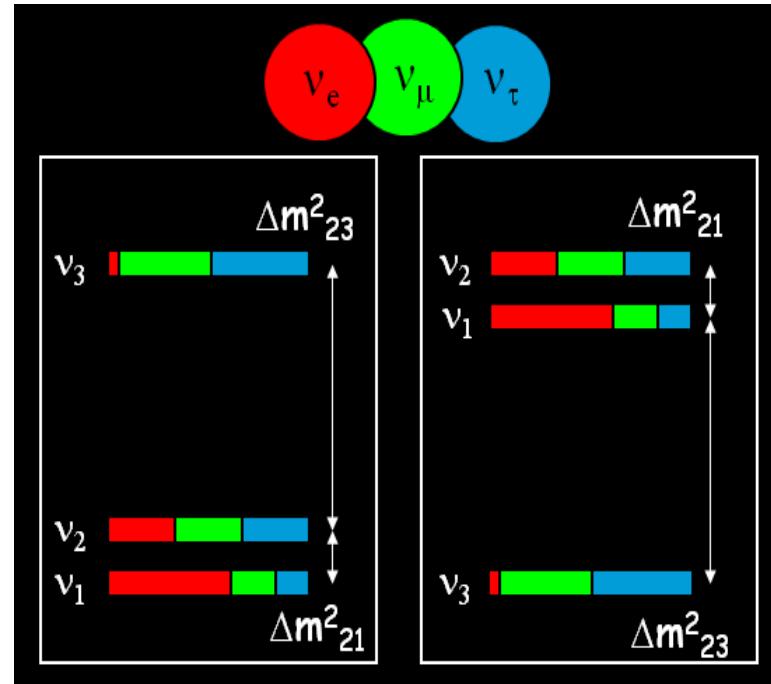
Understand at least the difference  
of neutrino mass and mixing from  
quark mixing and masses

What we  
really know

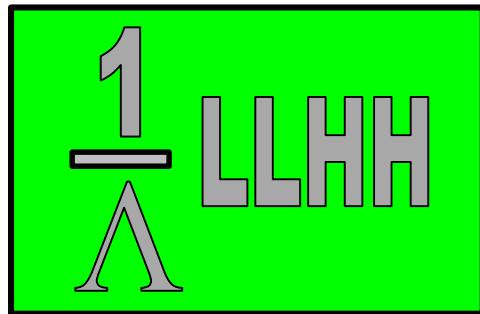
# $3\nu$ - paradigm

All well established/confirmed results fit well a framework

- **three neutrinos**
  - with interactions described by the standard model
  - *With masses and mixing*
- negligible feedback of neutrino mass generation on the standard model



# The theory?



S. Weinberg

Large scale  
of new physics

Violation of  
universality,  
Unitarity?

or maybe:

$$h L \bar{\nu}_R H$$

With very small  
coupling  $h \lll 1$

That's all?  
Will we learn more?

# Outline

- I. Data and interpretation: bottom-up
- II. Origins of  $\nu$ -mass and mechanisms of generation
- III. Mixing and flavor symmetries
- IV. Models of neutrino mass, Connections

# I. Data and interpretation bottom-up

Definitions and parameterization

# Dirac and Majorana masses

Dirac mass term

$$-m_D \bar{\nu}_R \nu_L + \text{h.c.}$$

Instead of independent  
RH component

$$\nu_R \rightarrow \nu_L^C \quad \nu_L^C = C(\bar{\nu}_L)^T \quad \begin{matrix} \text{charge conjugate} \\ C = i\gamma_0 \gamma_2 \end{matrix}$$

→  $- \frac{1}{2} m_L \nu_L^T C \nu_L + \text{h.c.} \rightarrow \text{two component massive neutrino}$

corresponds to Majorana neutrino:

$$\nu_M^C = e^{i\alpha} \nu_M \quad \nu_M = \nu_L + e^{-i\alpha} \nu_L^C \quad \alpha \text{ is the Majorana phase}$$

$$- \frac{1}{2} m_M \bar{\nu}_M \nu_M = - \frac{1}{2} m_M e^{i\alpha} \nu_L^T C \nu_L + \text{h.c.}$$

No invariance under  $\nu_L \rightarrow e^{i\alpha} \nu_L$

Lepton number of the mass operator:  $L = 2$  and  $-2$  (for h.c.)  
mass term violates lepton number by  $|\Delta L| = 2$

→ Processes with lepton number violation  
by  $|\Delta L| = 2$  with probabilities

$$\beta\beta_{\text{ov}}$$

$$\Gamma \sim m_L^2$$

# Vacuum mixing

$$\nu_f = U_{PMNS} \nu_m$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

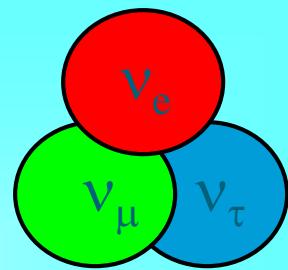
SM definition of flavor states may differ from "physical" ones,  
if e.g. ...

New heavy neutral leptons mix with neutrinos

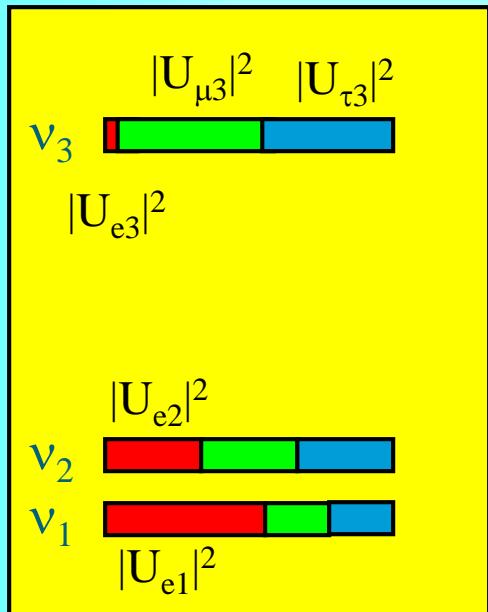
Physical flavor states produced e.g. in beta, pion,  
muon decays depend on kinematics of the process

# Mixing

Dual role

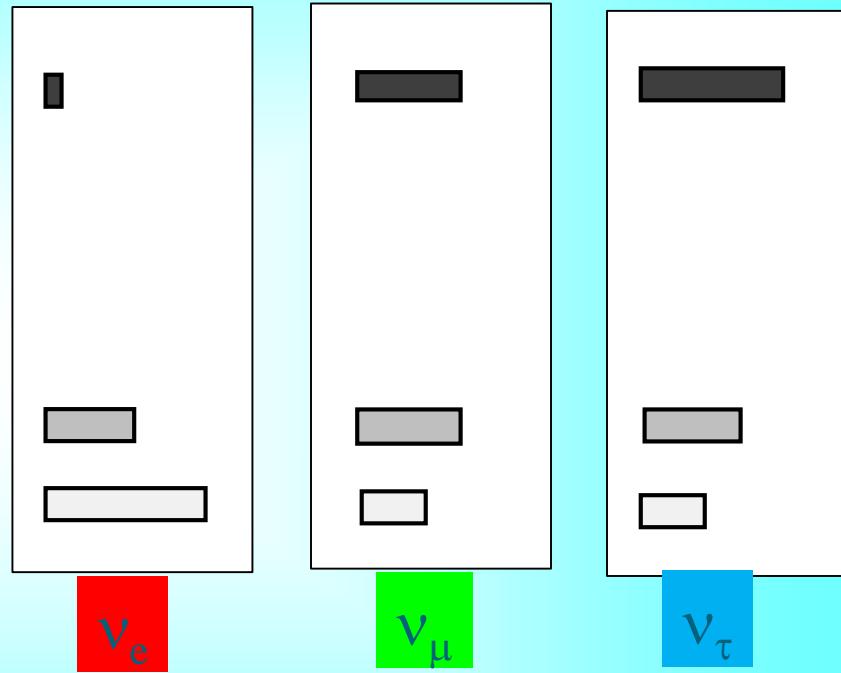


mass



Flavor content of mass states

$$\nu_{\text{mass}} = U_{\text{PMNS}} + \nu_f$$



Mass content of flavor states

$$\nu_f = U_{\text{PMNS}} \nu_{\text{mass}}$$

# Standard parametrization

$$U_{PMNS} = U_{23} I_\delta U_{13} I_{-\delta} U_{12} \quad I_M$$

$$I_\delta = \text{diag}(1, 1, e^{i\delta})$$

Dirac phase matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & -s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

$$\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}$$

$$\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$I_M$$

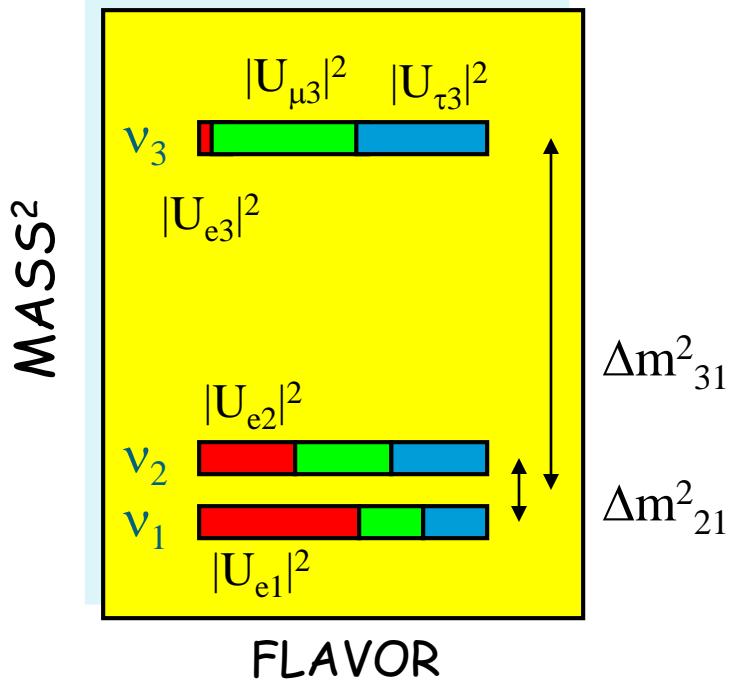
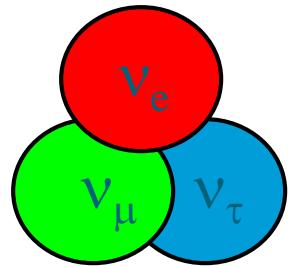
$$I_M = \text{diag}(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2}) \quad - \text{matrix of Majorana phases}$$

Not unique parametrization

Convenient for phenomenology, especially for oscillations in matter

Insightful for theory?

# Mixing angles



Normal mass hierarchy

$$\Delta m^2_{31} = m^2_3 - m^2_1$$

$$\Delta m^2_{21} = m^2_2 - m^2_1$$

Mixing determines the flavor composition of mass states

Mixing parameters

$$\tan^2 \theta_{12} = |U_{e2}|^2 / |U_{e1}|^2$$

$$\sin^2 \theta_{13} = |U_{e3}|^2$$

$$\tan^2 \theta_{23} = |U_{\mu 3}|^2 / |U_{\tau 3}|^2$$

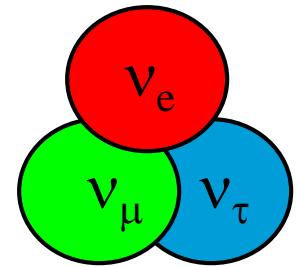
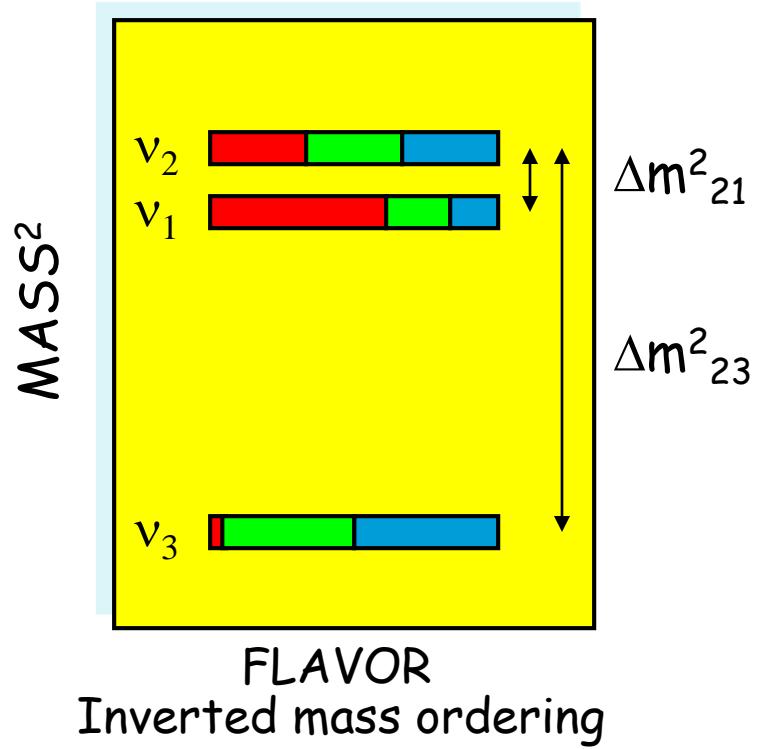
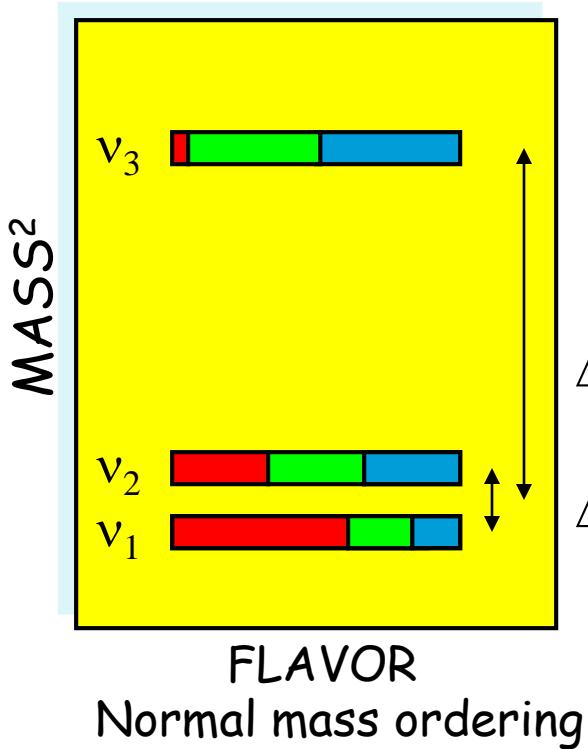
Mixing matrix:

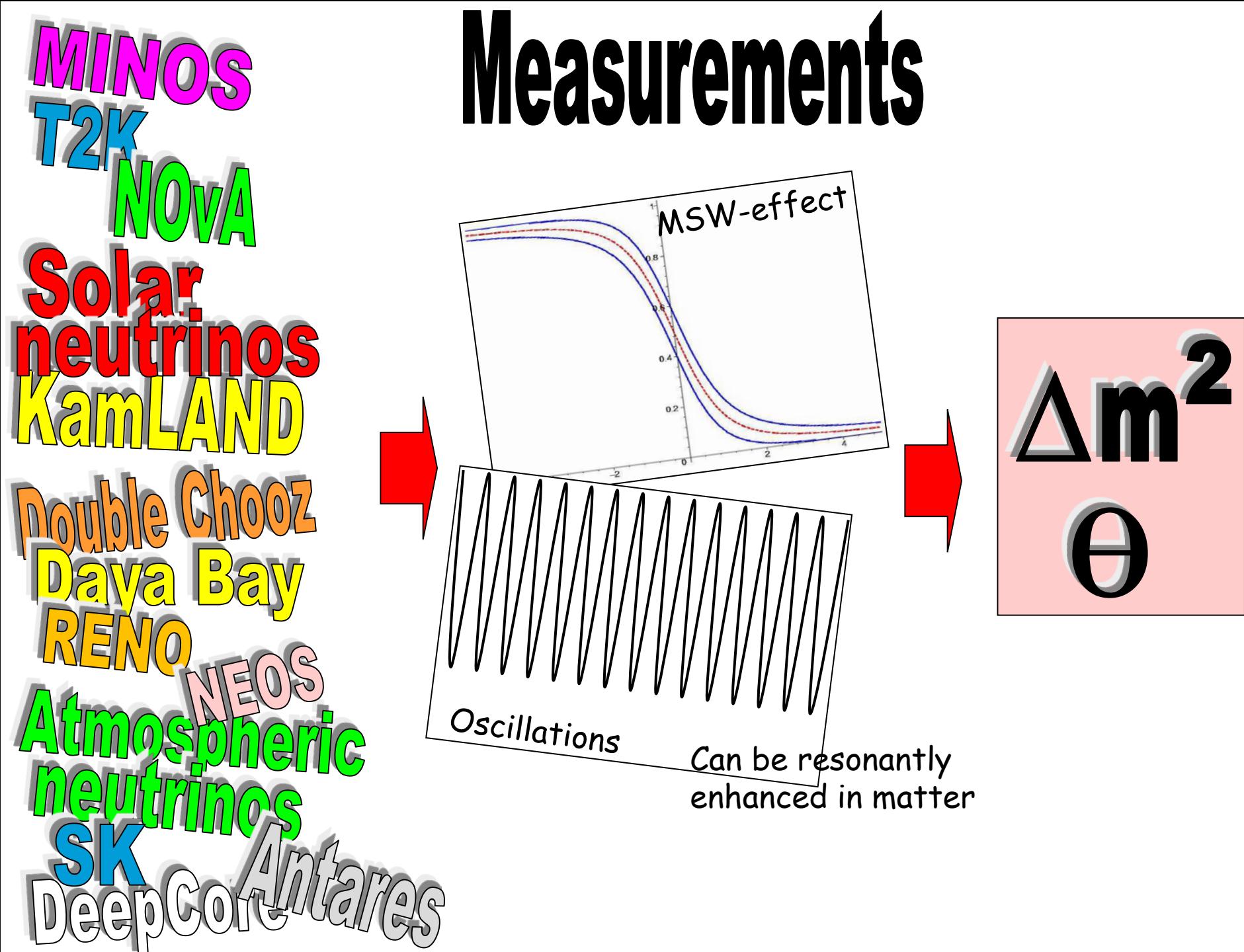
$$v_f = U_{PMNS} v_{mass}$$

$$\begin{pmatrix} v_e \\ v_\mu \\ v_\tau \end{pmatrix} = U_{PMNS} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

$$U_{PMNS} = U_{23} I_\delta U_{13} I_{-\delta} U_{12}$$

# Mass Ordering

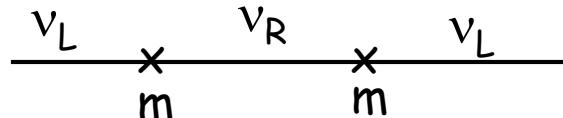




# Oscillations and masses

Oscillations and adiabatic conversion test the dispersion relations and not neutrino masses

$$p_i = \sqrt{E_i^2 - m_i^2}$$



In oscillations: no change of chirality, so e.g. V, A interactions with medium can reproduce effect of mass. Also interactions with scalar fields

It is consistency of results of many experiments in wide energy ranges and different environment: vacuum, matter with different density profiles that makes explanation of data without mass almost impossible.

proof of non-zero neutrino mass

Kinematical methods: distortion of the beta decay spectrum near end point - KATRIN

Neutrinoless double beta decay

Cosmology, Large scale structure of the Universe

# Probing Nature of neutrino mass

Determination of masses, mass squared differences from processes at different conditions

Searches for dependence of mass on external variables:

Vacuum - media with different densities, fields

Solar - KamLAND:  $\Delta m_{21}^2$   
2-3 mixing: T2K - NOvA

Energies (in medium, or if Lorentz is violated)

Epochs (red shifts)

MAVAN

Momentum transfer

Virtuality: On shell - off shell

Neutrinoless Double beta decay - unique?

