Topics in Neutrino Cosmology



VII International Pontecorvo Neutrino Physics School Prague, August 20-31 2017

> Gianpiero Mangano INFN Naples, Italy



Two approaches:

- I. Ask what cosmology can do for neutrinos! (particle physicists?)
- Ask what neutrinos can do for cosmology! (cosmologists?)



Summary

- review of neutrino properties and interactions (very brief)
- cosmology and statistical mechanics in an expanding universe (brief)
- v distribution and mixing in the early universe (CNB=Cosmic Neutrino Background)
- CNB chemical potentials (asymmetries)
- BBN and neutrinos: v asymmetries, extra radiation and N_{eff}
- sterile neutrinos
- exotic scenarios for sterile neutrinos



Summary

- v non standard interactions
- v properties from CMB anisotropies and LSS
- (keV) sterile neutrinos as warm dark matter
- v background today and detection perspectives. Tritium decay and v capture





Neutrino properties from cosmology: overview

Neutrinos impact expansion history:

Extremely high T regime (above EW scale) (Leptogenesis) Majorana vs. Dirac, see-saw mechanism, high scale physics (Leptogenesis)

```
High T regime (≈ MeV):
weak + gravitational effects (BBN)
observables: phase space density (in particular v<sub>e</sub> distribution), non
standard interactions, chemical potentials, number of species (active,
```

sterile)

Intermediate T regime (eV): gravitational effects including perturbations (CMB) observables: phase space density, non standard interactions, mass scale, number of species

Low T regime (< eV): gravitational effects including perturbations (LSS) observables: phase space density, non standard interactions, mass scale

Extremely low T regime (today): mass scale, local density (CNB direct detection)

$\left[\begin{array}{c} 0.005 \\ 0.02 \\ 0.23 \\ 0.24 \\ 0.23 \\ 0.24 \\ 0.23 \\ 0.22 \\ 0.24 \\ 0.23 \\ 0.22 \\ 0.24 $	-JOINE TO A LAND AND A LAND A	2df Obser Redshit Surver 15. Jenuery 2003 4 4 4 4 4 4 4 4 4 4 4 4 4
Primordial Nucleosynthesis BBN	Cosmic Microwave Background CMB	Formation of Large Scale Structures LSS
T ~ MeV	T < eV	
$\nu_{e} \ versus \nu_{\mu,\tau} N_{eff}$	No flavour sensitivity $N_{eff} \& m_v$	





Books Suggested References

The Early Universe, E. Kolb & M. Turner (Addison-Wesley, 1990)

Modern Cosmology, S. Dodelson (Academic Press, 2003)

Kinetic theory in the expanding Universe, J. Bernstein (Cambridge U., 1988)

Neutrino Cosmology, J. Lesgourgues, G.M, G. Miele & S. Pastor (Cambridge U., 2013)

Reviews

Neutrino Cosmology, A.D. Dolgov, Phys. Rept. 370 (2002) [hep-ph/0202122]

Massive neutrinos and cosmology, J. Lesgourgues & S. Pastor, Phys. Rep. 429 (2006) [astro-ph/0603494]

Primordial Nucleosynthesis: from precision cosmology to fundamental physics, F. locco et al, Phys. Rept. 472 (2009) arXiv:0809.0631 [astro-ph]

Review of neutrino properties and interactions

three active weakly interacting states (LEP, N= 2.984 ± 0.008)

$$\begin{split} L^{\text{int}} &= -\frac{g}{2\sqrt{2}} J^{\mu}_{W} W_{\mu} - \frac{g}{2\cos\theta_{W}} J^{\mu}_{Z} Z_{\mu} + h.c. \\ J^{\mu}_{W} &\supseteq 2\bar{v}_{iL} \gamma^{\mu} l^{i}_{L} \\ J^{\mu}_{Z} &\supseteq 2g_{\nu L} \bar{v}_{iL} \gamma^{\mu} v^{i}_{L} + 2g_{lL} \bar{l}_{iL} \gamma^{\mu} l^{i}_{L} + 2g_{lR} \bar{l}_{iR} \gamma^{\mu} l^{i}_{R} \end{split}$$

with couplings:

$$g_{aL} = T_{3a} - Q_a \sin^2 \theta_W, \quad g_{aR} = -Q_a \sin^2 \theta_W$$

Notice: fields are not in general mass eigenstates but flavour eigenstates (oscillations!) Similarly for quarks. Diagonalizing mass terms in the Higgs sector, charged (not neutral ones!!) currents in mass eigenstates become

$$J_W^{\mu} = 2\overline{v}_{iL}\gamma^{\mu}U_j^{PMNS,i}l_L^j + 2\overline{u}_{iL}\gamma^{\mu}U_j^{CKM,i}d_L^j$$

flavour oscillations!

In the following, low energy regime of neutrino interactions (well below W and Z mass scale) and the tree level process amplitudes can be described in terms of contact (Fermi) interactions

$$L_{eff}^{CC} = -\frac{G_F}{\sqrt{2}} J_W^{\mu+} J_{\mu W}$$
$$J_Z^{\mu} = -\frac{G_F}{\sqrt{2}} \frac{M_W^2}{M_Z^2 \cos \theta_W^2} J_Z^{\mu+} J_{\mu Z}$$

examples:

- lepton scatterings

$\text{Process } 1+2 \longrightarrow 3+4$	$3\pi G_F^{-2} \sigma_{12\to 34}$	
$\nu_e + \overline{\nu}_e \longrightarrow \nu_e + \overline{\nu}_e$	8	
$\nu_e+\nu_e \longrightarrow \nu_e+\nu_e$	$\frac{3}{2}s$	
$\nu_e + \overline{\nu}_e \longrightarrow \nu_{\mu(\tau)} + \overline{\nu}_{\mu(\tau)}$	$\frac{1}{4}s$	
$\nu_e + \overline{\nu}_{\mu(\tau)} \longrightarrow \nu_e + \overline{\nu}_{\mu(\tau)}$	$\frac{1}{4}s$	
$\nu_e + \nu_{\mu(\tau)} \longrightarrow \nu_e + \nu_{\mu(\tau)}$	$\frac{3}{4}s$	
$\nu_e + \overline{\nu}_e \longrightarrow e^+ + e^-$	$\frac{4}{\sqrt{s}}\sqrt{s-4m_e^2}[(s-m_e^2)(\tilde{g}_L^{l2}+g_R^{l2})+6m_e^2\tilde{g}_L^{l}g_R^{l}]$	
$e^+ + e^- \longrightarrow \nu_e + \overline{\nu}_e$	$\frac{\sqrt{s}}{2\sqrt{s-4m_e^2}}[(s-m_e^2)(\tilde{g}_L^{l2}+g_R^{l2})+6m_e^2\tilde{g}_L^{l}g_R^{l}]$	
$\nu_e + e^- \longrightarrow \nu_e + e^-$	$(3s ilde{g}_L^{l2} + sg_R^{l2} - 3m_e^2 ilde{g}_L^lg_R^l)$	
$\overline{ u}_e + e^- \longrightarrow \overline{ u}_e + e^-$	$(3sg_R^{l2}+s{ar g}_L^{l2}-3m_e^2{ar g}_L^lg_R^l)$	

-neutron β decay and n-p charged current processes. Crucial to set the final value of neutron to proton density ratio at the onset of Big Bang Nucleosynthesis. At tree level and in the infinite nucleon mass $(C_V=0.97425, C_A/C_V=-1.2701)$ $\cos \theta_C \langle p | \overline{u} \gamma^{\mu} (1-\gamma_5) d | n \rangle \approx \overline{u}_p \gamma^{\mu} (C_V - C_A \gamma_5) u_n$

EXERCISE I

Compute the neutron lifetime in the infinite nucleon mass limit (but with fixed $Q=M_n-M_P$, at tree level (Born amplitude). Verify that:

$$\tau_n^{-1} = \frac{G_F^2 (C_V^2 + 3C_A^2)}{2\pi^3} \int_{1}^{Q/m_e} x \, dx \left(x - \frac{Q}{m_e} \right)^2 \left(x^2 - 1 \right)^{1/2}$$

which using the values of C_V , C_A and $Q=M_n-M_P=1.29$ MeV, gives a value 10% off with respect with the experimental value, $\tau_n \approx 880.1$ sec.

What is wrong then?

