Neutrino Oscillation Phenomenology

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August, 2017

NASA Hubble Photo

What Are Neutrinos Good For?

Energy generation in the sun starts with the reaction —

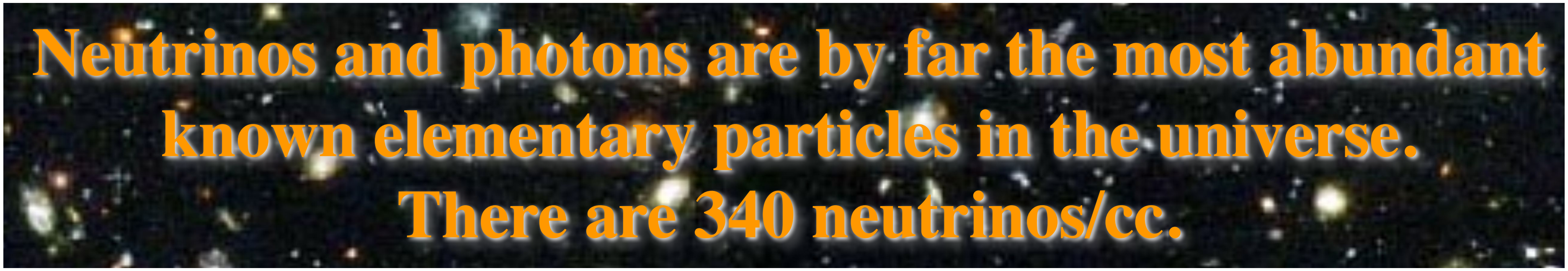
$$p + p \rightarrow d + e^{+} + \nu$$

Spin: $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$

Without the neutrino, angular momentum would not be conserved.

Uh, oh ……

The Neutrinos



The neutrinos are spin – 1/2, electrically neutral, leptons.

The only known forces they experience are the weak force and gravity.

This means that they do not interact with other matter very much at all. Thus, neutrinos are difficult to detect and study.

Their weak interactions are successfully described by the Standard Model.

The Neutrino Revolution
(1998 – …)

Neutrinos have nonzero masses!

Leptons mix!

The 2015 Nobel Prize in Physics went to **Takaaki Kajita** and **Art McDonald** for the experiments that proved this.

Super-Kamiokande, Japan

Sudbury Neutrino Observatory, Canada

The Origin of Neutrino Mass

The fundamental constituents of matter are the ***quarks***, the ***charged leptons***, and the ***neutrinos***.

The discovery and study of the ***Higgs boson*** at CERN's Large Hadron Collider has provided strong evidence that the ***quarks*** and ***charged leptons*** derive their masses from an interaction with the ***Higgs field***.

Most theorists strongly suspect that the origin of the neutrino masses is different from the origin of the quark and charged lepton masses.

The Standard-Model ***Higgs field*** is probably still involved, but there is probably something more — something way outside the Standard Model —

Majorana masses.

More later ……….

The discovery of neutrino mass
and leptonic mixing
comes from the observation of
neutrino flavor change
(neutrino oscillation).

The Physics of Neutrino Oscillation

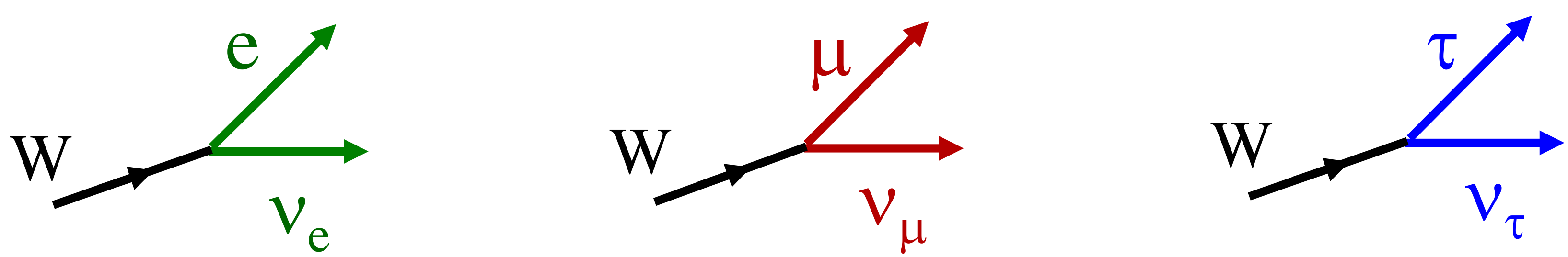
— Preliminaries

The Neutrino Flavors

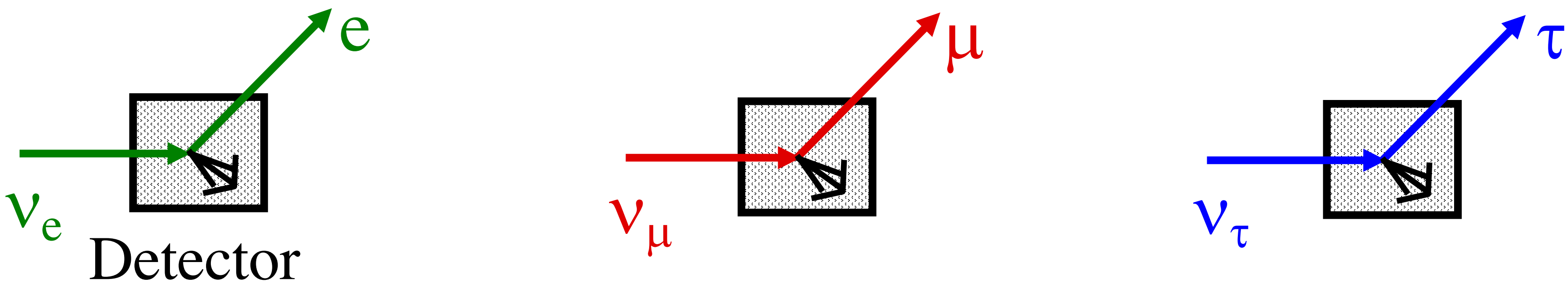
There are three flavors of charged leptons: e , μ , τ

There are three known flavors of neutrinos: ν_e, ν_μ, ν_τ

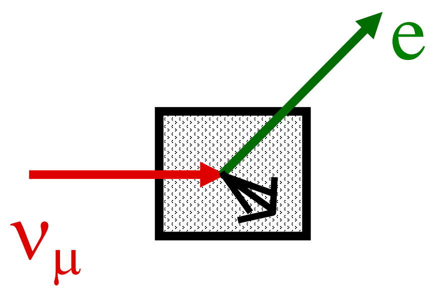
We ***define*** the neutrinos of specific flavor, ν_e, ν_μ, ν_τ, by W boson decays:



As far as we know, when a neutrino of given flavor interacts and turns into a charged lepton, that charged lepton will always be of the same flavor as the neutrino.



but not

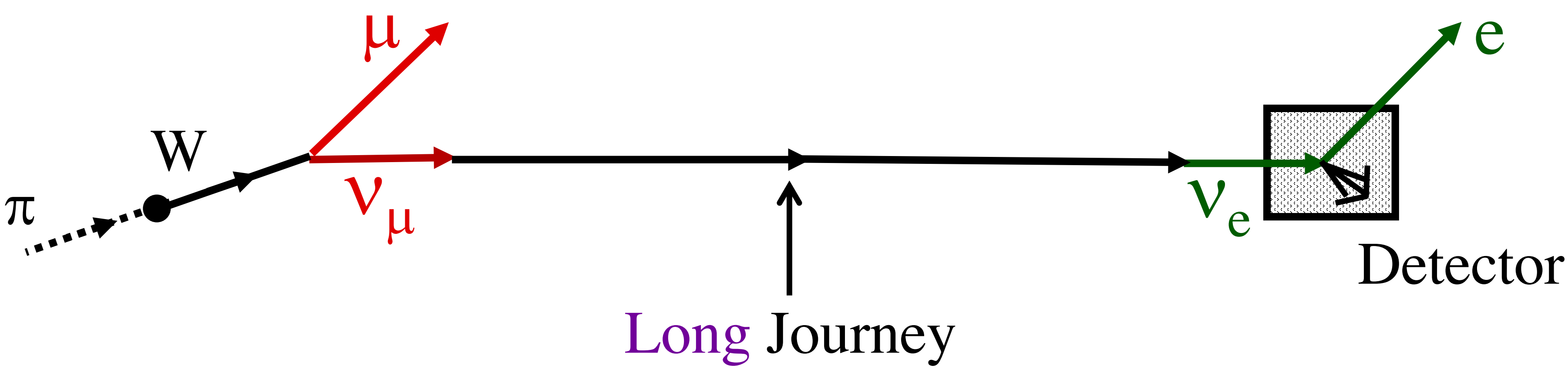


Lederman
Schartz
Steinberger

The weak interaction couples the neutrino of a given flavor only to the charged lepton of the same flavor.

Neutrino Flavor Change ("Oscillation")

If neutrinos have masses, and leptons mix, we can have —



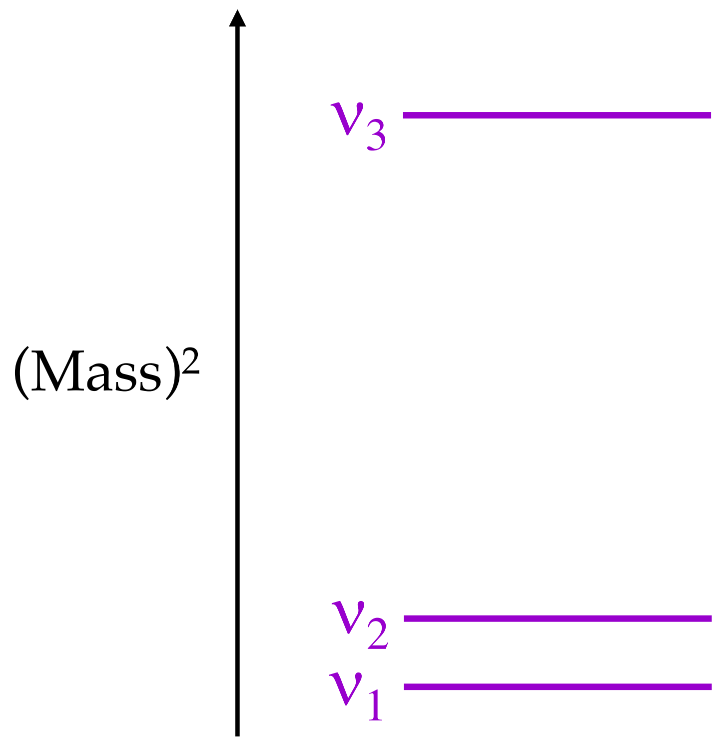
Give a ν time to change character, and you can have

for example: $\nu_\mu \longrightarrow \nu_e$

The last 19 years have brought us compelling evidence that such flavor changes actually occur.

Flavor Change Requires ***Neutrino Masses***

There must be some spectrum
of neutrino mass eigenstates ν_i:



Mass (ν_i) $\equiv m_i$

Flavor Change Requires ***Leptonic Mixing***

The neutrinos $\nu_{e,\mu,\tau}$ of definite flavor

$(W \rightarrow e\nu_e$ or $\mu\nu_\mu$ or $\tau\nu_\tau)$

must be superpositions of the mass eigenstates:

$$|\nu_\alpha> = \sum_i U^*_{\alpha i} |\nu_i> \quad .$$

Neutrino of flavor
α = e, μ, or τ

Neutrino of definite mass m_i

"PMNS" Leptonic Mixing Matrix

Pontecorvo

Notation: ℓ denotes a charged lepton. $\ell_e \equiv e$, $\ell_\mu \equiv \mu$, $\ell_\tau \equiv \tau$.

Since the only charged lepton ν_α couples to is ℓ_α,
the 3 ν_α must be orthogonal.

To make up 3 orthogonal ν_α, we must have at least 3 ν_i.
Unless some ν_i masses are degenerate,
all ν_i will be orthogonal.

Then —

$$\delta_{\alpha\beta} = \langle \nu_\alpha | \nu_\beta \rangle = \left\langle \sum_i U^*_{\alpha i} \nu_i \middle| \sum_j U^*_{\beta j} \nu_j \right\rangle$$

$$= \sum_{i,j} U_{\alpha i} U^*_{\beta j} \langle \nu_i | \nu_j \rangle = \sum_i U_{\alpha i} U^*_{\beta i}$$

This says that U is unitary, but note the unitary U may not be 3 x 3.

Leptonic mixing is easily incorporated into the Standard Model (SM) description of the $\ell\nu W$ interaction.