# Global 3nu analysis

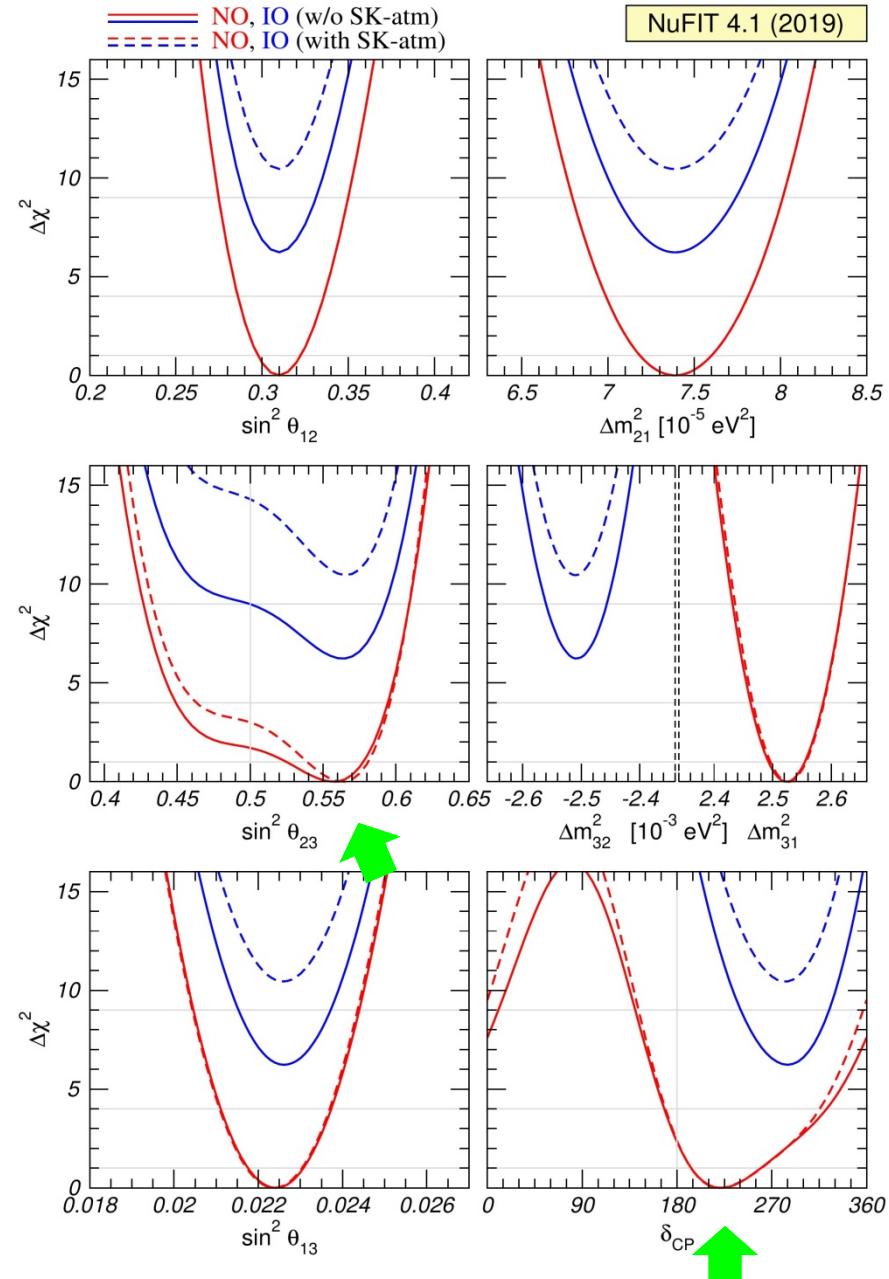
Esteban, Ivan et al.  
arXiv:1811.05487 [hep-ph]

NuFIT 4.1 (2019), [www.nu-fit.org](http://www.nu-fit.org)

$\Delta\chi^2$  profiles minimized with respect to all undisplaced parameters.

The red (blue) curves correspond to Normal (Inverted) Ordering. Solid (dashed) curves are without (with) adding the tabulated SK-atm  $\Delta\chi^2$ .

Mass-squared splitting:  
 $\Delta m_{31}^2$  for NO and  $\Delta m_{31}^2$  for IO



# Global 3nu analysis

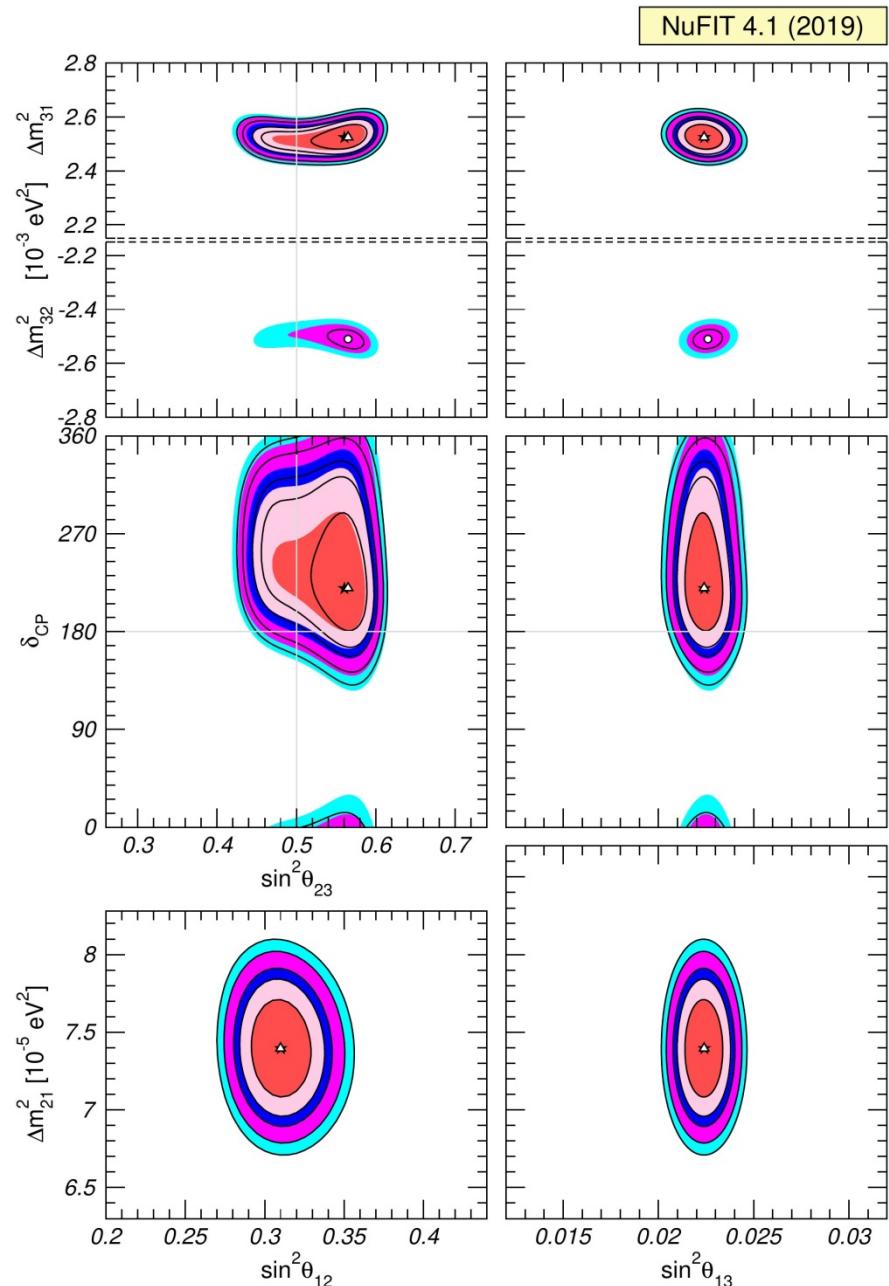
Esteban, Ivan et al. 1811.05487 [hep-ph]

NuFIT 4.1 (2019), [www.nu-fit.org](http://www.nu-fit.org)

The two-dimensional projection of the allowed six-dimensional region after minimization with respect to the undisplayed parameters.

The regions in the four lower panels are obtained from  $\Delta\chi^2$  minimized with respect to the mass ordering.

Contours correspond to  $1\sigma$ , 90%,  $2\sigma$ , 99%,  $3\sigma$  CL (2 dof). Coloured regions (black contour curves) are without (with) adding the tabulated SK-atm  $\Delta\chi^2$ .



# Summarizing results

Data are in a very good agreement with  $3\nu$  framework

Data are internally consistent within error bars  
Over determined: different experiments are  
sensitive to the same parameters

Some tensions:

Solar vs. KamLAND      OK within experimental uncertainties

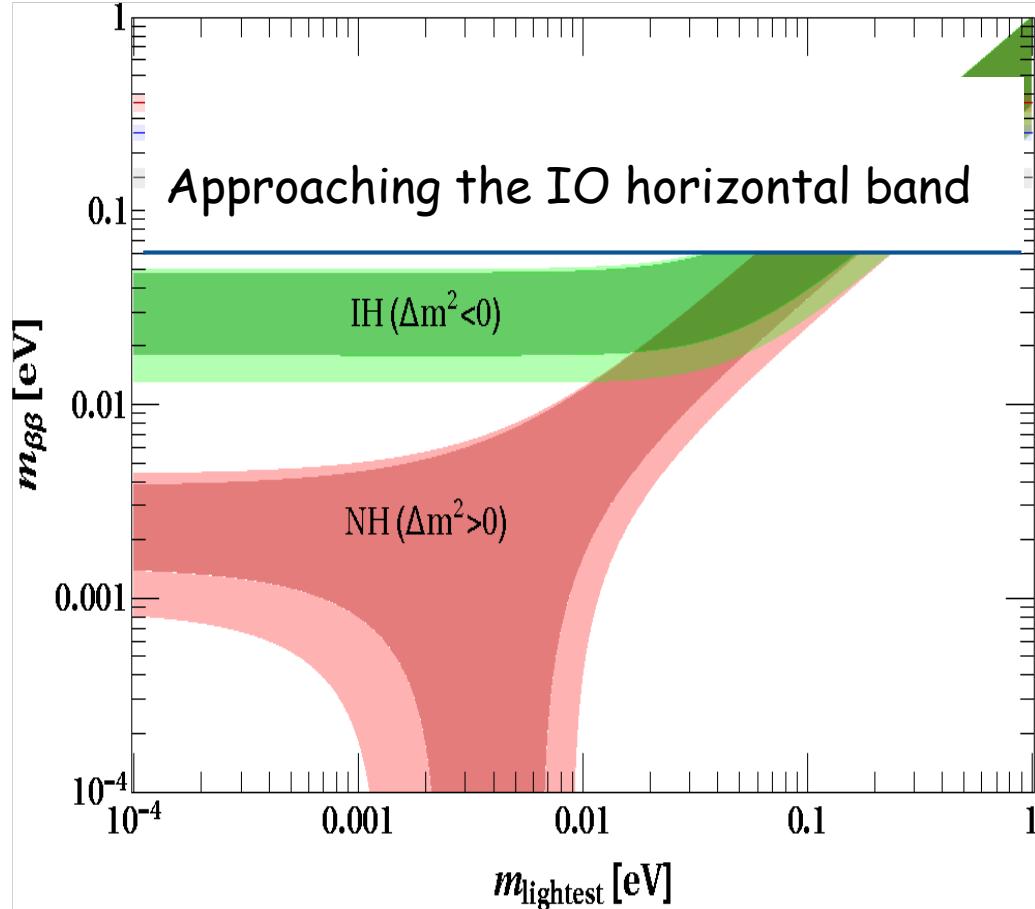
UNKNOWNs:

- CP-phase: best fit value deviates from  $3\pi/2$ ,  $\pi$  and  $3\pi/2$  are equally plausible.
- Mass ordering: NO is favored by  $2 - 3 \sigma$ ,  $\Delta\chi^2 = 7.5$
- Deviation of 2-3 mixing from maximal at  $1.5\sigma$ , second (high) octant is preferable

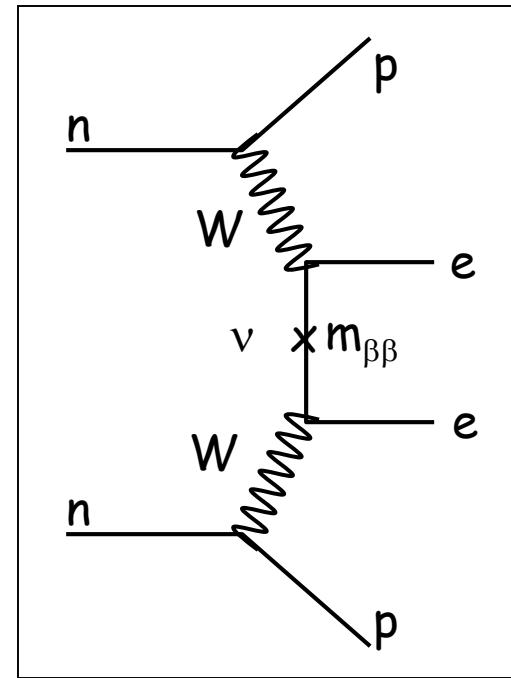
ANOMALIES:

LSND, MiniBooNE, Reactor, Gallium:  
Oscillations into steriles, new interactions, new  
particles - can dramatically affect our  
considerations

# $\beta\beta 0\nu$ - decay results



$$m_{\beta\beta} = U_{e1}^2 m_1 + U_{e2}^2 m_2 e^{i\alpha} + U_{e3}^2 m_3 e^{i\phi}$$



90% CL upper bound on  $m_{\beta\beta}$  depending on NME

KamLAND-Zen (61 - 165) meV

A.Gando, et al, 1605.02889 [hep-ex]

EXO-200

(93 - 286) meV

G. Anton et al. 1906.02723 [hep-ex]

CUORE

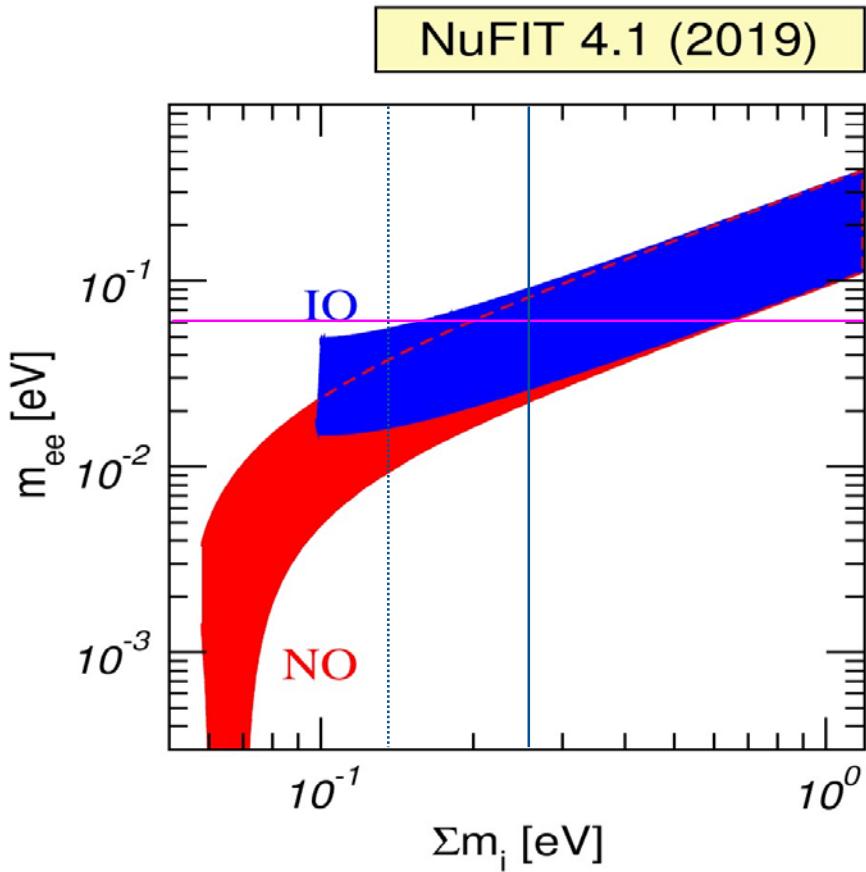
(110 - 520) meV

S. Dell'Oro et al. 1905.07667 [nucl-ex]

GERDA

(110 - 260) meV

# Bounds on masses



Allowed regions at  $2\sigma$  (2 dof) obtained by projecting the results of the global analysis of oscillation data (w/o SK-atm) over the plane  $(\sum m_\nu, m_{ee})$ . The region for each ordering is defined with respect to its local minimum

Cosmology:  $\sum_i m_i < 0.12 - 0.26$  eV

A. Loureiro et al,  
1811.02578 [astro-ph.CO]

# Observations and Implications

What these results show, hint?  
Regularities?

Relations?

# Smallness of mass?

Special

comparing within generation:

$$\frac{m_3}{m_\tau} \sim 3 \cdot 10^{-11}$$

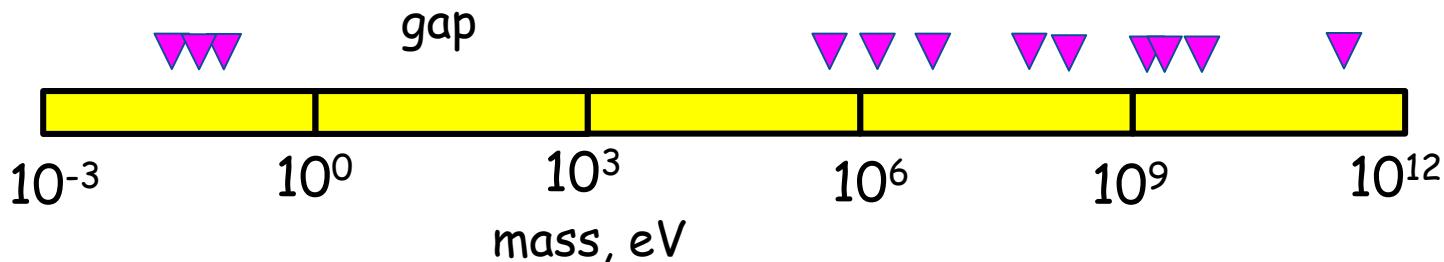
Similar for other generations  
if spectrum is hierarchical

Normal?

Neutrinos: no clear generation structure and correspondence light flavor - light mass,  
especially if the mass hierarchy is inverted or spectrum is quasi-degenerate

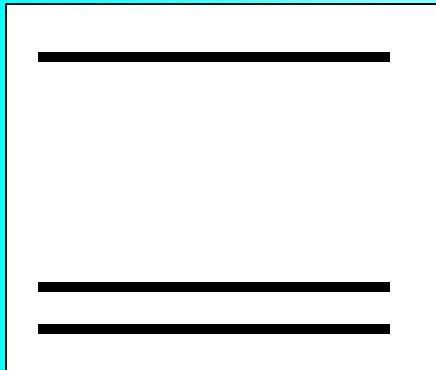
$$\frac{m_3}{m_e} \sim 3 \cdot 10^{-6}$$

$$\frac{m_e}{m_\tau} \sim 3 \cdot 10^{-6}$$

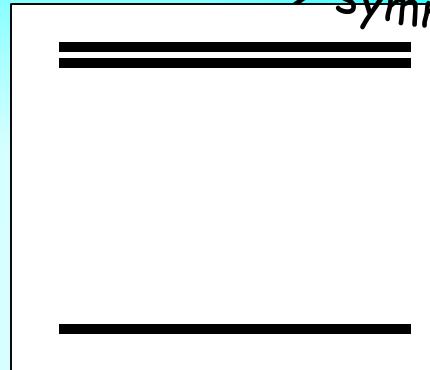


# Neutrino Mass Ordering

Fundamental:  
principle, symmetry



Accidental: selection of  
values of parameters



Quasi-degenerate  
→ symmetry

Correlation of masses  
of neutrinos and  
charged leptons  
in weak interactions:  
Light - likes light,  
heavy - likes heavy

$$\frac{m_2}{m_3} \sim \sqrt{\frac{\Delta m_{21}^2}{\Delta m_{32}^2}} = 0.18$$

$$\theta \sim \sqrt{\frac{m_2}{m_3}}$$

the weakest  
hierarchy

Similar to quark spectrum

rescaling

See-saw

Quark-lepton  
symmetry

Unification

$$\frac{\Delta m}{m} \sim \frac{\Delta m_{21}^2}{2 \Delta m_{32}^2} = 1.6 \cdot 10^{-2}$$

