

Can Water “Burn”?

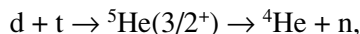
V. B. Belyaev, A. K. Motovilov, and W. Sandhas

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Nuclear states whose energy is close (in nuclear scale) to the threshold of their production usually have large spatial sizes. This occurs due to “long” tails of wave functions expressed in terms of corresponding variables. For example, the ground state of the ${}^8\text{B}$ nuclei, which are the main source of high-energy solar neutrinos [1], differs from the threshold of the two-particle channel $p + {}^7\text{Be}$ by only 130 KeV. This fact leads to the necessity of integrating in the transition matrix elements of the reaction $p + {}^7\text{Be} \rightarrow {}^8\text{B} + \gamma$ over the region up to 300 fm [2].

In a number of cases, the existence of the near-threshold nuclear resonance leads to a considerable increase in the probability of the transition into this state as compared to the transition into the first localized state. A good example of such a situation is the nuclear transition in the $d\text{t}\mu$ mesomolecule, namely,



where the difference between the threshold (in the $d + t$ channel) and resonance energies is 50 KeV. As is well known [5–8], the probability of this transition exceeds, by at least four orders of magnitude, those for $dd\mu$ or $p\text{d}\mu$ molecules, where nuclear resonances are absent.

We now discuss the spectrum of states for the ${}^{18}\text{Ne}$ nucleus [3, 4]. As follows from the diagram of nuclear levels presented in the figure, the energy of the state 4.522 MeV, for the ${}^{18}\text{Ne}(1^-)$ nucleus within the accuracy of measurements coincides with the threshold energy of the three-particle channel $p + p + {}^{16}\text{O}$. Since the coupling energy of the H_2O molecule is several electronvolts, we may ascertain that, with high accuracy, the rotational state 1^- for the molecule of water and the 1^- state for the ${}^{18}\text{Ne}$ nucleus are degenerated in energy, and the actual physical state characterized by these quantum numbers is a superposition of the molecular and nuclear states. In other words, in the 1^- state, the wave function of a molecule of water always contains a certain admixture of the 1^- state of

the ${}^{18}\text{Ne}$ nucleus. Evidently, the mixing factor is given by the integral of overlapping between pure states. This is the integral that determines the nuclear-transition probability in the H_2O molecule. Below, we show that the probability of the nuclear transition is not suppressed by the usual Coulomb factor due to the closeness of the threshold and resonance energies [9]. This conclusion follows from the dominant contribution of intermediate and long (in nuclear scale) distances to the overlap integral.

We now determine the wave functions of pure states. We make use of the expression

$$\Psi_{\text{mol}}(X) = \frac{1}{N_{\text{mol}}} \frac{F_{5/2}(\eta_0, \kappa\rho)}{\rho^{5/2}} e^{-\kappa\rho} Y_{l\lambda}^{1M}(\hat{x}, \hat{y}) \quad (1)$$

as a wave function of the molecule of water. Here, x and y are Jacobi variables for the system $p + p + {}^{16}\text{O}$, $X = \{x, y\}$;

$\rho = \sqrt{x^2 + y^2}$ is the hyperradius, F_ν is the regular solution to the Schrödinger equation with the Coulomb potential taken into account, $\kappa \sim \sqrt{|\epsilon_{\text{mol}}|}$, ϵ_{mol} is the coupling energy for the H_2O molecule, $Y_{l\lambda}^{JM}(\hat{x}, \hat{y})$ is the angular function for the three-particle system having the angular momentum J ; N_{mol} is the normalization factor. Finally, η_0 is the Sommerfeld parameter,

$$\eta_0 = \frac{v_0}{2\kappa},$$

where v_0 is the mean value of the angular part

$$v(\Omega) = \frac{1}{\rho} v(X)$$

of the total Coulomb potential $v(X)$ in the system ${}^{16}\text{O} + p + p$ for $|X| = \rho$. Expression (1) adequately takes into account both the Coulomb repulsion at small distances between the particles and geometric dimensions of the system.

To describe the resonance nuclear state of the three-particle system, we make use of the Coulomb distorted spherical wave [10] normalized to the volume of this nucleus. Such a normalization condition reflects the nuclear nature of this state. Thus, we consider the

*Joint Institute for Nuclear Research, Dubna, Moscow oblast, 141980 Russia.