For this interaction, we then have —

Semi-weak coupling

Left-handed

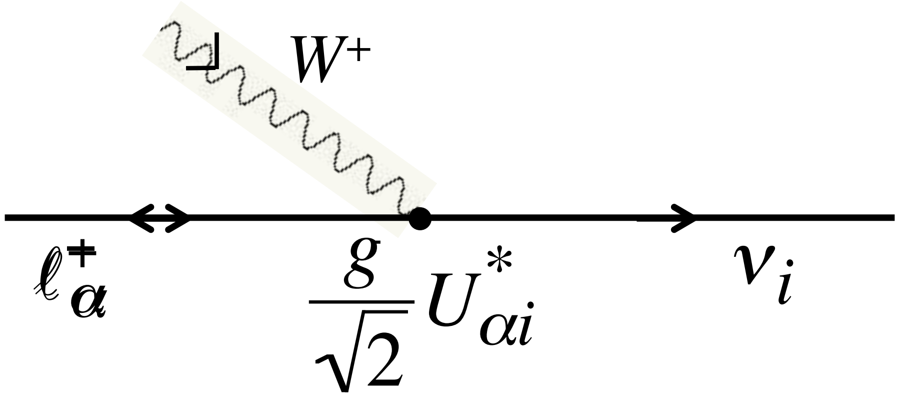
$$\mathcal{L}_{SM} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \left(\overline{\ell}_{L\alpha} \gamma^{\lambda} \nu_{L\alpha} W_{\lambda}^{-} + \overline{\nu}_{L\alpha} \gamma^{\lambda} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

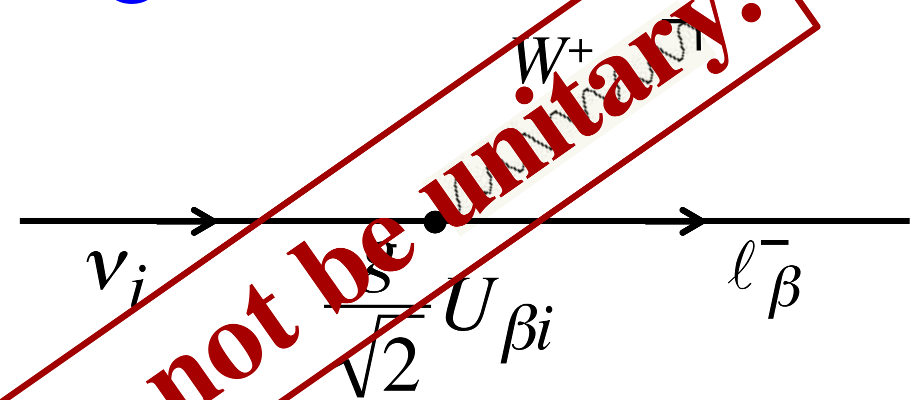
$$= -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} \left(\overline{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}^{-} + \overline{\nu}_{Li} \gamma^{\lambda} U_{\alpha i}^{*} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

Taking mixing into account

The SM interaction conserves the Lepton Number L, defined by $L(\nu) = L\left(\ell^{-}\right) = -L(\overline{\nu}) = -L\left(\ell^{+}\right) = 1$.

The Meaning of *U*





$$U = \begin{array}{c} \\ e \\ \mu \\ \tau \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \end{array}$$

Please talk about <u>***leptonic***</u> mixing, not neutrino mixing.

Caution: The 3 x 3 *U* may not be unitary.

The *e* row of *U*: The linear combination of neutrino mass eigenstates that couples to *e*.

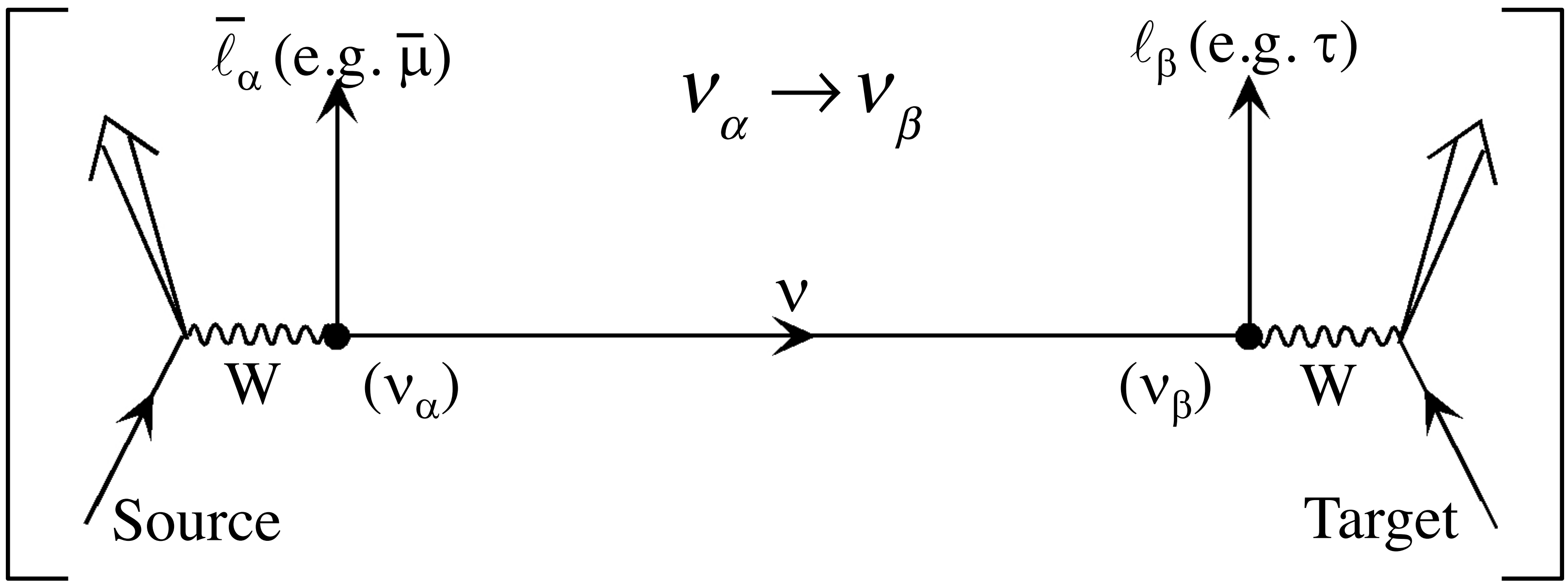
The ν_1 column of *U*: The linear combination of charged-lepton mass eigenstates that couples to ν_1 .

How Neutrino Oscillation In Vacuum Works

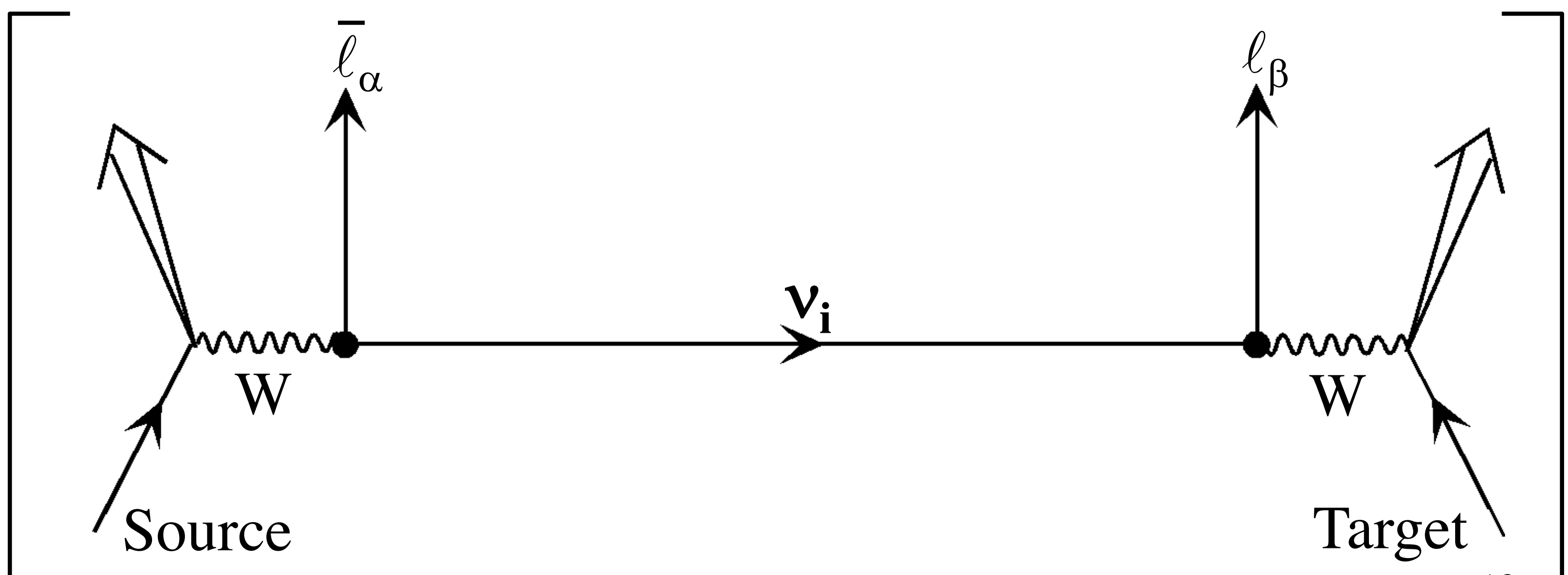
Neutrino Oscillation

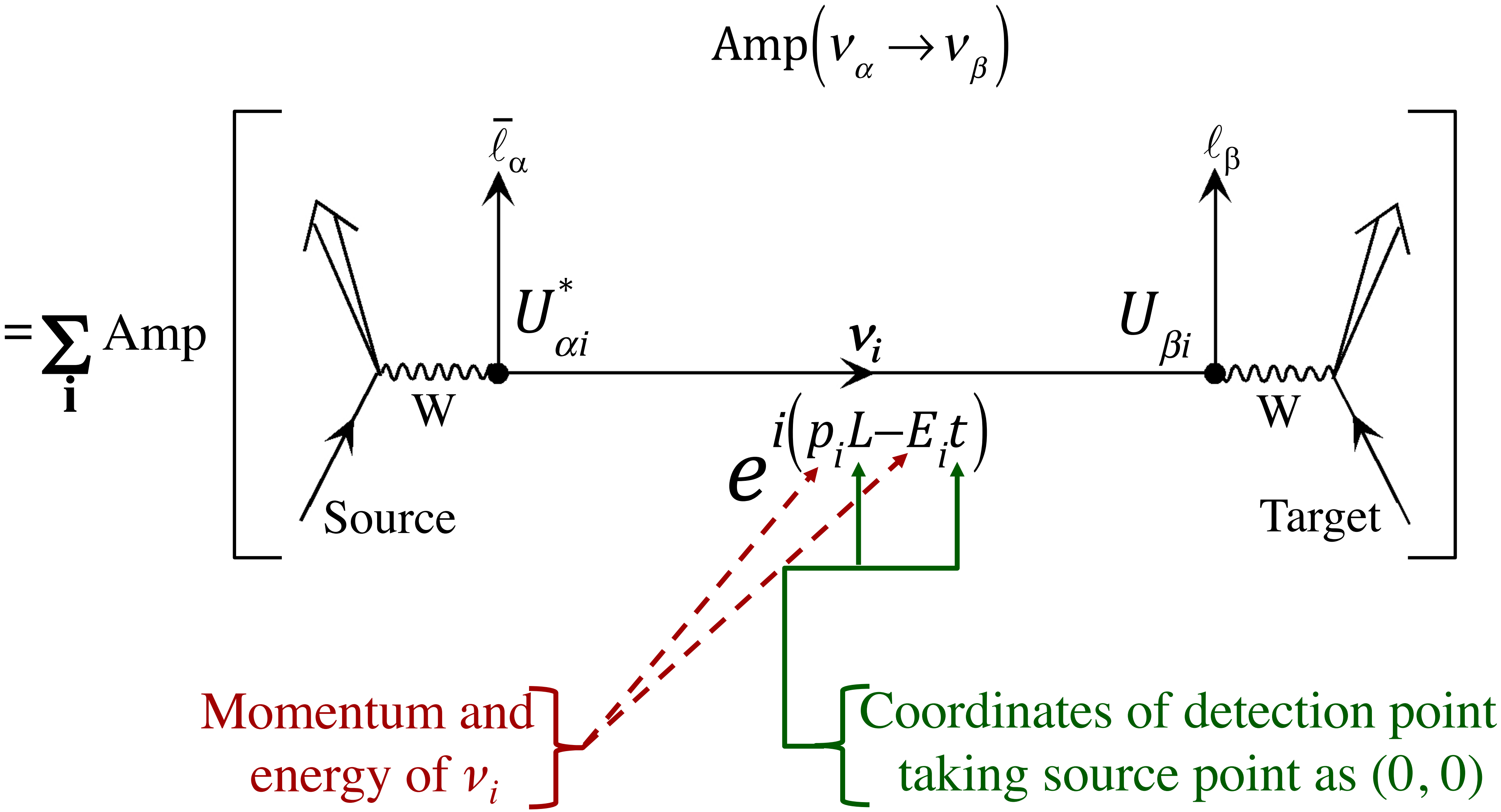
(Approach of B.K. and Stodolsky)

Amp



$= \sum_{\mathbf{i}}$ Amp





Amp$(\nu_\alpha \rightarrow \nu_\beta)$
$= \sum_i$ Amp
$\bar{\ell}_\alpha$
ℓ_β
$U^*_{\alpha i}$
ν_i
$U_{\beta i}$
W
W
Source
Target
$e^{i(p_i L - E_i t)}$
Momentum and energy of ν_i
Coordinates of detection point, taking source point as (0, 0)

Neutrino sources are ~ constant in time.

Averaged over time, the

$$e^{-iE_1 t} - e^{-iE_2 t} \quad \text{interference}$$

is —

$$\left\langle e^{-i(E_1 - E_2)t} \right\rangle_t = 0 \quad \textit{unless } E_2 = E_1.$$

Only neutrino mass eigenstates with a common energy E are coherent.

