Introduction to neutrino

S. Bilenky
JINR(Dubna)

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The award of the 2015 Nobel Prize to T. Kajita and A. McDonald "for the discovery of neutrino oscillations, which shows that neutrinos have mass" was a result of more than fifty years of efforts of many experimentalists and theoreticians

First idea of neutrino oscillations was pioneered in 1957-58 by B. Pontecorvo

First idea of neutrino mixing was discussed by Maki, Nakagawa and Sakata in 1962

First model independent evidence in favor of disappearance of atmospheric ν_μ's was obtained in 1998 by the Super-Kamiokande collaboration

First model independent evidence of the disappearance of solar ν_e's was obtained by the SNO collaboration in 2001

First model independent evidence of the disappearance of reactor $\bar{\nu}_e$'s was obtained by the KamLAND collaboration in 2002

The discovery of neutrino oscillations was confirmed by many experiments: accelerator K2K, MINOS, T2K and NOvA, reactor DayaBay, RENO and Double Chooz, atmospheric IceCube

I will discuss

- The Standard Model and neutrino
- Origin of neutrino masses (a plausible mechanism)
- Status of neutrino oscillations
- Neutrinoless double β-decay

After the discovery of the Higgs boson at LHC the Standard Model acquired the status of the theory of elementary particles in the electroweak energy range (up to ~ 300 GeV)

What is the role of neutrinos in the SM? What are neutrino masses in the SM? Neutrino interaction?

What general conclusions we can infer from the SM?

The Standard Model apparently started with the theory of the two-component neutrino

A bit of history. In 1928 for a relativistic spin 1/2 particle Dirac proposed his famous equation

$$(i\gamma^{\alpha}\partial_{\alpha} - m)\,\psi(x) = 0$$

From Lorenz invariance, linearity in $\frac{\partial}{\partial x^0}$ etc it followed that $\psi(x)$ must be four-component spinor. We know now that $\psi(x)$ is the field of particles and antiparticles

In 1929 H. Weil put the following question: can we have for a relativistic spin 1/2 particle a two-component equation? (like Pauli equation for a non relativistic particle)

Weil introduced left-handed and right-handed two-component spinors $\psi_{L,R} = \frac{1}{2}(1 \mp \gamma_5)\psi$

From Dirac equation we have two coupled equations

$$i\gamma^\alpha \partial_\alpha \psi_L(x) - m\psi_R(x) = 0, \quad i\gamma^\alpha \partial_\alpha \psi_R(x) - m\psi_L(x) = 0$$

Thus, if $m \neq 0$ we need $\psi_L(x)$ and $\psi_R(x)$. But if $m = 0$ we obtain two-component Weil equations

$$i\gamma^\alpha \partial_\alpha \psi_L(x) = 0, \quad \text{and} \quad i\gamma^\alpha \partial_\alpha \psi_R(x) = 0$$

These equations are not invariant, however, under the inversion

In fact, under the inversion

$$\psi'_R(x') = \eta\gamma^0 \psi_L(x), \quad \psi'_L(x') = \eta\gamma^0 \psi_R(x), \quad x' = (x^0, -\vec{x})$$

By this reason during many years Weil equations were forgotten

Pauli in the book on Quantum Mechanics "...because the equation for $\psi_L(x)$ ($\psi_R(x)$) is not invariant under space reflection it is not applicable to the physical reality".

H. Weil: "In my work I always tried to unite the truth with the beautiful, but when I had to choose one or the other, I usually choose the beautiful."

In 1958 soon after discovery of the parity violation in the β-decay Lee and Yang applied the Weil theory to neutrino. The two-component neutrino theory was proposed at the same time by Landau (CP invariance) and Salam (γ_5 invariance)

According to this theory helicity of neutrino (antineutrino) is equal to -1 (+1) in the case of $\nu_L(x)$ and, correspondingly, +1 (-1) in the case of $\nu_R(x)$

The neutrino helicity was measured in the spectacular Goldhaber et al experiment (1958). Authors concluded "... our result is compatible with 100% negative helicity of neutrino emitted in orbital electron capture".

The two-component Weyl field is the most economical possibility for a massless particle (2 dof instead of 4 for the Dirac field)

Success of the two-component neutrino theory signifies that nature chooses simplicity and economy by the prise of the non conservation of P (and C) in weak interaction

Symmetry is a manifestation of simplicity

Other fundamental fermions lepton and quarks like neutrinos at the stage of symmetry have to be also massless, two-component

The simplest symmetry is $SU(2)_L$ with left-handed doublets

$$\psi^{lep}_{eL} = \begin{pmatrix} \nu'_{eL} \\ e'_L \end{pmatrix},\ \psi^{lep}_{\mu L} = \begin{pmatrix} \nu'_{\mu L} \\ \mu'_L \end{pmatrix},\ \psi^{lep}_{\tau L} = \begin{pmatrix} \nu'_{\tau L} \\ \tau'_L \end{pmatrix}, \dots$$

Invariance under transformation $(\psi^{lep}_{lL}(x))' = e^{i\frac{1}{2}\ \vec{\tau}\cdot\vec{\Lambda}}\ \psi^{lep}_{lL}(x)$ means invariance under arbitrary rotation of the quantization axis

In QFT more natural to require that rotations at different $\vec{x}$ are different (local Yang-Mills invariance) $\vec{\Lambda}(x)$

The local $SU(2)_L$ can be insured if

$$\partial_\alpha \psi^{lep}_{lL}(x) \to (\partial_\alpha + ig\tfrac{1}{2}\ \vec{\tau}\cdot\vec{A}_\alpha(x))\ \psi^{lep}_{lL}(x)$$

Minimal interaction: $\mathcal{L}_I(x) = -g\ \vec{j}_\alpha \vec{A}^\alpha,\ \ \vec{j}_\alpha = \sum_l \bar{\psi}^{lep}_{lL}\gamma_\alpha \frac{1}{2}\vec{\tau}\psi^{lep}_{lL}$

The same interaction constant g for different doublets ($e-\mu-\tau$ universality).