but 1-2 mixing strongly  
deviates from maximal

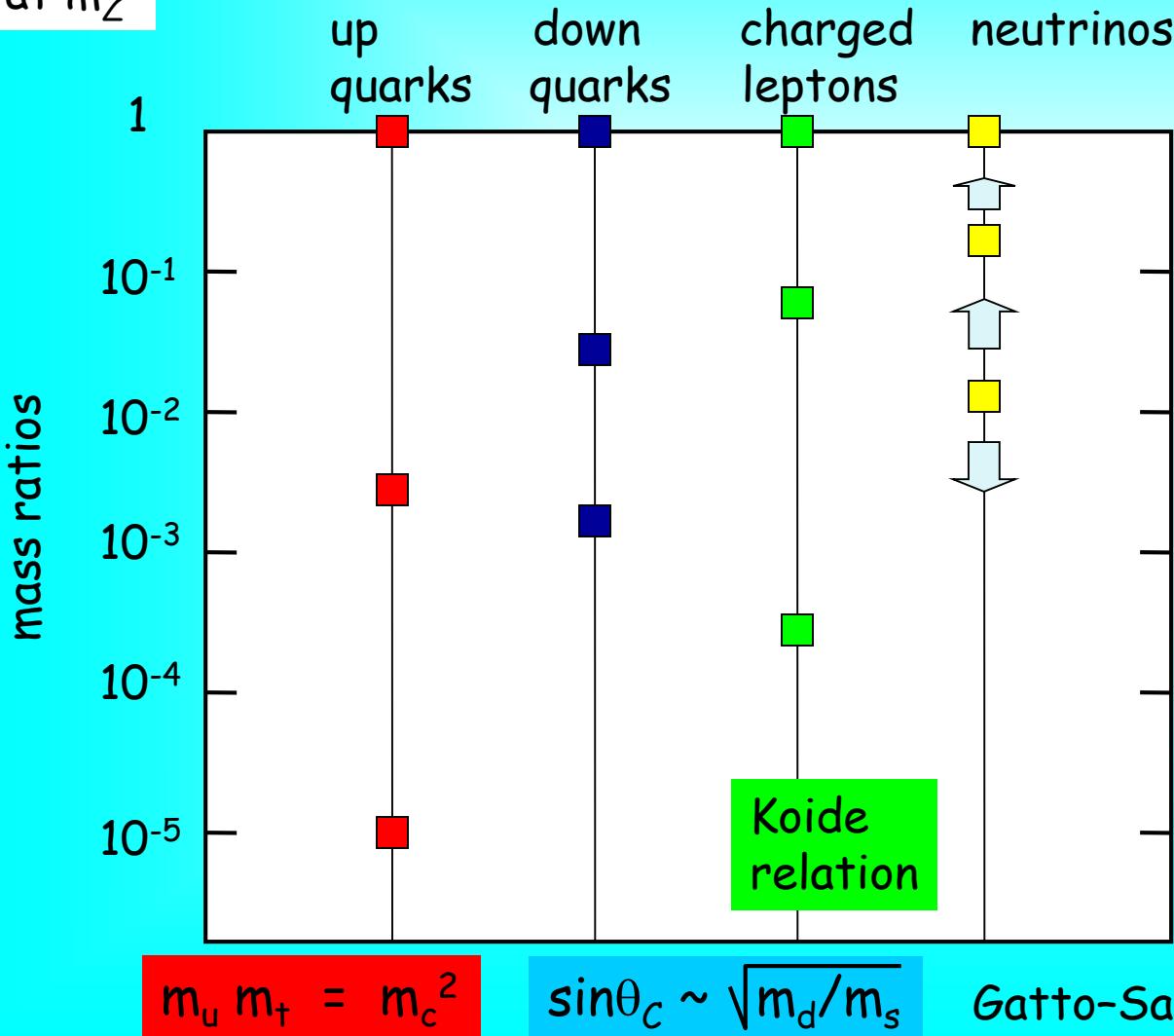
Pseudo-Dirac + 1 Majorana

Flavor symmetries

Broken  $L_e - L_\mu - L_\tau$  symmetry

# Mass hierarchies

at  $m_Z$



$$\frac{m_2}{m_3} \geq \sqrt{\frac{\Delta m_{21}^2}{\Delta m_{32}^2}}$$

$\sim 0.18$

Neutrinos have  
the weakest mass  
hierarchy (if any)  
among fermions

Related to large  
lepton mixing?

# TBM mixing: zero order approximation?

As the reference point

$$U_{\text{tbm}} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

L. Wolfenstein

P. F. Harrison

D. H. Perkins

W. G. Scott



$\nu_3$  is bi-maximally mixed

$\nu_2$  is tri-maximally mixed

$$U_{\text{tbm}} = U_{23}(\pi/4) U_{12}(\theta_{12})$$

$$\sin^2 \theta_{12} = 1/3$$

- maximal 2-3 mixing
- zero 1-3 mixing
- no CP-violation

Uncertainty related to sign of 2-3 mixing:  
 $\theta_{23} = \pi/4 \rightarrow -\pi/4$

# Mixing and mass matrices

Origin of mixing:  
off-diagonal mass matrices

$$M_I \neq M_\nu$$

Diagonalization:



Mixing matrix

Mass spectrum

$$M_I = U_{IL} m_I^{\text{diag}} U_{IR}^+$$

$$m_I^{\text{diag}} = (m_e, m_\mu, m_\tau)$$

$$M_\nu = U_{vL} m_\nu^{\text{diag}} U_{vL}^T$$

$$m_\nu^{\text{diag}} = (m_1, m_2, m_3)$$

CC in terms of mass eigenstates:  $\bar{l} \gamma^\mu (1 - \gamma_5) U_{PMNS} \nu_{\text{mass}}$



$$U_{PMNS} = U_{IL}^+ U_{vL}$$

Flavor basis:  $M_I = m_I^{\text{diag}}$        $U_{PMNS} = U_{vL}$

for Majorana  
neutrinos

# The TBM- mass matrix

Mixing from diagonalization of mass matrix in the flavor basis

$$m_{TBM} = U_{TBM} m^{\text{diag}} U_{TBM}^T$$

$$m^{\text{diag}} = \text{diag}(|m_1|, |m_2|e^{i2\alpha}, |m_3|e^{i2\beta})$$

$$m_{TBM} = \begin{pmatrix} a & b & b \\ \dots & \frac{1}{2}(a+b+c) & \frac{1}{2}(a+b-c) \\ \dots & \dots & \frac{1}{2}(a+b+c) \end{pmatrix}$$

$$a = (2m_1 + m_2)/3, \quad b = (m_2 - m_1)/3, \quad c = m_3$$

The matrix has  $S_2$  permutation symmetry

$$\nu_\mu \leftrightarrow \nu_\tau$$

Mixing is determined by relations between the matrix elements:

$$m_{12} = m_{13} \quad m_{22} = m_{33} \quad m_{11} + m_{12} = m_{22} + m_{23}$$

Eigenvalues -by absolute values of elements

# Deviations & observations

Deviations  
from TBM

bf

$$\delta(\sin \theta_{12}) = -0.017$$

$$|\delta(\sin \theta_{23})| = 0.035$$

$$\delta(\sin \theta_{13}) = \sin \theta_{13} = 0.15$$

Certain hierarchy  
of deviations =  
additional rotations

Problematic for a number  
of models, require further  
assumptions/ complications

Deviation from  
maximal 1-2 mixing:

$$\delta(\sin \theta_{12}) = 0.15$$



$$\delta(\sin \theta_{12}) = \delta(\sin \theta_{13}) = \sin \theta_{13}$$

1-3 mixing is in  
agreement  
with prediction

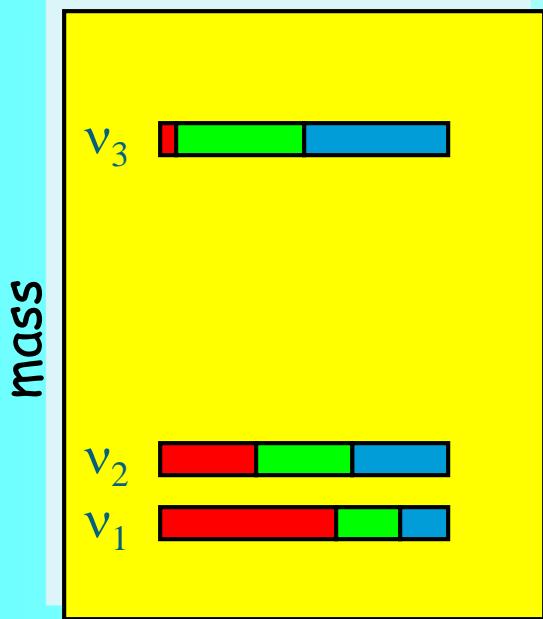
$$\theta_{13} \sim \frac{1}{\sqrt{2}} \theta_C$$

$\theta_C$  Cabibbo angle

$$\sin^2 \theta_{13} \sim \frac{1}{2} \sin^2 \theta_C$$

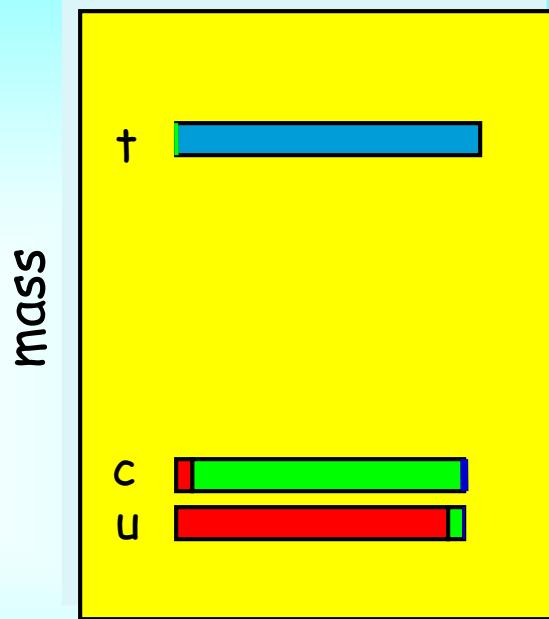
# Leptons versus quarks

Leptons



$$v_f = U_{PMNS} v_{\text{mass}}$$

Quarks



$$U_d = V_{CKM}^+ U \quad U = (u, c, t)$$

Mixings of quarks and leptons are strongly different but still related

Observation:

$$\theta_{12}^l + \theta_{12}^q \sim \pi/4$$

$$\theta_{23}^l + \theta_{23}^q \sim \pi/4$$

Sum up to maximal mixing angle  
kind of complementarity

# Quark-lepton complementarity

based on relations:

$$\theta^l_{12} + \theta^q_{12} \sim \pi/4$$

$$\theta^l_{23} + \theta^q_{23} \sim \pi/4$$

A.S.  
M. Raidal  
H. Minakata

``Lepton mixing = bi-maximal mixing - quark mixing''

## Implications

Quark-lepton symmetry

Existence of structure  
which produces  
bi-maximal mixing

Grand Unification or  
family symmetry

See-saw?  
Properties of  
the RH neutrinos

# Bi-maximal mixing

Another zero order reference structure

$$U_{\text{bm}} = \begin{pmatrix} \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \sqrt{\frac{1}{2}} \\ \frac{1}{2} & -\frac{1}{2} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

*F. Vissani  
V. Barger et al*

Two maximal rotations

$$U_{\text{bm}} = U_{23}^m U_{12}^m$$

- maximal 2-3 mixing
- zero 1-3 mixing
- maximal 1-2 mixing
- no CP-violation

# In general: lepton mixing

The data are in very good agreement with the ansatz

$$U_{PMNS} = U_{CKM}^+ U_X$$

$$U_{CKM}^+ = V_{CKM}$$

$$U_X = U_{TBM} \text{ or } U_{BM}$$

This  
reproduces QLC approximately  
can be realized in the seesaw type I  
gives prediction for 1-3 mixing

# 13-mixing

$$\sin \theta_{13} \sim \sqrt{\frac{1}{2}} \sin \theta_c$$

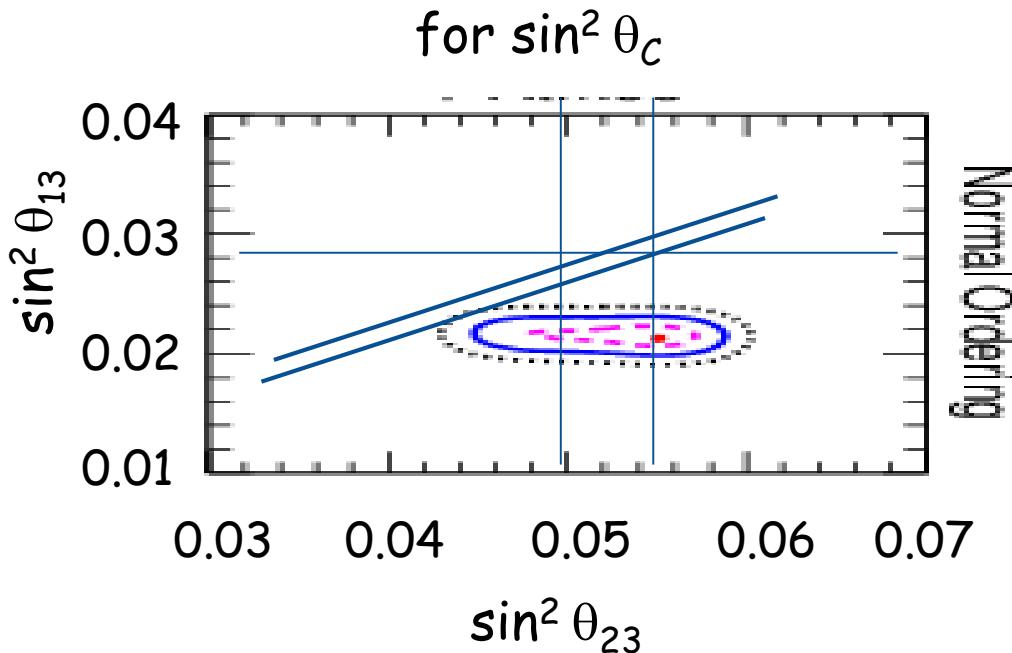
- Phenomenological level *C. Giunti, M. Tanimoto*
- From QLC  
(Quark-Lepton Complementarity) *H. Minakata, A Y S*
- From TBM-Cabibbo scheme *S. F. King et al*

Now accuracy of measurements  
permits detailed comparison

# Experimental status

From global fit

*F. Capozzi, et al. Prog.Part.Nucl.Phys.  
102 (2018) 48, arXiv:1804.09678 [hep-ph]*



~ 20% deviation in  $\sin^2 \theta_{13}$

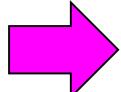
can be due to deviation  
of  $\theta_{12}^{(1)}$  from  $\theta_c$

Renormalization (RGE)  
effects from GUT  
scales to low energies

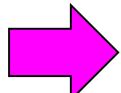
$$\sin^2 \theta_{13} = \sin^2 \theta_{23} \sin^2 \theta_c (1 + O(\lambda^2))$$

lines: predictions from QLC

# Implications



Quarks and leptons know about each other,  
Q L unification, GUT or/and  
Common flavor symmetries



Some additional physics is involved in the lepton sector  
which explains smallness of neutrino mass and difference  
of the quark and lepton mixing patterns

# But may be ... 1-3 mixing of different origin

``Naturalness" : absence of fine tuning of mass matrix

Connecting solar and atmospheric neutrino sectors

$$\sin^2\theta_{13} = O(1) \frac{\Delta m_{21}^2}{\Delta m_{32}^2}$$

0.75

E. K. Akhmedov, G.C. Branco,  
M.N. Rebelo Phys.Rev.Lett. 84  
(2000) 3535, [hep-ph/9912205]

Very small 1-3 mixing would be something special (symmetry)

Yet another "normal" relation:

almost the same relation in the quark and lepton sectors

$$\sin^2\theta_{13} = C \sin^2\theta_{12} \sin^2\theta_{23}$$

K. Patel, A. Y. S.

$$C_q = 0.380 +/- 0.020$$

$$C_l = 0.407 +/- 0.033$$

→ Similar structure of the mass matrices → Abelian symmetry

# III. Mechanisms of neutrino mass generation

## Nature of neutrino mass

is the neutrino mass of  
the same origins as masses  
of other particles?

# Two aspects

Similar to cosmological constant

Smallness:

Suppression wrt.  
the EW scale

Why there is no usual  
scale Dirac masses?

No RH component  
→ Dirac mass can  
not be formed

symmetry

See-saw or multi-  
singlet mechanisms  
- suppression only  
- finite contribution  
negligible

Finite value

Mechanisms unrelated  
to suppression of  
usual Dirac masses

Seesaw type II  
Radiative mechanisms

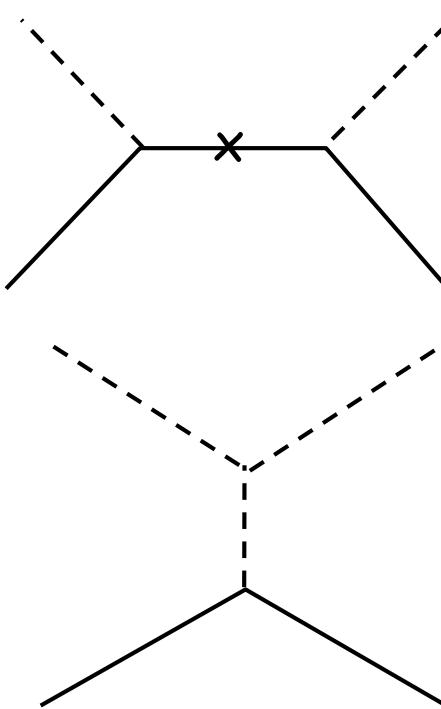
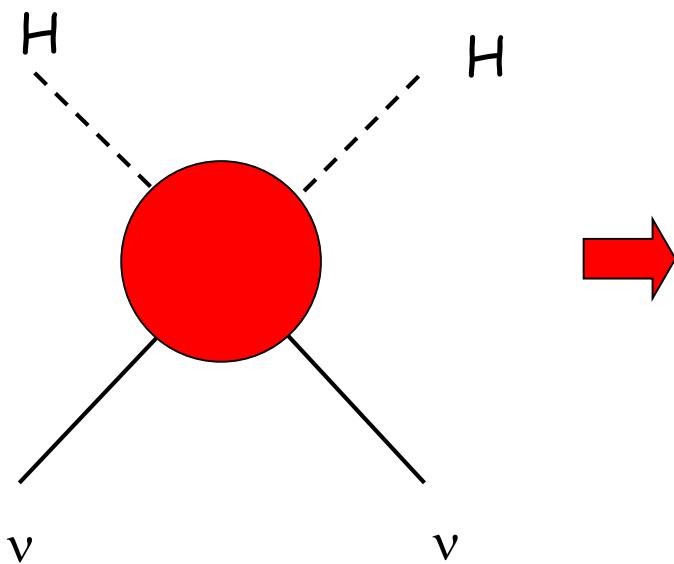
see-saws type-I does  
both things  
simultaneously:  
incomplete suppression

# D5 operator

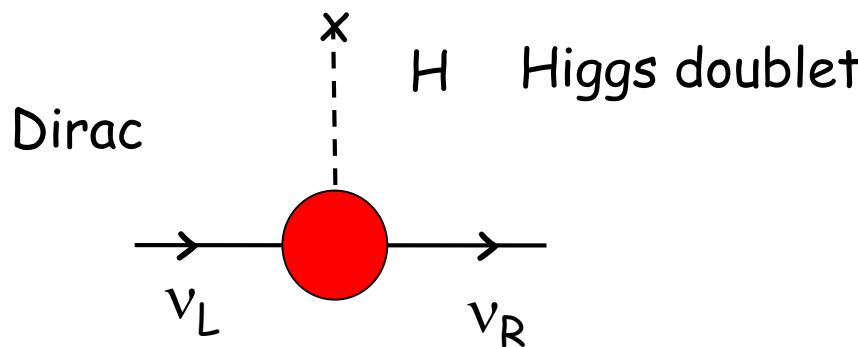
S. Weinberg

If no new particles at the EW scale, after decoupling of heavy degrees of freedom → set of non-renormalizable operators