(Stodolsky)

For each mass eigenstate ν_i,

$$p_i = \sqrt{E^2 - m_i^2} \cong E - \frac{m_i^2}{2E} \; .$$

Then the plane-wave factor $e^{i(p_i L - E_i t)}$ is —

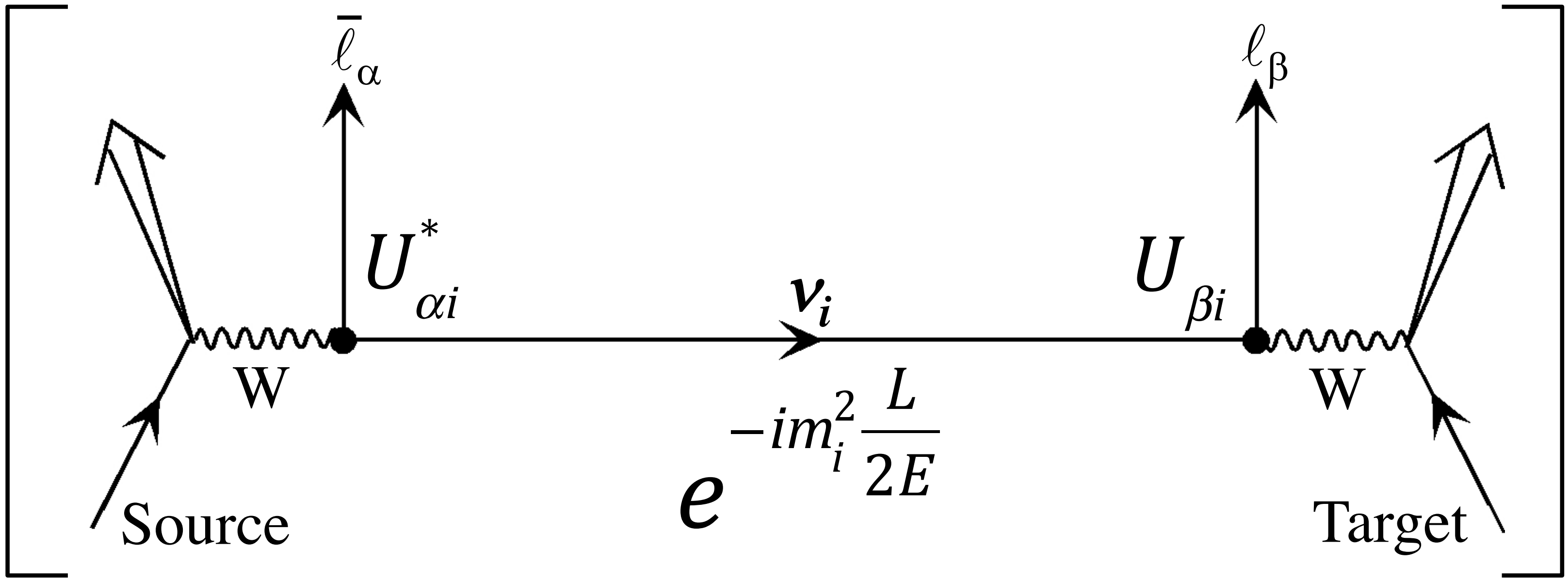
$$e^{i(p_i L - E_i t)} \cong e^{i\left\{\left(E - \frac{m_i^2}{2E}\right)L - Et\right\}} = e^{iE(L-t)} e^{-im_i^2 \frac{L}{2E}}$$

Irrelevant overall phase factor → $e^{iE(L-t)}$

Then —

$$\text{Amp}\left(\nu_\alpha \to \nu_\beta\right)$$

$$= \sum_{\mathbf{i}} \text{Amp}$$



$$= \sum_i U^*_{\alpha i} e^{-im_i^2 \frac{L}{2E}} U_{\beta i}$$

Probability of Neutrino Oscillation in Vacuum

$$P\left(\nu_\alpha \rightarrow \nu_\beta\right) = \left|\text{Amp}\left(\nu_\alpha \rightarrow \nu_\beta\right)\right|^2 =$$

$$= \delta_{\alpha\beta} - 4\sum_{i>j} \text{Re}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin^2\left(\Delta m^2_{ij} \frac{L}{4E}\right)$$

$$+ 2\sum_{i>j} \text{Im}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin\left(\Delta m^2_{ij} \frac{L}{2E}\right)$$

where $\Delta m^2_{ij} \equiv m^2_i - m^2_j$.

Neutrino flavor change implies neutrino mass!

Neutrinos vs. Antineutrinos

$$\left[\bar{\nu}_\alpha(\mathrm{RH}) \to \bar{\nu}_\beta(\mathrm{RH})\right] = \mathrm{CP}\left[\nu_\alpha(\mathrm{LH}) \to \nu_\beta(\mathrm{LH})\right]$$

A difference between the probabilities of these two oscillations in vacuum would be a leptonic violation of CP invariance.

Assuming CPT invariance —

$$P\left[\bar{\nu}_\alpha(\mathrm{RH}) \to \bar{\nu}_\beta(\mathrm{RH})\right] = P\left[\nu_\beta(\mathrm{LH}) \to \nu_\alpha(\mathrm{LH})\right]$$

↑ Probability

$$P\left(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta\right) =$$

$$= \delta_{\alpha\beta} - 4\sum_{i>j} \mathrm{Re}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin^2\left(\Delta m^2_{ij} \frac{L}{4E}\right)$$

$$\overset{}{\underset{(-)}{+}} 2\sum_{i>j} \mathrm{Im}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin\left(\Delta m^2_{ij} \frac{L}{2E}\right)$$

In neutrino oscillation, CP non-invariance comes from phases in the leptonic mixing matrix U.

Note: Including $\hbar$ and c, $\Delta m^2_{ij} \frac{L}{4E} = 1.27 \Delta m^2_{ij}\left(\mathrm{eV}^2\right) \frac{L(\mathrm{km})}{E(\mathrm{GeV})}$

(Lectures by Gary Feldman)

The plane-wave treatment of neutrino oscillation is not completely correct,

since $\Delta x \Delta p \geq \frac{\hbar}{2}$.

The expression for the oscillation probability resulting from this treatment is wrong at very large L.

But a wave packet treatment shows that the plane-wave expression is correct under almost all circumstances.

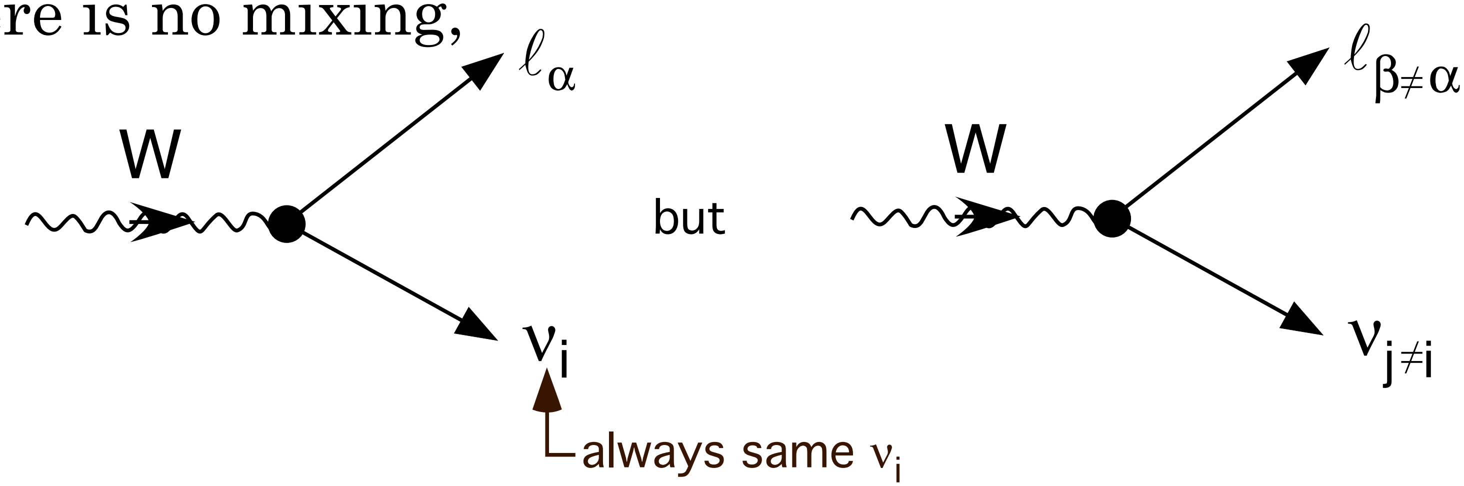
— Comments —

1. If all $m_i = 0$, so that all $\Delta m_{ij}^2 = 0$,

$$P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) = \delta_{\alpha\beta}$$

Flavor *change* $\Rightarrow \nu$ Mass

2. If there is no mixing,



$$\Rightarrow U_{\alpha i} U_{\beta\neq\alpha, i} = 0, \text{ so that } P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) = \delta_{\alpha\beta}.$$

3. One can detect ($\nu_\alpha \to \nu_\beta$) in two ways:

See $\nu_{\beta\neq\alpha}$ in a ν_α beam (Appearance)

See some of known ν_α flux disappear (Disappearance)

4. Including $\hbar$ and c

$$\Delta m^2 \frac{L}{4E} = 1.27 \Delta m^2 (\mathrm{eV}^2) \frac{L(\mathrm{km})}{E(\mathrm{GeV})}$$

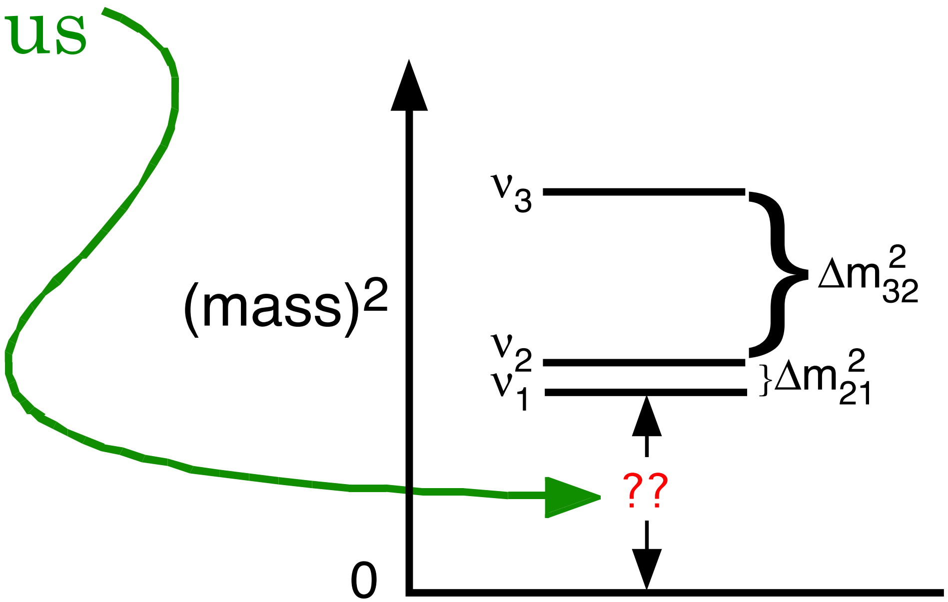
$\sin^2[1.27 \Delta m^2 (\mathrm{eV})^2 \frac{L(\mathrm{km})}{E(\mathrm{GeV})}]$ becomes appreciable when its argument reaches $\mathcal{O}(1)$.

An experiment with given L/E is sensitive to

$$\Delta m^2 (\mathrm{eV}^2) \gtrsim \frac{E(\mathrm{GeV})}{L(\mathrm{km})} \quad .$$

5. Flavor change in vacuum oscillates with L/E. Hence the name "neutrino oscillation". {The L/E is from the proper time τ.}

6. P $(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta)$ depends only on squared-mass splittings. Oscillation experiments cannot tell us



7. Neutrino flavor change does not change the total flux in a beam.

It just redistributes it among the flavors.

$$\sum_{\text{All } \beta} P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) = 1$$

But some of the flavors $\beta \neq \alpha$ could be sterile.

Then some of the *active* flux disappears:

$$\phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau} < \phi_{\text{Original}}$$

Important Special Cases

Three Flavors

For $\beta \neq \alpha$,

$$e^{-im_1^2 \frac{L}{2E}} \mathrm{Amp}^*(\nu_\alpha \to \nu_\beta) = \sum_i U_{\alpha i} U_{\beta i}^* e^{im_i^2 \frac{L}{2E}} e^{-im_1^2 \frac{L}{2E}}$$

$$= U_{\alpha 3} U_{\beta 3}^* e^{2i\Delta_{31}} + U_{\alpha 2} U_{\beta 2}^* e^{2i\Delta_{21}} \underbrace{- (U_{\alpha 3} U_{\beta 3}^* + U_{\alpha 2} U_{\beta 2}^*)}_{\text{Unitarity}}$$

$$= 2i [U_{\alpha 3} U_{\beta 3}^* e^{i\Delta_{31}} \sin \Delta_{31} + U_{\alpha 2} U_{\beta 2}^* e^{i\Delta_{21}} \sin \Delta_{21}]$$

$$\text{where } \Delta_{ij} \equiv \Delta m_{ij}^2 \frac{L}{4E} \equiv (m_i^2 - m_j^2) \frac{L}{4E} \quad .$$

$$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) = \left| e^{-im_1^2 \frac{L}{2E}} \text{Amp}^*(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) \right|^2$$

$$= 4[|U_{\alpha 3}U_{\beta 3}|^2 \sin^2 \Delta_{31} + |U_{\alpha 2}U_{\beta 2}|^2 \sin^2 \Delta_{21}$$

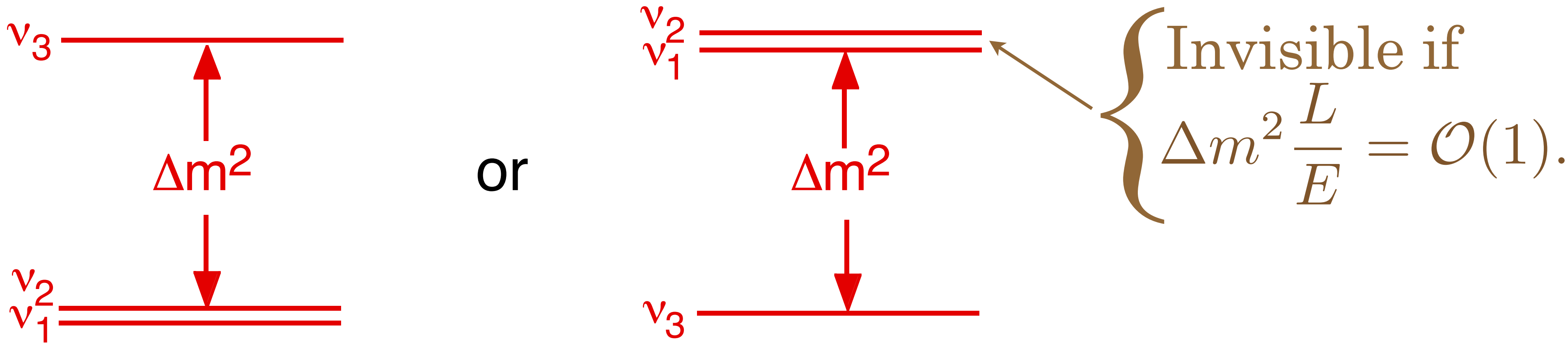
$$+2|U_{\alpha 3}U_{\beta 3}U_{\alpha 2}U_{\beta 2}| \sin \Delta_{31} \sin \Delta_{21} \cos(\Delta_{32} \underset{(-)}{+} \delta_{32})] \quad .$$

Here $\delta_{32} \equiv \arg(U_{\alpha 3}U^*_{\beta 3}U^*_{\alpha 2}U_{\beta 2})$, a CP – violating phase.