This is connected with the fact that local $SU(2)$ group is a non abelian group

Local gauge symmetry is a very powerful symmetry

- existence of gauge vector bosons is predicted ($W^{\pm}$ and Z^0 were predicted by SM)
- minimal interaction is predicted (from agreement of SM with experiment we can conclude that nature chooses minimal interactions)

The Standard Model is the unified theory of the weak and electromagnetic interactions

Electromagnetic current of charged leptons is the sum of a left-handed and right-handed terms

$$j_{\alpha}^{\mathrm{EM}} = \sum_l (-1)\, \bar{l}' \gamma_\alpha l' = \sum_l (-1)\, \bar{l}'_L \gamma_\alpha l'_L + \sum_l (-1)\, \bar{l}'_R \gamma_\alpha l'_R$$

The $SU(2)_L$ symmetry group must be enlarged. A new symmetry group must include transformations of L and R components of charged fields

The simplest enlargement is $SU(2)_L \times U(1)_Y$ group $U(1)_Y$ is the hypercharge group

The invariance can be provided if we change

$$\partial_\alpha \psi_{lL}^{lep}(x) \to \left(\partial_\alpha + ig\,\tfrac{1}{2}\;\vec{\tau}\cdot\vec{A}_\alpha(x) + ig'\,\tfrac{1}{2}\,Y_L B_\alpha(x)\right)\,\psi_{lL}^{lep}(x)$$

$$\partial_\alpha l'_R(x) \to \left(\partial_\alpha + ig'\,\tfrac{1}{2}\,Y_R B_\alpha(x)\right)\,l'_R(x)$$

Hypercharges can be chosen in such a way that $Q = T_3 + \frac{1}{2}Y$ (the Gell-Mann-Nishijima relation)

We come to the Lagrangian of minimal CC, NC and EM (electroweak) interaction

$$\mathcal{L}_I = \left(-\frac{g}{2\sqrt{2}}\,j_\alpha^{CC}\,W^\alpha + \mathrm{h.c}\right) - \frac{g}{2\cos\theta_W}\,j_\alpha^{\mathrm{NC}}\,Z^\alpha - e\;j_\alpha^{EM}\,A^\alpha$$

$$j_\alpha^{\mathrm{CC}} = 2\;\sum_{l=e,\mu,\tau}\bar{\nu}_{lL}\gamma_\alpha l_L$$

$$j_\alpha^{\mathrm{NC}} = 2\;j_\alpha^3 - 2\;\sin^2\theta_W\,j_\alpha^{\mathrm{EM}}$$

Unification of weak and electromagnetic interactions allowed to predict

- Neutral current interaction, a new weak interaction. It was discovered after the SM was proposed
- the unification constraint $g\sin\theta_W = e$, $\frac{g'}{g} = \tan\theta_W$

Neutrinos are the only neutral fermions. There is no electromagnetic current of neutrinos. The weak-electromagnetic unification does not require right-handed neutrino fields.

A minimal, most economical possibility: there is no right-handed neutrino fields in the SM

The SM mechanism of the mass generation is the Brout-Englert-Higgs mechanism.

To provide masses of $W^{\pm}$ and Z^0 we need three dof.

Minimal assumption: Higgs field is a doublet (four dof)

$$\phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix}$$

After spontaneous symmetry breaking

$$\phi(x) = \begin{pmatrix} 0 \\ \frac{v+H(x)}{\sqrt{2}} \end{pmatrix}$$

v is vev of Higgs field. Characterizes scale of the symmetry breaking . Dimension M

One neutral, scalar boson (Higgs boson) is predicted. Corresponds to the LHC finding

From the minimal assumption (one Higgs doublet) for the masses of $W^{\pm}$ and Z^0 we have

$$m_W^2 = \frac{1}{4} g^2 \ v^2, \quad m_Z^2 = \frac{1}{4}(g^2 + g'^2) \ v^2$$

Two additional relations

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 \ m_W^2} \ (\mathrm{CC}), \quad g \ \sin\theta_W = e \ (\mathrm{unification})$$

From these relations fundamental quantities v, m_W, m_Z can be predicted

$$\frac{G_F}{\sqrt{2}} = \frac{1}{2 \ v^2}, \quad v = (\sqrt{2} \ G_F)^{-1/2} \simeq 246 \ \mathrm{GeV}$$

$$m_W^2 = \frac{\pi\alpha}{\sqrt{2} G_F \sin^2\theta_W}, \ m_Z^2 = \frac{m_W^2}{\cos^2\theta_W} = \frac{\pi\alpha}{\sqrt{2} G_F \sin^2\theta_W \cos^2\theta_W}$$

Perfect agreement with experiment (radiative corrections must be included)

Dirac mass terms of leptons and quarks are generated by the $SU(2)_L \times U(1)_Y$ invariant Yukawa interactions

$$\mathcal{L}_Y^{lep} = -\sqrt{2} \sum_{l_1, l_2} \bar{\psi}_{l_1 L}^{lep} Y_{l_1 l_2} l'_{2R} \ \phi + \mathrm{h.c}$$