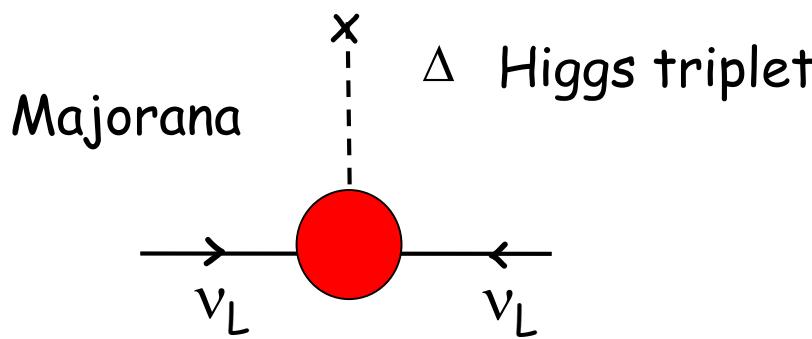
$$\frac{1}{\Lambda} \text{ LLHH}$$



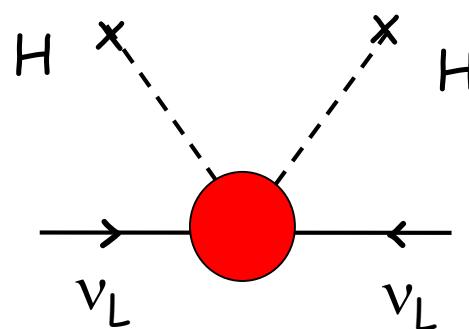
# Neutrino mass and EW Symmetry breaking



$$m_D = h \langle H \rangle$$



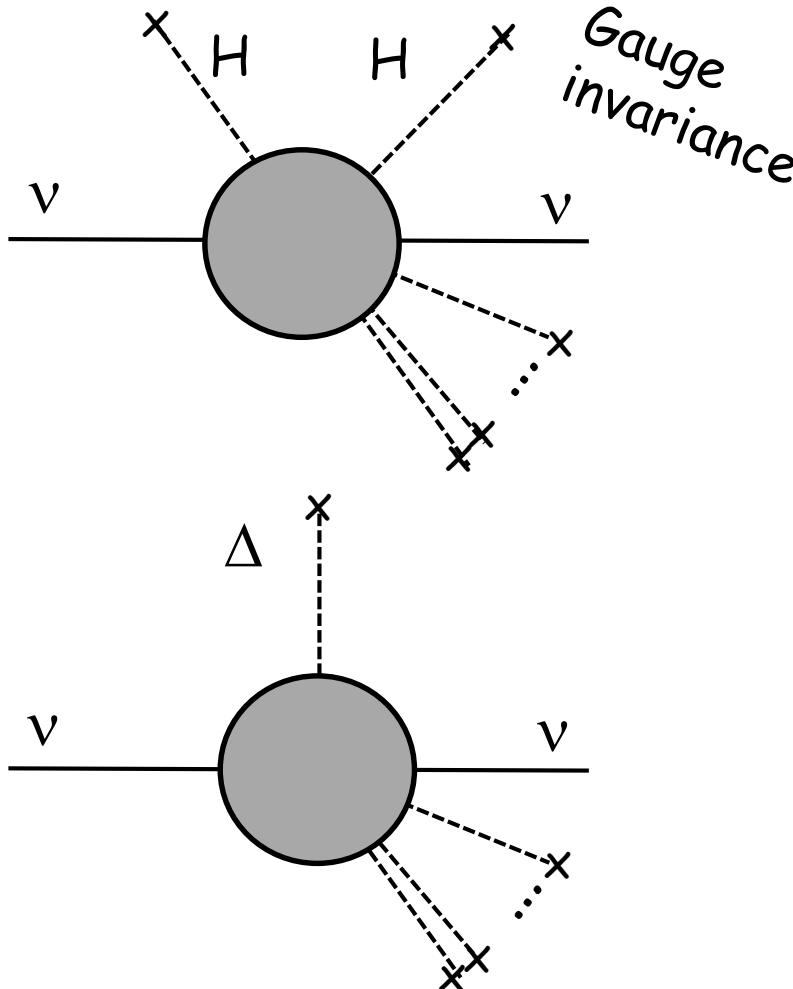
$$m_L = f \langle \Delta \rangle$$



$$m_L = f \langle H \rangle \langle H \rangle$$

Elementary or composite  
operator with  $I_W = 1$

# Origins of (finite) mass



Hard mass related to the EW scale  
small effective coupling  
small induced VEV formed  
by large VEV's (seesaw II)

Soft mass      VEV created at  
                  small scales  
                  melting at  $T \sim \text{VEV}$

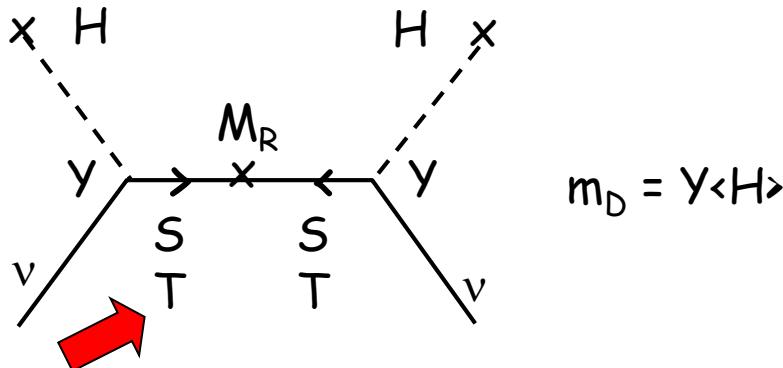
MAVAN      Environment dependent  
                  masses; relic neutrinos

Gravitationally induced mass  
Melting couplings

Similarly for Dirac neutrinos

# See-saw

Type 1



Type 3 (  $SU(2)$  triplet intermediate state)

*R Foot, H Lew, X G He, G C Joshi*

Mass matrix:

$$\begin{matrix} \nu & N \\ \nu & \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \end{matrix}$$

if  $M_R \gg m_D$

$$m_\nu = - m_D^T M_R^{-1} m_D$$

$$M_R \sim 10^{14} \text{ GeV}$$

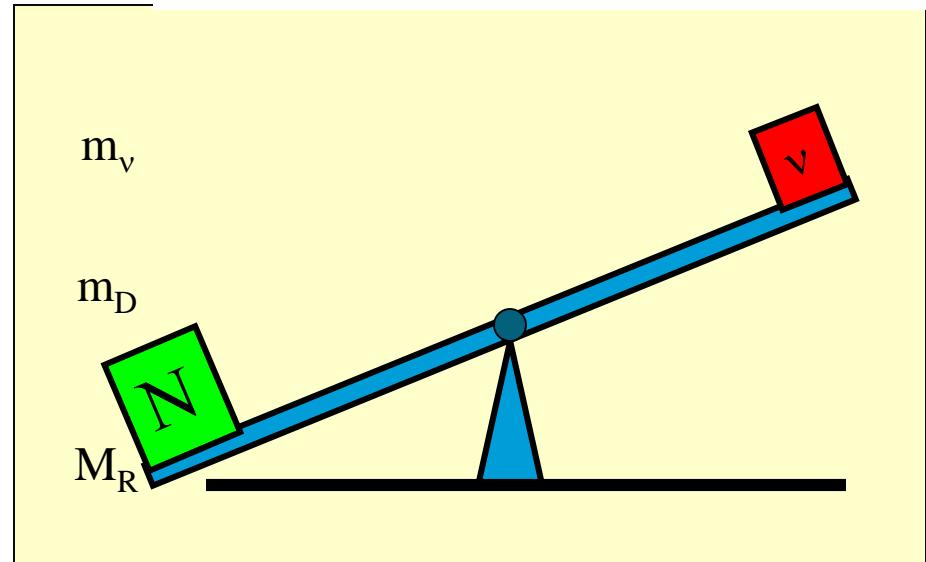
*P. Minkowski*

*T. Yanagida*

*M. Gell-Mann, P. Ramond, R. Slansky*

*S. L. Glashow*

*R.N. Mohapatra, G. Senjanovic*



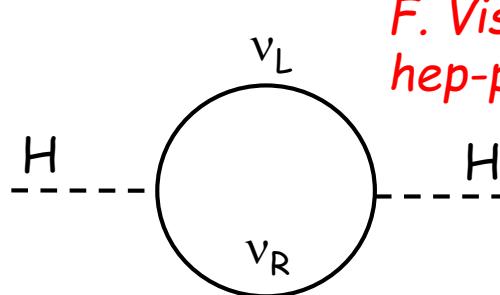
# High scale line: the problem

Simplest seesaw implies new physical scale

$$M_R \sim m_D^2 / m_\nu \sim 10^{14} \text{ GeV} \ll M_{\text{Pl}}$$

(Another indication: unification  
of gauge couplings)

$v_R$



F. Vissani  
hep-ph/9709409

J Elias-Miro et al,  
1112.3022 [hep-ph]

$$\delta m_H^2 \sim \frac{y^2}{(2\pi)^2} M_R^2 \log(q/M_R)$$

$$\sim \frac{M_R^3 m_\nu}{(2\pi v)^2} \log(q/M_R)$$

M. Fabbrichesi  
Cancellation?



New physics below Planck scale  
 $M_R < 10^7 \text{ GeV}$

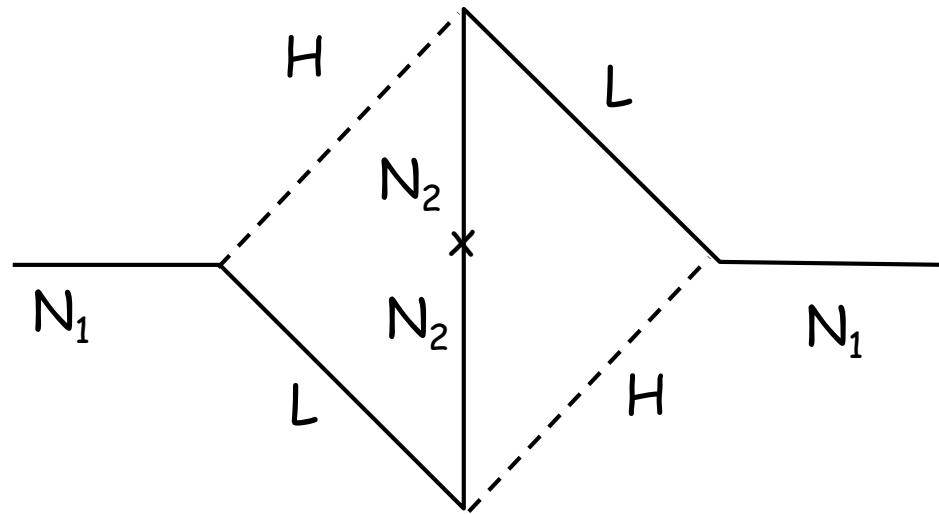
Small Yukawas,  
Leptogenesis ?

"Partial" SUSY?

# Planck-scale lepton number violation

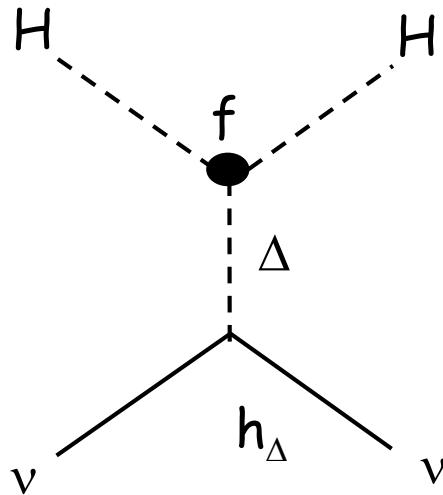
A Ibarra, et al  
1802.09997 [hep-ph]

$$M_2 \sim M_{\text{Pl}}$$



$$M_1 \sim M_2 \frac{4 \gamma_1^2 \gamma_2^2}{(16 \pi^2)^2} \log M_2 / M_{\text{Pl}} \sim 10^{14} \text{ GeV}$$

# See-saw type-III



M. Magg and C. Wetterich  
G. Lazarides, Q Shafi and C Wetterich  
R. Mohapatra, G. Senjanovic,

Seesaw for VEV's:

$$\langle \Delta \rangle = \langle H \rangle^2 f / M_\Delta^2$$

$$m_\nu = h_\Delta \langle \Delta \rangle = h f \langle H \rangle^2 / M_\Delta^2$$

Natural smallness of VEV

Light triplet?

# Double Seesaw

Three additional singlets  $S$  which couple with RH neutrinos

$$\begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & M_D^T \\ 0 & M_D & \mu \end{pmatrix} \begin{pmatrix} v \\ v^c \\ S \end{pmatrix}$$

*R.N. Mohapatra  
J. Valle*

Beyond SM:  
many heavy singlets  
...string theory

$\mu$  - scale of B-L violation

$$\mu = M_S$$



$$m_v = m_D^T M_D^{-1T} \mu M_D^{-1} m_D$$

$$\mu = 0$$

massless neutrinos

violation of universality,  
unitarity

$$\mu \ll M_D$$

Inverse seesaw

lower the scales of neutrino  
mass generation

$$\mu \gg M_D$$

Cascade seesaw

explains intermediate scale  
for the RH neutrinos

$$\mu \sim M_{\text{Pl}}, M \sim M_{\text{GU}}$$



$$M \sim M_{\text{GU}}^2/M_{\text{Pl}} \sim 10^{-14} \text{ GeV}$$

# Screening of Dirac structures

A.Y.S  
M. Lindner,  
M.A. Schmidt  
A.Y.S

$$d = M_D^{-1} m_D \quad \rightarrow \quad m_\nu = d^T M_S d$$

screening factor

## 1. Complete screening of the Dirac structure

$$m_D = A M_D \text{ as a consequence of symmetry } A = v_{EW}/V_{GUT}$$

$$d = A I \quad \rightarrow \quad m_\nu = A^2 M_S$$

Light neutrino mass matrix is determined by the heaviest one  $M_S$

## 2. Partial screening of the Dirac structure

$$m_D, M_D = \text{diag} \quad \rightarrow \quad d = \text{diagonal} \quad \text{e.g. } d = \text{diag}(a, 1, 1)$$

$$d = U_{23}^{\max} \quad \text{or } U_\omega$$

Affect mixing

S belong to Hidden sector

# Linear Seesaw

Three additional singlets  $S$  which couple with RH neutrinos

$$\begin{pmatrix} 0 & m_D^T & m_L^T \\ m_D & 0 & M_D^T \\ m_L & M_D & 0 \end{pmatrix} \begin{pmatrix} v \\ v^c \\ S \end{pmatrix}$$

*E. Witten,  
E. K. Ahmedov et al*

→  $m_v = m_D^T M_D^{-1} m_L + m_L^T M_D^{-1T} m_D$

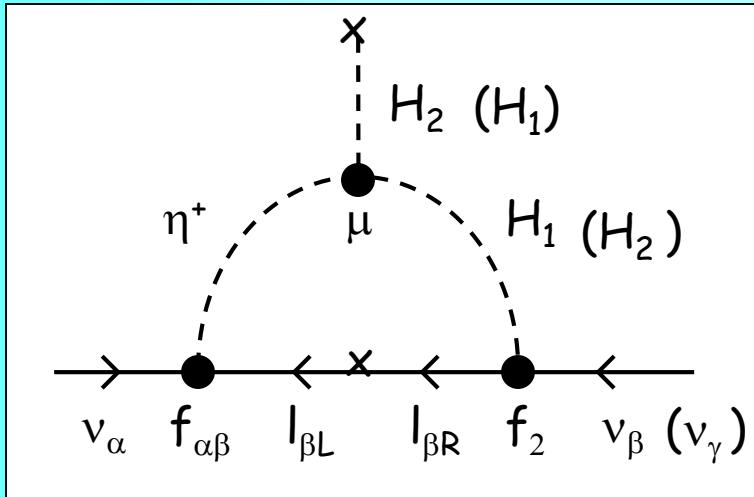
Linear in  $m_L$  - can produce weaker hierarchy than the double or inverse seesaw

if  $\sim \mu$  nonzero - both linear and double seesaw contributions

$$\frac{m_v^{\text{lin}}}{m_v^{\text{dss}}} = \frac{m_L M_D}{m_D \mu}$$

for  $m_L = m_D$  linear seesaw dominates over the inverse seesaw,

# Zee-mechanism



No RH neutrinos  
new bosons: singlet  $\eta^+$ , doublet  $H_2$

$$m_\nu = A [(f m^2 + m^2 f^T) - v (\cos \beta)^{-1} (f m f_2 + f_2^T m f^T)]$$

$$A = \sin 2\theta_Z \ln (M_2/M_1) / (8\pi^2 v \tan \beta)$$

$$m = (m_e, m_\mu, m_\tau)$$

If only  $H_1$  couples with leptons

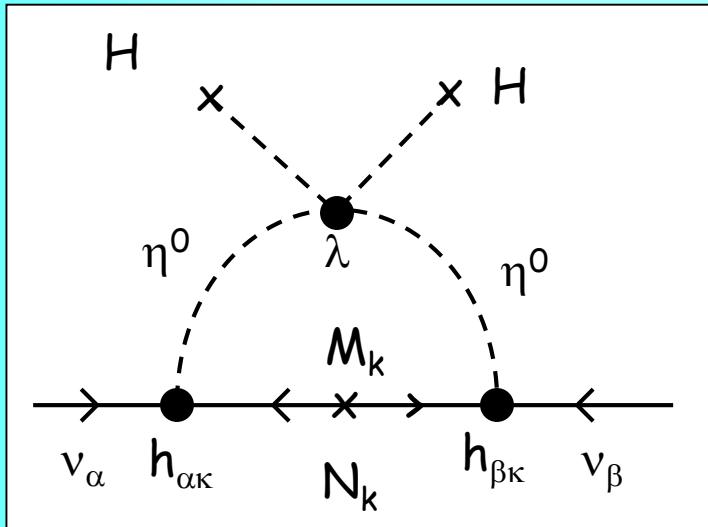
$$\begin{pmatrix} 0 & m_{e\mu} & m_{e\tau} \\ 0 & m_{\mu\tau} & 0 \end{pmatrix}$$

Can not reconcile  
two large mixings  
one small mixing and  
hierarchy of  $\Delta m^2$

X-G He  
P. Frampton, M. C. Oh  
T. Yoshikawa

- inverse hierarchy of  $f_{\alpha\beta}$
- $f_{\alpha\beta} < 10^{-4}$

# Scotogenic mechanism



E. Ma, hep-ph 0601225

No RH neutrinos  
new higgs doublet ( $\eta^+$ ,  $\eta^0$ )  
and fermionic singlets  $N_k$   
odd under discrete symmetry  $Z_2$

SM particles are  $Z_2$  even

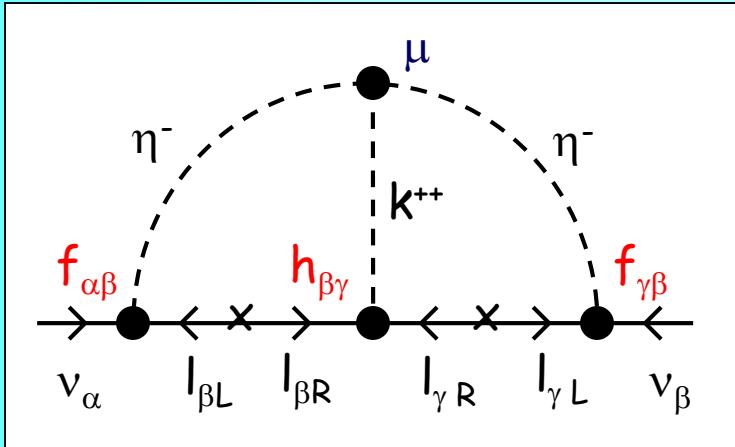
$\eta^0$  has zero VEV

If  $H$  gives mass to charged leptons

If  $Z_2$  is exact  
 $\eta^0$  or lightest  $N_k$  are stable and can be Dark Matter particles

Neutrino mass - DM connection

# Zee-Babu mechanism



Features:

- the lightest neutrino mass is zero
- neutrino data require inverted hierarchy of couplings  $h$
- $f, h \sim 0.1$

No RH neutrinos  
new scalar singlets  $\eta^-$  and  $k^{++}$

$$m_\nu \sim 8 \mu f m_l h m_l f I$$

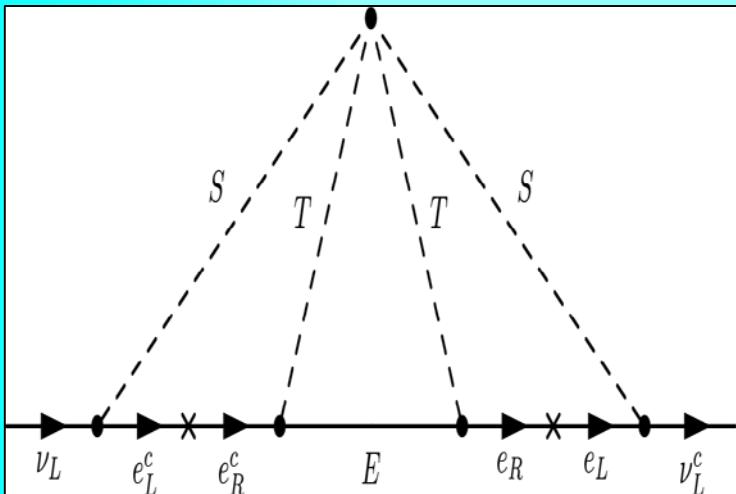
$$m_l = \text{diag}(m_e, m_\mu, m_\tau)$$

$f$  and  $h$  are matrices of the coupling in the flavor basis

Testable:

- new charged bosons
- decays  $\mu \rightarrow \gamma e$ ,  $\tau \rightarrow 3 \mu$  within reach of the forthcoming experiments

# Three loops



A. Ahriche et al,  
1404.2696 hep-ph

$Z_2$  symmetry

Classification of the  
effective operators  
→ dressing  
→ Multiloop mechanisms

$S \sim (1,1,2)$  and  $T \sim (1,3,2)$  are scalars

$E \sim (1,3,0)$  is a fermionic triplet.