Review of neutrino properties and interactions Oscillations in vacuum

 $|\boldsymbol{v}_a\rangle(t) = U_{ai}^*|\boldsymbol{v}_i\rangle e^{-iE_it}$

a flavour eigenstates (production and detection) *i* mass eigenstates (evolution)

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}$$

for relativistic states, $E_i(p) \approx p + m_i^2/2p$ oscillation/survival probabilities

$$P(v_a \rightarrow v_b) = U_{ai}^* U_{bi} U_{aj} U_{bj}^* \exp(-i \Delta m_{ij}^2 L/2p)$$

"solar" δ m² ≈ 7 10⁻⁵ eV² $sin^2 θ_{12} ≈ 0.3$ "atmospheric" Δ m² ≈ 2 10⁻³ eV² $sin^2 θ_{23} ≈ 0.5$ (?) "absolute" ?? M_β < 2 eV (³H decay) $sin^2 θ_{13} ≈ 0.02$ hierarchy ??





Oscillations in matter

$$H_a^{eff} = \overline{v}_a \gamma^{\mu} (1 - \gamma_5) v_a Tr(\rho_f J_{f,\mu})$$
$$\rho_f = \int \frac{d^3 p}{2E(p)} \sum_h f(p,h) |f(p,h)\rangle \langle f(p,h)|$$

 $J_{f,\mu}$ charged/neutral current of fermions in the medium (leptons, baryons)

Ex. only (unpolarized) electrons and baryons (Sun)

$$Tr(\rho_{f}J_{f,\mu}) = \int \frac{d^{3}p}{2E(p)} f(p)Tr[(p+m_{f})\gamma_{\mu}(g'_{V}-g'_{A}\gamma_{5})] = 4g'_{V,f} \langle p_{\mu} \rangle$$

g'_{V,f} = I(CC) + g_{V,f}(NC)

matter potential

$$V_{\text{CC+NC}}(\nu_{e}) = \sqrt{2} \text{ G}_{\text{F}} (n_{e} - n_{n}/2) \quad V_{\text{NC}}(\nu_{\mu,\tau}) = -\sqrt{2} \text{ G}_{\text{F}} n_{n}/2$$
$$i\frac{d}{dL}|\nu_{a}\rangle = (U_{ai}^{*}E_{i}U_{ib} + V_{a}\delta_{ab})|\nu_{b}\rangle$$

In the early universe, at the MeV epoch (neutrino decoupling) baryons and almost the same number of electrons and positrons (from BBN, CMB and charge neutrality we know that $n_{e_{-}} - n_{e^{+}} \approx 10^{-9} n_{\gamma}$, so the two contribution of opposite sign almost cancel!) More relevant the contribution expanding W propagator at second order

$$D_{\mu\nu}(p) = \frac{g_{\mu\nu}}{M_W^2} + \frac{1}{M_W^4} (g_{\mu\nu} p^2 - p_{\mu} p_{\nu})$$

$$\Rightarrow V_{CC} = \sqrt{2} G_F \left(n_{e^-} - n_{e^+} + \frac{8p}{3M_W^2} (\rho_{e^-} + \rho_{e^+}) \right)$$

Resonance in matter (MSW)

removing the diagonal neutral current contribution via a phase redefinition, in a two flavour scheme we have (atmospheric or solar)

$$\begin{aligned} \mathcal{H}_F &= \frac{1}{4 \left| \vec{p} \right|} (\Delta m^2 + 2 \left| \vec{p} \right| V_{CC}) \mathbb{I} \\ &+ \frac{1}{4 \left| \vec{p} \right|} \begin{pmatrix} -\Delta m^2 \cos(2\theta) + 2 \left| \vec{p} \right| V_{CC} & \Delta m^2 \sin(2\theta) \\ \Delta m^2 \sin(2\theta) & \Delta m^2 \cos(2\theta) - 2 \left| \vec{p} \right| V_{CC} \end{pmatrix} \end{aligned}$$

resonance condition: first term is diagonal, second term diagonal via an orthogonal trasformation

$$\tan 2 \theta_M = \tan 2 \theta \left[1 - \frac{2 |\vec{p}| V_{CC}}{\Delta m^2 \cos 2\theta}\right]^{-1}$$

$$\Delta m_M^2 = \sqrt{(\Delta m^2 \cos 2\theta - 2 |\vec{p}| V_{CC})^2 + (\Delta m^2 \sin 2\theta)^2}$$

$$n_e^{res} = \frac{\Delta m^2 \cos 2\theta}{2\sqrt{2}G_F p}$$

Cosmology and statistical mechanics in an expanding universe

The FLRW Model describes the evolution of the isotropic and homogeneous expanding Universe

$$ds^{2} = g_{\mu\nu}dx^{\mu}dx^{\nu} = dt^{2} - a(t)^{2} \left(\frac{dr^{2}}{1 - kr^{2}} + r^{2}d\theta^{2} + r^{2}\sin^{2}\theta d\phi^{2}\right)$$

a(t) is the scale factor and k=-1,0,+1 the curvature

Einstein eqs
$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}$$

Energy-momentum tensor of a perfect fluid

$$T_a^b = (\rho + P)u_a u^b + P\delta_a^b$$

Cosmological constant

Eqs in the SM of Cosmology

00 component (Friedmann eq)

$$H(t)^{2} = \left(\frac{a}{a}\right)^{2} = \frac{8\pi G_{N}}{3}\rho - \frac{k}{a^{2}}$$

H(t) is the Hubble parameter $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$ $\frac{k}{H(t)^2 a^2} = \Omega - 1$ $\Omega = \rho / \rho_{crit}$ 1 + z = a(today)/a redshift $\rho = \frac{d\rho}{dt} = -3H(p+p)$ $\rho_{crit}=3H^2/8\pi G$ is the critical density $\rho = \text{const } \mathbf{a}^{-3(1+\alpha)}$ Eq of state $p=\alpha\rho$ Radiation $\alpha = 1/3$ Matter $\alpha = 0$ Cosmological constant $\alpha = -1$ $\rho_R \sim I/a^4$ $\rho_M \sim I/a^3$ $\rho_A \sim const$



Evolution of the Universe

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$





definition of momentum

$$P^{a} = \frac{dx^{a}}{ds}, \quad \frac{dP^{a}}{ds} = -\Gamma^{a}_{bc}P^{b}P^{c} \qquad \Gamma^{c}_{ab} = \frac{1}{2}g^{cd}(\partial_{a}g_{db} + \partial_{b}g_{ad} - \partial_{d}g_{ab})$$
$$g_{ab}P^{a}P^{b} = -m^{2}$$

EXERCISE II

Compute the Levi Civita connection for a spatially flat (k=0) FLRW universe and show that P^i decreases as a^{-2}

comoving momentum: $p_a = g_{ab}P^b$, $y = \sum_i p_i^2$ y = constant

"physical momentum": rate of change in physical distances: p = a(t) P, $-E^2 + p^2 = -m^2$ p behaves as I/a current density $n_a^{}$ and T^a_{b}

macroscopic approach: perfect fluid (no viscosity) $T_{b}^{a}=(\rho +P)u^{a}u_{b} + P \delta_{b}^{a}$ $n_{a}=N u_{a}$ (N = number density in the fluid comoving frame, $u^{a}=(1,0,0,0)$

microscopic description $T_{a}^{b} = g \int \frac{d^{4}P}{(2\pi)^{4}} (-g)^{1/2} P^{a} P_{b} 2(2\pi) \ \theta(P_{0}) \delta(P_{c}P^{c} + m^{2}) f(P,t)$ $n_{a} = g \int \frac{d^{4}P}{(2\pi)^{4}} (-g)^{1/2} P_{a} 2(2\pi) \ \theta(P_{0}) \delta(P_{c}P^{c} + m^{2}) f(P,t)$



or in physical momentum

$$T_{0}^{0} = -g \int \frac{d^{3}p}{(2\pi)^{3}} E(p)f(p,t) = -\rho$$
$$T_{j}^{i} = g\delta_{j}^{i} \int \frac{d^{3}p}{(2\pi)^{3}} \frac{p^{2}}{3E(p)} f(p,t) = \delta_{j}^{i}P$$

for equilibrium distribution we can define a temperature parameter T. For relativistic species and covariant conservation of ${\sf T}^{\sf b}_{\sf a}$

$$\nabla_a T_b^a = 0 = \partial_a T_b^a + \Gamma_{ac}^a T_b^c - \Gamma_{ab}^c T_c^a \Longrightarrow Ta = const$$



definition of comoving particle horizon

$$\chi = \int_{a_{em}}^{a} \frac{da'}{{a'}^2 H} = \int_{z}^{z_{em}} \frac{dz}{H} \text{ massless particles}$$
$$d\chi = \frac{p}{aE} dt \text{ particle with mass m}$$

$$\chi = \int_{a_{em}} \frac{y \, da}{a'^2 \, H(a') \sqrt{y^2 + m^2 a'^2}}$$

EXERCISE III

The last scattering surfaces of photons and neutrinos are at redshifts $\approx 10^3$ and $\approx 10^9$ respectively. Compute the comoving distances from earth of these surfaces for a neutrino mass of 1 eV

Equilibrium vs kinetic theory

Along the history of the universe many phases are characterized by equilibrium conditions.