**Physikalisches Institut, Universität Bonn, Endenicher Allee 11-13, D-53115 Bonn, Germany

following wave function for the 1^- state of the ^{18}Ne nucleus:

$$\Psi_{\text{res}}(X) = \frac{1}{N_{\text{res}}} \frac{f^1(\rho, \omega)}{\rho^{5/2}} Y_{l\lambda}^{1M}(\hat{x}, \hat{y}), \quad (2)$$

where $\omega = \arctan \frac{|y|}{|x|}$ is the hyperangle, and

$$f^J(\rho, \omega) = \int d\hat{x} d\hat{y} \times \exp \left\{ iK\rho - i \frac{v(\Omega)}{2K} \ln(2K\rho) \right\} Y_{l\lambda}^{JM}(\hat{x}, \hat{y}). \quad (3)$$

Here, $K \sim \sqrt{E}$, where E is the energy of diverging particles in the system $^{16}\text{O} + p + p$.

Using expressions (1) and (2), we find that the overlap integral I is

$$I = \int dX \Psi_{\text{res}}(X) \Psi_{\text{mol}}(X) \sim \exp \left\{ -\frac{\pi}{2} \eta_K^0 \right\} \exp \{ i\eta_K^0 S(\sigma, \xi) \}, \quad (4)$$

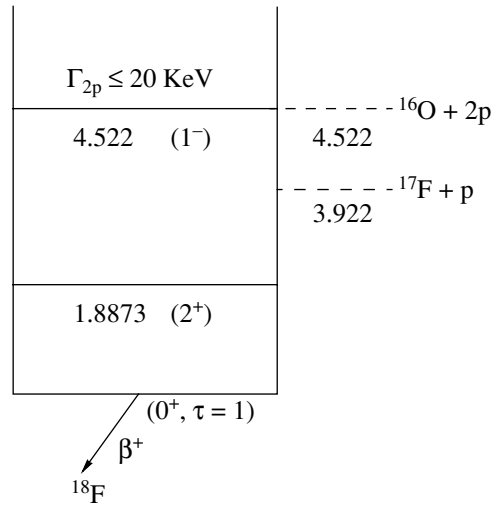
where $\eta_K^0 = \frac{v(\Omega_0)}{2K}$ is the generalized Sommerfeld parameter corresponding to the point Ω_0 of the minimum for the function $v(\Omega)$, while $\xi = \frac{K}{\kappa}$ and $\sigma = \frac{v(\Omega_0)}{v_0}$. According to its definition, the variable σ varies within the limits $0 < \sigma < 1$. The variation of the second parameter ξ is caused by the nonzero width of the 4.522 MeV (1^-) level of the ^{18}Ne nucleus. According to the definition of this parameter, it can vary within the range

$$0 \leq \xi \leq \sqrt{\frac{\Gamma_{2p}}{\epsilon_{\text{mol}}}},$$

where Γ_{2p} is the decay width of the nuclear resonance into the channel $^{16}\text{O} + p + p$. The numerical investigation of the function $S(\sigma, \xi)$ demonstrates the existence of the subregion of parameters σ and ξ , for which $\text{Im}S < 0$ and, simultaneously, $|\text{Im}S| > \pi/2$. Thus, the overlap integral (4) and the velocity W of the transition $\text{H}_2\text{O} \rightarrow ^{18}\text{Ne} (1^-)$ into the resonance state of the ^{18}Ne nucleus, which is defined by the expression

$$W = \kappa c |I|^2, \quad (5)$$

exponentially increases with decreasing K (at low energies E) in contrast to the probability of the transition into localized nuclear states. Such behavior of the velocity of the transition is explained by the contribution into inte-



Fragment of the spectrum for states of the ^{18}Ne nucleus.

gral (4) from the region of intermediate and large distances between the particles.

Thus, it follows from (4) that water must gradually transform from the molecular state into the excited state of the ^{18}Ne nucleus. The latter one, in its turn, will decay into either the channel

$$^{17}\text{F} + p + Q_1, \quad Q_1 \approx 0.6 \text{ MeV},$$

or according to the chain $^{18}\text{Ne} (1^-) \rightarrow ^{18}\text{Ne} + \gamma + Q_2 \rightarrow ^{18}\text{F} + e^+ + \nu, Q_2 \approx 4.522 \text{ MeV}$.

We now discuss possible modifications of the wave functions (1) and (2) as well as their effect on the overlap integral (4).

The internal structure of a molecule of water (for example, the fixed angle between the directions from the oxygen nucleus to the protons) could be taken into account more adequately when multiplying (1) by an appropriate polynomial of variables ρ and ω . However, as follows from the derivation of the asymptotic formula (4), this leads only to exponential (in terms of η_0) corrections to the asymptotic formula (4) but does not affect its exponential behavior.

In essence, the modification of expression (2) for the wave function of the nuclear resonance should be performed only in the region of small distances between the ^{16}O nucleus and protons. It is clear that any such modification can lead to only exponentially small corrections to the result (4).

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