There are three distinct diagrams with the sets

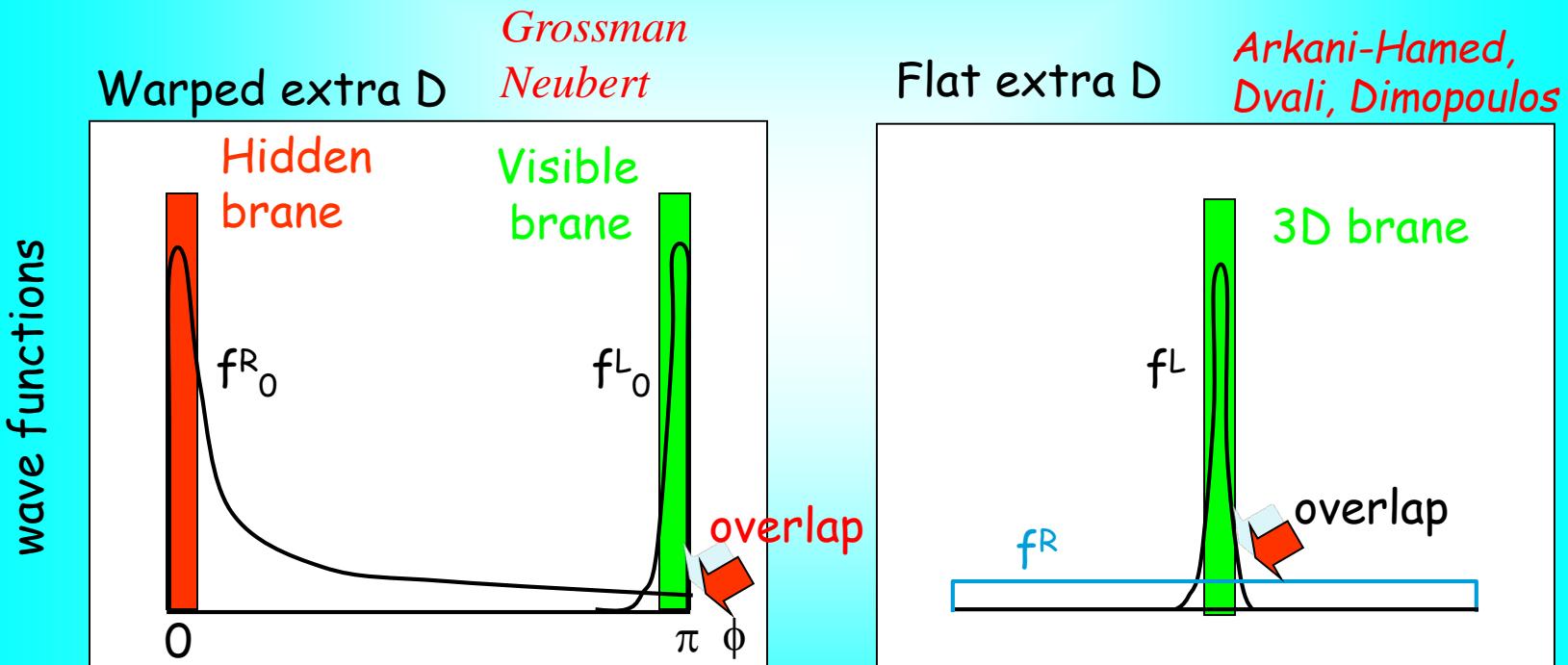
$\{T+, E^0, T-\}$ ,  $\{T^+, (E+)_c, T0\}$  and  $\{T0, E+, T--\}$

propagating in the inner loop.

# Overlap in extra dimensions

Right handed components are localized differently in extra dimensions

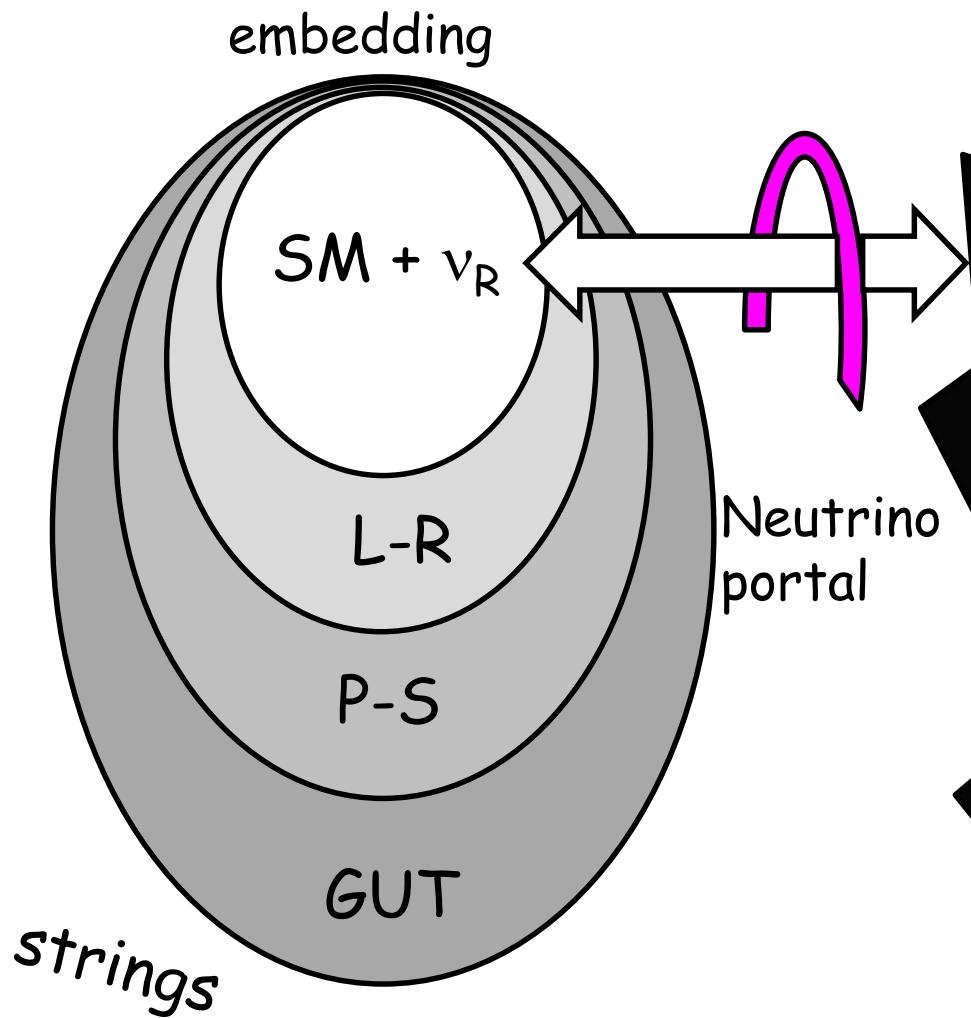
small Dirac masses due to overlap suppression:



$$m \in \overline{f^L} f^R + \text{h. c.}$$

↑ amount of overlap in extra D

# Mass and mixing from Hidden world



Neutrinos due to neutrality play special role

Hidden sector

Singlets (fermions, bosons) of GUT

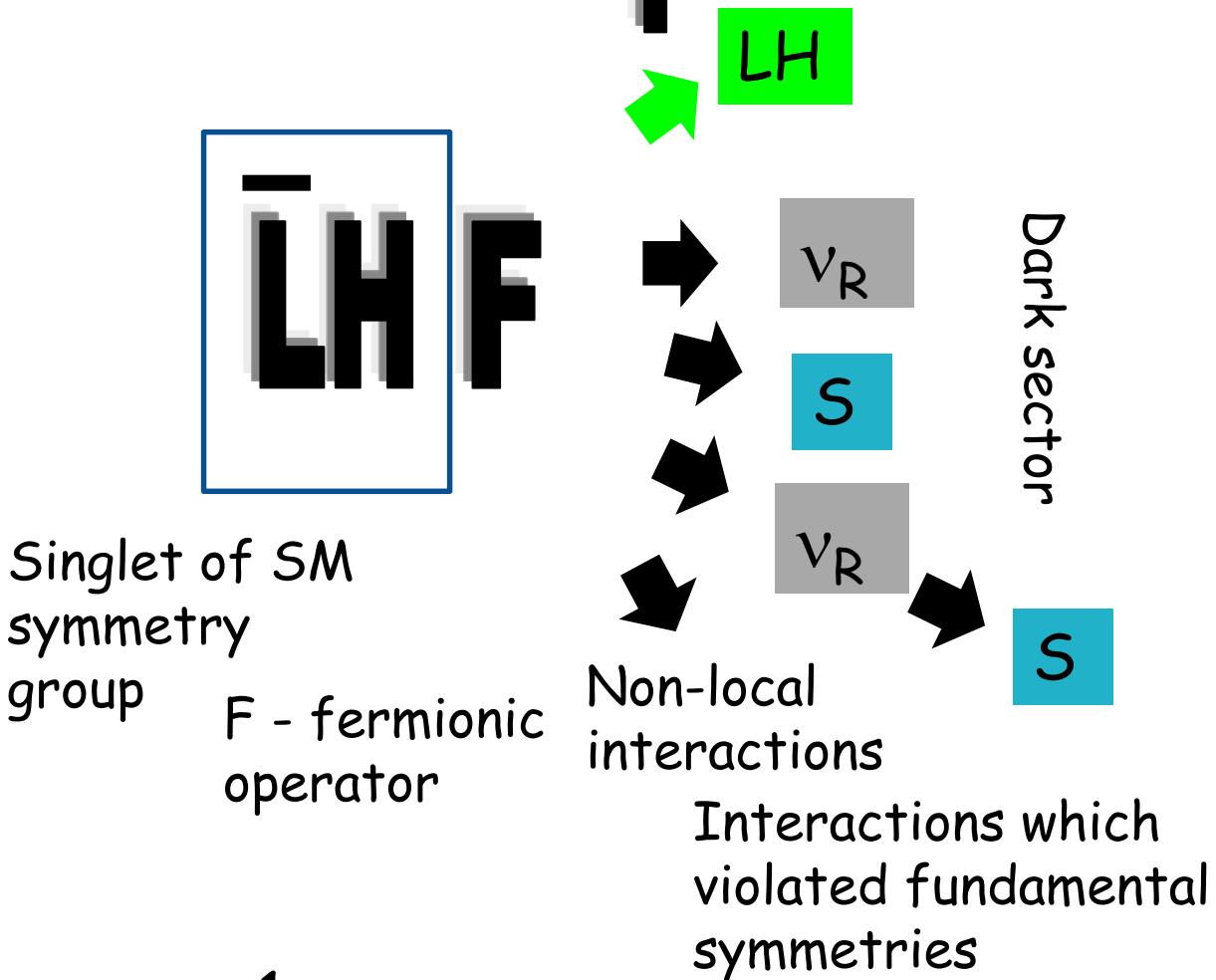
Sterile neutrinos

Axions, Majorons, DM

Realize double seesaw

Origins of smallness of neutrino mass and large (maximal mixing)

# Neutrino portal



$$\frac{1}{\Lambda^{n(F) - 3/2}} LHF$$

Neutrino are special

via the portal:

Neutrino mass - seesaw  
Large lepton mixing  
Non Standard  
Interactions

SM is well protected

Singlet of symmetry group of hidden sector

Connection to the Higgs portal:  $H^+H^-$

# Clockwork mechanism

A. Ibarra et all  
1711.02070  
[hep-ph]

fast rotation



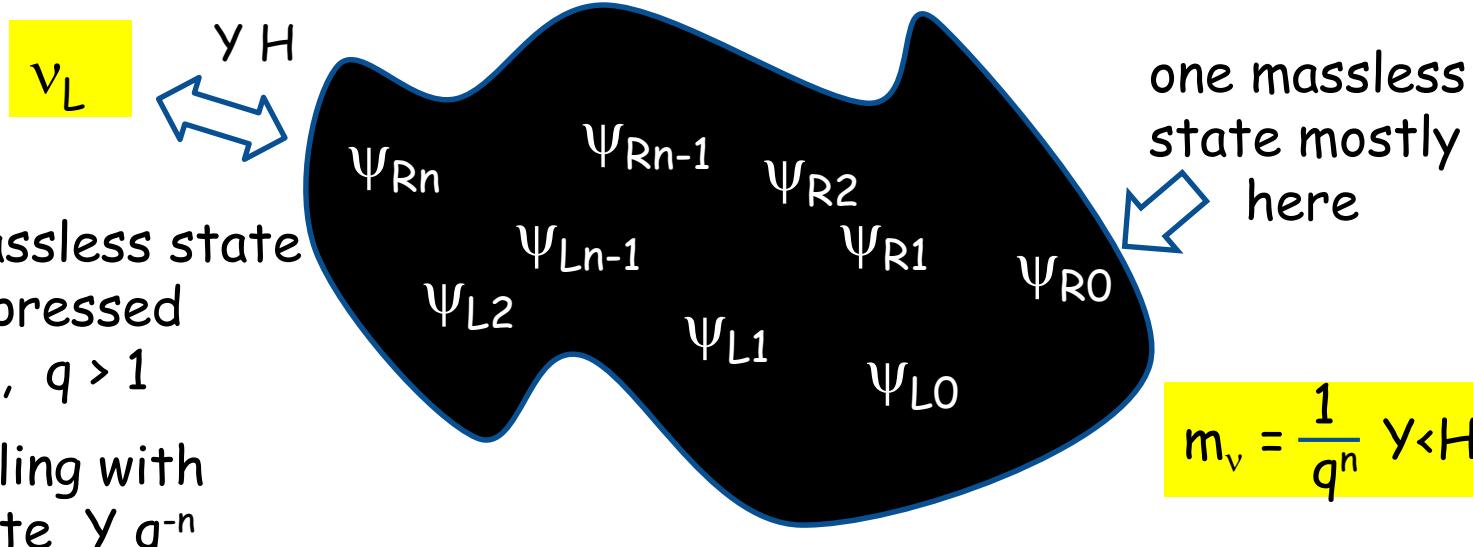
slow rotation

Strong hierarchy (small quantities) without small parameters

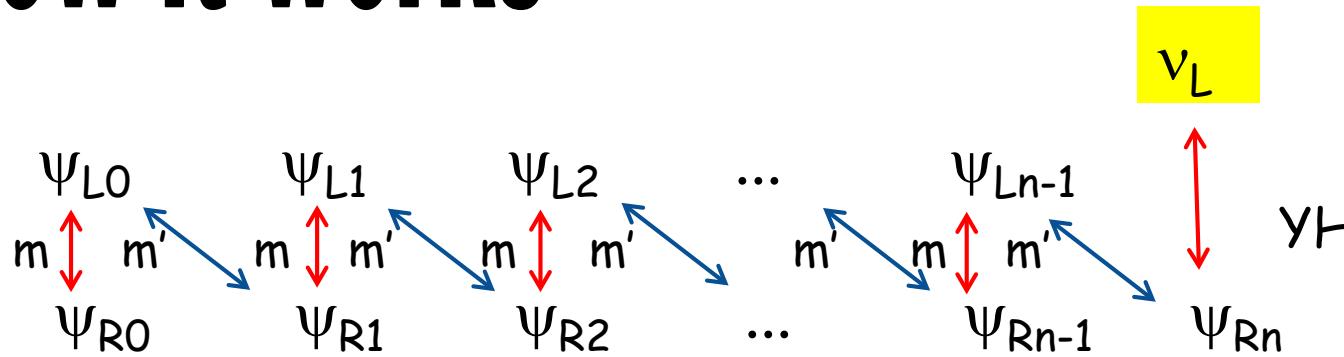
G. Giudice, et al

It can be considered as special case neutrino mass with multiple RH neutrinos, or neutrino via hidden sector

Resembles generation due to extra dimension in deconstruction mode



# How it works



$$q = m'/m$$

gear

$$\begin{pmatrix} \psi_{R0} \\ \psi_{R1} \\ \psi_{R2} \\ \dots \\ \psi_{Rn} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 & \dots \\ -q & 1 & 0 & 0 & \dots \\ 0 & -q & 1 & 0 & \dots \\ \dots & \dots & \dots & \dots & \\ 0 & \dots & & & 0 & -q \end{pmatrix} \sum q^n q^{n-1} q^{n-2} \dots 1$$

Massless state

$$(q^n \psi_{R0} + q^{n-1} \psi_{R1} + q^{n-2} \psi_{R2} + \dots + \psi_{Rn})/N$$

Normalization:

$$N^2 = \sum_{j=0}^n q^{2j}$$

Mixing of massless state in  $\psi_{Rn}$

$$1/N$$

$$m_v = \gamma \langle H \rangle / N$$

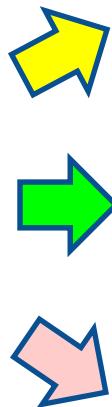
Suppression factor

$$\frac{1}{q^n} \sqrt{\frac{q^2 - 1}{q^2 - q^{-2n}}}$$

# Medium generated mass

Due to interactions with new light scalar fields

$m_v$



Interactions with usual matter  
(electrons, quarks) due to  
exchange by very light mediator

Interactions with scalar field  
sourced by DM particles

Interaction with "Fuzzy" dark matter

*Strongly  
restricted*

# Effective mass due to interactions with dark matter

$$L = -g_X \phi \bar{X} X - g_\nu \phi \bar{\nu}_L \nu_R + \text{h.c.}$$

H. Davoudiasl, et al  
1803.0001 [hep-ph]

$\phi$  - very light scalar field producing long (astronomical) range forces  
 $X$  - Dark matter particle (fermion of GeV mass scale) source of the scalar field

$$m_\nu = g_\nu \phi$$

From equation of motion for  $\phi$  neglecting neutrino contribution to generation of  $\phi$

$$\dot{\phi} = -\frac{g_X n_X}{m_\phi^2}$$

$m_\phi = 10^{-20} - 10^{-26}$  eV is mass of scalar

$n_X = \langle \bar{X} X \rangle$  is the number density of  $X$

$\rho_X$  - energy density of DM

$$g_X = g_\nu = 10^{-19} \rightarrow m_\nu = 0.1 \text{ eV}$$

$$m_\nu = \frac{g_X g_\nu \rho_X}{m_\phi^2 m_X}$$

Mass depends on local density of DM and different in different parts of the Galaxy and outside

# Interactions with fuzzy dark matter

A. Berlin,  
B. 1608.01307 [hep-ph]

Ultra-light scalar DM, huge density  $\rho$  - as a classical field, solution

$$\phi(t, x) \sim \frac{\sqrt{2\rho(x)}}{m_\phi} \cos(m_\phi t)$$

Coupling to neutrinos  $g_\phi \phi \nu_i \nu_j + \dots$

gives contribution to neutrino mass and modifies mixing

Mass states oscillate

$$\delta m(t) = g_\phi \phi(t)$$

$$\Delta\theta_m(t) = g_\phi \phi(t) / \Delta m_{ij}$$

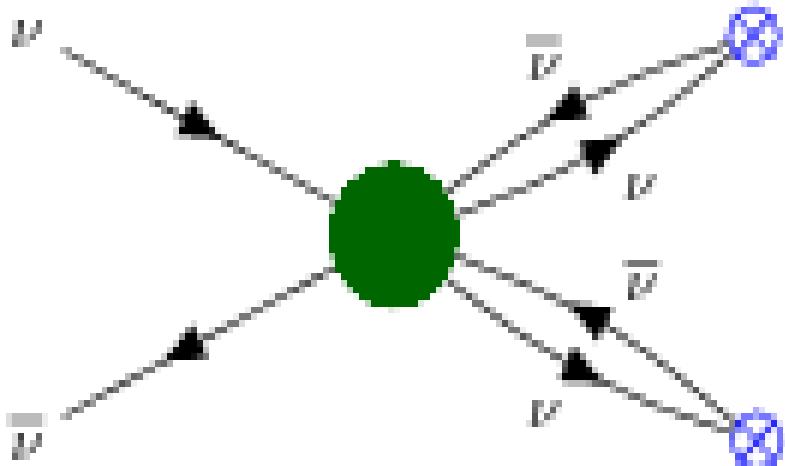
Neutrinos propagating in this field will experience time variations of mixing in time with frequency given by  $m_\phi$

Period  $\sim$  month, bounds from solar neutrinos, lab. experiments

Observable new effects (and not just renormalization of SM Yukawa and VEV) if the field has

- spatial dependence
- different sign for neutrinos and antineutrinos

# Soft couplings and small VEV's



Neutrino mass generation through the condensate (crossed blue circles) via non-perturbative interaction (green circle).