Simple criterium: interaction rate (Γ) >> expansion rate (H) Ex. two body scattering (a+b \rightarrow c+d) $\Gamma = \sigma \vee n_a$

 $\sigma v n_a > (8 \pi G_N \rho / 3)^{1/2}$





Kinetic theory

Boltzmann (Stosszahlansatz: no correlations among different momenta)

 $\pounds f(p,t) = C(f(p,t))$

Liouville operator (evolution in phase space of particle distribution)
 C collisional integral (change of momentum and/or

particle species)

In a spatially flat FLRW universe

$$\left(\frac{d}{dt} - Hp\frac{d}{dp}\right)f_{v}(p,t) = C(f_{v}(p,t))$$



Statistical Factor



9-dim Phase Space

 $\Sigma \boldsymbol{P}_i$ conservation

Process

 $F = f_3 f_4 [1 + f_1] [1 - f_2] - f_1 f_2 [1 + f_3] [1 - f_4]$

Notice: collisional integral can be zero either because interactions are **very efficient** or **too weak**!

EXERCISE IV

Show that for example, for a 2-2 scattering process collisional integral vanishes for kinetic equilibrium distribution and for chemical equilibrium $\mu_a + \mu_b = \mu_c + \mu_d$





Neutrino decoupling

As the Universe expands, particle densities are diluted and temperatures fall. Weak interactions become ineffective to keep neutrinos in good thermal contact with the e.m. plasma

Rough, but quite accurate estimate of the decoupling temperature

Rate of weak processes ~ Hubble expansion rate

$$\Gamma_{w} \approx \sigma_{w} |v| n$$
, $H^{2} = \frac{8\pi\rho_{R}}{3M_{p}^{2}} \rightarrow G_{F}^{2}T^{5} \approx \sqrt{\frac{8\pi\rho_{R}}{3M_{p}^{2}}} \rightarrow T_{dec}^{v} \approx 1 MeV$

Since v_e have both CC and NC interactions with e^{\pm} $T_{dec}(v_e) \sim 2 \text{ MeV}$ $T_{dec}(v_{\mu,\tau}) \sim 3 \text{ MeV}$

EXERCISEV

Prove that for a relativistic specie, any function of the comoving momentum y is a solution of the collisionless Boltzmann equation. Use this to show that if neutrino decoupling is assumed to be instantaneous neutrino distribution keeps a Fermi-Dirac shape after decoupling (like photon keep a almost perfect black body distribution after last scattering) with a temperature falling as I/a

Discuss then the case of a very massive particle (see Bernstein book)



Neutrino and Photon (CMB) temperatures




The Cosmic Neutrino Background

Neutrinos decoupled at T~MeV, keeping a spectrum as that of a relativistic species

$$f_{v}(p,T) = \frac{1}{e^{p/T_{v}} + 1}$$

$$n_{v} = \int \frac{d^{3}p}{(2\pi)^{3}} f_{v}(p,T_{v}) = \frac{3}{11} n_{\gamma} = \frac{6\zeta(3)}{11\pi^{2}} T_{CMB}^{3} \approx 112 \text{ cm}^{-3}$$

$$\rho_{v_{i}} = \int \sqrt{p^{2} + m_{v_{i}}^{2}} \frac{d^{3}p}{(2\pi)^{3}} f_{v}(p,T_{v}) \rightarrow \begin{cases} \frac{7\pi^{2}}{120} \left(\frac{4}{11}\right)^{4/3} T_{CMB}^{4} & \Omega_{v} h^{2} = 1.7 \times 10^{-5} \\ m_{v_{i}} n_{v} & \Omega_{v} h^{2} = \frac{\sum_{i=1}^{i} m_{i}}{94.1 \text{ eV}} \end{cases}$$

Massive $m_{\nu} >> T$

Relativistic particles in the Universe

At T<m_e, the radiation content of the Universe is

$$\rho_{\rm r} = \rho_{\gamma} + \rho_{\nu} = \frac{\pi^2}{15} T_{\gamma}^4 + 3 \times \frac{7}{8} \times \frac{\pi^2}{15} T_{\nu}^4 = \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} 3 \right] \rho_{\gamma}$$

$$\rho_{\rm r} = \rho_{\gamma} + \rho_{\nu} = \left[1 + \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} N_{\rm eff}\right] \rho_{\gamma}$$



Extra relativistic particles

• Extra radiation can be:

scalars, pseudoscalars, sterile neutrinos (totally or partially thermalized, bulk), neutrinos in very low-energy reheating scenarios, relativistic decay products of heavy particles...

• Particular cases: relic neutrino asymmetries, sterile v's

Actually N_{eff} is slightly larger than 3 for standard active neutrinos



Non-instantaneous neutrino decoupling

At T~m_e, e⁺e⁻ pairs annihilate heating photons $e^+e^- \rightarrow \gamma\gamma$

But, since $T_{dec}(v)$ is close to m_e , neutrinos share a small part of the entropy release





Evolution of f_v for a particular momentum p=10T



Neutrino oscillations in the Early Universe

Neutrino oscillations are effective when medium effects get small enough

Compare oscillation term with effective potentials (see Exercise VI) and neglecting large neutrino asymmetries (see later)

$$\left(i\partial_{t} - Hp\partial_{p}\right)\rho = \left[\frac{M^{2}}{p} - \frac{8\sqrt{2}G_{F}}{M_{W}^{2}}p\rho_{e\pm} + \frac{\sqrt{2}G_{F}}{3M_{Z}^{2}}(\rho_{v} - \overline{\rho}_{v}) - \frac{8\sqrt{2}G_{F}}{M_{Z}^{2}}p\rho_{v,\overline{v}},\rho\right] + C(\rho)$$

with ρ the neutrino density matrix (similarly for antineutrinos)

$$\langle a_b^+(p)a_a(p')\rangle = (2\pi)^3\delta^3(p-p')\rho_{ab}$$

p description to account for scatterings AND oscillations

EXERCISE VI

Recalling that:

- I. vacuum oscillation term goes like $\Delta m^2 \cos 2\theta/2$ p
- 2. neutrino momentum redshift as the photon temperature T
- 3. the energy density of electrons/positrons in the non relativistic regime scales as T^3

draw the evolution of vacuum and matter terms in the neutrino equation of motion versus T. Consider the two cases of atmospheric and solar square mass difference and mixing angle θ . Find the two corresponding resonance temperatures

Standard case: all neutrino flavours equally populated

oscillations are effective below a few MeV, but have no effect (except for mixing the small distortions δf_v)

Cosmology is insensitive to neutrino flavour after decoupling!

Non-zero neutrino asymmetries: flavour oscillations lead to (almost) equilibrium for all μ_v

Effects of flavour neutrino oscillations on the spectral distortions





Results

	T_{fin}^{γ} / T_0^{γ}	δρ _{ve} (%)	δ $ρ_{ν\mu}$ (%)	δ $ρ_{v^{\tau}}$ (%)	N _{eff}
Instantaneous decoupling	1.40102	0	0	0	3
SM	1.3978	0.94	0.43	0.43	3.045
+3v mixing (θ ₁₃ =0)	1.3978	0.73	0.52	0.52	3.045
+3v mixing (sin²θ ₁₃ =0.02)	1.3978	0.72	0.54	0.52	3.045



Neutrino asymmetries

neutrino distribution with a flavour dependent chemical potential

$$f_a(p,T) = \frac{1}{e^{p/T_v - \xi_a} + 1}$$

Total lepton asymmetry expected quite small in (standard) leptogenesis

$$\sum_{a} \eta_{a} = \sum_{a} \frac{n_{a} - n_{\bar{a}}}{n_{\gamma}} = \sum_{a} \frac{1}{12\zeta(3)} \left(\pi^{2}\xi_{a} + \xi_{a}^{3}\right) \approx \frac{n_{B}}{n_{\gamma}} \eta_{B} = 6 \times 10^{-10}$$

unless leptogenesis takes place well below the EW breaking scale

$$\exp\!\left(-M_W(T)/g^2T\right) << 1$$

but for each flavour in principle they could be large!