Two waves of different frequencies,

and their ~~CP~~ interference.

When the Spectrum Is—



For $\beta \neq \alpha$,

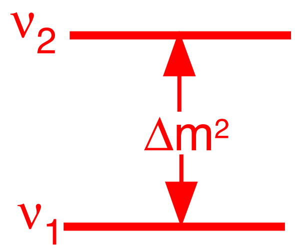
$$P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) \cong 4|U_{\alpha 3}U_{\beta 3}|^2 \sin^2(\Delta m^2 \tfrac{L}{4E}) \quad .$$

For no flavor change,

$$P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\alpha) \cong 1 - 4|U_{\alpha 3}|^2(1 - |U_{\alpha 3}|^2) \sin^2(\Delta m^2 \tfrac{L}{4E}) \quad .$$

Experiments with $\Delta m^2 \frac{L}{E} = \mathcal{O}(1)$ can determine the flavor content of ν_3.

When There are Only Two Flavors and Two Mass Eigenstates



Majorana ~~CP~~ phase

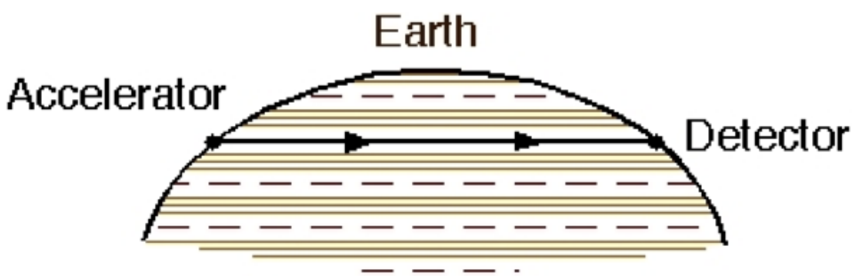
$$U = \begin{bmatrix} U_{\alpha 1} & U_{\alpha 2} \\ U_{\beta 1} & U_{\beta 2} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} e^{i\xi} & 0 \\ 0 & 1 \end{bmatrix}$$

Mixing angle

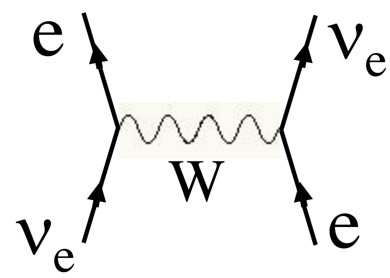
For $\beta \neq \alpha$, $$P(\overset{(-)}{\nu}_\alpha \leftrightarrow \overset{(-)}{\nu}_\beta) = \sin^2 2\theta \sin^2(\Delta m^2 \frac{L}{4E}) \quad .$$

For no flavor change, $$P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\alpha) = 1 - \sin^2 2\theta \sin^2(\Delta m^2 \frac{L}{4E}).$$

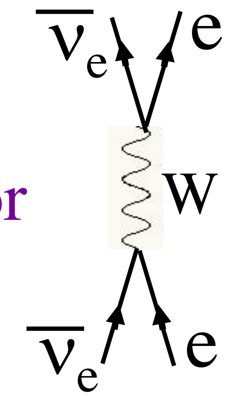
Neutrino Flavor Change In Matter



involves



or



Coherent forward scattering via this W-exchange interaction leads to an extra interaction potential energy —

$$V_W = \begin{cases} +\sqrt{2}G_F N_e, & \nu_e \\ -\sqrt{2}G_F N_e, & \overline{\nu_e} \end{cases}$$

Fermi constant (G_F) — Electron density (N_e)

This raises the effective mass of ν_e, and lowers that of $\overline{\nu_e}$.

The fractional importance of matter effects on an oscillation involving a vacuum splitting Δm^2 is —

$$\underbrace{[\sqrt{2}G_F N_e]}_{\text{Interaction energy}} \, / \, \underbrace{[\Delta m^2/2E]}_{\text{Vacuum energy}} \, .$$

The matter effect —

More later

— Grows with neutrino energy E

— Is sensitive to Sign(Δm^2)

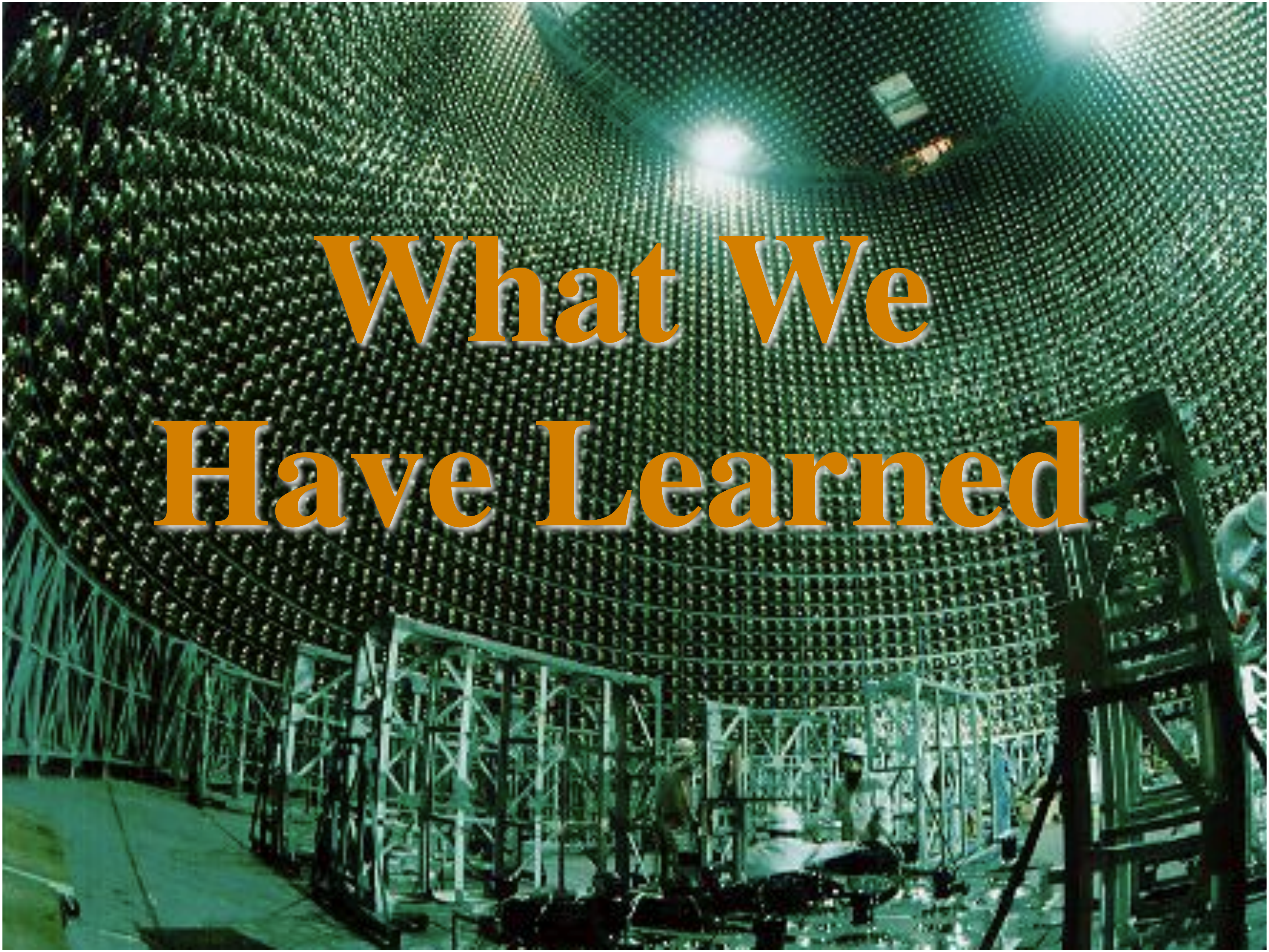
— Reverses when ν is replaced by $\bar{\nu}$

This last is a "fake CP violation" that has to be taken into account in searches for genuine CP violation.

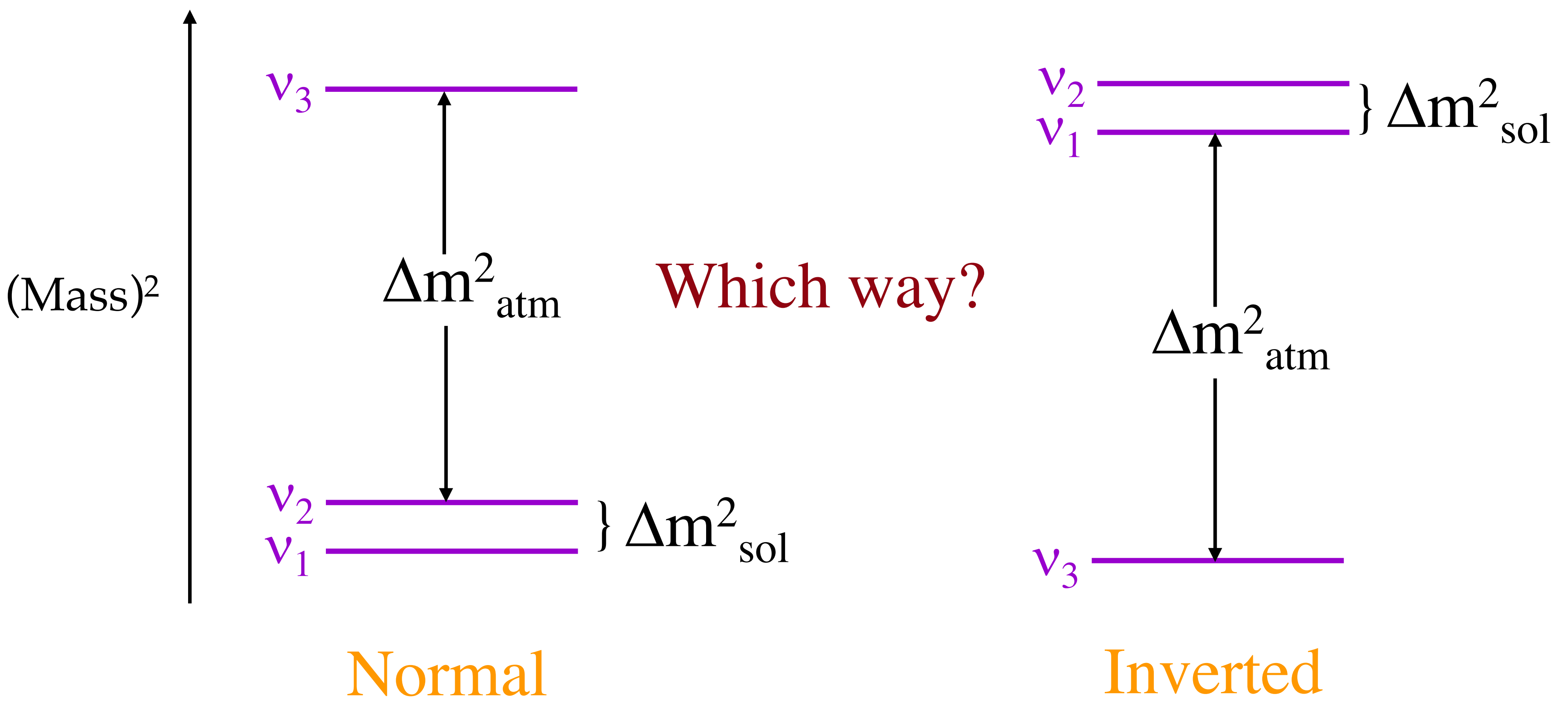
Evidence For Flavor Change

Neutrinos	Evidence of Flavor Change
Solar	Compelling
Reactor (Long-Baseline)	Compelling
Atmospheric	Compelling
Accelerator (Long-Baseline)	Compelling
Accelerator, Reactor, and Radioactive Sources (Short-Baseline)	"Interesting"

What We Have Learned



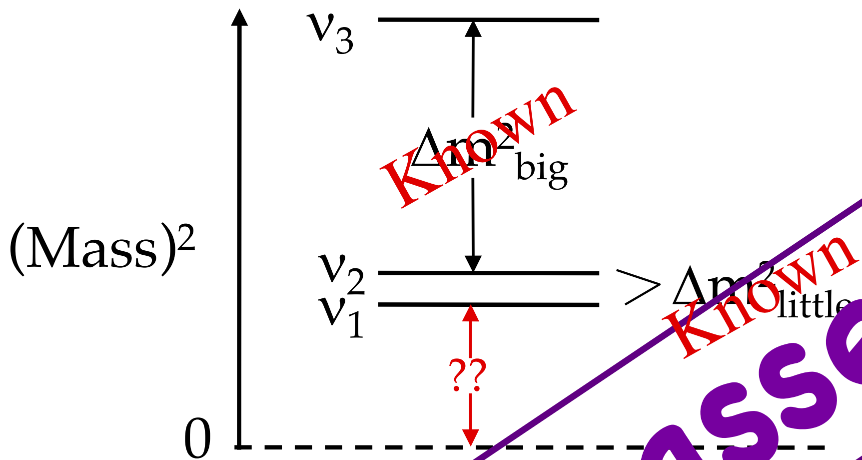
The (Mass)^2 Spectrum



$\Delta m^2_{sol} \cong 7.5 \times 10^{-5}\ eV^2$, $\Delta m^2_{atm} \cong 2.5 \times 10^{-3}\ eV^2$

Are there ***more*** mass eigenstates?

