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LEP1 and LEP2 $2f$ review

Present Status of Theoretical Calculations

for the process $e^+e^- \to f\overline{f}$

at LEP1 and LEP2 energies

D. Bardin, JINR, Dubna

The numerical results are based on the work of TOPAZ0 and ZFITTER teams

At present, the following physicists are members of teams:
TOPAZ0[1]:
G. Montagna, O. Nicrosini, G. Passarino and F. Piccinini
ZFITTER[2]:
D. Bardin, P. Christova, M. Jack,
L. Kalinovskaya, A. Olshevski, S. Riemann, T. Riemann

Compiled and summarized in:

Precision Calculation Project Report

Dmitri Bardin, Martin Grünewald, Giampiero Passarino

hep-ph/9902452, 24 Feb 1999

Abstract

The complete list of definitions for quantities relevant in the analysis of SLD/LEP-1 results around the Z-resonance is given. The common set of conventions adopted by the programs TOPAZ0 and ZFITTER, following the recommendations of the LEP electroweak working group, is reviewed. The relevance of precision calculations is discussed in detail both for pseudo-observables (PO) and for realistic observables (RO). The model-independent approach is also discussed. A critical assessment is given of the comparison between TOPAZ0 and ZFITTER.

[1] version 4.4: hep-ph/9804211
[2] version 6.10: DESY 99-070

The Z resonance observables measured at LEP1 (CERN) and SLC (SLAC) with

$$\text{experimental precision} \ \sim\ 10^{-3}$$

Therefore, one needs

$$\text{theoretical precision} \ \sim\ 2 \div 3 \times 10^{-4}$$

Due to excellent performance of LEP in last three years even at LEP2 energies

$$\text{experimental precision} \ \sim\ 10^{-2}$$

Therefore, one needs

$$\text{theoretical precision} \ \sim\ 2 \div 3 \times 10^{-3}$$

This is **challenging** precision! Are these numbers within the reach?

<u>Number of free parameters in calculations (fits) of LEP observables</u>

All masses of fundamental particles, fermions and bosons
All mixings, V_{ij} (irrelevant for precision tests at Z-resonance)
All coupling constants of fundamental interactions: α and α_S

<u>Lepton masses</u> are known very precise:

$$\begin{aligned} m_e &= 0.51099907(15)\,\text{MeV} \\ m_\mu &= 105.658389(34)\,\text{MeV} \end{aligned}$$

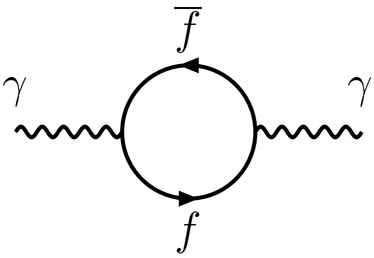
even the worst

$$m_\tau = 1777.05^{+0.29}_{-0.26}\ \text{MeV} \ < 10^{-4}$$

is known infinitely precise in typical LEP1 precision scale 10^{-3}
The same for the fine structure constant: $\alpha(0)$.

Quark masses, especially *of light quark* masses are not well defined quantities.

Their contribution to the vacuum polarization



leads to

$$\sum_f \ln \frac{s}{m_f^2}$$

and can not be calculated in perturbation theory.

Light quark masses are being replaced with the aid of an experimentally measured quantity

$$\sigma\left(e^+e^- \to \text{hadrons}\right) \quad \to \quad \alpha\left(M_Z^2\right)$$

$$\text{then} \quad 1/\tau_\mu \quad \to \quad M_W \quad \to \quad G_F \ \text{ infinitely precise}$$

So, we are left with 5 parameters only – **LEP1, IPS**:

$$\alpha\left(M_Z^2\right) \qquad \alpha_S\left(M_Z^2\right) \qquad m_t \qquad M_Z \qquad \mathbf{M_H}$$

With M_Z measured infinitely precise at LEP1:

$$M_Z = 91.1867 \pm 0.0021 \quad \text{relative error} \sim 2 \times 10^{-5}$$

And for three of them

$$\alpha\left(M_Z^2\right) \qquad \alpha_S\left(M_Z^2\right) \qquad m_t$$

an information is available from the other then LEP1 measurements. This allows fits *constrained on these parameters*.