After spontaneous breaking of the symmetry and standard diagonalization

$$\mathcal{L}_Y^{lep} = -\sum_l \bar{l} y_l l \ (v + \frac{1}{v} H) = -\sum_l m_l \bar{l} \ l - \sum_l y_l \bar{l} \ lH$$

Lepton mass $m_l = y_l \ v \quad l = e, \mu, \tau$

The second term is the Lagrangian of the interaction of leptons and Higgs boson

Interaction constant $y_l = \frac{m_l}{v}$ is predicted by the SM. Confirmed by the LHC data

Yukawa constants of leptons (and quarks) are parameters

For particles of the third family

$$y_t \simeq 7 \cdot 10^{-1}, \quad y_b \simeq 2 \cdot 10^{-2}, \quad y_\tau \simeq 7 \cdot 10^{-3}$$

Remark. Assume that neutrino masses are generated by the standard Higgs mechanism.

$$y_3 = \frac{m_3}{v} \simeq \frac{\sqrt{\Delta m_A^2}}{v} \simeq 2 \cdot 10^{-13}$$

Neutrino masses and quark and lepton masses can not be of the same (SM) origin

The most natural, plausible and economic assumption: neutrinos after spontaneous symmetry breaking remain two-component, massless Weyl particles. If correct, neutrino masses and mixing are due to a beyond the SM physics

What is the most economical, simplest possibility?

Neutrino mass term is a source of neutrino masses and mixing. A mass term is a sum of Lorenz-invariant products of left-handed and right-handed components. Can we build neutrino mass term if we have only flavor left-handed fields ν_{lL} $(l=e,\mu,\tau)$?

The answer to this question was given by Gribov and Pontecorvo in 1969 in the case of two neutrinos and later generalized by Pontecorvo and SB. Important that $(\nu_{lL})^c = C(\bar\nu_{lL})^T$ is a right-handed component $(C\gamma_\alpha^T C^{-1} = -\gamma_\alpha, \quad C^T = -C)$

If we assume that the total lepton number L is violated we can built the following Majorana mass term

$$\mathcal{L}^M = -\frac{1}{2}\sum_{l_1,l_2}\bar\nu_{l_1L}\ M^M_{l_1l_2}\ (\nu_{l_2L})^c + \mathrm{h.c.}$$

M^M is a general symmetric matrix

After diagonalization the mass term takes the standard form

$$\mathcal{L}^M = -\frac{1}{2}\sum_{i=1}^{3} m_i\ \bar{\nu}_i \nu_i$$

$\nu_i = \nu_i^c = C\ (\bar{\nu}_i)^T$ is the Majorana field with mass m_i $(\nu \equiv \bar{\nu})$

Neutrino mixing

$$\nu_{lL} = \sum_{i=1}^{3} U_{li}\nu_{iL}$$

This is the most economical possibility: in the neutrino interaction and in the mass term only flavor fields ν_{lL} enter

The number of flavor fields (three) is equal to the number of massive neutrinos

The economy is reached because of non conservation of L.

Neutrino masses are deeply connected with this assumption

However, in this phenomenological approach neutrino masses m_i are parameters. No possibility to explain (understand) smallness of neutrino masses with respect to the SM masses of leptons and quarks.

Modern approach is the effective Lagrangian approach

This is a general method which allows to describe effects of a beyond the Standard Model physics

The effective Lagrangian is a non renormalizable Lagrangian invariant under $SU(2)_L \times U(1)_Y$ transformations and built from the Standard Model fields. In general the effective Lagrangian is a sum of operators of dimension five and more.

Let us consider $SU(2)_L \times U(1)_Y$ invariant

$$(\tilde{\phi}^\dagger\ \psi_{lL}^{lep}) \quad (l = e, \mu, \tau)$$

$$\psi_{lL}^{lep} = \left(\begin{array}{c} \nu'_{lL} \\ e'_L \end{array} \right), \quad \phi = \left(\begin{array}{c} \phi_+ \\ \phi_0 \end{array} \right)$$

are lepton and Higgs doublets

After spontaneous symmetry breaking

$$(\tilde{\phi}^\dagger\ \psi_{lL}^{lep}) \to \frac{v}{\sqrt{2}} \nu'_{lL}$$

left-handed flavor field $\times$ v, dimension $M^{5/2}$

Effective Lagrangian which generate the neutrino mass term has the form (Weinberg)

$$\mathcal{L}_I^{\rm eff} = -\frac{1}{\Lambda}\ \sum_{l_1,l_2}(\bar{\psi}^{lep}_{l_1 L}\tilde{\phi})\ Y'_{l_1 l_2}\ (\tilde{\phi}^T(\psi^{lep}_{l_2 L})^c) + \mathrm{h.c.}$$

Parameter Λ has a dimension M. It characterizes a scale of a beyond the SM physics, Y' is 3×3 dimensionless, symmetrical matrix

- It is natural to assume that $\Lambda \gg v$
- The Lagrangian $\mathcal{L}_I^{\rm eff}$ is the only effective Lagrangian which generate the neutrino mass term. It is a dimension five operator.
- The Lagrangian $\mathcal{L}_I^{\rm eff}$ does not conserve the total lepton number L

After spontaneous symmetry breaking we come to the Majorana mass term

$$\mathcal{L}^{\rm M} = -\frac{1}{2}\,\frac{v^2}{\Lambda}\sum_{l_1,l_2}\bar{\nu}_{l_1 L}\ Y_{l_1 l_2}(\nu_{l_2 L})^c + \mathrm{h.c.}$$

After diagonalization the mass term takes the standard form

$$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2}\sum_{i=1}^{3} m_i \bar{\nu}_i \nu_i, \quad \nu_{lL} = \sum_{i=1}^{3} U_{li}\ \nu_{iL}, \quad \nu_i = \nu_i^c$$

Majorana neutrino masses

$$m_i = \frac{v^2}{\Lambda} y_i$$

Small Majorana neutrino masses is a signature of a new L-violating physics

Can we estimate Λ ?