The role of oscillation!



The presence of v chemical potential

- shift the n-p chemical equilibrium and thus BBN (see later)
- change the neutrino energy density (N_{eff})

$$\Delta N_{\nu} = \frac{15}{7} \left[2 \left(\frac{\xi_{\nu}}{\pi} \right)^2 + \left(\frac{\xi_{\nu}}{\pi} \right)^4 \right]$$

Oscillations and neutrino asymmetries

The value of θ_{13} is crucial (and to a minor extent the mass hierarchy)



BBN and neutrinos

- BBN: almost seventy years after $\alpha\beta\gamma$ seminal paper(Alpher, Bethe & Gamow 1948)
- Theory reasonably under control (per mille level for ⁴He (neutron lifetime), I-2 % for ²H);
- Increased precision in nuclear reaction cross sections at low energy (underground lab's);
- Ωbh^2 (baryon density). measured by WMAP/Planck with high precision (1% level).

crucial parameter! $\eta = n_B/n_{\gamma} \approx 274 \,\Omega_{\,b} h^2$ 10 ⁻¹⁰ = 6 10 ⁻¹⁰

• Still some systematics on ⁴He, ²H fixes Ω_bh^2 value in good agreement with CMB data, ⁷Li not understood, ⁶Li too small, yet some claim.



Freezing of weak rates and so of n/p ratio

 $G_F^2 T_{fr}^5 = H(T_{fr}) \approx (8 \pi G_N g T_{fr}^4/3)^{1/2}$

 $n/p = \exp[(-M_n - M_p)/T_{fr}] \exp[-(t(T_D) - t(T_{fr}))/\tau_n] \approx 1/7$

EXERCISE VII

How the freezing temperature changes as function of G_N ? What would be the ratio n/p if at the MeV epoch the Newton constant was a 10% larger than today?

 2 H forms at T_D~0.08 MeV; Photodisintegration prevents earlier formation for temperatures closer to nuclear binding energies



n p**→**d γ

EXERCISE VIII

Consider a two body interaction process $a+b \rightarrow c+d$. Writing for simplicity the particle distribution in the Boltzmann limit

 $n_i = \exp(\mu_i / T) \int \frac{d^3 p}{(2\pi)^3} e^{-E_i / T}$

show that chemical equilibrium, i.e.

 $\mu_a + \mu_b = \mu_c + \mu_d$

gives the "Saha" equation

$$\frac{n_c n_d}{n_a n_b} = \frac{\int d^3 p \, \exp(-E_c / T) \int d^3 p \, \exp(-E_d / T)}{\int d^3 p \, \exp(-E_a / T) \int d^3 p \, \exp(-E_b / T)}$$

Use this result to get the order of magnitude of deuterium formation i.e. when $n_D \approx n_B$



Phase III: 700 - 30 keV Formation of light nuclei starting from D



Weak rates:

radiative corrections $O(\alpha)$ finite nucleon mass $O(T/M_N)$ plasma effects $O(\alpha T/m_e)$ neutrino decoupling $O(G_F^2 T^3 m_{Pl})$

N_{eff}=3.045

 $\begin{array}{l} \mbox{Main uncertainty: neutron lifetime} \\ \tau_n = 885.6 \pm 0.8 \mbox{ sec (old PDG mean)} \\ \tau_n = 878.5 \ \pm 0.8 \mbox{ sec (Serebrov et al 2005)} \end{array}$

Presently:

 τ_n =880.2 ± 1.0 sec (PDG)

 ^{4}He mass fraction Y_{P} linearly increases with $\tau_{n} : 0.246$ - 0.249



Nico & Snow 2006





CA02

GR63

WA63 🛆 GE67

10⁻²

 $^{2}H(p,\gamma)^{3}He$

SC95+96+97+Exfor

LUNA

E (MeV)

LUNA

Weitzmann Inst.

10⁻¹

(<mark>1</mark>]0m

R (cm³ s⁻¹ r

10-19

SN50

CSICRS

△ HA03

10^2

 $n(p,\gamma)^2H$

RU00

10⁻¹

E (MeV)

1000 1200

SU95

A NA97

BI50

Rupak

10-3

0.6

[q 0.5 (He A p)

0.3

0.2

200

400

600

 $^{3}\text{He}(\alpha,\gamma)^{7}\text{Be}$

Energy [keV]

800



Nuclear rate error budget:

- ⁴He $\tau_n \approx 100\%$ (0.0003)
- ²H/H d(p,γ)³He 78% (0.06) d(d,n)³He 19% (0.02) d(d,p)³H 3% (0.013)
- ⁶Li $d(\alpha, \gamma)$ ⁶Li (almost 100%)

Nuclear rates: for $d(p,\gamma)$ ³He also available ab initio calculations (Viviani et al 2000 PRC, Marcucci et al 2005 PRC, ..., Marcucci et al 2016 PRL)



Larger cross section than present data fit (Adelberger et al, 2011, Rev. Mod. Phys.)

$$R = \langle S \rangle_{TH} / \langle S \rangle_{exp} > |!$$

Important to check experimentally this result! LUNA 2018?

 $d(\alpha, \gamma)$ ⁶Li crucial for ⁶Li production, see later



non minimal models: extra radiation $g= 5.5 + 7 N_{eff}/4$ boosts the expansion rate H

 $\xi_i = \mu_i / T$ i= e, μ , τ boosts the expansion rate H change chemical equilibrium of n/p (v_e)

DATA

The quest for primordiality

 Observations in systems negligibly contaminated by stellar evolution (e.g. high redshift);

Careful account for galactic chemical evolution.

He recombination lines in ionized H_{II} regions in BCG & regression to zero metallicity. Small statistical error but large systematics



Aver, Olive & Skillmann 2015





New recent analysis use also the infrared I $\lambda 10830$

 $Y_{p} = 0.2551 \pm 0.0022$ Izotov et al 2014

 $Y_{p} = 0.2449 \pm 0.0040$ Aver et al 2015

 $Y_{p} = 0.245 \pm 0.0040$ PDG 2016

²H measures baryon fraction. Quite good agreement with Planck determination:

 $\Omega_b h^2 = 0.02225 \pm 0.00032$

Observations: absorption lines in clouds of light from high redshift background QSO





 $^{2}H/H(10^{-5})=2.53\pm0.04$

Cooke et al, 2014, ApJ

 $^{2}H/H(10^{-5})=2.55\pm0.03$

Riemer-Sorensenet al, 2017, MNRAS

³He

observed on Earth (nuclear weapons) observed in the Solar System (Sun): ²H \longrightarrow ³He observed in the ISM ³He/H= 0.1 observed in planetary nebulae and H_{II} regions outside the solar system (³He⁺ spin flip 3.46 cm wavelength band)





³He/H<(1.1±0.2) 10⁻⁵

⁷Li (and ⁶Li) still a puzzle. Spite plateau in metal poor dwarfs questioned



$[^{7}Li/H] = 12 + log_{10}(^{7}Li/H)$

 $\begin{array}{ll} (\text{Bonifacio et al. 97}) & [^7\text{Li}/\text{H}] = 2.24 \pm 0.01 \\ (\text{Ryan et al. 99, 00}) & [^7\text{Li}/\text{H}] = 2.09 \, {}^{+0.19} {}_{-0.13} \\ (\text{Bonifacio et al. 02}) & [^7\text{Li}/\text{H}] = 2.34 \pm 0.06 \\ (\text{Melendez et al. 04}) & [^7\text{Li}/\text{H}] = 2.37 \, \pm 0.05 \\ (\text{Charbonnel et al. 05}) & [^7\text{Li}/\text{H}] = 2.21 \, \pm 0.09 \\ (\text{Asplund et al. 06}) & [^7\text{Li}/\text{H}] = 2.095 \pm 0.055 \\ (\text{Korn et al. 06}) & [^7\text{Li}/\text{H}] = 2.54 \, \pm 0.10 \end{array}$

A factor 2 or more below BBN prediction, trusting ²H +PLANCK 2015 baryon density and ³He upper bound



• Nuclear rates under control $({}^{3}\text{He}(\alpha,\gamma){}^{7}\text{Be} \& {}^{7}\text{Be}$ (d,p)2 α)

• Systematics in measurements?