Constraints On the Absolute Scale of Neutrino Mass



How far above zero is the whole pattern?

Neutrino masses are tiny

Cosmology, *under certain assumptions* ⇒ $\sum m(\nu_i) < 0.17$ eV

All *i*

Tritium beta decay ⇒ $\sqrt{0.69m^2(\nu_1)+0.29m^2(\nu_2)+0.02m^2(\nu_3)} < 2$ eV

Oscillation ⇒ Mass[Heaviest ν_i] > $\sqrt{\Delta m^2_{big}}$ > 0.05 eV

Measurements of the tritium β energy spectrum bound the average neutrino mass —

$$\langle m_\beta \rangle = \sqrt{\sum_i |U_{ei}|^2 m_i^2}$$ (Farzan & Smirnov)

Presently: $\langle m_\beta \rangle < 2$ eV (Mainz & Troitzk)

(Lectures by Igor Tkachev & Loredana Gastaldo)

Leptonic Mixing

Mixing means that —

$$|\nu_\alpha> = \sum_i U^*_{\alpha i}\, |\nu_i> \quad .$$

Neutrino of flavor

α = e, μ, or τ

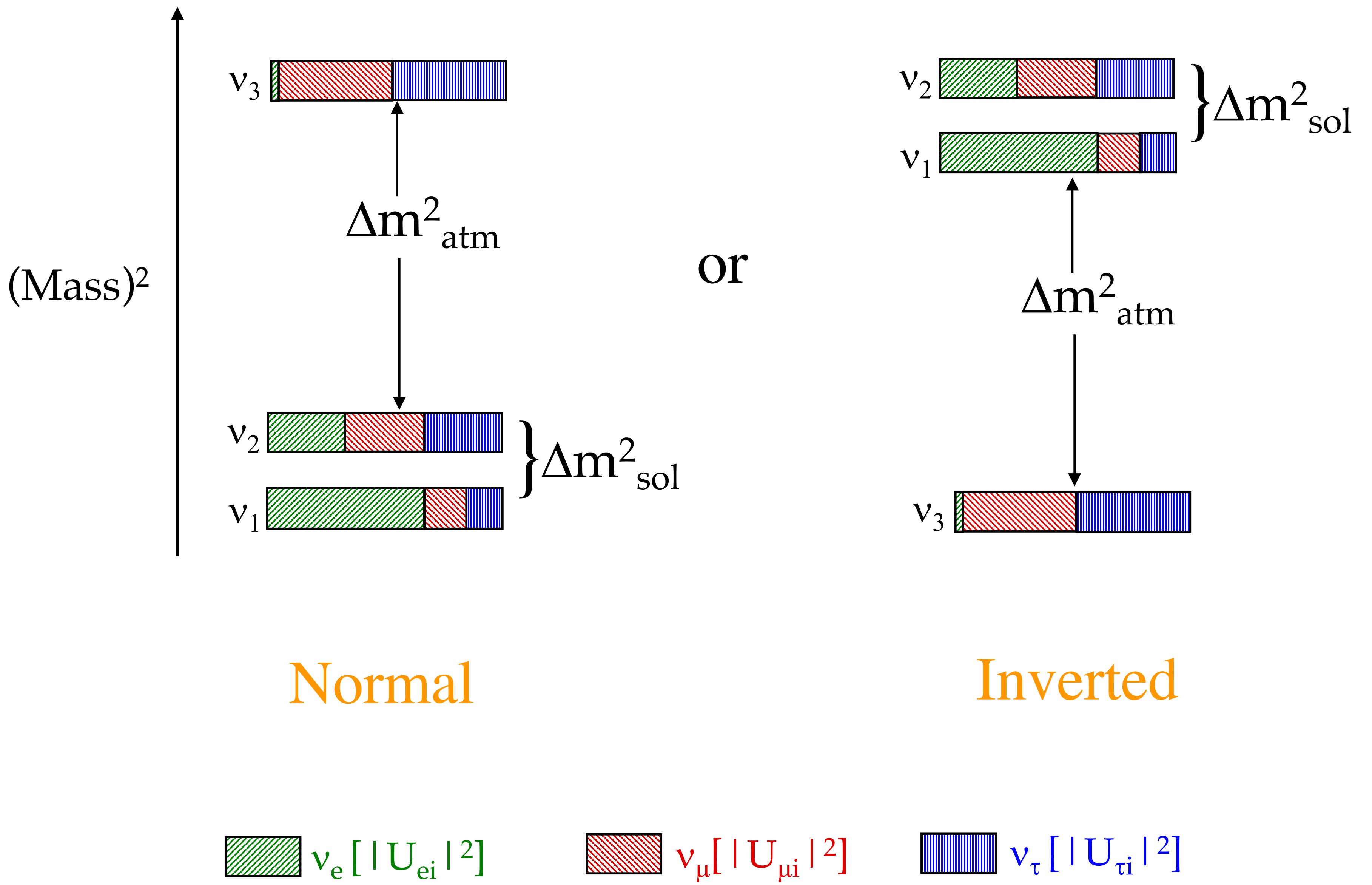
Neutrino of definite mass m_i

Inversely, $|\nu_i> = \sum_\alpha U_{\alpha i}\, |\nu_\alpha> \; .$ (***if*** U is unitary)

Flavor-α fraction of $\nu_i = |U_{\alpha i}|^2$.

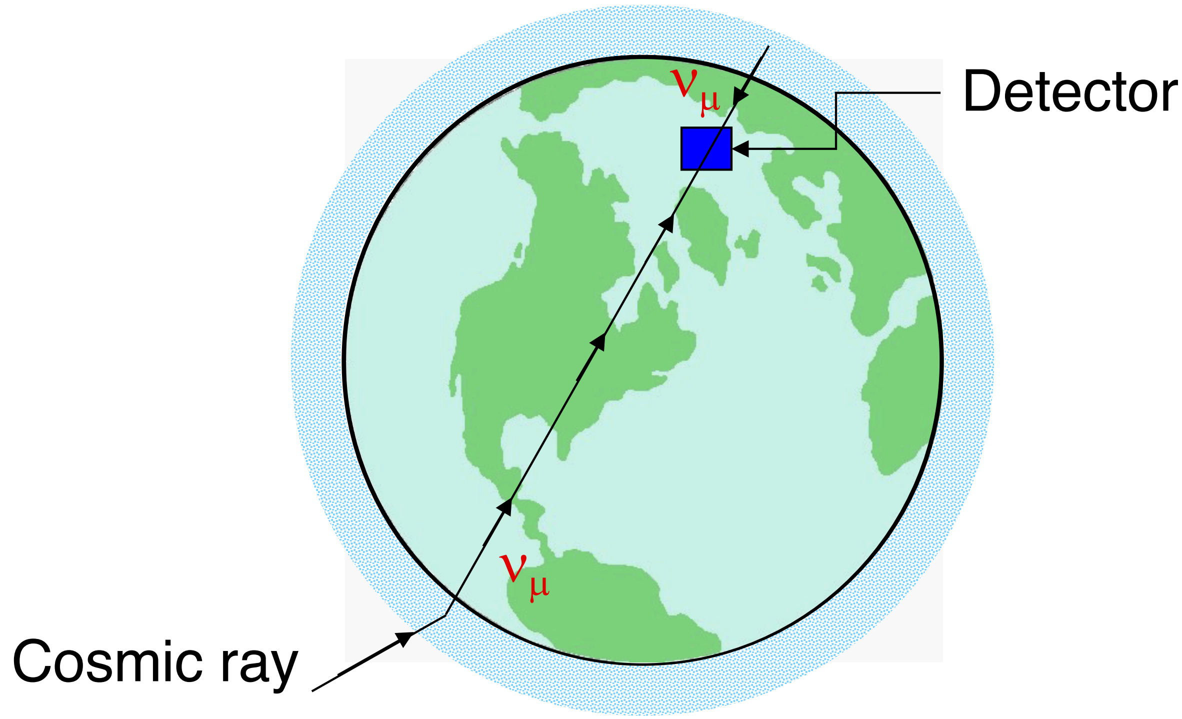
When a ν_i interacts and produces a charged lepton, the probability that this charged lepton will be of flavor α is $|U_{\alpha i}|^2$.