Small neutrino masses from gravitational  $\Theta$ -term

*G. Dvali and L. Funcke,  
Phys.Rev. D93 (2016) no.11, 113002  
arXiv:1602.03191 [hep-ph]*

No  $\beta\beta_{0\nu}$  decay due to large  $q^2$   
the vertex does not exist

$\beta\beta_{0\nu}$  decay - unique process where neutrinos are highly virtual

Certain generic features independent on specific scenario can be considered on phenomenological level

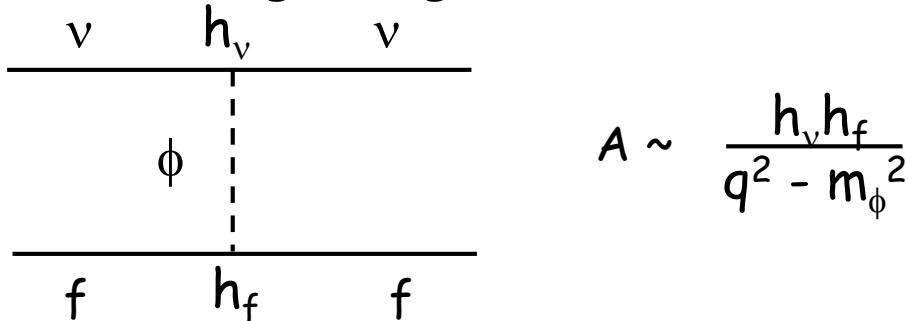
# Back-up

# **Definitions and parameterization**

# Refraction due to long range forces

Light dark sector scalars, vectors ...

Scattering via light mediators exchange:



$$A \sim \frac{h_v h_f}{q^2 - m_\phi^2}$$

With decrease of  $m_\phi$  and the same decrease of  $h$

refraction ( $q^2 = 0$ )  $\sim h_v h_f / m_\phi^2$  does not change  
inelastic scattering is suppressed as  $h_v h_f / q^2$

Refraction effects dominate at small  $m_\phi$

Potential

$$V = \frac{h_v h_f}{m_\phi^2} n_f$$

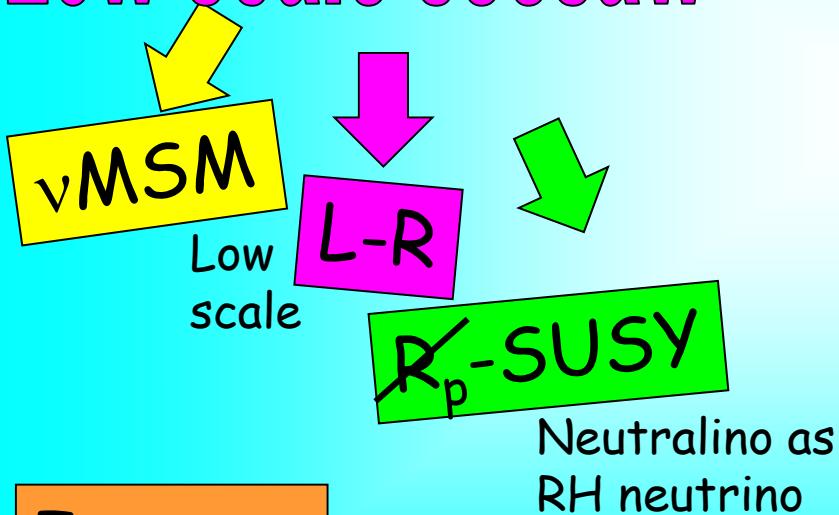
in scalar case  
contributes to mass

number density of scatterers

# EW - LHC scale

- No hierarchy problem (even without SUSY)
- testable at LHC, new particles at 0.1 - few TeV scale
- LNV decays

## Low scale seesaw



Inverse seesaw

## Radiative

- One loop
- Two loops
- Three loops
- Four loops

## Small VEV

Higgs Triplet

New Higgs doublets

## Radiative seesaw

High dimensional operators

Connection to Dark Matter

# Remarks on sterile neutrinos

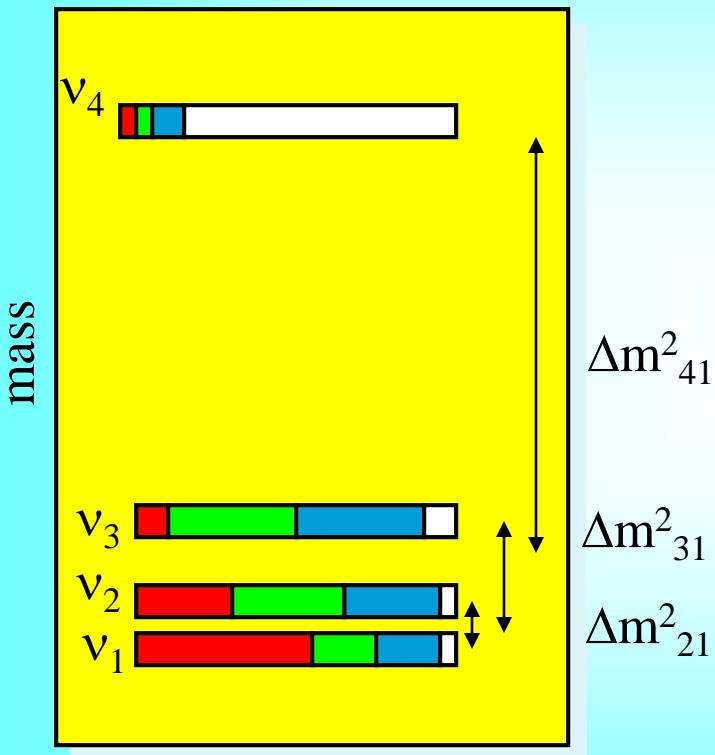
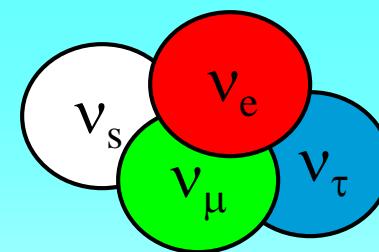
Most of discussion in 3 neutrino framework

Rather plausible that new neutrino states exist

Depending on mass and mixing they may or may not affect our consideration significantly

# (3 + 1) scheme

## Interpretation



Strong perturbation of 3ν pattern:

$$m_{\alpha\beta}^{\text{ind}} \sim m_4 U_{\alpha 4} U_{\beta 4} \sim \sqrt{\Delta m_{32}^2}$$

Effect of possible sterile neutrinos can be neglected if

$$m_{\alpha\beta}^{\text{ind}} \ll \frac{1}{2} \sqrt{\Delta m_{21}^2} \sim 3 \cdot 10^{-3} \text{ eV}$$

$$|U_{\alpha 4}|^2 < 10^{-3} (1 \text{ eV}/m_4)$$

# Large flavor mixing from steriles

Mass matrix

$$\begin{array}{c} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_S \end{array} \left( \begin{array}{ccc|c} m_{ee} & m_{e\mu} & m_{e\tau} & m_{eS} \\ \dots & m_{\mu\mu} & m_{\mu\tau} & m_{\mu S} \\ \dots & \dots & m_{\tau\tau} & m_{\tau S} \\ \dots & \dots & \dots & m_{SS} \end{array} \right)$$

no contribution from S to  $\beta\beta_{0\nu}$  decay, but S do contribute to oscillations

$$m_\nu = m_a + m_{\text{ind}}$$

$$m_a = \begin{pmatrix} 0.2 & 0.4 & 0.4 \\ \dots & 2.8 & 2.0 \\ \dots & \dots & 3.0 \end{pmatrix} 10^{-2} \text{ eV}$$

$$m_{\text{ind}} = \frac{m_{SS}}{1 \text{ eV}}$$

eV scale seesaw

$$\begin{pmatrix} 2.0 & 2.0 & 4.5 \\ \dots & 2.0 & 4.5 \\ \dots & \dots & 10.0 \end{pmatrix} 10^{-2} \text{ eV}$$

produce dominant  $\mu\tau$ -block with small determinant

Enhance lepton mixing  
Generate TBM mixing

$m_{eS}$   $m_{\mu S}$   $m_{\tau S}$  may have certain symmetry

# Summarizing results

# Quark and Lepton Mixing

No immediate relations,  
equalities

→ Different mechanism  
of generation of masses  
of quarks and neutrinos

Partially  
connected

e.g. in seesaw  
type-II

Still some relations can be obtained within GUT since  
the same 126 contributes to quark masses

$$\theta_{12}^l \sim \pi/2 - \theta_{12}^q = \theta_c$$

QLC -relations

$$\theta_{23}^l \sim \pi/2 - \theta_{23}^q$$

$$\theta_{13}^l \sim \frac{1}{\sqrt{2}} \theta_c$$

Predicted from QLC

Other quark mixing angles can be involved  
But they give small corrections to these relations

# Neutrino weak states

$$\begin{bmatrix} \nu_e \\ e \end{bmatrix}_L \quad \begin{bmatrix} \nu_\mu \\ \mu \end{bmatrix}_L \quad \begin{bmatrix} \nu_\tau \\ \tau \end{bmatrix}_L$$

$$I_W = 1/2$$

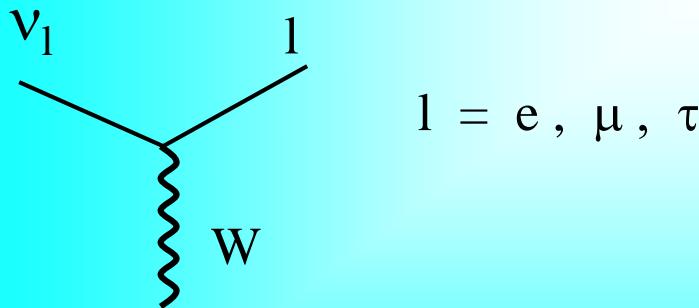
$$I_{3W} = 1/2$$

Chiral components

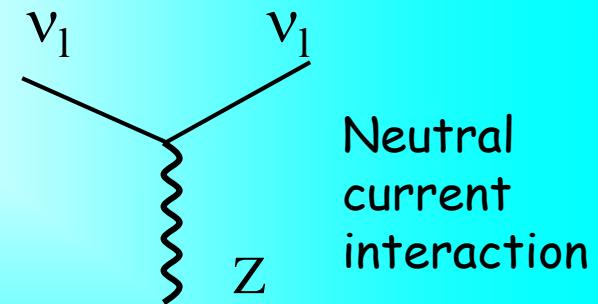
$$\nu_L = \frac{1}{2}(1 - \gamma_5) \nu$$

$$\nu_R = \frac{1}{2}(1 + \gamma_5) \nu$$
?

$\nu_e \quad \nu_\mu \quad \nu_\tau$  neutrino weak states, form doublets  
(charged currents) with definite charged leptons,



$$\frac{g}{2\sqrt{2}} \bar{l} \gamma^\mu (1 - \gamma_5) \nu_l W_\mu^+ + \text{h.c.}$$



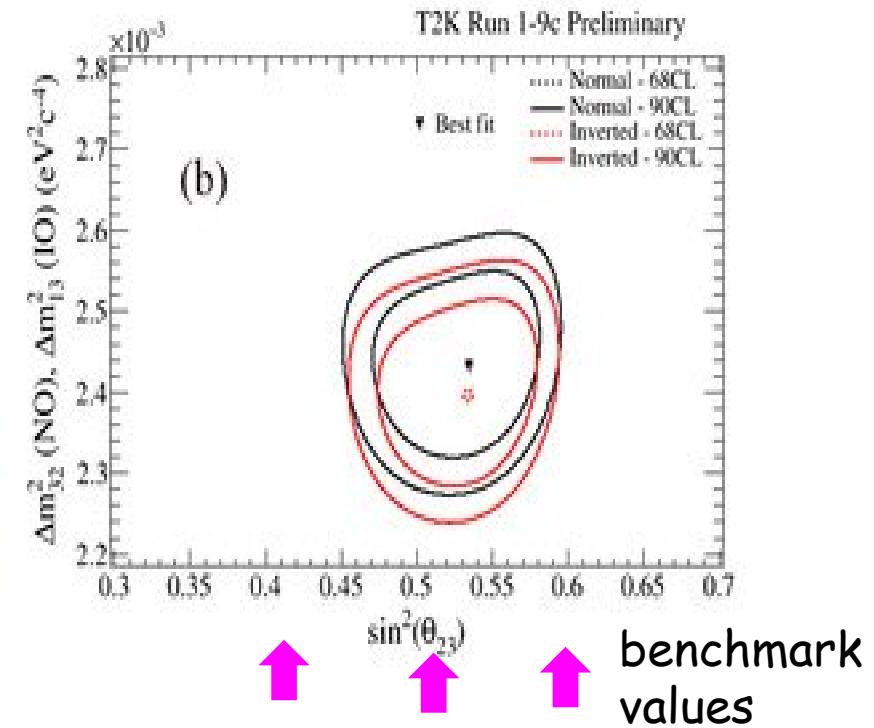
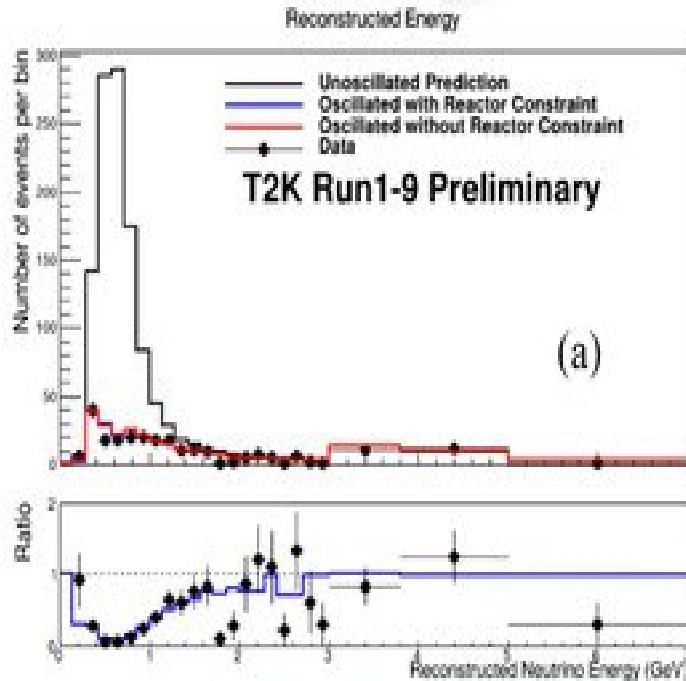
$$\frac{g}{4} \bar{\nu}_l \gamma^\mu (1 - \gamma_5) \nu_l Z_\mu$$

Conservation of lepton numbers  $L_e, L_\mu, L_\tau$

# T2K results

Update 2.2 →  $2.6 \times 10^{21}$  POT

D. Karlen, (T2K Collaboration)  
Universe 2019, 5(1), 21.

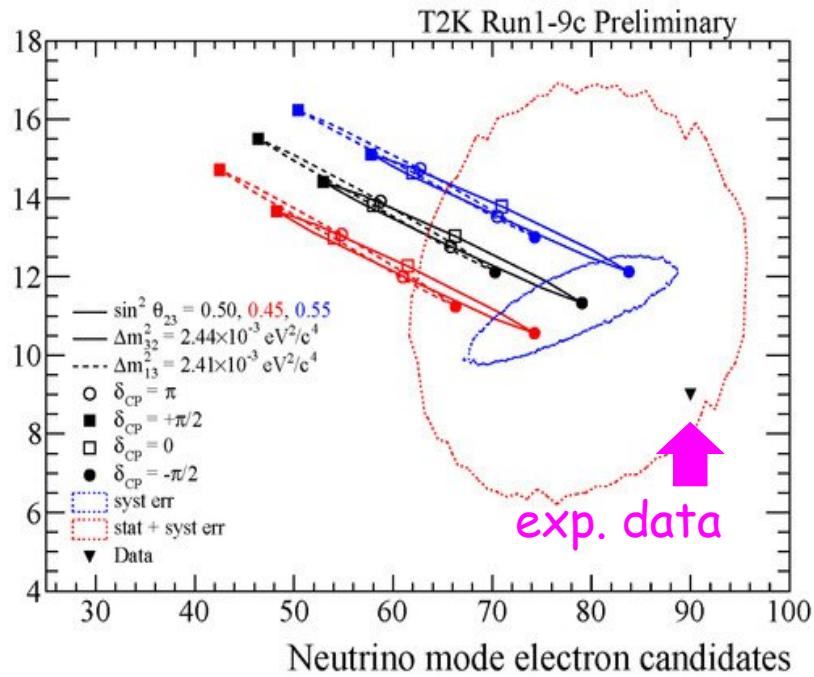


The rate of muon-neutrinos in the far detector. Data vs. expected rate for the best fit oscillation parameters.

Confidence intervals for the atmospheric oscillation parameters for the normal and inverted mass ordering .

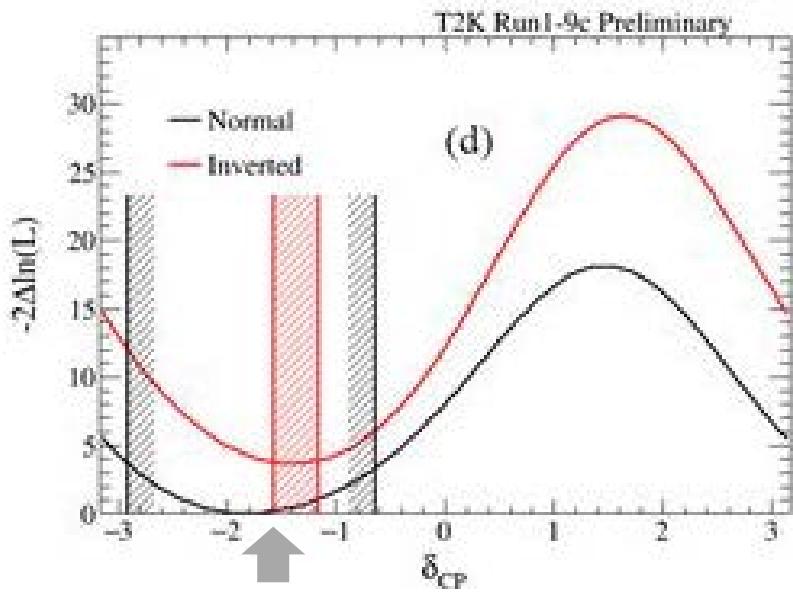
# T2K results

Antineutrino mode electron candidates



The expected numbers of  $\nu_e$  and  $\bar{\nu}_e$  events for optimized systematic parameter values. The solid (dashed) ellipses are for NO (IO)

Jagged - expected  $1\sigma$  regions for  $\sin^2 \theta_{23} = 0.5$ ,  $\delta_{CP} = -\pi/2$  with different treatment of systematics: random with external data (blue) or Poisson random (red)



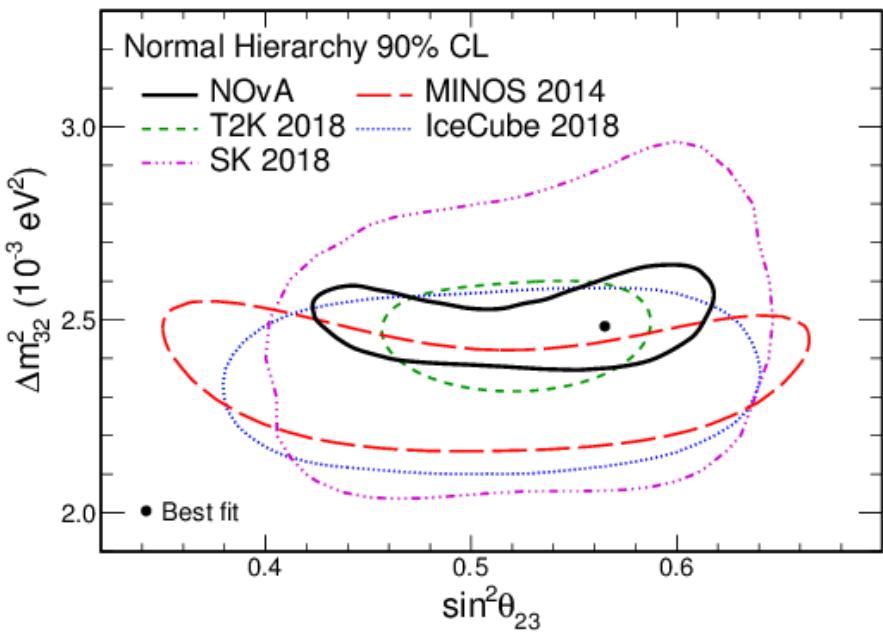
The frequentist  $2\sigma$  confidence intervals on  $\delta_{CP}$