- Non standard BBN (catalyzed BBN)?
- Observed values NOT primordial

⁶Li/⁷Li = 0.05 (Asplund et al 2006), expected much smaller!!

Convective motions might generate asymmetries in the line shape and mimic the presence of ⁶Li

MINIMAL SCENARIO: ALL FIXED!

 $\Omega_{b}h^{2}=0.0223 \pm 0.0002$ $Y_{p}=0.2467\pm 0.0001 \pm 0.0003$ $^{2}H/H=2.60 \pm 0.03 \pm 0.07$

PLANCK 2015

EXP: $Y_p = 0.2551 \pm 0.0022 !!!$ $Y_p = 0.2449 \pm 0.0040 !$ ${}^{2}H/H(10^{-5}) = 2.55 \pm 0.03 !!$


Discrepancies at worst 2 σ: ✓ New physics? ✓ systematics/uncertainties

Example: increasing $d(p,\gamma)^3$ He (as from by ab initio calculations) deuterium decreases, better agreement with Planck $\Omega_b h^2$

 $R = \Gamma(d(p,\gamma)^{3}He)/\Gamma(d(p,\gamma)^{3}He)_{exp}$









Extra neutrinos

For several cosmological observables, all in a single parameter

$$\rho_{rad} = \left(1 + \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} N_{eff}\right) \frac{\pi^2}{15} T_{\gamma}^4$$

Instantaneous v decoupling value for T_v / T_{γ}

CMB and BBN scrutinize different "mass" scales!



Room for extra light particles?

⁴He grows with N_{eff}



Figure 4:

(Left) In blue (solid), the 68% and 95% contours in the N_{ν} - η_{10} plane derived from a comparison of the observationally-inferred and BBN-predicted primordial abundances of D and ⁴He. In red (dashed), the 68% and 95% contours derived from the combined WMAP 5-year data, small scale CMB data, SNIa, and the HST Key Project prior on H_0 along with the LSS matter power spectrum data. (Right) The 68% and 95% joint BBN-CMB-LSS contours in the N_{ν} – η_{10} plane.

Steigman 2008

2-3 σ claim ! (Izotov & Thuan 2010,2014)



FIG. 1.— Linear regressions of the helium mass fraction Y vs. oxygen abundance for H II regions in the HeBCD sample. The Ys are derived with the He I emissivities from Porter et al. (2005). The electron temperature $T_e(\text{He}^+)$ is varied in the range $(0.95 - 1) \times T_e(\text{O III})$. The oxygen abundance is derived adopting an electron temperature equal to $T_e(\text{He}^+)$ in a) and to $T_e(\text{O III})$ in b).





Izotov et al 2014

 $N_{eff} = 3.7 \pm 0.2$

But using Aver et al. 2015 (larger error)

 $N_{eff} = 2.9 \pm 0.3$

Planck 2015: N_{eff} = 3.04 ±0.18 !!

Remember: CMB and BBN scrutinize different "mass" scales!



Planck 2015



Deuterium constraint: crucial the $d(p,\gamma)^{3}He$!

Present data fit (Adelberger et al) leads to a slightly deuterium overproduction which might be compensated by a smaller expansion rate (N_{eff} =2.84)

Ab initio calculation gives a larger cross section and lower deuterium yield! In this case better a larger expansion rate (N_{eff} =3.2)



What could it be this putative extra radiation?

Sterile neutrinos?

Succesfull picture of 3-active neutrino mixing in terms of 2 mass differences and 3 mixing angles.

Few parameters describe a lot of data: solar v flux, atmospheric v's, accelerator v beams!

Yet, few anomalies (2-3 σ) :

I) LSND-MiniBooNE (short baseline exp's);

- 2) Reactor anomaly;
- 3) Gallium anomaly.



LSND+ MiniBooNE: evidence for $\overline{v}_{\mu} \rightarrow \overline{v}_{e}$

MiniBooNE: excess of $v_{\mu} \rightarrow v_{e}$

Interpretation: order 1 eV massive extra sterile neutrino with large mixing angle

 $\Delta m^2 \approx eV^2$ sin² 2 $\theta \approx 10^{-3} - 1$

 $P_{e\mu} = \sin^2 2\theta \sin^2 (1.27 \Delta m^2 L/E)$

(L in meters, E in MeV)



sterile-active oscillations

consider | active + | sterile

$$\rho = \begin{pmatrix} \rho_{aa} & \rho_{as} \\ \rho_{sa} & \rho_{ss} \end{pmatrix}$$

$$\begin{pmatrix} i\partial_{t} - Hp\partial_{p} \end{pmatrix} \rho = \begin{bmatrix} \frac{M^{2}}{2p} + V, \rho \end{bmatrix} + C(\rho)$$

$$V = (V_{CC} + V_{NC} + V_{asym}) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + V_{st} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$C(\rho) = \begin{pmatrix} C(G_{F}^{2}) & 0 \\ 0 & C(G_{X}^{2}) \end{pmatrix}$$

For large mixing angles sterile neutrino too much produced ($N_{eff} = I$)





Lepton asymmetry suppresses sterile production



Possible way out? active neutrino large (> 10^{-3}) chemical potential, but then v_e distortion

sterile neutrino "secret interactions" ?

 $g_X \overline{v}_s \gamma^\mu X_\mu v_s$

Fermi type lagrangian term with coupling G_X^2 and a sterile potential term linear in G_X

$$V_s = -\sqrt{2}G_x \frac{8p}{3M_x^2}\rho_s$$

"small" G_X (<10⁴ G_F) problem with BBN "large" G_X (>10⁵ G_F) problem with N_{eff} (smaller than 3 and neutrino mass bounds from CMB)



BBN sensitive to $\nu_{\rm e}$ distribution







 $\mathbf{Y}_{\mathbf{p}}$

Degenerate Big Bang Nucleosynthesis

If $\xi_v \neq 0$, for any flavor

$$\Delta N_{\nu} = \frac{15}{7} \left[2 \left(\frac{\xi_{\nu}}{\pi} \right)^2 + \left(\frac{\xi_{\nu}}{\pi} \right)^4 \right] \rho(\xi_{\nu}) > \rho(0) \rightarrow \uparrow {}^4\text{He}$$

Plus the direct effect on $n \leftrightarrow p$ if $\xi(v_e) \neq 0$

$$\left(\frac{n}{p}\right)_{eq} = \exp\left(-\frac{m_n - m_p}{T} - \xi_e\right) \left| \xi_e > 0 \rightarrow \psi^4 \text{He}\right|$$

Pairs $(\xi_e, \Delta N_v)$ that produce the same observed abundances for larger η_B Kang & Steigman 1992 oscillations and v decoupling almost at the same temperature scale

Example: $\xi_{a} = -\xi_{b} = \xi$ $T_{mix} >> T_{dec}$ $f_{a} = f_{b} = \frac{1}{e^{p/T} + 1}$ $N_{eff} = 3.045$ $T_{mix} << T_{dec}$ $f_{a} = f_{b} = \cos^{2}\theta \frac{1}{e^{p/T - \xi} + 1} + \sin^{2}\theta \frac{1}{e^{p/T + \xi} + 1}$

 $N_{eff} > 3$

unless $\xi = 0$





the bounds: scanning all asymmetries compatible with BBN

 N_{eff} < 3.2

-0.2 (-0.1) $\leq \eta_{\nu} \leq 0.15$ (0.05)



 N_{eff} ≥ 3.2 some extra "dark" radiation required or highly non-thermal neutrino distribution, or both

Planck 2015: a (large) neutrino asymmetry is still viable and can saturate the N_{eff} (68 % C.L.) upper bound