Experimentally, the flavor fractions are —



Observations We Can Use To Understand The Flavor Fractions

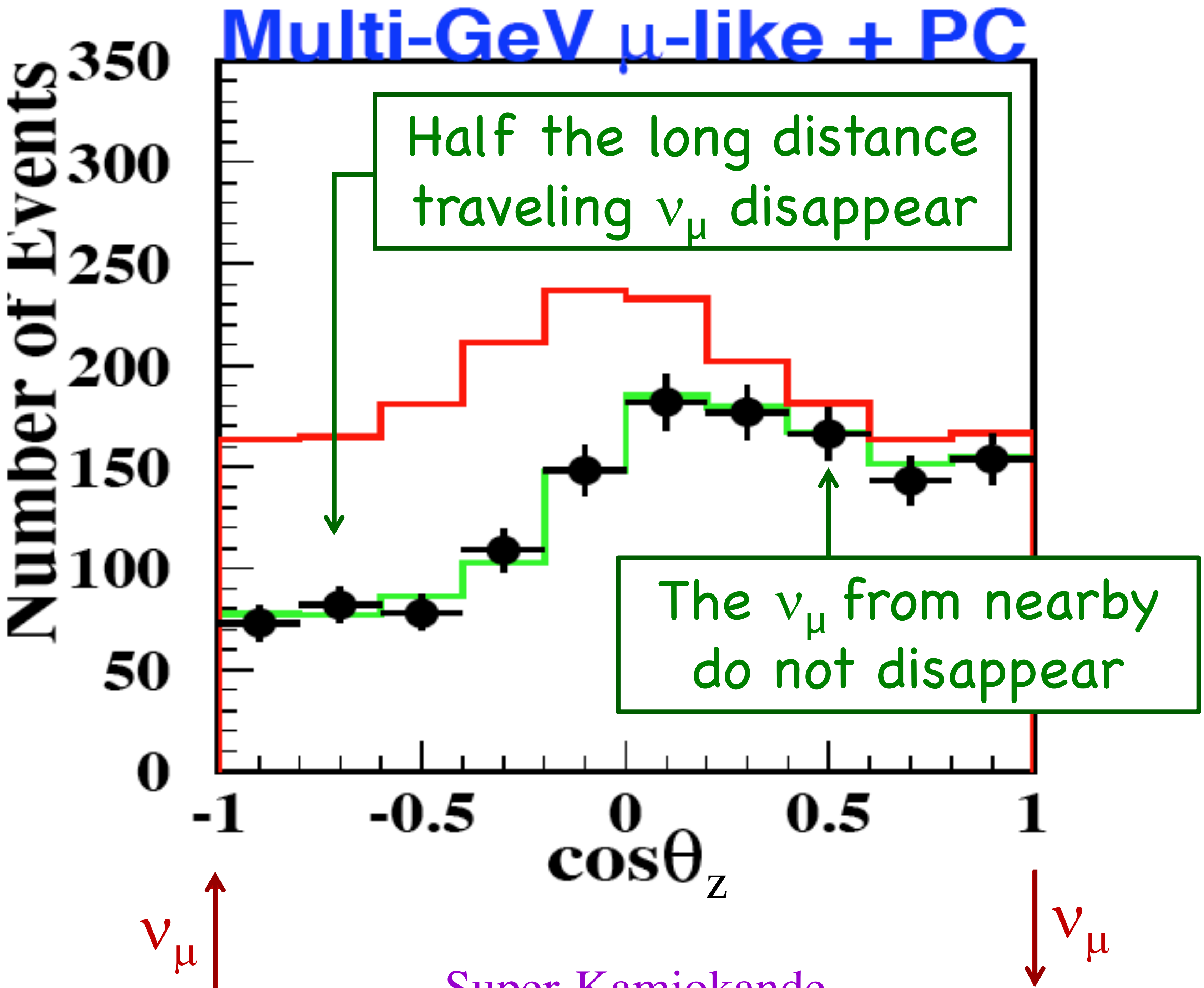
The Disappearance of Atmospheric ν_μ



Isotropy of the $\gtrsim$ 2 GeV cosmic rays + Gauss' Law + No ν_μ disappearance

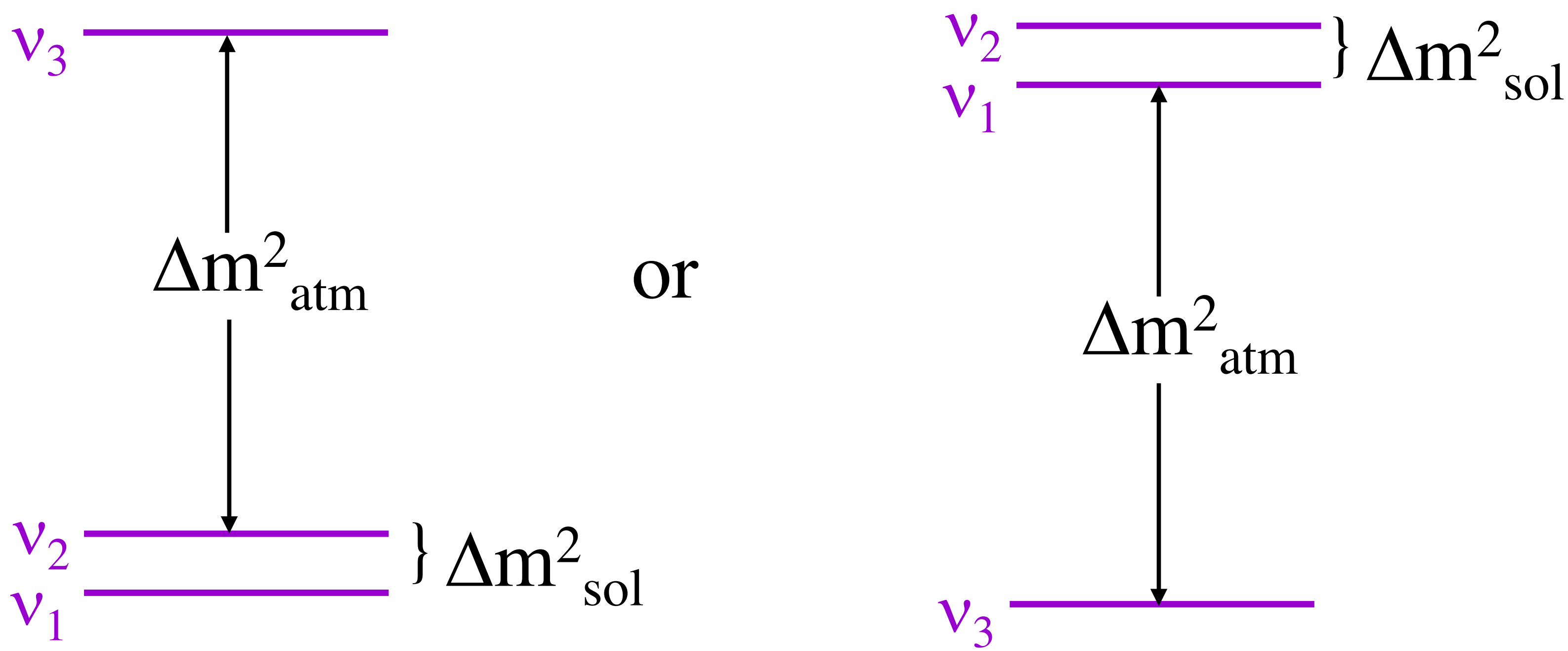
$$\Rightarrow \quad \frac{\phi_{\nu_\mu}(\text{Up})}{\phi_{\nu_\mu}(\text{Down})} = 1 \; .$$

But Super-Kamiokande finds for $E_\nu > 1.3$ GeV —



Super-Kamiokande

At $E_\nu > 1.3$ GeV, in —



the solar splitting is largely invisible. Then—

$$\underbrace{P(\nu_\mu \to \nu_\mu)}_{\frac{1}{2}} \cong 1 - \underbrace{4|U_{\mu 3}|^2 \left(1 - |U_{\mu 3}|^2\right)}_{1} \underbrace{\sin^2\left[1.27 \Delta m^2_{\text{atm}} \frac{L(\text{km})}{E(\text{GeV})}\right]}_{\frac{1}{2}}$$

$$1 \Rightarrow |U_{\mu 3}|^2 = \frac{1}{2}$$

At large L/E

Reactor – Neutrino Experiments and $|U_{e3}|^2 = \sin^2\theta_{13}$

Reactor $\overline{\nu}_e$ have $E \sim 3$ MeV, so if $L \sim 1.5$ km,

$$\sin^2\left[1.27\Delta m^2\left(\text{eV}^2\right)\frac{L(\text{km})}{E(\text{GeV})}\right]$$ will be sensitive to —

$$\Delta m^2 = \Delta m^2_{\text{atm}} = 2.5 \text{ x } 10^{-3}\,\text{eV}^2 = \frac{1}{400}\text{eV}^2$$

but not to —

$$\Delta m^2 = \Delta m^2_{\text{sol}} = 7.5 \text{ x } 10^{-5}\,\text{eV}^2 \approx \frac{1}{13{,}000}\text{eV}^2 \ .$$

Then —

$$P\left(\bar{\nu}_e \to \bar{\nu}_e\right) \cong 1 - 4\left|U_{e3}\right|^2\left(1 - \left|U_{e3}\right|^2\right)\sin^2\left[1.27\Delta m^2_{\text{atm}}\frac{L(\text{km})}{E(\text{GeV})}\right]$$

Measurements by the Daya Bay, RENO,
and Double CHOOZ reactor neutrino experiments,
(and by the T2K accelerator neutrino experiment)

⟹ $|U_{e3}|^2 \cong 0.02$

(Lectures by Yifang Wang)

The Change of Flavor of Solar ν_e

Nuclear reactions in the core of the sun produce ν_e. Only ν_e.

The Sudbury Neutrino Observatory (SNO) measured, for the high-energy part of the solar neutrino flux:

$$\nu_{sol}\, d \to e\, p\, p \;\Rightarrow \phi_{\nu_e}$$

$$\nu_{sol}\, d \to \nu\, n\, p \;\Rightarrow \phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau} \qquad (\nu \text{ remains a } \nu)$$

From the two reactions,

$$\frac{\phi_{\nu_e}}{\phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau}} = 0.301 \pm 0.033$$

For solar neutrinos, $P(\nu_e \to \nu_e) = 0.3$

The Significance of P($\nu_e \rightarrow \nu_e$)

For SNO-energy-range solar neutrinos,
there is a very pronounced solar matter effect.

(Lectures by Alexei Smirnov)

At these energies —

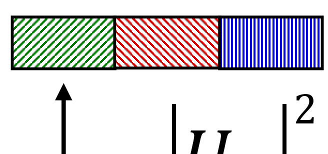
A solar neutrino is born in the core of the sun as a $\boldsymbol{\nu_e}$.

But by the time it emerges from the outer edge of the sun, with 91% probability it is a $\boldsymbol{\nu_2}$.

(Nunokawa, Parke, Zukanovich-Funchal)

Then $P\left(\nu_e \rightarrow \nu_e\right)$ at earth $= \left|\left\langle \nu_e \middle| \nu_2 \right\rangle\right|^2 = \left|U_{e2}\right|^2$.

ν_2



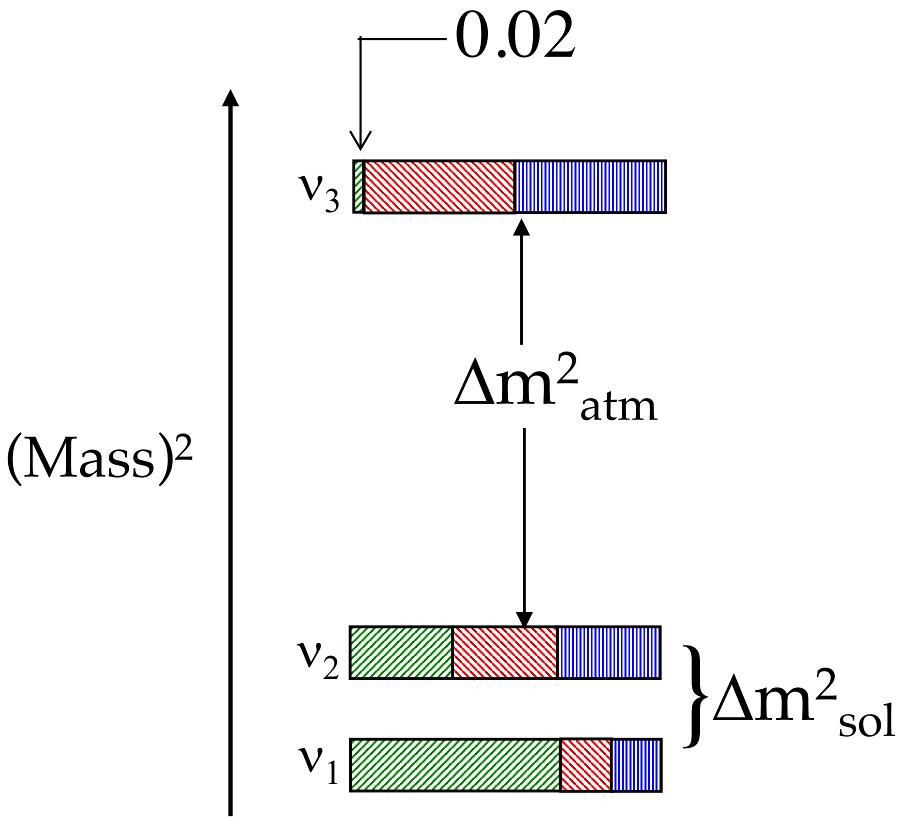
$\left|U_{e2}\right|^2 = 0.3.$

Constructing the Approximate Mixing Matrix
(A Blackboard Exercise)

The result —

$$U \approx \begin{array}{c} \\ \\ \end{array} \begin{matrix} \nu_1 & \nu_2 & \nu_3 & \\ \end{matrix}$$

$$U \approx \begin{bmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{bmatrix} \begin{matrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{matrix}$$



$$
\begin{array}{c} \\ U \approx \end{array}
\begin{array}{c}
\begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\
\begin{bmatrix}
\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\
-\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\
\sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}}
\end{bmatrix}
\end{array}
\begin{array}{c} \\ \nu_e \\ \nu_\mu \\ \nu_\tau \end{array}
$$

The Lepton Mixing Matrix *U*

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \times \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \times \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\times \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Majorana phases

$c_{ij} \equiv \cos\theta_{ij}$

$s_{ij} \equiv \sin\theta_{ij}$

Note big mixing!

$\theta_{12} \approx 34°$, $\theta_{23} \approx 38\text{-}53°$, $\theta_{13} \approx 8.4°$ ← *Not very small!*

The phases violate CP. δ would lead to $P(\overline{\nu}_\alpha \rightarrow \overline{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$.

↑ ~~CP~~

But note the crucial role of $s_{13} \equiv \sin\theta_{13}$.

There is already a hint of ~~CP~~ ($\sin\delta \neq 0$).

The Majorana ~~CP~~ Phases

The phase α_i is associated with neutrino mass eigenstate ν_i:

$U_{\alpha i} = U^0{}_{\alpha i} \exp(i\alpha_i/2)$ for all flavors α.

$$\mathrm{Amp}(\nu_\alpha \to \nu_\beta) = \sum_i U_{\alpha i}^* \exp(-\,i m_i^2 L/2E)\, U_{\beta i}$$

is insensitive to the Majorana phases α_i .

Only the phase δ can cause CP violation in neutrino oscillation.

There Is Nothing Special About θ_{13}

All mixing angles must be nonzero for $\not{CP}$ in oscillation.

For example —

$$P(\bar{\nu}_\mu \to \bar{\nu}_e) - P(\nu_\mu \to \nu_e) = 2\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin\delta$$
$$\times \sin\left(\Delta m^2{}_{31}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{32}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{21}\frac{L}{4E}\right)$$

In the factored form of U, one can put δ next to θ_{12} instead of θ_{13}.


Looking to the Future
Open
Questions

•Is the physics behind the masses of neutrinos different from that behind the masses of all other known particles?

•Are neutrinos their own antiparticles?

•What is the absolute scale of neutrino mass?

•Is the spectrum like  or  ?

• Is θ_{23} maximal?

• Do neutrino interactions violate CP?

Is $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$?