Main uncertainty Yukawa constants y_i. Values of neutrino masses are also unknown. Assuming normal hierarchy $m_1 \ll m_2 \ll m_3$ we have $m_3 \simeq \sqrt{\Delta m_A^2} \simeq 5 \cdot 10^{-2}$ eV

$$\Lambda \simeq 1.2 \cdot 10^{-15}\ y_3\ \text{GeV}$$

Extreme cases

$y_3 \simeq 1$ (like in the case of quarks and leptons)

$\Lambda \simeq 1 \cdot 10^{-15}$ GeV (GUT scale)

Assume $\Lambda \simeq 1\ \mathrm{TeV}$. In this case $y_3 \simeq 10^{-12}$ (to small, fine tuning)

$\Lambda \gg v$ is a plausible possibility

Effective $V-A$, four-fermion Lagrangian which describes low-energy SM processes has the form

$$\mathcal{L}_I = -\frac{G_F}{\sqrt{2}} j^{\alpha}\, j^{\dagger}_{\alpha}, \quad j^{\alpha} = 2(\sum_l \bar{\nu}_{lL}\gamma^{\alpha} l_L + \mathrm{quarks})$$

This Lagrangian is generated by the CC SM Lagrangian

$$\mathcal{L}_I^{CC} = -\frac{g}{2\sqrt{2}} j^{\alpha} W^{\alpha} + \mathrm{h.c.}$$

in the second order in g. The effective Lagrangian is applicable to processes with virtual $W^{\pm}$ at $Q^2 \ll m_W^2$

The local Weinberg effective Lagrangian is generated by the interaction

$$\mathcal{L}_I^{CC} = -\sqrt{2}\sum_{li}(\bar{\psi}_{lL}^{lep}\tilde{\phi}) y_{li} N_{iR} + \mathrm{h.c.}$$

$N_i = N_i^c$ is the field of a heavy Majorana leptons with mass M_i

Taking into account that at $q^2 \ll M_i^2$ the propagator is given by

$$\langle 0|T(N_{iR}(x_1 N_{iR}^T(x_2)|0\rangle = i\frac{1}{M_i}\delta(x_1 - x_2)\frac{1+\gamma_5}{2}C$$

in the second order of the perturbation theory we come to the Weinberg Lagrangian with

$$Y'_{l'l} = \sum_i y_{l'i}\frac{\Lambda}{M_i} y_{l'i}$$

The Weinberg effective Lagrangian is applicable if $M_i \gg v$

Summarizing

- ν_i are Majorana particles. Neutrinoless double β-decay $(A, Z) \to (A, Z+2) + e^- + e^-$ is allowed. Probability is extremely small (second order in G_F and suppression due to small neutrino masses). Many experiments
- The number of ν_i is equal to three. Means no sterile (noninteracting) neutrinos. Exist several indications (LSND, MiniBooNE, reactor, source), in favor of sterile neutrinos. A contradiction between appearance and disappearance data, in the latest experiments (Daya Bay, MINOS, IceCube) no sterile neutrinos were found. Many experimnts are going on
- If heavy Majorana leptons exist their production in early Universe and subsequent CP-violating decays into Higgs-lepton pairs could explain barion asymmetry of the Universe. (bariogenesis through leptogenesis)

Neutrino oscillations

Mixing is a relation between fields

$\nu_{lL}(x) = \sum_i U_{li}\, \nu_{iL}(x), \quad U^\dagger U = 1$

$j^\alpha(x) = 2 \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}(x) \gamma^\alpha l_L(x)$

Neutrino produced together with μ^+ in $\pi^+ \to \mu^+ + \nu_\mu$ is called flavor muon neutrino ν_μ etc

the state of flavor neutrino is a coherent superposition

$|\nu_l\rangle = \sum_i U^*_{li}\, |\nu_i\rangle \quad (l = e, \mu, \tau)$

$|\nu_i\rangle$ is the state with mass m_i, momentum $\vec{p}$, energy $E_i \simeq E + \frac{m_i^2}{2E}$

Can we distinguish production of neutrinos with different masses?

$|\Delta p_{ki}| = |(p_k - p_k)| \simeq \frac{|\Delta m^2_{ki}|}{2E} = \frac{1}{L_{ki}}, \quad \Delta m^2_{ki} = m_i^2 - m_k^2.$

We are interested in $E \gtrsim 1$ MeV (reactors), $E \gtrsim 1$ GeV (atmospheric, accelerator). From neutrino oscillation experiments

$\Delta m^2_{23} \simeq 2.5 \cdot 10^{-3}\ \mathrm{eV}^2$, $\Delta m^2_{12} \simeq 8 \cdot 10^{-5}\ \mathrm{eV}^2$

$L_{ki} = \frac{2E}{|\Delta m^2_{ki}|}$ is a large macroscopic quantity: $\sim 10^3\ \mathrm{km}$ (atmospheric and accelerator neutrinos), $\sim 10^2\ \mathrm{km}$ (reactor neutrinos)