Bounds on non standard neutrino interactions

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \sum_{\alpha,\beta} \mathcal{L}_{\rm NSI}^{\alpha\beta}$$

$$\mathcal{L}_{\mathrm{NSI}}^{\alpha\beta} = -2\sqrt{2}G_F \sum_{P} \varepsilon_{\alpha\beta}^{P} \big(\bar{v}_{\alpha} \gamma^{\mu} L v_{\beta} \big) (\bar{e} \gamma_{\mu} P e)$$

New effective interactions between electron and neutrinos

$$\mathcal{L}_{\rm SM} = -2\sqrt{2}G_F \left\{ \left(\bar{\nu}_e \gamma^{\mu} L \nu_e \right) (\bar{e}\gamma_{\mu} L e) + \sum_{P,\alpha} g_P \left(\bar{\nu}_{\alpha} \gamma^{\mu} L \nu_{\alpha} \right) (\bar{e}\gamma_{\mu} P e) \right\}$$
$$P = L, R = (1 \mp \gamma_5)/2 \qquad g_L = -\frac{1}{2} + \sin^2 \theta_W \text{ and } g_R = \sin^2 \theta_W$$



Analytical calculation of T_{dec} in presence of NSI



Contours of equal $\mathsf{T}_{\mathsf{dec}}$ in MeV with diagonal NSI parameters

N_{eff} varying the neutrino decoupling temperature





Effects of NSI on the neutrino spectral distortions





Results

	T_{fin}^{γ} / T_0^{γ}	δρ _{νe} (%)	δρ _{νμ} (%)	δ $ρ_{ντ}$ (%)	N _{eff}
Instantaneous decoupling	1.40102	0	0	0	3
$\epsilon^{L}_{ee} = 4.0$ $\epsilon^{R}_{ee} = 4.0$	1.3812	9.47	3.83	3.83	3.357

Very large NSI parameters, FAR from allowed regions



Results

	T_{fin}^{γ} / T_0^{γ}	δρ _{νe} (%)	δρ _{νμ} (%)	δ $ρ_{v^{\tau}}$ (%)	N _{eff}
Instantaneous decoupling	1.40102	0	0	0	3
$\epsilon^{L}_{ee} = 0.12$ $\epsilon^{R}_{ee} = -1.58$ $\epsilon^{L}_{\tau\tau} = -0.5$ $\epsilon^{R}_{\tau\tau} = 0.5$ $\epsilon^{R}_{\tau\tau} = 0.5$ $\epsilon^{L}_{e\tau} = -0.85$ $\epsilon^{R}_{e\tau} = 0.38$	1.3937	2.21	1.66	0.52	3.120

Large NSI parameters, still allowed by present lab data

Neutrino properties from CMB and LSS

LSS: neutrino mass scale (free streaming and suppression of perturbation growth on scales smaller than free streaming length

CMB: N_{eff} and neutrino mass scale (gravitational lensing)

EXERCISE IX

Compute the comoving neutrino free streaming length as function of neutrino mass

$$d_{FS} = \int_{t_{in}}^{t} c_{v}(t') \frac{dt}{a(t')} = \int_{a_{in}}^{a} c_{v}(a') \frac{da'}{a'^{2} H(a')}$$

Direct laboratory bounds on m $_{\nu}$

Searching for non-zero neutrino mass in laboratory experiments

• Tritium beta decay: measurements of endpoint energy

$${}^{3}\text{H} \rightarrow {}^{3}\text{He} + e^{-} + \overline{v}_{e}$$

 $m(v_e) < 2.2 \text{ eV} (95\% \text{ CL})$ Mainz

Future experiments (KATRIN) $m(v_e) \sim 0.2-0.3 \text{ eV}$

Neutrinoless double beta decay: if Majorana neutrinos

$$(A,Z) \rightarrow (A,Z+2)+2e^{-}$$

experiments with ⁷⁶Ge and other isotopes: $Im_{ee}I < 0.4h_N eV$





Power Spectrum of density fluctuations







Neutrinos as Hot Dark Matter

Massive Neutrinos can still be subdominant DM: limits on m_v from Structure Formation (combined with other cosmological data)

• Effect of Massive Neutrinos: suppression of Power at small scales

The small-scale suppression is given by

$$\left(\frac{\Delta P}{P}\right) \approx -8 \frac{\Omega_{\nu}}{\Omega_{m}} \approx -0.8 \left(\frac{m_{\nu}}{1 \text{ eV}}\right) \left(\frac{0.1N}{\Omega_{m}h^{2}}\right)$$

$$\mathbf{f}_{\mathbf{v}}$$



Structure formation after equality





baryons and CDM experience gravitational clustering





growth of $\delta \rho / \rho$ (k,t) fixed by « gravity vs. expansion » balance $\Rightarrow \delta \rho / \rho \propto a$


baryons and CDM experience gravitational clustering



neutrinos experience free-streaming with v = c or /m



neutrinos experience free-streaming with <u>v = c or <p</u>>/m

neutrinos cannot cluster below their diffusion length

 $\lambda = \int v dt/a < \int c dt/a$





neutrinos experience free-streaming with v = c or /m

$$\ddot{\delta}_{\rm cdm} + \frac{2}{\tau} \dot{\delta}_{\rm cdm} - \frac{6}{\tau^2} (1 - f_{\nu}) \,\delta_{\rm cdm} = 0 \,.$$

free-streaming supresses growth of structures during MD

for $(2\pi/k) < \lambda$,

with $f_v = \rho_v / \rho_m \approx (\Sigma m_v) / (15 \text{ eV})$





CMB bounds



Planck 2015







Are there neutrinos in the universe?

CMB

fixing the angular scale of acoustic peaks and z_{eq} , a larger amount of dark radiation (and a larger H₀) gives a higher expansion speed, a shorter age of the universe T at recombination.

Diffusion length $\approx \sqrt{T}$ (Brownian motion)

Sound horizon $\approx T$



J. Lesgourgues, Planck 2014, Ferrara



Planck 2015 results, XIII

 N_{eff} > 0 at 10 σ

Perturbation effects:

- gravitational feedback of neutrino free streaming damping
- anisotropic stress contributions

c_{vis} :velocity/metric shear – anisotropic stress relation (Hu 1998)



Trotta & Melchiorri 2005

Neutrino perturbations in terms of two phenomenological parameters:

$$\delta P = c_{eff}^2 \delta \rho (I/3)$$
$$c_{vis}^2 (I/3)$$



How many of them? (the long tale of N_{eff}) Planck 2013 : a narrower 95 % C.L. range for N_{eff} , but still inconclusive. H_0 problem:



3.4±0.7 3.3±0.5 3.6±0.5 3.5±0.5



Ade et al. 2013 (Planck XVI)



Planck 2015 :

$$\begin{split} N_{\rm eff} &= 3.13 \pm 0.32 \quad Planck \, {\rm TT+lowP}\,; \\ N_{\rm eff} &= 3.15 \pm 0.23 \quad Planck \, {\rm TT+lowP+BAO}\,; \\ N_{\rm eff} &= 2.99 \pm 0.20 \quad Planck \, {\rm TT}, {\rm TE}, {\rm EE+lowP}\,; \\ N_{\rm eff} &= 3.04 \pm 0.18 \quad Planck \, {\rm TT}, {\rm TE}, {\rm EE+lowP+BAO}\,. \end{split}$$

Standard expectation (3.045)

Caveat: discrepancy with SNIa value of H_0 at 2.2 σ level

 $\sigma_8 \approx 0.83$





CMB and BBN are quite consistent



Planck 2015 results, XIII



Neutrino mass from CMB

CMB:

For the expected mass range the main effect is around the first acoustic peak due to the early integrated Sachs-Wolfe (ISW) effect;

Planck: gravitational lensing. Increasing neutrino mass, increases the expansion rate at z > 1 and so suppresses clustering on scales smaller than the horizon size at the nonrelativistic transition (Kaplinghat et al. 2003; Lesgourgues et al. 2006). Suppression of the CMB lensing potential. Total neutrino mass also affects the angular-diameter distance to last scattering, and can be constrained through the angular scale of the first acoustic peak. Degenerate with Ω_{Λ} (and so the derived H_0)

Including BAO constraint is much tighter:

$$\sum m_{\nu} < 0.72 \text{ eV} \quad Planck \text{ TT+lowP};$$

$$\sum m_{\nu} < 0.21 \text{ eV} \quad Planck \text{ TT+lowP+BAO};$$

$$\sum m_{\nu} < 0.49 \text{ eV} \quad Planck \text{ TT}, \text{TE}, \text{EE+lowP};$$

$$\sum m_{\nu} < 0.17 \text{ eV} \quad Planck \text{ TT}, \text{TE}, \text{EE+lowP+H}$$



Planck 2015 results, XIII

(keV) sterile neutrinos as warm dark matter

viable candidate as warm dark matter: not hot (decoupled when relativistic)neither cold (massive particles such as WIMP's).