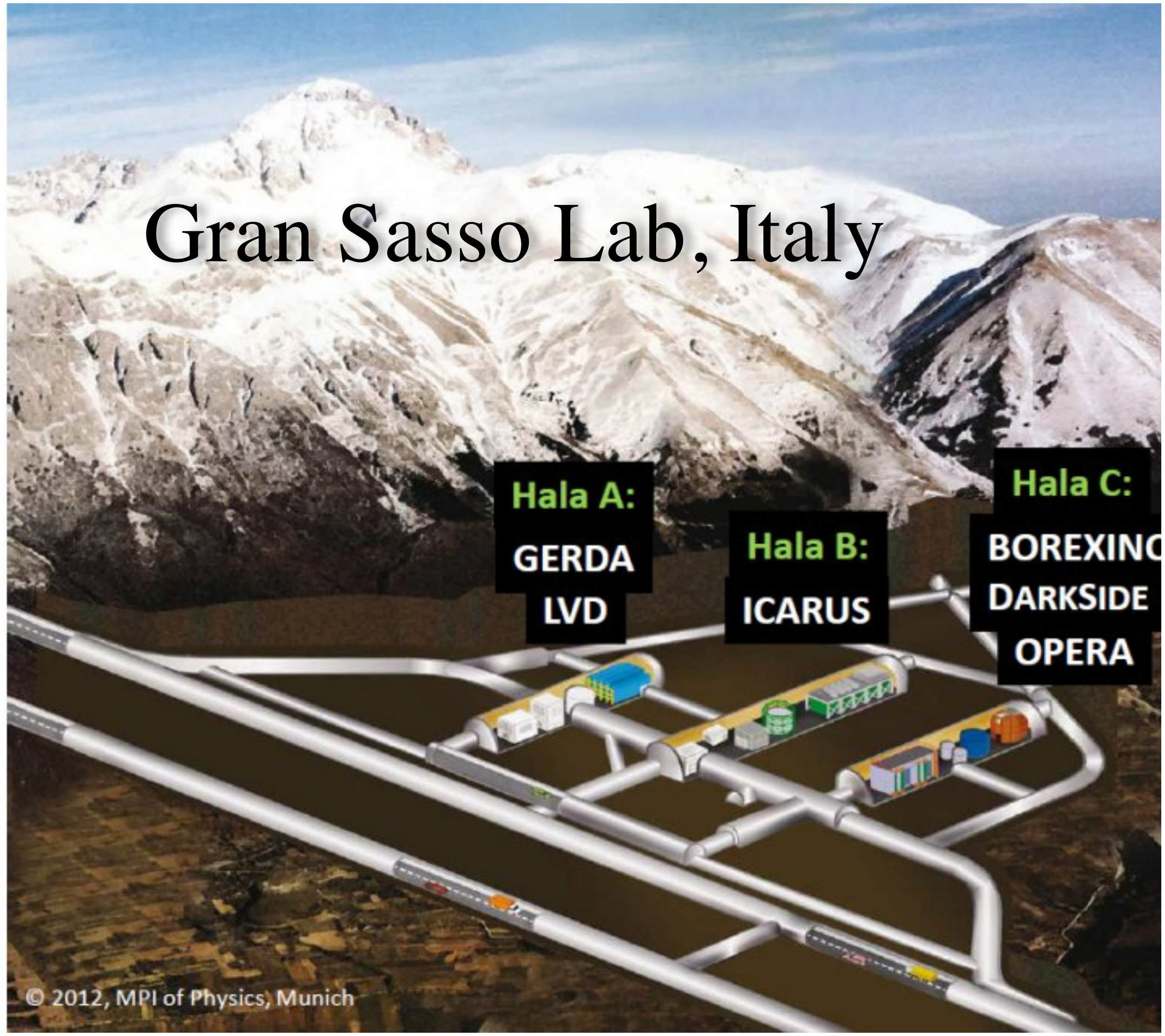
• Is CP violation involving neutrinos the key to understanding the matter – antimatter asymmetry of the universe?

- What can neutrinos and the universe tell us about one another?

- Are there ***more*** than 3 mass eigenstates?
  - Are there "sterile" neutrinos that don't couple to the W or Z?

- Do neutrinos have Non-Standard-Model interactions?

- Do neutrinos break the rules?
  - Violation of Lorentz invariance?
  - Violation of CPT invariance?
  - Departures from quantum mechanics?



Is the Origin of Neutrino Mass Different?

Neutrino Masses Without Field Theory

We will describe what the quantum field theory does, but without equations.

For simplicity, let us treat a world with just one flavor, and correspondingly, just one neutrino mass eigenstate.

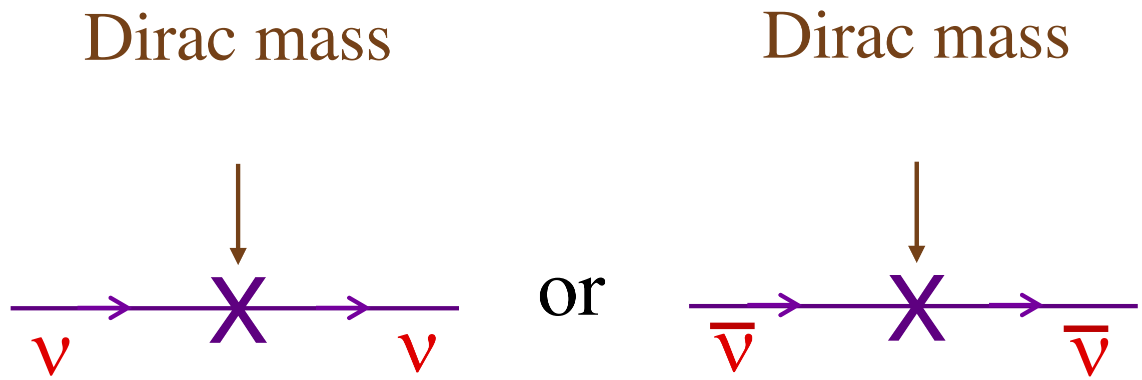
We start with underlying neutrino states ν and $\bar{\nu}$ that are distinct from each other, like other familiar fermions, and are not the mass eigenstates.

We will have to see what the mass eigenstates are later.

We can have two types of masses:

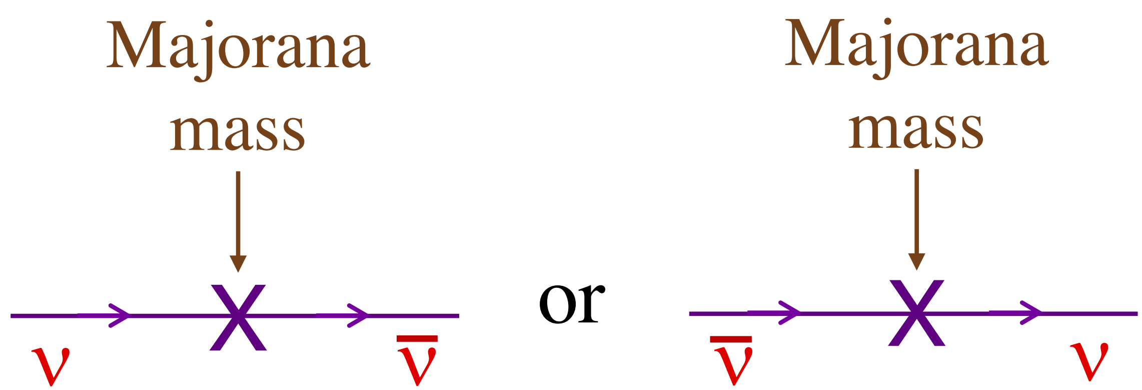
Dirac Mass

A Dirac mass
has the effect:



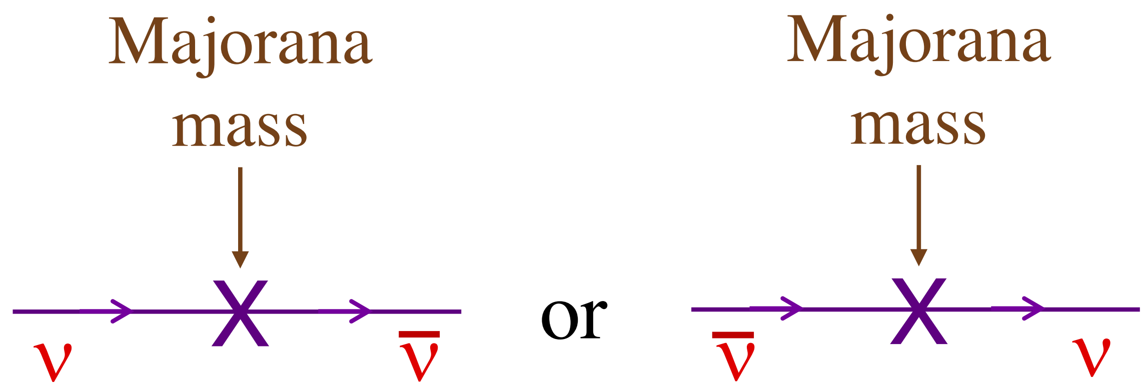
Majorana Mass

A Majorana mass
has the effect:



Majorana Mass

A Majorana mass has the effect:



Majorana masses mix ν and ν̄, so they do not conserve the Lepton Number L that distinguishes leptons from antileptons:

$$L(\nu) = L(\ell^-) = -L(\bar{\nu}) = -L(\ell^+) = 1$$

If there are no visibly large non-SM interactions that violate lepton number L, any violation of L that we might discover would have to come from Majorana neutrino masses.

A Majorana mass for any fermion f causes $f \longleftrightarrow \bar{f}$.

Quark and ***charged-lepton*** Majorana masses are **forbidden** by electric charge conservation.

But ***neutrinos*** are electrically neutral, so they **can** have Majorana masses.

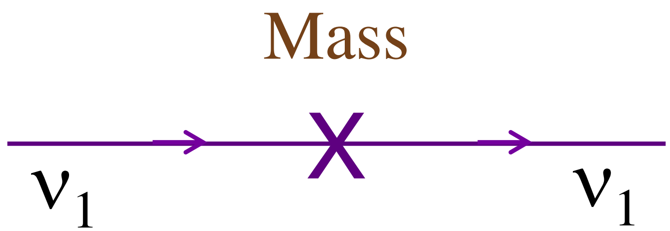
Neutrino Majorana masses would make the neutrinos *very* distinctive, because —

Majorana neutrino masses have a different origin than the quark and charged-lepton masses.

(Lectures by Steve King)

The Mass Eigenstates When There Are Majorana Masses

For any fermion mass eigenstate, e.g. ν_1, the action of its mass must be —

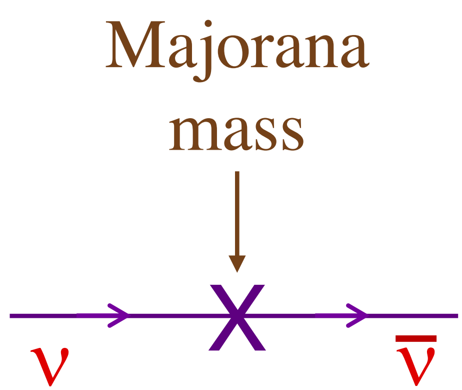


The mass eigenstate must be sent back into itself:

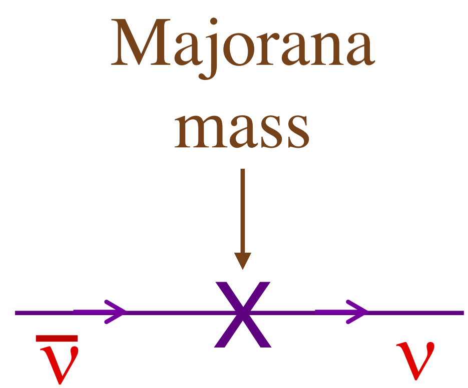
$$H|\nu_1\rangle = m_1|\nu_1\rangle$$

Recall that —

A ***Majorana*** mass has the effect:



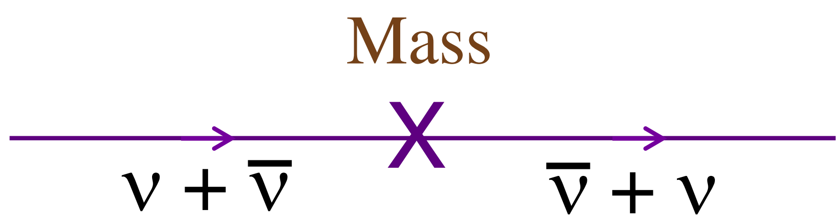
or



Then the mass eigenstate neutrino ν_1 must be —

$$\nu_1 = \nu + \bar{\nu} ,$$

since this is the neutrino that the Majorana mass term sends back into itself, as required for any mass eigenstate particle:



Consequence: The neutrino mass eigenstates ν_1, ν_2, ν_3 are their own antiparticles.

$\bar{\nu}_i = \nu_i$ For given helicity

The Terminology

Suppose ν_i is a ***mass eigenstate,***

with given helicty h.

- $\overline{\nu_i}(h) = \nu_i(h)$ *Majorana neutrino*

or

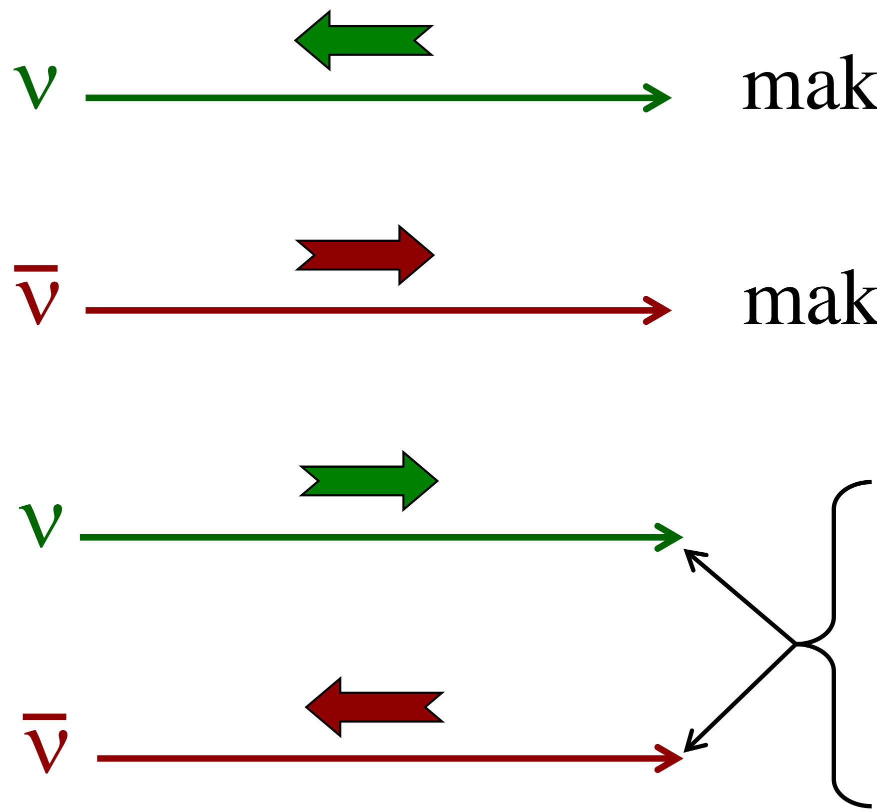
- $\overline{\nu_i}(h) \neq \nu_i(h)$ *Dirac neutrino*

We have just shown that if the underlying neutrino masses are *Majorana masses*, then the mass eigenstates are *Majorana neutrinos*.