On the other side from the Heisenberg uncertainty relation

$$(\Delta p)_{\mathrm{QM}} \simeq \frac{1}{d}$$

d is a microscopic size of a source

$$L_{ki} \gg d \qquad |\Delta p_{ki}| \ll (\Delta p)_{\mathrm{QM}}$$

Impossible to resolve emission of neutrinos with different masses in weak decays ($|\nu_l\rangle$ is a coherent state)

Small neutrino mass-squared differences can be resolved in special experiments with large distances between neutrino sources and detectors

based on the time-energy uncertainty relation

$$\Delta E \Delta t \gtrsim 1$$

Δt is the time interval necessary to resolve ΔE

$$\Delta E_{ki} = |E_i - E_k| \simeq \frac{|\Delta m^2_{ki}|}{2E}$$

for ultrarelativistic neutrinos $\Delta t \simeq L$

The condition to resolve $|\Delta m^2_{ki}|$:

$$\frac{|\Delta m^2_{ki}|}{2E}\ L \gtrsim 1$$

$$\frac{|\Delta m^2_{ki} c^4|}{2E\hbar c}\ L \simeq 10^3 \frac{|\Delta m^2_{ki}(\mathrm{eV}^2)|}{E(\mathrm{MeV})})\ L(\mathrm{km}) \gtrsim 1$$

At $t = 0$ ν_l is produced. At time t

$$|\nu_l\rangle_t = e^{-iH_0 t}|\nu_l\rangle = \sum_i e^{-iE_i t} U^*_{li}\ |\nu_i\rangle$$

Superposition of states with different energies, nonstationary state

Flavor neutrinos are detected

$$|\nu_l\rangle_t = \sum_{l'} |\nu_{l'}\rangle\ \sum_i U_{l'i} e^{-iE_i t} U^*_{li}$$

The probability of the $\nu_l \to \nu_{l'}$ transition

$$P(\nu_l \to \nu_{l'}) = |\sum_i U_{l'i} e^{-iE_i t} U^*_{li}|^2$$

Simple meaning: U^*_{li} is the amplitude $\nu_l \to \nu_i$, $e^{-iE_i t}$ propagation in state with E_i, $U_{l'i}$ is the amplitude $\nu_i \to \nu_{l'}$. Sum over all i

$$P(\nu_l \to \nu_{l'}) = |\delta_{l'l} - 2i \sum_i U_{l'i} U^*_{li} e^{-i\Delta_{pi}} \sin\Delta_{pi}|^2$$

$\Delta_{pi} = \frac{\Delta m^2_{pi} L}{4E}$ $\quad$ p is an arbitrary, fixed index, $i \neq p$

$$P(\overset{(-)}{\nu}_l \to \overset{(-)}{\nu}_{l'}) = \delta_{l'l} - 4 \sum_i |U_{li}|^2 (\delta_{l'l} - |U_{l'i}|^2) \sin^2\Delta_{pi}$$

$$+8 \sum_{i>k} \mathrm{Re}\,(U_{l'i} U^*_{li} U^*_{l'k} U_{lk}) \cos(\Delta_{pi} - \Delta_{pk}) \sin\Delta_{pi} \sin\Delta_{pk}$$

$$\pm 8 \sum_{i>k} \mathrm{Im}\,(U_{l'i} U^*_{li} U^*_{l'k} U_{lk}) \sin(\Delta_{pi} - \Delta_{pk}) \sin\Delta_{pi} \sin\Delta_{pk}$$

Two-neutrino oscillations.

We can choose $p = 1$, $(i \neq p)$ $i = 2$, no interference terms

$$P(\nu_l \to \nu_{l'}) = P(\bar{\nu}_l \to \bar{\nu}_{l'}) = \delta_{l'l} - 4|U_{l2}|^2(\delta_{l'l} - |U_{l'2}|^2)\sin^2\Delta_{12}$$

From the unitarity of the mixing matrix

$|U_{l2}|^2 = \sin^2\theta \quad |U_{l'2}|^2 = \cos^2\theta, \quad (l' \neq l)$ (θ is the mixing angle)

$$P(\nu_l \to \nu_l) = 1 - \sin^2 2\theta \ \sin^2\frac{\Delta m_{12}^2 L}{4E} = 1 - \frac{1}{2}\sin^2 2\theta(1 - \cos\frac{2\pi L}{L_{osc}})$$

$$P(\nu_l \to \nu_{l'}) = \sin^2 2\theta \ \sin^2\frac{\Delta m_{12}^2 L}{4E} = \frac{1}{2}\sin^2 2\theta(1 - \cos\frac{2\pi L}{L_{osc}}), \quad l' \neq l$$

$\sin^2 2\theta$ is the amplitude of oscillations,

$$L_{osc} = 4\pi\frac{E}{\Delta m_{12}^2} \simeq 2.47\frac{E(\mathrm{MeV})}{\Delta m_{12}^2(\mathrm{eV}^2)}\mathrm{m}$$

is the oscillation length. Describe periodical transitions $\nu_l \to \nu_{l'}$

In the three-neutrino case six parameters: three mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$, CP phase δ and two neutrino mass-squared differences. One mass-squared difference is about 30 times smaller than another one

Two neutrino mass spectra are possible. Usually, neutrino masses are labeled in such a way that $m_2 > m_1$. $\Delta m^2_{12} = \Delta m^2_S$ is small (solar) mass-squared difference

Possible neutrino mass spectra are determined by the mass m_3. Two possibilities

1. Normal ordering (NO) $m_3 > m_2 > m_1$
2. Inverted ordering (IO) $m_2 > m_1 > m_3$

Large (atmospheric) mass-squared difference

$$\Delta m^2_A = \Delta m^2_{23} \quad (\mathrm{NO}) \qquad \Delta m^2_A = |\Delta m^2_{13}| \quad (\mathrm{IO})$$