Bounds

I) $m_s > 0.4$ keV (Tremain-Gunn): since they're fermions their local density cannot exceed the thermal Fermi degenerate gas density

Non thermal production! Otherwise too much energy density!

$$\rho_{s} > 45 \text{ keV cm}^{-3}$$

 $\rho_{cr} = 10.5 \text{ h}^{2} \text{ keV cm}^{-3}$



Production via oscillations:

Resonant mode: a large active neutrino asymmetry can give a resonant matter effect



3) bounds from LSS. For warm dark matter the free streaming length is smaller: suppression of structure at a smaller scale with respect to hot dark matter: Ly α forest





III International Pontecorvo Neutrino Physics School (2009)

Neutrinos in Cosmology







Several indirect effects of the neutrino background on cosmological observables

Informations on neutrino properties: mass oscillations, extra relativistic species, lifetime, magnetic moments,.....

DIRECT OBSERVATION?

Pauli to his friend Baade:

"Today I did something a physicist should never do. I predicted something which will never be observed experimentally..."



Detection I: Stodolsky effect

Energy split of electron spin states in the v background

Requires v chemical potential (Dirac) or net helicity (Majorana) Requires breaking of isotropy (Earth velocity) Results depend on Dirac or Majorana, relativistic/non relativistic, clustered/unclustered

$$\Delta E \approx G_F g_A \vec{s} \cdot \vec{\beta}_{\oplus} (n_v - \overline{n_v}) \qquad \text{Duda et al '01}$$

Torque on frozen magnetized macroscopic piece of material of dimension **R**

$$a \approx 10^{-27} \left(\frac{100}{A}\right) \left(\frac{cm}{R}\right) \left(\frac{\beta_{\oplus}}{10^{-3}}\right) \left(\frac{n_v - \overline{n_v}}{100 \text{ cm}^{-3}}\right) cm \text{ s}^{-2}$$

Presently Cavendish torsion balances $a \approx 10^{-12} \text{ cm s}^{-2}$ The only well established linear effect in G_F

Coherent interaction of large De Broglie wavelength

Energy (and angular momentum unless target is polarized and CNB has a net neutrino – antineutrino asymmetry) transfer at order G_F^2

Cabibbo and Maiani '82

I. target is static

Langacker et al '83

2. time average of incident neutrinos is spatially homogeneous over the detector size

Detection II: G_F²

V-Nucleus collision: net momentum transfer due to Earth peculiar motion

$$\sigma_{vN} = G_F^2 E_v^2 \qquad a = n_v v_v \frac{N_A}{A} \sigma_{vN} \Delta p$$
$$\Delta p = \beta_{\oplus} E_v$$

$$\Delta p = \beta_{\oplus} m_{v}$$

$$\Delta p = \beta_{\oplus} T_{v}$$

$$a \approx (10^{-46} - 10^{54}) \frac{A}{100} \text{ cm s}^{-2}$$

Coherence enhances $\lambda_{v} \approx 1/T_{v} - 1/m_{v} \approx mm$ $N_{c} = \frac{N_{A}}{A}\rho\lambda_{v}^{3}$ Zeldovich and Khlopov '81 Smith and Lewin '83

Backgrounds: solar v + Dark matter (WIMPS)

Detection III

Accelerator: vN scattering hopeless

$$R \approx 10^{-8} \, \mathrm{yr}^{-1}$$

LHC

Cosmic Rays (indirect): resonant v annihilation at m_z

$$E = \frac{m_Z^2}{2m_v} \approx 4 \ 10^{21} \left(\frac{eV}{m_v}\right) eV$$

Absorption dip (sensitive to high z)

Emission: Z burst above GZK (sensitive to GZK volume, (50 Mpc)³)



A '62 paper by S.Weinberg and v chemical potential

PHYSICAL REVIEW

VOLUME 128, NUMBER 3

NOVEMBER 1, 1962

Universal Neutrino Degeneracy

STEVEN WEINBERG* Imperial College of Science and Technology, London, England (Received March 22, 1962)

In the original idea a large neutrino chemical potential produces a distortion of the electron (positron) spectrum near the endpoint energy



FIG. 1. Shape of the upper end of an allowed Kurie plot to be expected in a β^+ decay if neutrinos are degenerate up to energy E_F , or in a β^- decay if antineutrinos are degenerate.



FIG. 2. Shape of the upper end of an allowed Kurie plot to be expected in a β^- decay if neutrinos are degenerate up to energy E_F , or in a β^+ decay if antineutrinos are degenerate.

Massive neutrinos and neutrino capture on beta decaying nuclei



This process has no energy threshold !



A 2 m_v gap in the electron spectrum centered around Q_β

Two issues:

Rate

Background

NCB Cross Section

Beta decay rate

$$\lambda_{\beta} = \frac{G_{\beta}^2}{2\pi^3} \int_{m_e}^{W_o} p_e E_e F(Z, E_e) C(E_e, p_{\nu})_{\beta} E_{\nu} p_{\nu} \, dE_e$$

NCB

$$\sigma_{\rm NCB} v_{\nu} = \frac{G_{\beta}^2}{\pi} p_e E_e F(Z, E_e) C(E_e, p_{\nu})_{\nu}$$

The nuclear shape factors \pmb{C}_β and \pmb{C}_ν both depend on the same nuclear matrix elements

Defining

$$\mathcal{A} = \int_{m_e}^{W_o} \frac{C(E'_e, p'_\nu)_\beta}{C(E_e, p_\nu)_\nu} \frac{p'_e}{p_e} \frac{E'_e}{E_e} \frac{F(E'_e, Z)}{F(E_e, Z)} E'_\nu p'_\nu dE'_e$$

$$\sigma_{\rm \scriptscriptstyle NCB} v_{\nu} = \frac{2\pi^2 \ln 2}{\mathcal{A} t_{1/2}}$$

In a large number of cases A can be evaluated in an exact way and NCB cross section depends only on Q_{β} and $t_{1/2}$ (measurable)

NCB Cross Section Evaluation using measured values of Q_{β} and $t_{1/2}$



selected from 14543 decays listed in the ENSDF database

NCB Cross Section Evaluation The case of Tritium

$$\sigma_{\rm ncb} v_{\nu} = \frac{G_{\beta}^2}{\pi} p_e E_e F(Z, E_e) C(E_e, p_{\nu})_{\nu}$$

$$\sigma_{\text{\tiny NCB}}(^{3}\text{H}) \frac{v_{\nu}}{c} = \left[(7.7 \pm 0.2) \times 10^{-45} \text{ cm}^{2} \right]$$
$$\lim \beta \to \mathbf{0}$$

where the error is due to Fermi and Gamow-Teller matrix element uncertainties

Using shape factors ratio

$$\sigma_{\text{NCB}} v_{\nu} = 2\pi^2 \ln 2 \frac{p_e E_e F(Z, E_e)}{f t_{1/2}}$$

$$\sigma_{\text{NCB}} {}^{(3}\text{H}) \frac{v_{\nu}}{c} = \left[(7.84 \pm 0.03) \times 10^{-45} \text{ cm}^2 \right]$$

$$\lim \beta \to \mathbf{0}$$

where the error is due only to uncertainties on \boldsymbol{Q}_β and $\boldsymbol{t}_{1/2}$



NCB Cross Section Evaluation specific cases

Isotope

Decay

Isotope	Q_eta	Half-life	$\sigma_{ m \scriptscriptstyle NCB}(v_{ u}/c)$
	(keV)	(sec)	(10^{-41} cm^2)
10 ~			
¹⁰ C	885.87	1320.99	5.36×10^{-3}
^{14}O	1891.8	71.152	1.49×10^{-2}
26m Al	3210.55	6.3502	3.54×10^{-2}
^{34}Cl	4469.78	1.5280	5.90×10^{-2}
38m K	5022.4	0.92512	7.03×10^{-2}
42 Sc	5403.63	0.68143	7.76×10^{-2}
^{46}V	6028.71	0.42299	9.17×10^{-2}
50 Mn	6610.43	0.28371	1.05×10^{-1}
54 Co	7220.6	0.19350	1.20×10^{-1}