Best fit close to maximal CP violation

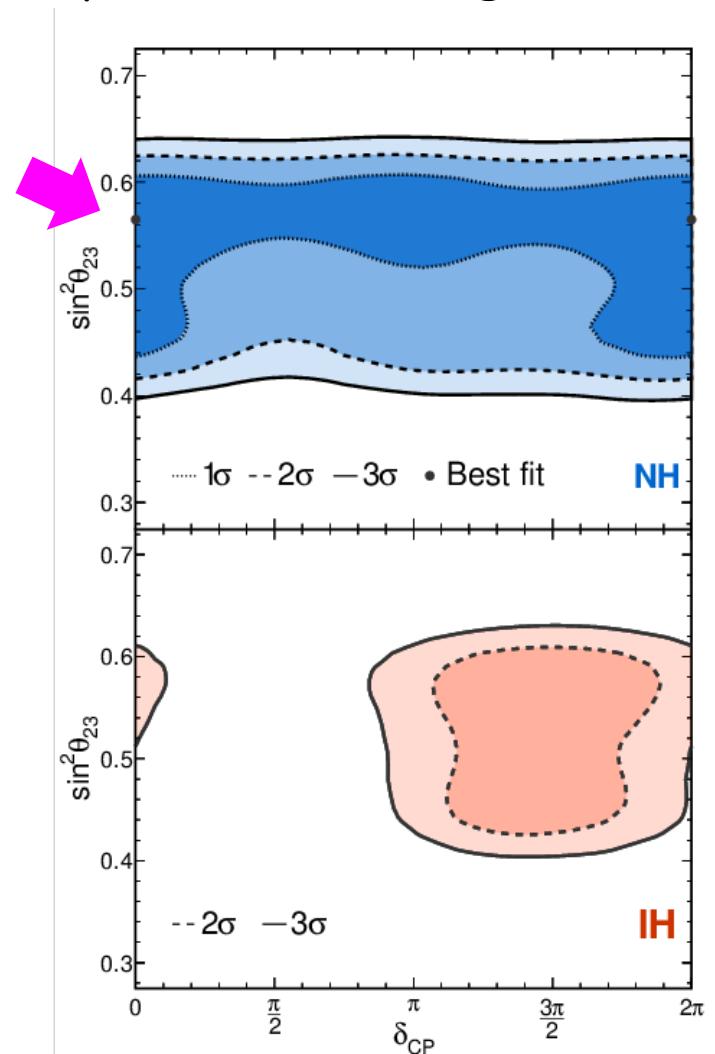
# NOvA results

NOvA Collaboration (Acero, M.A. et al.)  
arXiv:1906.04907 [hep-ex]

First measurement of neutrino oscillation parameters using neutrinos and antineutrinos by NOvA



Best fit point (NO):  
no CP violation,  $\delta_{CP} = 0$ .  
At  $1\sigma$  any value is allowed



# 1-3 mixing from reactors

First Double Chooz  $\theta_{13}$  Measurement via Total Neutron Capture Detection - Double Chooz Collaboration (de Kerret, H. et al.)  
arXiv:1901.09445 [hep-ex]

Double Chooz IV

TnC MD (n-H⊕n-C⊕n-Gd)

Daya Bay

PRD 95, 072006 (2017) n-Gd  
PRD 93, 072011 (2016) n-H

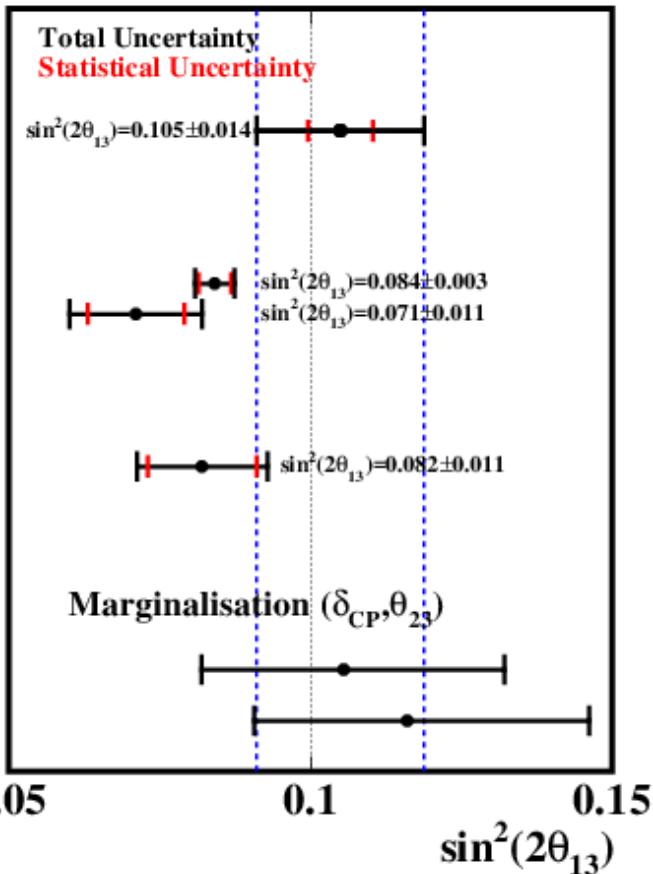
RENO

PRL 116, 211801(2016) n-Gd

T2K

PRD 96, 092006 (2017)

$\Delta m_{32}^2 > 0$   
 $\Delta m_{32}^2 < 0$



The most precise published reactor measurements of  $\theta_{13}$  from DC MD TnC , DYB and RENO .

DC result shows a [25,48] % higher central value whose significance ranges [1.3,1.9] $\sigma$  compared to other reactor measurements.

The T2K larger uncertainty is due to the marginalisation over  $\theta_{23}$  and CP violation.

# Atmospheric neutrinos

ANTARES: measurements of 2-3 mass and mixing

IceCube Deep Core: tentative attempts to extract mass hierarchy

ORCA: 2 strings employed

Super-Kamiokande -IV

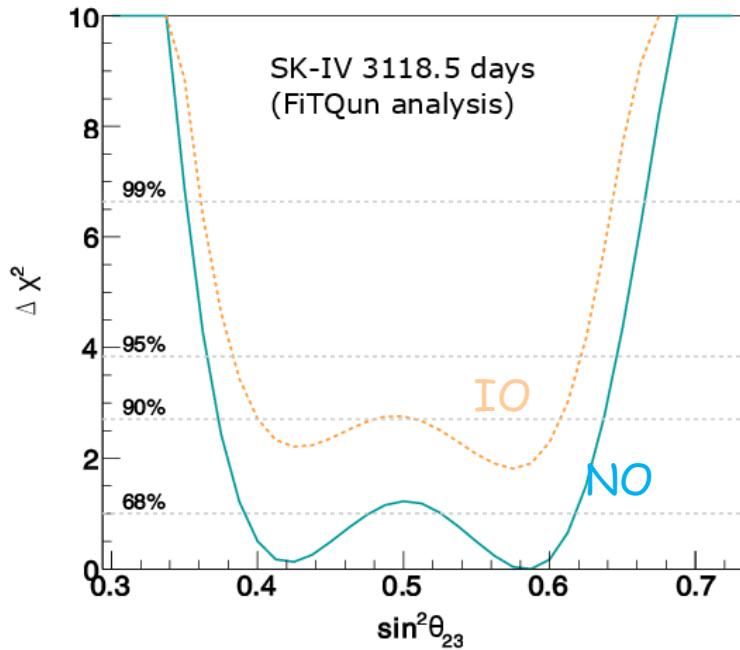
*Super-Kamiokande Collaboration  
(Jiang, M. et al.) PTEP 2019 (2019)  
no.5, 053F01, 1901.03230 [hep-ex]*

Atmospheric Neutrino Oscillation Analysis With Improved Event Reconstruction

- A new event reconstruction algorithm based on a maximum likelihood method developed .
- Improves kinematic and particle identification capabilities,
- Enable to increase fiducial volume by 32%
- increase the sensitivity to the neutrino mass hierarchy.

# Super-Kamiokande results

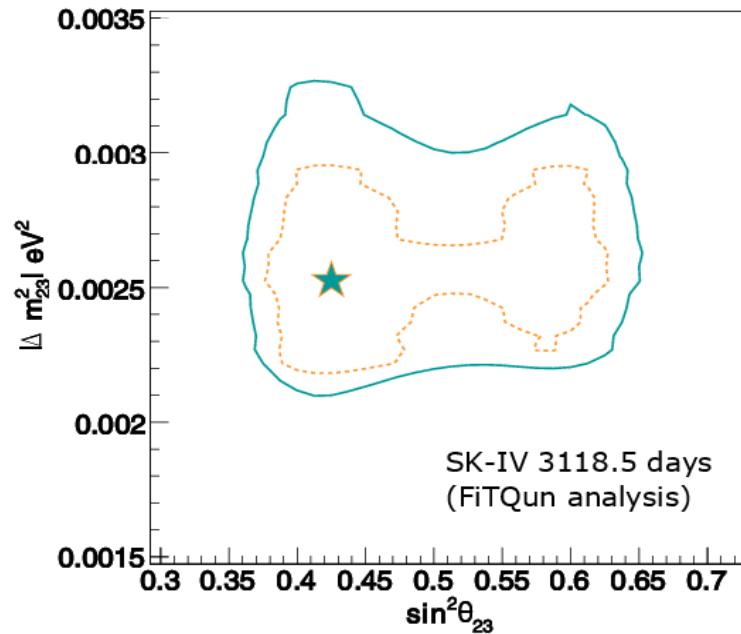
253.9 kton·year exposure



Super-K constraint with no assumed bounds on 13 mixing

Weak preference for the NO, disfavoring the IO at 74%

Super-Kamiokande Collaboration  
(Jiang, M. et al.) PTEP 2019 (2019)  
no.5, 053F01, 1901.03230 [hep-ex]



The best-fit value, (star) is the same for NO and IO.  $\sin^2\theta_{13} = 0.0210 \pm 0.0011$ .  
The contours - relative to the global bf.

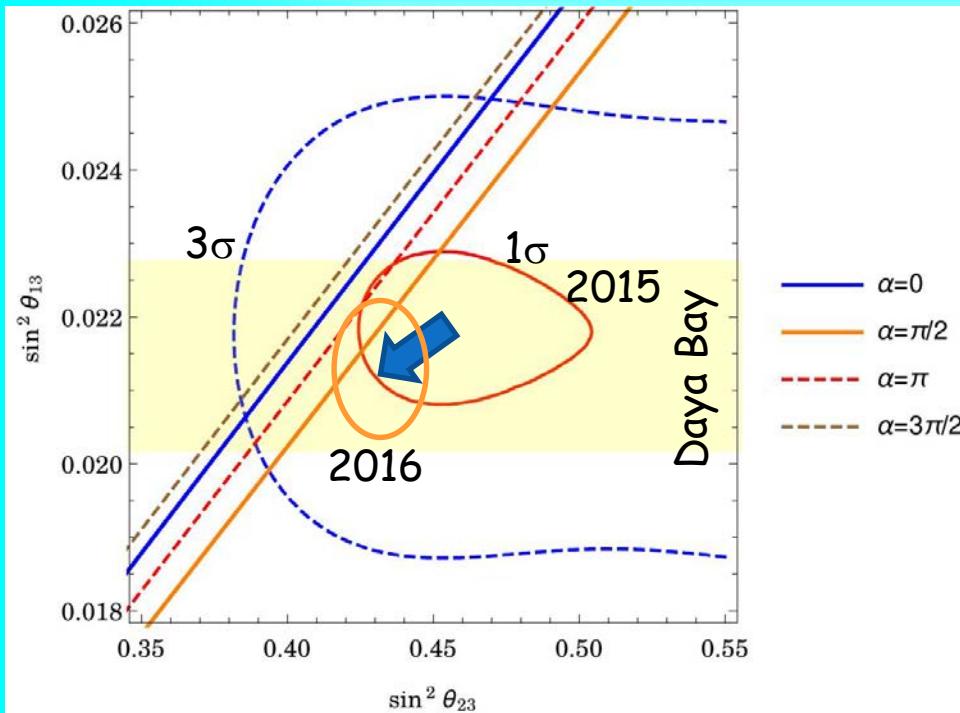
bf - substantial deviation from maximal:  $\sin^2\theta_{23} = 0.42$ .  
At 1 maximal mixing and high octant are allowed

# General relation

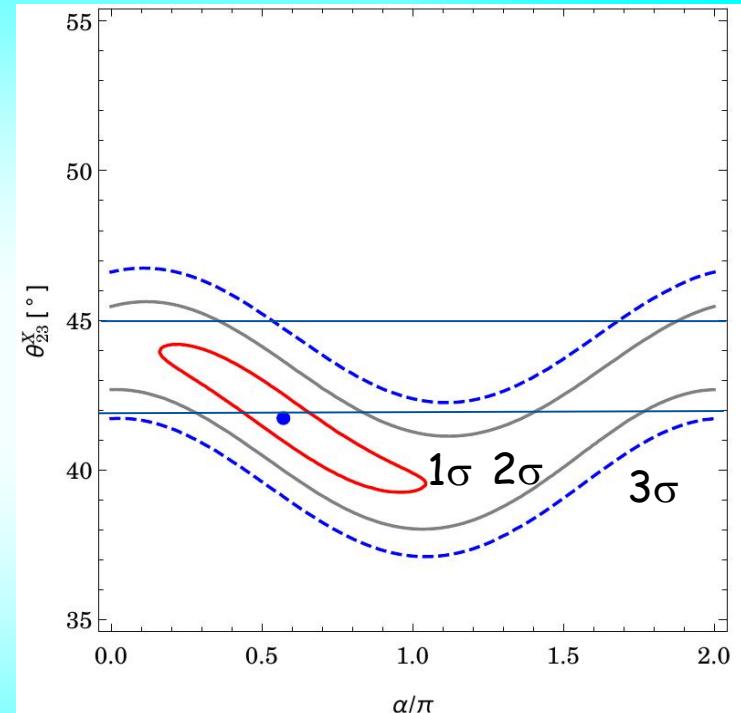
$$\sin^2 \theta_{13} = \sin^2 \theta_{23} \sin^2 \theta_C (1 + O(\lambda^2))$$

Normal mass ordering

$$\lambda = \sin \theta_C$$



Dependence of 1-3 mixing on 2-3 mixing for different values of the phase  $\alpha$ . Allowed regions from the global fit NuFIT 2015



Allowed values of parameters of  $U_x$   
Best fit value:  $\theta_x^{23} = 42^\circ$

RGE effect from maximal mixing value at high scale

# Summarizing results

NuFIT 4.1 (2019)

$$|U|_{3\sigma}^{\text{w/o SK-atm}} = \begin{pmatrix} 0.797 \rightarrow 0.842 & 0.518 \rightarrow 0.585 & 0.143 \rightarrow 0.156 \\ 0.244 \rightarrow 0.496 & 0.467 \rightarrow 0.678 & 0.646 \rightarrow 0.772 \\ 0.287 \rightarrow 0.525 & 0.488 \rightarrow 0.693 & 0.618 \rightarrow 0.749 \end{pmatrix}$$

$$|U|_{3\sigma}^{\text{with SK-atm}} = \begin{pmatrix} 0.797 \rightarrow 0.842 & 0.518 \rightarrow 0.585 & 0.143 \rightarrow 0.156 \\ 0.243 \rightarrow 0.490 & 0.473 \rightarrow 0.674 & 0.651 \rightarrow 0.772 \\ 0.295 \rightarrow 0.525 & 0.493 \rightarrow 0.688 & 0.618 \rightarrow 0.744 \end{pmatrix}$$

TBM

$$\begin{pmatrix} 0.866 & 0.577 & 0 \\ 0.408 & 0.577 & 0.707 \\ 0.408 & 0.577 & 0.707 \end{pmatrix}$$

But there are correlations between elements

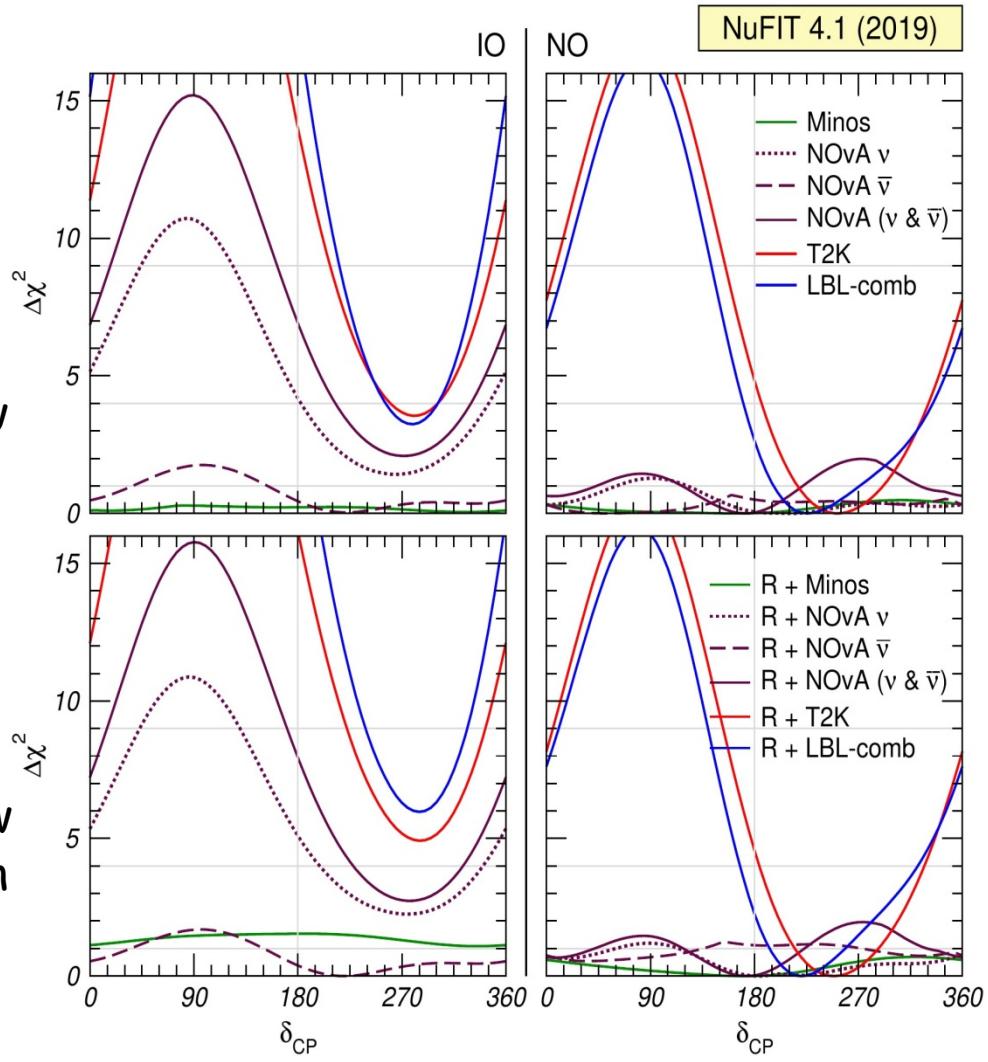
# Summarizing

NuFIT 4.1 (2019)

		Normal Ordering (best fit)		Inverted Ordering ( $\Delta\chi^2 = 6.2$ )	
		bfp $\pm 1\sigma$	$3\sigma$ range	bfp $\pm 1\sigma$	$3\sigma$ range
without SK atmospheric data	$\sin^2 \theta_{12}$	$0.310^{+0.013}_{-0.012}$	$0.275 \rightarrow 0.350$	$0.310^{+0.013}_{-0.012}$	$0.275 \rightarrow 0.350$
	$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \rightarrow 36.27$	$33.82^{+0.78}_{-0.76}$	$31.61 \rightarrow 36.27$
	$\sin^2 \theta_{23}$	$0.558^{+0.020}_{-0.033}$	$0.427 \rightarrow 0.609$	$0.563^{+0.019}_{-0.026}$	$0.430 \rightarrow 0.612$
	$\theta_{23}/^\circ$	$48.3^{+1.1}_{-1.9}$	$40.8 \rightarrow 51.3$	$48.6^{+1.1}_{-1.5}$	$41.0 \rightarrow 51.5$
	$\sin^2 \theta_{13}$	$0.02241^{+0.00066}_{-0.00065}$	$0.02046 \rightarrow 0.02440$	$0.02261^{+0.00067}_{-0.00064}$	$0.02066 \rightarrow 0.02461$
	$\theta_{13}/^\circ$	$8.61^{+0.13}_{-0.13}$	$8.22 \rightarrow 8.99$	$8.65^{+0.13}_{-0.12}$	$8.26 \rightarrow 9.02$
	$\delta_{\text{CP}}/^\circ$	$222^{+38}_{-28}$	$141 \rightarrow 370$	$285^{+24}_{-26}$	$205 \rightarrow 354$
	$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \rightarrow 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \rightarrow 8.01$
with SK atmospheric data	$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.523^{+0.032}_{-0.030}$	$+2.432 \rightarrow +2.618$	$-2.509^{+0.032}_{-0.030}$	$-2.603 \rightarrow -2.416$
		Normal Ordering (best fit)		Inverted Ordering ( $\Delta\chi^2 = 10.4$ )	
		bfp $\pm 1\sigma$	$3\sigma$ range	bfp $\pm 1\sigma$	$3\sigma$ range
	$\sin^2 \theta_{12}$	$0.310^{+0.013}_{-0.012}$	$0.275 \rightarrow 0.350$	$0.310^{+0.013}_{-0.012}$	$0.275 \rightarrow 0.350$
	$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \rightarrow 36.27$	$33.82^{+0.78}_{-0.75}$	$31.61 \rightarrow 36.27$
	$\sin^2 \theta_{23}$	$0.563^{+0.018}_{-0.024}$	$0.433 \rightarrow 0.609$	$0.565^{+0.017}_{-0.022}$	$0.436 \rightarrow 0.610$
	$\theta_{23}/^\circ$	$48.6^{+1.0}_{-1.4}$	$41.1 \rightarrow 51.3$	$48.8^{+1.0}_{-1.2}$	$41.4 \rightarrow 51.3$
	$\sin^2 \theta_{13}$	$0.02237^{+0.00066}_{-0.00065}$	$0.02044 \rightarrow 0.02435$	$0.02259^{+0.00065}_{-0.00065}$	$0.02064 \rightarrow 0.02457$
	$\theta_{13}/^\circ$	$8.60^{+0.13}_{-0.13}$	$8.22 \rightarrow 8.98$	$8.64^{+0.12}_{-0.13}$	$8.26 \rightarrow 9.02$
$\Delta m_{21}^2$	$\delta_{\text{CP}}/^\circ$	$221^{+39}_{-28}$	$144 \rightarrow 357$	$282^{+23}_{-25}$	$205 \rightarrow 348$
	$\frac{10^{-5} \text{ eV}^2}{10^{-5} \text{ eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \rightarrow 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \rightarrow 8.01$
	$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.528^{+0.029}_{-0.031}$	$+2.436 \rightarrow +2.618$	$-2.510^{+0.030}_{-0.031}$	$-2.601 \rightarrow -2.419$

# Summarizing results

Bounds on  $\delta_{CP}$  from MINOS (green), NOvA (dark-redwood), T2K (red) and their combination (blue). Left (right) panels are for IO (NO); for each experiment  $\Delta\chi^2$  is defined with respect to the global minimum of the two orderings. For NOvA we also show as dotted (dashed) lines the results obtained using only neutrino (antineutrino) data. The upper panels show the 1-dimensional  $\Delta\chi^2$  from LBL accelerator experiments after imposing a prior on  $\Theta_{13}$  to account for reactor bounds. The lower panels show the corresponding determination when the full information of LBL and reactor experiments is used in the combination. In all panels solar and KamLAND data are included to constrain  $\Delta m^2_{21}$  and  $\Theta_{12}$ .