Do not depend on the mass ordering. Confusion in literature

In the case of NO (choosing $p = 2$)

$$P^{\rm NS}(\overset{(-)}{\nu}_l \to \overset{(-)}{\nu}_{l'}) = \delta_{l'l} - 4\ |U_{l3}|^2(\delta_{l'l} - |U_{l'3}|^2)\ \sin^2\Delta_A$$
$$-4\ |U_{l1}|^2(\delta_{l'l} - |U_{l'1}|^2)\ \sin^2\Delta_S$$
$$-8\ [{\rm Re}\ (U_{l'3}U_{l3}^*U_{l'1}^*U_{l1})\ \cos(\Delta_A + \Delta_S)$$
$$\pm 8\ {\rm Im}\ (U_{l'3}U_{l3}^*U_{l'1}^*U_{l1})\ \sin(\Delta_A + \Delta_S)\ ]\sin\Delta_A \sin\Delta_S$$

In the case of IO ($p = 1$)

$$P^{\rm IS}(\overset{(-)}{\nu}_l \to \overset{(-)}{\nu}_{l'}) = \delta_{l'l} - 4\ |U_{l3}|^2(\delta_{l'l} - |U_{l'3}|^2)\ \sin^2\Delta_A$$
$$-4\ |U_{l2}|^2(\delta_{l'l} - |U_{l'2}|^2)\ \sin^2\Delta_S$$
$$-8\ [{\rm Re}\ (U_{l'3}U_{l3}^*U_{l'2}^*U_2)\ \cos(\Delta_A + \Delta_S)\sin\Delta_A \sin\Delta_S$$
$$\mp 8\ {\rm Im}\ (U_{\alpha'3}U_{\alpha 3}^*U_{\alpha'2}^*U_{\alpha 2})\ \sin(\Delta_A + \Delta_S)]\sin\Delta_A \sin\Delta_S.$$

$$\Delta_A = \frac{\Delta m_A^2 L}{4E}, \quad \Delta_S = \frac{\Delta m_S^2 L}{4E}$$

Two neutrino oscillation parameters are small

$$\frac{\Delta m^2_S}{\Delta m^2_A} \simeq 3 \cdot 10^{-2}, \quad \sin^2\theta_{13} \simeq 2.5 \cdot 10^{-2}$$

If we neglect contribution of these parameters we will obtain simple two-neutrino formulas which describe basic feature of the three-neutrino oscillations

For both mass spectra

$\nu_\mu \to \nu_\mu$ in the atmospheric range of $\frac{L}{E}$ $(\Delta_A \simeq 1, \ \ \Delta_S \ll 1)$ due to two-neutrino $\nu_\mu \rightleftarrows \nu_\tau$ oscillations

$$P(\nu_\mu \to \nu_\mu) = P(\bar\nu_\mu \to \bar\nu_\mu) \simeq 1 - \sin^2 2\theta_{23} \ \sin^2 \frac{\Delta m^2_A L}{4E}$$

$\bar\nu_e \to \bar\nu_e$ in the solar range of $\frac{L}{E}$ (KamLAND experiment, $\Delta_S \simeq 1$, $\Delta_A \gg 1$) due to $\bar\nu_e \rightleftarrows \bar\nu_{\mu,\tau}$ oscillations

$$P(\bar\nu_e \to \bar\nu_e) \simeq 1 - \sin^2 2\theta_{12} \ \sin^2 \frac{\Delta m^2_S L}{4E}$$

In the leading approximation $P(\nu_\mu \to \nu_e) = P(\bar\nu_\mu \to \bar\nu_e)$. Effects of the CP violation can not be observed

The results of the global analysis of present-day data

Table I.

Parameter	Normal Ordering	Inverted Ordering
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.304^{+0.013}_{-0.012}$
$\sin^2\theta_{23}$	$0.452^{+0.052}_{-0.028}$	$0.579^{+0.025}_{-0.037}$
$\sin^2\theta_{13}$	$0.0218^{+0.0010}_{-0.0010}$	$0.0219^{+0.0011}_{-0.0010}$
δ (in $^\circ$)	(306^{+39}_{-70})	(254^{+63}_{-62})
Δm_S^2	$(7.50^{+0.19}_{-0.17})\cdot 10^{-5}\ \mathrm{eV}^2$	$(7.50^{+0.19}_{-0.17})\cdot 10^{-5}\ \mathrm{eV}^2$
Δm_A^2	$(2.457^{+0.047}_{-0.047})\cdot 10^{-3}\ \mathrm{eV}^2$	$(2.449^{+0.048}_{-0.047})\cdot 10^{-3}\ \mathrm{eV}^2$

Neutrino oscillations parameters are known with accuracies from $\sim$ 3 % ($\Delta m^2_{S,A}$) to $\sim$ 10% ($\sin^2\theta_{23}$). Existing data do not allow to distinguish normal and inverted neutrino mass ordering. *CP* phase δ is practically unknown

In future neutrino oscillation experiments

1. Oscillation parameters will be measured with % accuracy
2. Neutrino mass ordering will be determined
3. The *CP* phase δ will be measured

The most important unsolved problem: are ν_i Majorana or Dirac particles?