$^{3}\mathrm{H}$	β^-	18.591	3.8878×10^{8}	7.84×10^{-4}
⁶³ Ni	β^-	66.945	3.1588×10^{9}	1.38×10^{-6}
93 Zr	β^{-}	60.63	4.952×10^{13}	2.39×10^{-10}
106 Ru	β^{-}	39.4	3.2278×10^{7}	5.88×10^{-4}
107 Pd	β^{-}	33	2.0512×10^{14}	2.58×10^{-10}
187 Re	β^{-}	2.64	1.3727×10^{18}	4.32×10^{-11}
$^{11}\mathrm{C}$	β^+	960.2	1.226×10^{3}	4.66×10^{-3}
${ m ^{11}C}{{ m ^{13}N}}$	$egin{smallmatrix} eta^+\ eta^+ \end{pmatrix}$	$960.2 \\ 1198.5$	1.226×10^{3} 5.99×10^{2}	4.66×10^{-3} 5.3×10^{-3}
$^{11}{ m C}$ $^{13}{ m N}$ $^{15}{ m O}$	$egin{array}{c} eta^+ \ eta^+ \ eta^+ \ eta^+ \ eta^+ \end{array}$	$960.2 \\ 1198.5 \\ 1732$	1.226×10^{3} 5.99×10^{2} 1.224×10^{2}	4.66×10^{-3} 5.3×10^{-3} 9.75×10^{-3}
^{11}C ^{13}N ^{15}O ^{18}F	$\beta^+ \\ \beta^+ \\ \beta^+ \\ \beta^+ \\ \beta^+$	$960.2 \\ 1198.5 \\ 1732 \\ 633.5$	$\begin{array}{c} 1.226 \times 10^{3} \\ 5.99 \times 10^{2} \\ 1.224 \times 10^{2} \\ 6.809 \times 10^{3} \end{array}$	$\begin{array}{c} 4.66 \times 10^{-3} \\ 5.3 \times 10^{-3} \\ 9.75 \times 10^{-3} \\ 2.63 \times 10^{-3} \end{array}$
${ m ^{11}C}{ m ^{13}N}{ m ^{15}O}{ m ^{18}F}{ m ^{22}Na}$	$\beta^+ \\ \beta^+ \\ \beta^+ \\ \beta^+ \\ \beta^+ \\ \beta^+$	$960.2 \\ 1198.5 \\ 1732 \\ 633.5 \\ 545.6$	$\begin{array}{c} 1.226 \times 10^{3} \\ 5.99 \times 10^{2} \\ 1.224 \times 10^{2} \\ 6.809 \times 10^{3} \\ 9.07 \times 10^{7} \end{array}$	$\begin{array}{c} 4.66 \times 10^{-3} \\ 5.3 \times 10^{-3} \\ 9.75 \times 10^{-3} \\ 2.63 \times 10^{-3} \\ 3.04 \times 10^{-7} \end{array}$

Half-life

(sec)

 $\sigma_{\rm NCB}(v_{\nu}/c)$ (10⁻⁴¹ cm²)

Q

(keV)

Superallowed $0^+ \rightarrow 0^+$ decays used for CVC hypotesis testing (very precise measure of Q_β and $t_{1/2}$)

Nuclei having the highest product $\sigma_{NCB} t_{1/2}$

The cosmological relic neutrino capture rate

$$\lambda_{\nu} = \int \sigma_{\rm \scriptscriptstyle NCB} v_{\nu} \, \frac{1}{\exp(p_{\nu}/T_{\nu}) + 1} \, \frac{d^3 p_{\nu}}{(2\pi)^3}$$

$$T_v = 1.7 \cdot 10^{-4} \text{ eV}$$

$$2.85 \cdot 10^{-2} \frac{\sigma_{\rm NCB} v_{\nu}/c}{10^{-45} {\rm cm}^2} {\rm yr}^{-1} {\rm mol}^{-1}$$



Relic Neutrino Detection signal to background ratio

The ratio between capture (λ_{ν}) and beta decay rate (λ_{β}) is obtained using the previous expressions

$$\frac{\lambda_{\nu}}{\lambda_{\beta}} = \frac{2\pi^2 n_{\nu}}{\mathcal{A}}$$

In the case of Tritium:

$$\lambda_{\nu}(^{3}\mathrm{H}) = 0.66 \cdot 10^{-23} \lambda_{\beta}(^{3}\mathrm{H})$$

Taking into account the beta decays occurring in the last bin of width Δ at the spectum end-point we have that

$$\frac{\lambda_{\nu}}{\lambda_{\beta}(\Delta)} = \frac{9}{2}\zeta(3)\left(\frac{T_{\nu}}{\Delta}\right)^{3}\frac{1}{\left(1+2m_{\nu}/\Delta\right)^{3/2}} \sim \mathbf{IO}^{-10}$$



where the last term is the probability for a beta decay electron at the endpoint to be measured beyond the $2m_v$ gap

It works for $\Delta < m_v$



Discovery potential

As an example, given a neutrino mass of 0.7 eV and an energy resolution at the beta decay endpoint of 0.2 eV a signal to background ratio of 3 is obtained

In the case of 100 g mass target of Tritium it would take one and a half year to observe a 5 σ effect

In case of neutrino gravitational clustering we expect a significant signal enhancement

$m_{\nu} (\mathrm{eV})$	FD (events yr^{-1})	NFW (events yr^{-1})	MW (events yr^{-1})
0.6	7.5	90	150
0.3	7.5	23	33
0.15	7.5	10	12

FD = Fermi-Dirac NFW= Navarro, Frenk and White MW=Milky Way (Ringwald, Wong)
KATRIN Karlsruhe Tritium Neutrino Experiment

Aim at direct neutrino mass measurement through the study of the ³H endpoint($Q_{\beta} = 18.59 \text{ keV}, t_{1/2} = 12.32 \text{ years}$)

Phase I: Energy resolution: 0.93 eV Tritium mass: ~ 0.1 mg Noise level 10 mHz Sensitivity to v_e mass: 0.2 eV



Magnetic Adiabatic Collimator + Electrostatic filter

MARE

Aim at direct neutrino mass measurement through the study of the ^{187}Re endpoint (Q_{\beta} =2.66 keV, t_{1/2}=4.3 \times 10^{10} years)

Using TEs+micro-bolometers @ 10 mK temperature





PTOLEMY

Development of a Relic Neutrino Detection Experiment at PTOLEMY: Princeton Tritium Observatory for Light, Early-Universe, Massive-Neutrino Yield

S. Betts¹, W. R. Blanchard¹, R. H. Carnevale¹, C. Chang², C. Chen³, S. Chidzik³, L. Ciebiera¹, P. Cloessner⁴, A. Cocco⁵, A. Cohen¹, J. Dong¹, R. Klemmer³, M. Komor³, C. Gentile¹, B. Harrop³, A. Hopkins¹, N. Jarosik³, G. Mangano⁵, M. Messina⁶, B. Osherson³, Y. Raitses¹, W. Sands³, M. Schaefer¹, J. Taylor¹, C. G. Tully³, R. Woolley¹, and A. Zwicker¹



- A large area surface deposition target
- MAC-E filter
- RF signal from cyclotron electron motion in a magnetic field
- cryogenic calorimetry

MAC-E filter (magnetic adiabatic collimation combined with an electrostatic filter

used by Katrin exp.

The beta electrons, isotropically emitted at the source, are transformed into a broad beam of electrons flying almost parallel to the magnetic field lines (adiabatically). This parallel beam of electrons is running against an electrostatic potential formed by a system of cylindrical electrodes. All electrons with enough energy to pass the electrostatic barrier are reaccelerated and collimated onto a detector, all others are reflected. Therefore the spectrometer acts as an integrating high-energy pass filter.





Conclusions I

• Cosmology can:

constrain neutrino physics (optimistic) guide lab experiments to constrain neutrino physics (conservative)

 many other neutrino properties not discussed here (but quite weak bounds)



Conclusions II

• Neutrino physics can:

constrain or give hints to understand unresolved problems of cosmology (ex. keV neutrinos as warm dark matter, leptogenesis if v's are Majorana particles)