The Interactions of Dirac, and Especially Majorana, Neutrinos

SM Interactions Of A Dirac Neutrino

We have 4 mass-degenerate states:

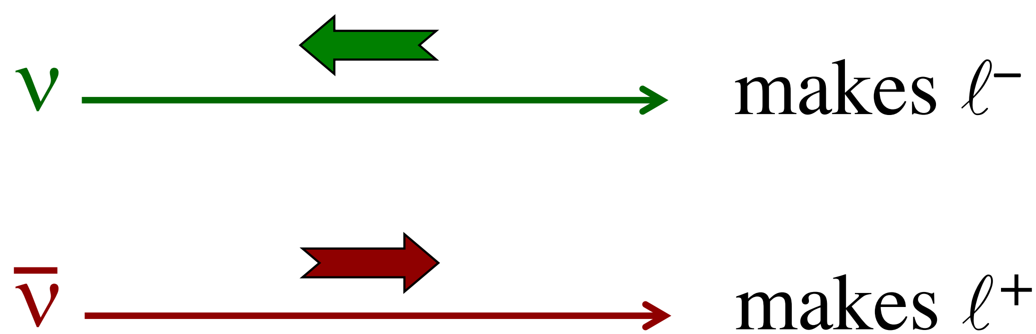


		Conserved L
ν	makes ℓ^-	+1
$\bar{\nu}$	makes ℓ^+	–1

These states, *when Ultra Rel*., do not interact.
(The weak interaction is Left Handed.)

SM Interactions Of A Majorana Neutrino

We have only 2 mass-degenerate states:



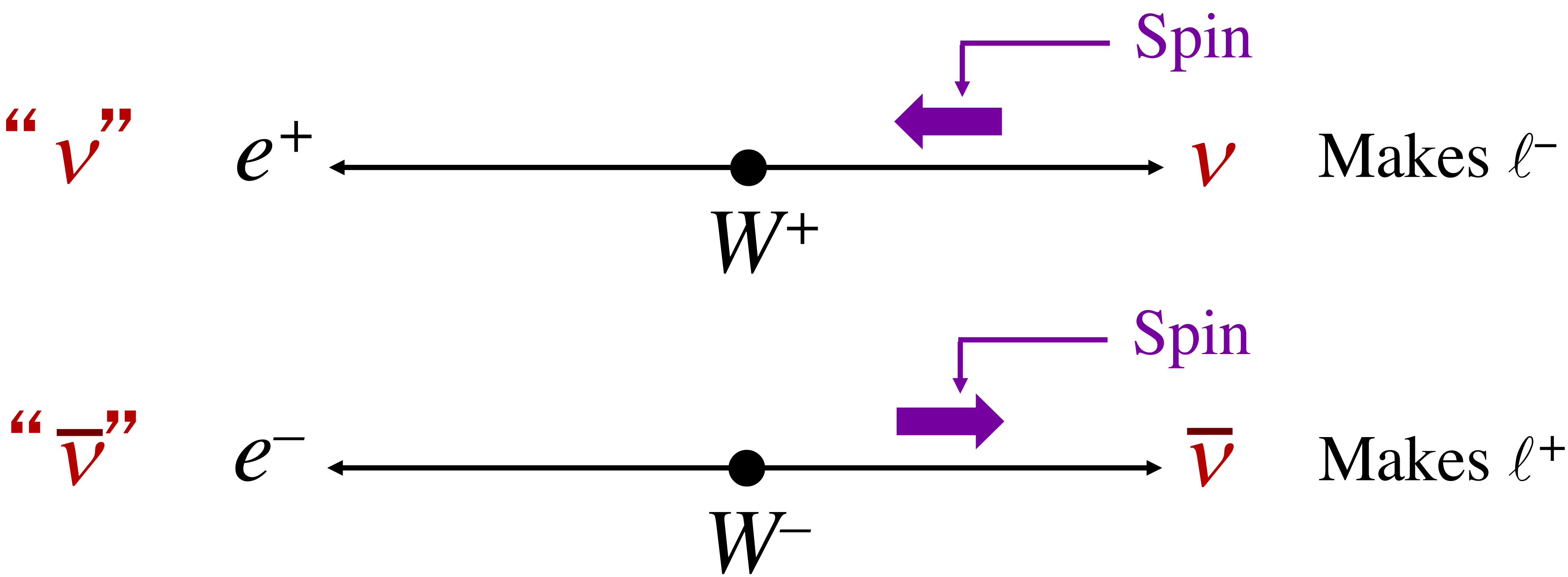
The SM weak interactions violate *parity*.

(They can tell *Left* from *Right*.)

An incoming left-handed neutral lepton makes ℓ^-.

An incoming right-handed neutral lepton makes ℓ^+.

Note: " ν " and " $\bar{\nu}$ " are ***produced*** with opposite helicity.



The weak interactions violate ***Parity***. ***Particles with left-handed and right-handed helicity can behave differently.***

For ***ultra-relativistic Majorana*** neutrinos,
helicity is a "substitute" for lepton number.

Majorana neutrinos behave
indistiguishably from Dirac neutrinos.

However, for ***non-relativistic*** neutrinos,
there can be a big difference between the behavior
of Majorana neutrinos and Dirac neutrinos.

To Determine Whether Majorana Masses Occur in Nature, So That $\bar{\nu} = \nu$

The Promising Approach — Seek

Neutrinoless Double Beta Decay [$0\nu\beta\beta$]

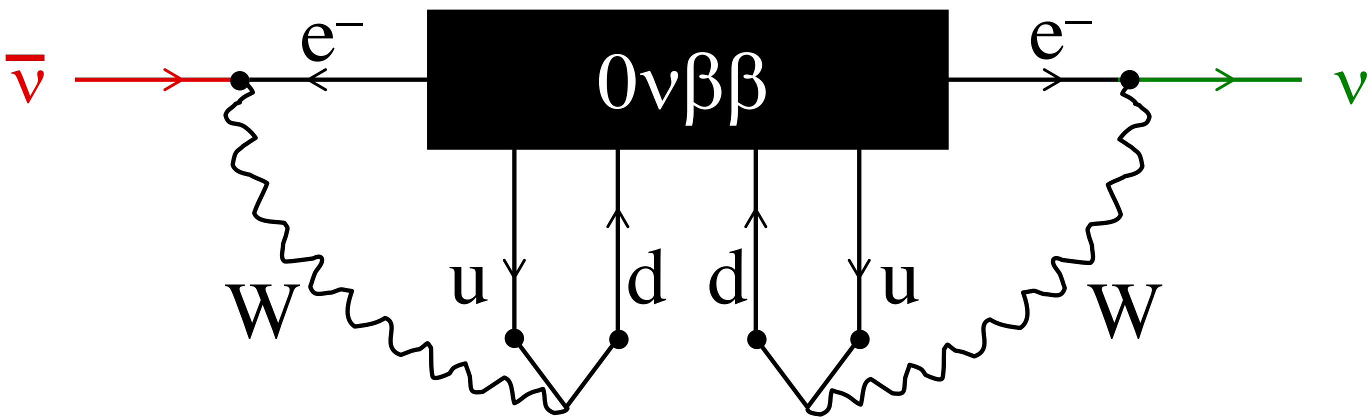
(Lectures by Alexander Barabash and Petr Vogel)

Observation at any non-zero level would imply —

- ➢Lepton number L is not conserved ($\Delta L = 2$)
- ➢Neutrinos have Majorana masses
- ➢Neutrinos are Majorana particles (self-conjugate)

Whatever diagrams cause 0νββ, its observation would imply the existence of a Majorana mass term:

(Schechter and Valle)



$\bar{\nu} \rightarrow \nu$: A (tiny) Majorana mass term

$\therefore$ 0νββ ⟹ $\bar{\nu}_i = \nu_i$

The Search for CP Violation When It Might Be That

$$\bar{\nu} = \nu$$

A major open question is whether neutrino interactions violate CP invariance.

The experimental approach to testing for this violation is almost always described as the attempt to see whether —

$$P(\bar{\nu}_\mu \to \bar{\nu}_e) \neq P(\nu_\mu \to \nu_e).$$

This description is valid if $\bar{\nu} \neq \nu$, but not if $\bar{\nu} = \nu$.

However, the present and future experimental probes of leptonic CP-invariance violation are valid probes of this violation whether $\bar{\nu} \neq \nu$ *or* $\bar{\nu} = \nu$.

These experiments are ***completely insensitive*** to whether $\bar{\nu} \neq \nu$ or $\bar{\nu} = \nu$.

For any process $i \rightarrow f$, and its CP-mirror image $\mathrm{CP}\left(i\right) \rightarrow \mathrm{CP}\left(f\right)$, CP invariance means that —

$$\left|\left\langle f\middle|T\middle|i\right\rangle\right|^2 = \left|\left\langle \mathrm{CP}\left(f\right)\middle|T\middle|\mathrm{CP}\left(i\right)\right\rangle\right|^2.$$

So, compare two CP-mirror-image processes.

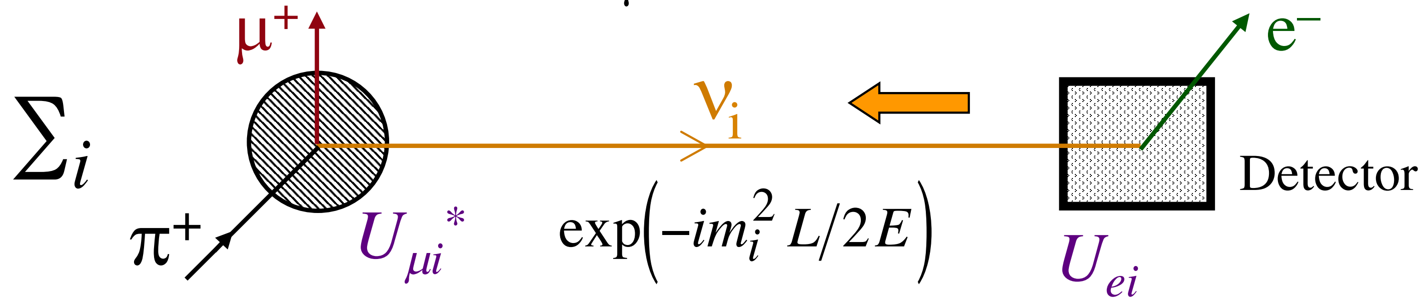
If they have different rates, CP invariance is violated.

Acting on a particle ψ with momentum $\vec{p}$ and helicity h,

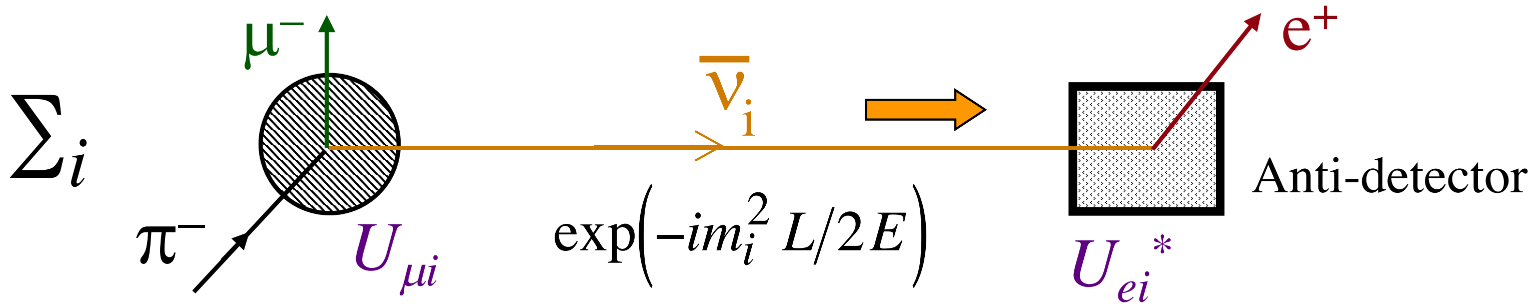
$$\mathrm{Rotation}\left(\pi\right)\mathrm{CP}\left|\psi\left(\vec{p},h\right)\right\rangle = \eta\left|\bar{\psi}\left(\vec{p},-h\right)\right\rangle$$

Irrelevant phase factor

Compare $\nu_\mu \to \nu_e$



with "$\bar{\nu}_\mu \to \bar{\nu}_e$"



If these two CP-mirror-image processes have different rates, CP invariance is violated.

Important Notice

To correct for our not using an anti-detector, we must know how the cross sections for left-handed and right-handed neutrinos to interact in a detector compare.

Experiments to determine these cross sections are very important.

(Lectures by Jan Sobczyk)

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K. Nakamura and S. Petcov, in C. Patrignani et al. (PDG), Chin. Phys. C40, 100001 (2016). Also at http://pdg.lbl.gov/2017/reviews/rpp2016-rev-neutrino-mixing.pdf

"Neutrino Mass, Mixing, and Flavor Change," B. K., in **Neutrino Mass**, eds. G. Altarelli and K. Winter (Springer, Berlin/Heidelberg, 2003). Also eprint hep-ph/0211134. This paper discusses quite a few of the topics covered in the lectures.

"Neutrino Oscillation Physics," B. K., in **Proceedings of the International School on AstroParticle Physics**, eds. G. Bellini and L. Ludhova (IOS Press, Amsterdam, 2012), and in **Proceedings of the 2011 European School of High-Energy Physics**, eds. C. Grojean and M. Mulders (CERN, Geneva, 2014). Also arXiv:1206.4325. This paper derives the probability for neutrino oscillation without assuming that all neutrino mass eigenstates in a beam have the same energy, or else the same momentum.

"Light Sterile Neutrinos: A White Paper," K. Abazajian et al., arXiv:1204.5379.

Good luck!