Experiment on the search for $(A,Z) \to (A,Z) + e^- + e^-$ are the most sensitive experiments

L is violated. If $0\nu\beta\beta$-decay will be observed, ν_i are Majorana neutrinos

However, many problems

Interaction $\mathcal{H}_I = \frac{G_F}{\sqrt{2}} 2\bar{e}_L \gamma^\alpha \nu_{eL}\; j^{CC}_\alpha + \mathrm{h.c.} \quad \nu_{eL} = \sum_i U_{eL}\nu_{iL}$

Second order $(\sim G_F^2)$ with virtual neutrino

Neutrino propagator

$$\langle 0|T(\nu_{eL}(x_1)\nu^T_{eL}(x_2)|0\rangle \simeq \frac{-i}{(2\pi)^4}\int \frac{e^{-ip(x_1-x_2)}}{p^2} d^4x\; \frac{1-\gamma_5}{2}\; C\; m_{\beta\beta}$$

$m_{\beta\beta} = \sum_i U^2_{ei} m_i$ small quantity, strongly depends on neutrino mass spectrum

$$2\times 10^{-2} \lesssim |m_{\beta\beta}| \lesssim 5\times 10^{-2}\,\mathrm{eV} \quad \text{(IO)}$$

$$2\times 10^{-3}\;\mathrm{eV} \lesssim |m_{\beta\beta}| \lesssim 4\times 10^{-3}\,\mathrm{eV} \quad \text{(NO)}$$

Large uncertainties in (calculated) nuclear matrix elements

Sterile neutrinos?

From the point of view of theory. In the most economical theory of neutrino masses (Weinberg effective Lagrangian) no sterile neutrinos. Sterile neutrinos with small masses (~ 1 eV) require right-handed fields and non conservation of L. Great discovery, if exist.

From the point of view of experiment. Experimental indications (observation in short baseline appearance experiments (LSND etc) and non observation in disappearance experiments) can not be described in the framework of one mixing scheme. In the latest experiments (IceCube, Daya Bay, MINOS) sterile neutrinos were not observed

Many new experiments (more than 20) are in preparation or going on

Hope that in a few years the problem will be solved

Bruno Pontecorvo came to an idea of neutrino oscillations in 1957 soon after parity violation in β-decay and μ-decay was discovered and two-component theory of massless neutrino was proposed by Landau, Lee and Yang and Salam and confirmed in the classical Goldhaber et al experiment on the measurement of the neutrino helicity

At that time only one type of neutrino (ν_e) was known (not only as Pauli proposal but discovered in the Reins and Cowan experiment)

According to the two-component theory only massless ν_L and $\bar{\nu}_R$ exist. Transitions between them are strictly forbidden

What was B. Pontecorvo motivations? What he had in mind when we introduced neutrino oscillations?

B. Pontecorvo believed in a similarity (analogy) of weak interactions of hadrons and leptons, very popular idea at that time

He was impressed by $K^0 \rightleftarrows \bar{K}^0$ oscillations, suggested by Gell-Mann and Pais, and looked for a similar phenomenon in the lepton world

Basics of $K^0 \rightleftarrows \bar{K}^0$ oscillations

1. K^0 ($\bar{K}^0$) is a particle with the strangeness $S = 1$ ($S = -1$). S conserved in the strong interaction.
2. Weak interaction, in which S is not conserved, induce transitions between K^0 and $\bar{K}^0$.
3. Particles with definite masses and life-times, eigenstates of the total Hamiltonian, are K_1, K_2. K^0, $\bar{K}^0$ are "mixed particles"
$$|K^0(\bar{K}^0)\rangle = \frac{1}{\sqrt{2}}(|K_1^0\rangle \pm |\bar{K}_2^0\rangle)$$

B. Pontecorvo (1957) raised the following question "...whether there exist other "mixed" neutral particles (not necessarily elementary ones) which are not identical to their corresponding antiparticles and for which particle $\leftrightarrows$ antiparticle transitions are not strictly forbidden".

B. Pontecorvo understood that such "neutral particles" could be muonium $(\mu^+ - e^-)$ and antimuonium $(\mu^- - e^+)$

B. Pontecorvo (1957) wrote that muonium $\leftrightarrows$ antimuonium transitions are allowed and "are induced by the same interaction which is responsible for μ-decay" (in the second order on G_F)

$$(\mu^+ - e^-) \to \nu + \bar{\nu} \to (\mu^- - e^+)$$

It was unknown at that time that ν_e and ν_μ are different particles muonium $\leftrightarrows$ antimuonium transition requires an interaction which provides $|\Delta L_e| = 2$ and $|\Delta L_\mu| = 2$

In the 1957 paper Pontecorvo made the following remark about neutrino oscillations: "If the theory of the two-component neutrino is not valid (which is hardly probable at present) and if the conservation law for the neutrino charge does not hold, neutrino $\to$ antineutrino transitions in vacuum in principle be possible."

The paper on neutrino oscillations was published by B. Pontecorvo in 1958

He wrote :"...neutrino may be a particle mixture and consequently there is a possibility of real transitions neutrino $\to$ antineutrino in vacuum, provided that the lepton (neutrino) charge is not conserved. This means that the neutrino and antineutrino are *mixed* particles, i.e., a symmetric and antisymmetric combination of two truly neutral Majorana particles ν_1 and ν_2 "

In other words B. Pontecorvo assumed the mixing

$$\nu_L = \frac{1}{\sqrt{2}}(\nu_{1L} + \nu_{2L}), \quad \nu_R^c = \frac{1}{\sqrt{2}}(\nu_{1L} - \nu_{2L})$$

which provides transition $\bar{\nu}_R \leftrightarrows \nu_R$

According to the two-component theory ν_R does not interact (sterile)

B. Pontecorvo: "...this possibility became of some interest in connection with new investigations of inverse β-processes."