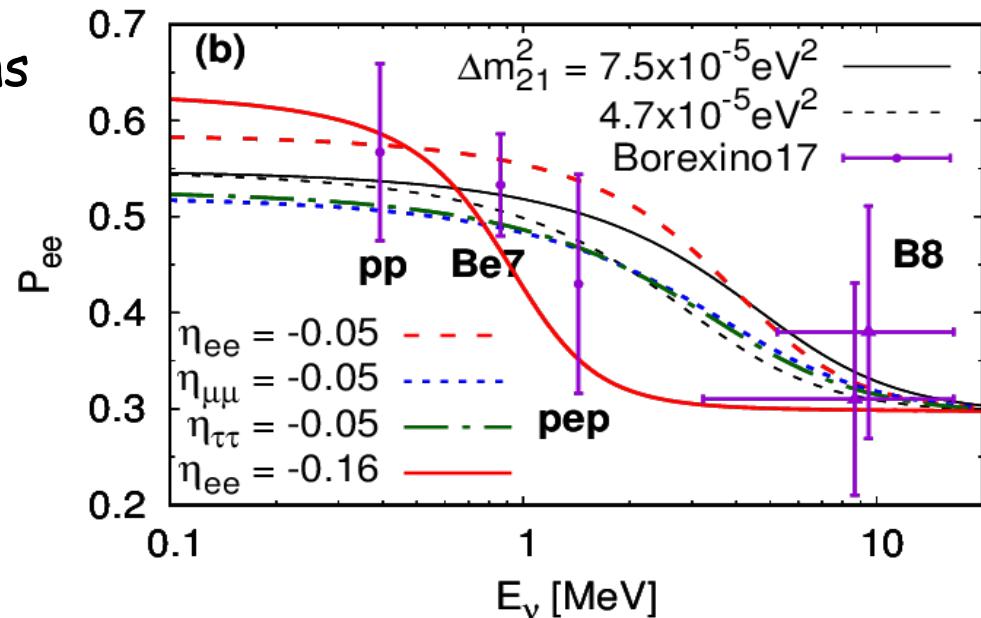


# Refraction due to very light scalar mediator

Shao-Feng Ge,  
S. Parke, 1812.08376  
[hep-ph]

Neutrino scattering on electrons  
via very light scalar exchange

The solar neutrino conversion  
probabilities with scalar  
NSIs vs. Borexino results.



To satisfy bounds on  $h_\nu$ ,  $h_e$  (especially from searches of 5th force):

$$1/m_\phi \gg R_{\text{Earth}}$$

→ strong suppression of the potential  $V = V_0 m_\phi R_{\text{Earth}}$

To avoid bounds - cancellations in 5<sup>th</sup> force experiments - not shown if this is possible

After more than 40 years of theoretical studies, thousands of papers written we are not far from the beginning: "ground zero" determined by experimental measurements

Big temptation to give such lectures as collection of jokes, if not one point

Enormous efforts in determination of matrix elements, cross-sections, systematics, backgrounds...

And all this is to measure neutrino parameters

Determination of neutrino parameters is not the end of story

We measure neutrino parameters to establish the underlying physics.

In spite of scepticism searches for true theory of mass and mixing is the must

# High scale seesaw, unification

$$m_\nu = - m_D^\top \frac{1}{M_R} m_D$$

q - l similarity:  $m_D \sim m_q \sim m_l$

Lepton number violation

$$M_R \sim \begin{cases} M_{\text{GUT}} \sim 10^{16} \text{ GeV} & \text{for the heaviest in the presence of mixing} \\ 10^8 - 10^{14} \text{ GeV} & \frac{M_{\text{GUT}}^2}{M_{\text{Pl}}} \text{ double seesaw} \\ 10^{16} - 10^{17} \text{ GeV} & \text{many heavy singlets (RH neutrinos)} \\ & \dots \text{string theory} \quad N \sim 10^2 \end{cases}$$

In favor

Gauge coupling unification

Leptogenesis

Seesaw sector is responsible for inflation  
(scalar which breaks B-L and gives masses  
of RH neutrinos), dark matter

# Double Seesaw

R.N. Mohapatra  
J. Valle

Three additional singlets  $S$  which couple with RH neutrinos

$$\begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & M_D^T \\ 0 & M_D & M_S \end{pmatrix} \begin{pmatrix} v \\ v^c \\ S \end{pmatrix}$$

$M_S$  - scale of B-L violation

$M_S \sim M_{\text{Pl}}, M_D \sim M_{\text{GU}}$



$$m_\nu = m_D^T M_D^{-1T} M_S M_D^{-1} m_D$$

$M_S \gg M_D$

$$M_R = M_D^T M_S^{-1} M_D$$

can be very hierarchical

Important feature:

if  $m_D = A M_D$



$$m_\nu \sim M_S$$

hierarchical Dirac structures disappear

# More than usual see-saw?

Scale of see-saw

$$M_R = - m_D^T \frac{1}{m_\nu} m_D$$

q - l similarity:  $m_D \sim m_q \sim m_l$

for one third generations  $M_R \sim 2 \cdot 10^{14} \text{ GeV}$

$M_R$  - hierarchy

$$M_R = - m_D^{\text{diag}} (m_{\text{TBM}})^{-1} m_D^{\text{diag}}$$

Quadratic hierarchy

Flavor structure

Difficult to reproduce

Can be explained in the framework of double seesaw

# Double Seesaw

R.N. Mohapatra  
J. Valle

Three additional singlets  $S$  which couple with RH neutrinos

$$\begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & M_D^T \\ 0 & M_D & M_S \end{pmatrix} \begin{pmatrix} v \\ v^c \\ S \end{pmatrix}$$

$$M_S \gg M_D$$

$M_S$  - scale of B-L violation

RH neutrinos get mass via see-saw

$$M_R = M_D^T M_S^{-1} M_D$$

This explains

1. strong mass hierarchy  $M_D \sim m_D$  and  $M_S$  has no strong hierarchy
2. intermediate scale of masses if  $M_S \sim M_{\text{Pl}}$ ,  $M_D \sim M_{\text{GU}}$
3. Flavor structure:

$$\rightarrow m_\nu = m_D^T M_D^{-1 T} M_S M_D^{-1} m_D$$

$$\text{if } m_D = A M_D \rightarrow$$

$$m_\nu \sim M_S$$

may have certain symmetries

A.Y.S  
M. Lindner,  
M.A. Schmidt  
A.Y.S

# Mass and Mixing

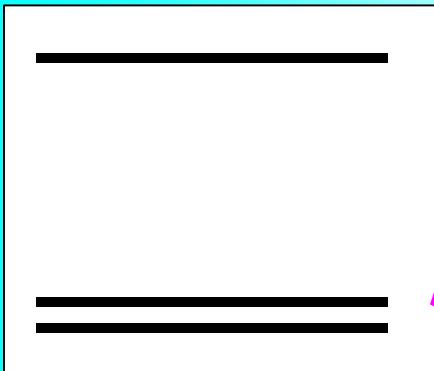
In general

Maximal  
mixing

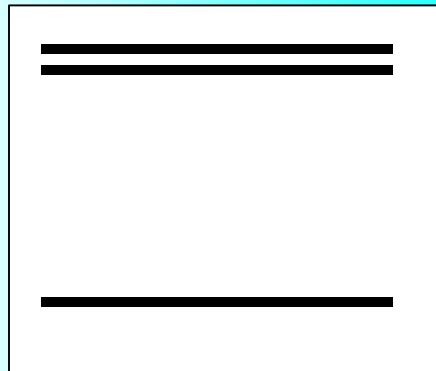


degeneracy  
of masses

This works for 1-2  
Sector for both  
hierarchies:



Charged Lepton  
mixing explains  
deviation from  
maximal



Still  
degeneracy  
is possible

quark sector relation  
Still possible

$$\theta \sim \sqrt{\frac{m_2}{m_3}}$$

2-3 mixing is close to maximal but 2-3 mass  
splitting is large. Complete degeneracy is  
disfavored by cosmology

Simple symmetries →  
degeneracy, massless states

# Standard parametrization

$$U_{PMNS} = U_{23} I_\delta U_{13} I_{-\delta} U_{12}$$

$$I_\delta = \text{diag}(1, 1, e^{i\delta})$$

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$c_{12} = \cos \theta_{12}, \text{ etc.}$$

$\delta$  is the Dirac CP violating phase

$\theta_{12}$  is the ``solar'' mixing angle

$\theta_{23}$  is the ``atmospheric'' mixing angle

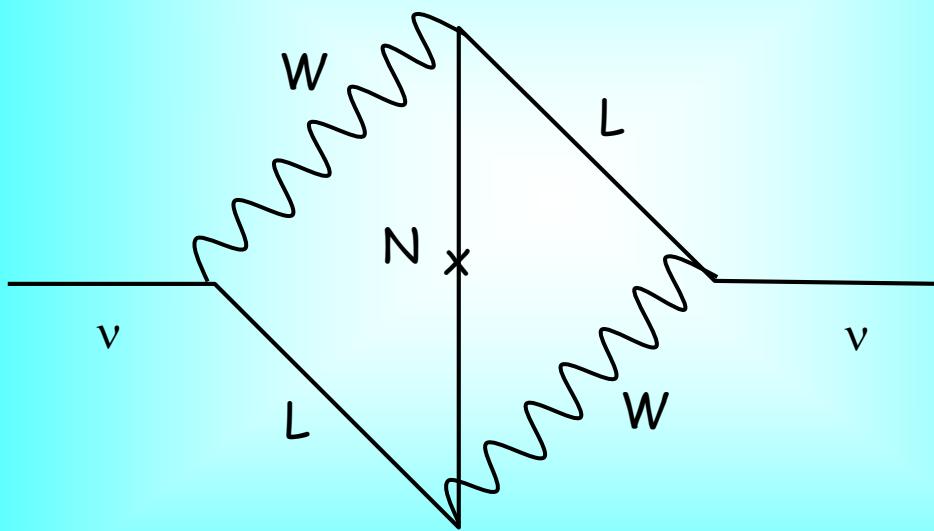
$\theta_{13}$  is the reactor mixing angle

# Summarizing results

# **Definitions and parameterization**

# Two loop mechanism

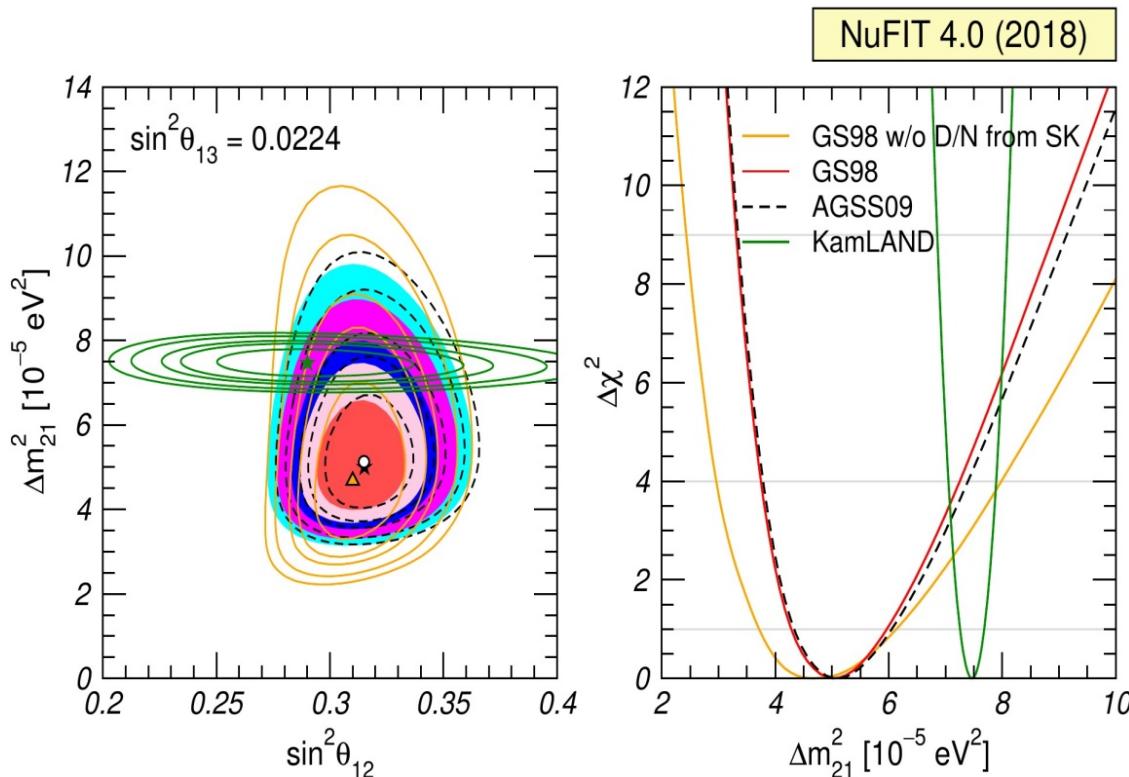
K S Babu, E Ma



If usual neutrinos mix with heavy Majorana lepton  $N$

4<sup>th</sup> generation of fermions  $\rightarrow$  main contribution

# Solar neutrinos: $\Delta m_{21}^2$ - tension



68%, 90%, 95%, 99%, 3 $\sigma$   
CL contours

Contours for solar models with  
different metallicity) also with  
and without DN effect

Origin of tension:

- Absence of the upturn of spectrum (SNO, SK)
- 50% larger than expected D-N asymmetry for the bf  $\Delta m_{21}^2$

Yellow lines - without the DN effect

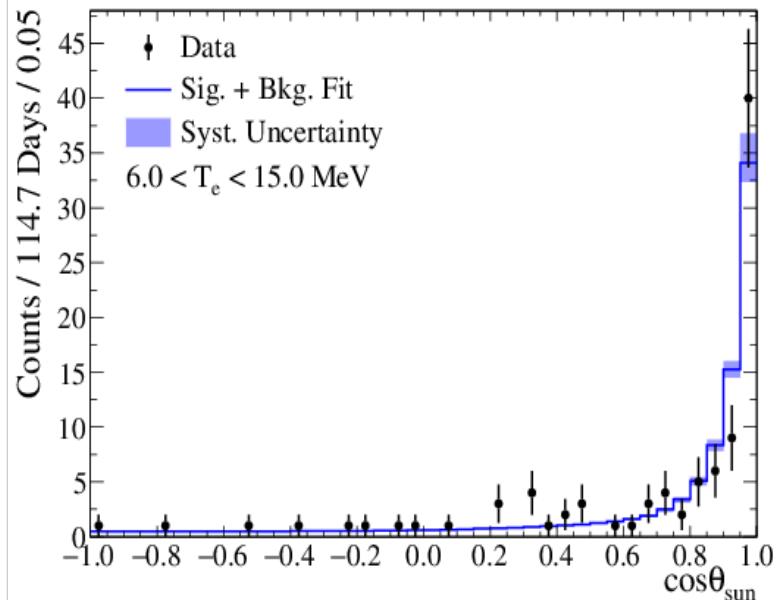
tension starts to disappear?

# Solar neutrinos SNO+ results

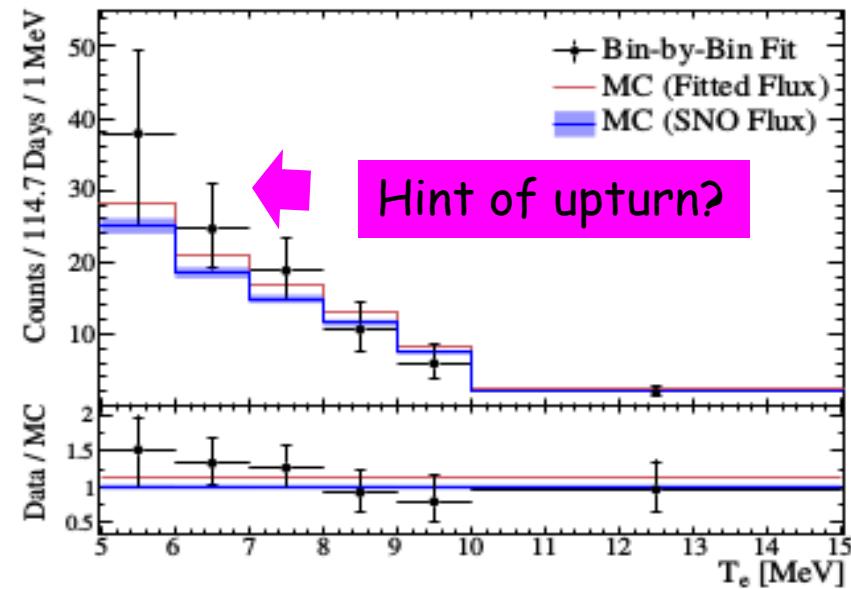
*SNO+ Collaboration (Anderson, M. et al.)*  
*Phys. Rev. D99 (2019) no.1, 012012*  
*1812.03355 [hep-ex]*

Water phase: Measurement of the 8B solar neutrino flux in SNO+ with very low backgrounds S/B  $\sim 4$ ,  $E > 6$  MeV

114.7 days of data



Distribution of event directions wrt. solar direction



The extracted event rate as function of reconstructed electron kinetic energy

69.2 kt-day dataset

Flux:  $2.53 [-0.28+0.31(\text{stat}) \ -0.10+0.13(\text{syst})] \times 10^{-6} \text{ cm}^{-2} \text{ s}^{-1}$

# Deviations from TBM

$$D_{13} = 0 - \sin^2\theta_{13} \quad D_{12} = 1/3 - \sin^2\theta_{12} \quad D_{23} = \frac{1}{2} - \sin^2\theta_{23}$$

Deviations - consequences of symmetry  
(complicated groups) → "direct"

Deviations - violation of (simple) symmetries → "semi-direct"

"Sum rules"

Ref. Nothing  
fundamental model  
dependent

Deviations related to mass ratios?

$Z_2 \times Z_2$  - TBM

$Z_2$  - only one column in the mixing matrix is fixed, e.g. TBM<sub>1</sub>

# Quark and Lepton Mixing

Patterns of mixing are strongly different

Un  
connected

Different mechanism  
of generation of masses  
of quarks and neutrinos

e.g. in seesaw  
type-II

In general:

Partially  
connected

QLC -relations

$$\theta_{12}^l \sim \pi/2 - \theta_{12}^q = \theta_c$$

$$\theta_{23}^l \sim \pi/2 - \theta_{23}^q$$

$$\theta_{13}^l \sim \frac{1}{\sqrt{2}} \theta_c \quad \text{Predicted from QLC}$$



$$U_{PMNS} = U_{CKM}^+ U_X$$

$$U_{CKM} = V_{CKM}$$

Bi-maximal or TBM matrix

Fully  
connected

by symmetry:  
difficult to  
realize