In 1957 R. Davis performed a reactor experiment in which he searched for

$$\bar{\nu}_R +^{37}\mathrm{Cl} \to \mathrm{e}^- +^{37}\mathrm{Ar}$$

A rumor reached B.Pontecorvo that Davis observed $^{37}\mathrm{Ar}$ events

B.Pontecorvo assumed that these "events" could be due to transitions of reactor antineutrinos into right-handed neutrinos in vacuum (neutrino oscillations)

The Reines and Cowan reactor experiment, in which neutrino was discovered in the process $\bar{\nu} + p \to e^+ + n$, was going on at that time. B.Pontecorvo suggested

"...the cross section of the process $\bar{\nu} + p \to e^+ + n$ would be smaller than the expected cross section. This is due to the fact that the neutral lepton beam which at the source is capable of inducing the reaction changes its composition on the way from the reactor to the detector."

"It would be extremely interesting to perform the Reins-Cowan experiment at different distances from reactor"

At a later stage of the Davis experiment the anomalous "events" disappeared

B. Pontecorvo soon understood that ν_R is a sterile particle. The terminology "sterile neutrino", which is standard nowadays, was introduced by him in the next publication on neutrino oscillations

Summarizing, in the 1958 pioneer paper B. Pontecorvo considered $\bar{\nu}_L \leftrightarrows \nu_R$ oscillations (the only possible oscillations in the case of one neutrino)

In 1958 paper B. Pontecorvo wrote "Effects of transformation of neutrino into antineutrino and vice versa may be unobservable in the laboratory but will certainly occur, at least, on an astronomical scale."

The second Pontecorvo paper on neutrino oscillations (1967) was written after ν_μ was discovered

"If the lepton charge is not an exactly conserved quantum number, and the neutrino mass is different from zero, oscillations similar to those in K^0 beams become possible in neutrino beams"

B. Pontecorvo considered oscillations between flavor neutrinos $\nu_\mu \rightleftarrows \nu_e$ (very natural and easy for him (no sterile neutrinos are needed)) but also transitions $\nu_\mu \rightleftarrows \bar{\nu}_{\mu L}$ etc."which transform active particles into particles, which from the point of view of ordinary weak processes, are sterile..."

In the 1967 paper B.Pontecorvo discussed the effect of solar neutrino oscillations. “From an observational point of view the ideal object is the sun. If the oscillation length is smaller than the radius of the sun region effectively producing neutrinos, direct oscillations will be smeared out and unobservable. The only effect on the earth's surface would be that the flux of observable sun neutrinos must be two times smaller than the total neutrino flux.”

In 1970 the first results of the Davis experiment were obtained. It occurred that the detected flux of solar neutrinos was about (2-3) times smaller than the flux predicted by the SSM ("the solar neutrino problem")

Pontecorvo neutrino oscillations, based on neutrino masses and mixing, was accepted as a natural explanation of the problem.

Later it was discovered that combination of effects of neutrino masses and mixing and coherent neutrino scattering in matter (MSW) provides explanation of suppression of the solar neutrino flux

The general phenomenological theory of neutrino mixing and oscillations (B. Pontecorvo and SB)

In series of papers all possible neutrino mass terms were considered:

Dirac (flavor ν_{lL} and sterile ν_{lR} fields, conservation of L, analogy with quarks)

Majorana (flavor ν_{lL} fields, non conservation of L)

The most general Dirac and Majorana (flavor ν_{lL} and sterile ν_{lR} fields, non conservation of L, basis for the seesaw mechanism)

All possible mixing

$\nu_{lL} = \sum_{i=1}^{3} U_{li}\nu_{iL}$ (ν_i Dirac) $\nu_{lL} = \sum_{i=1}^{3} U_{li}\nu_{iL}$ (ν_i Majorana)

$\nu_{lL} = \sum_{i=1}^{6} U_{li}\nu_{iL}$ $(\nu_{lR})^c = \sum_{i=1}^{6} U_{\bar{l}i}\nu_{iL}$ (ν_i Majorana)

After the success of the two-component theory during many years there was a general belief than neutrinos are massless particles

Our main arguments for neutrino masses

1. There is no principle (like gauge invariance for γ-quanta) which requires neutrino masses to be equal to zero
2. After $V - A$ theory (in the weak Hamiltonian enter left-handed components of *all fields*) it is natural to consider neutrinos not as a special massless particles but as a particles with some masses

In Dubna papers possible values of the neutrino mixing angles were discussed:

- there is no reason for the lepton and Cabibbo mixing angles to be the same.
- "it seems to us that the special values of the mixing angles $\theta = 0$ and $\theta = \frac{\pi}{4}$ (maximum mixing) are of the greatest interest."

In 1977 we wrote first review on neutrino oscillations which attracted attention of many physicists to the problem
Not only reactor experiments but also accelerator, solar and atmospheric experiments were discussed. It was stressed that experiments at different neutrino facilities are sensitive to different values of neutrino mass-squared differences and must be performed
The history of neutrino oscillations is an illustration of the importance of analogy in physics. It is also an illustration of the importance of new courageous ideas which are not always in agreement with general opinion
The discovery of neutrino oscillations was a great triumph of Bruno Pontecorvo, the founder and father of modern neutrino physics. Bruno Pontecorvo came to the idea of neutrino oscillations at a time when the common opinion favored massless neutrinos and no neutrino oscillations. He pursued the idea of neutrino oscillations over decades