In principle, we are approaching one-parameter fit:

$$\mathbf{M_H}$$

The running QED coupling $\alpha(s)$

$$\alpha(s) = \frac{\alpha(0)}{1-\Delta\alpha^{\rm fer}(s)} = \frac{\alpha(0)}{1-\Delta\alpha_l(s)-\Delta\alpha_h^{(5)}(s)-\Delta\alpha_t(s)-\Delta\alpha^{\alpha\alpha_S}(s)}$$

Hadronic $\Delta\alpha_h^{(5)}(s)$ – by making use of a dispersion relation

$$\Delta\alpha_h^{(5)}(s) = -\frac{\alpha}{3\pi} s \,{\rm Re}\int_{4m_\pi^2}^{\infty} ds' \frac{R_\gamma(s')}{s'\left(s'-M_Z^2-i\epsilon\right)}$$

with

$$R_\gamma(s) = \frac{\sigma\left(e^+e^-\to\gamma^*\to \text{hadrons}\right)}{\sigma\left(e^+e^-\to\gamma^*\to\mu^+\mu^-\right)}$$

as an **experimental** input. For hadronic contribution at M_Z (*Eidelman and Jegerlener, 1995*)

$$\begin{aligned} \Delta\alpha_h^{(5)}(M_Z^2) &= 0.02804 \pm 0.00064 \\ \text{or} \quad \alpha^{-1}(M_Z^2) &= 128.89 \pm 0.09 \end{aligned}$$

Typical precision of a **theory driven** analysis (*Davier and Höcker, 1998*, *Kühn and Steinhauser, 1998*, *Jegerlehner, 1999*)

$$\begin{aligned} \Delta\alpha_h^{(5)}(M_Z^2) &= \pm 0.00016 \\ \text{or} \quad \alpha^{-1}(M_Z^2) &= \pm 0.02 \end{aligned}$$

Leptonic $\Delta\alpha_l(s)$ – by calculating up to three loops (2-loop: *Källen 1955*; 3-loops: *Steinhauser 1998*)

$$\begin{aligned} \Delta\alpha_l &= 314.97637\cdot 10^{-4} \\ &= \left[314.18942_{1-\rm loop} + 0.77616_{2-\rm loop} + 0.01079_{3-\rm loop}\right]\cdot 10^{-4} \end{aligned}$$

(with above leptonic and Z masses [central values]).

The two remaining are given for $m_t = 173.8$ GeV and $\alpha_S = 0.119$:

$$\begin{aligned} \Delta\alpha_t(M_Z^2) &= -0.585844\cdot 10^{-4} \\ \Delta\alpha^{\alpha\alpha_S}(M_Z^2) &= -0.103962\cdot 10^{-4} \end{aligned}$$

The mixed two-loop $\mathcal{O}(\alpha\alpha_S)$ correction arising from $t\bar{t}$ loop with gluon exchange is calculated using formulae from *Kniehl 1990*.

More on coupling constants, typical scales

$$\begin{aligned} \text{LEP1 typical scale: } \sqrt{s} &\sim M_{Z} \\ \text{LEP2 typical scale: } \sqrt{s} &\sim 200 \text{ GeV} \\ M_{W} &\sim 80 \text{ GeV} \\ M_{Z} &\sim 91 \text{ GeV} \\ m_t &\sim 175 \text{ GeV} \\ M_{H} &\leq 300 \text{ GeV} \end{aligned}$$

All are of the order of typical EW scale: $100 \div 300$ GeV.

One cannot construct a small parameter out of

$$\sqrt{s}\,, \quad M_{W}\,, \quad M_{Z}\,, \quad m_t\,, \quad M_{H}$$

and calculation must be, in principle, exact in all these quantities. In real life the notion of m_t^2 *enchanced terms* is introduced:

$$\begin{aligned} \mathcal{O}\left(G_{F} m_t^2\right) &= \mathcal{O}\left(\frac{\pi}{\sqrt{2}}\frac{\alpha}{s^2_{W}}\frac{m_t^2}{M^2_{W}}\right) \\ \text{with} \quad s^2_{W} &= 1 - \frac{M^2_{W}}{M^2_{Z}} \\ \frac{m_t^2}{M^2_{W}} &\approx 4 \ \ \text{not so pronounced enchancement!} \end{aligned}$$

Since likely:

$$100 \text{ GeV} \leq M_{H} \leq 300 \text{ GeV},$$

popular expansions in M^2_{H}/m_t^2 and m_t^2/M^2_{H} may have only academic meaning.

Complete 2-loop EW corrections are welcome!

For light fermion masses it is sufficient to keep first order in

$$\frac{m^2_\tau}{s}\,, \qquad \frac{m^2_c(s)}{s}\,, \qquad \frac{m^2_b(s)}{s}$$

all other masses and higher than m^2/s terms may be ignored at LEP energies.

Coupling constants

$$
\begin{array}{llcll}
\text{QED} & \alpha\,(0) & = & \dfrac{1}{137}L & \text{up to } \mathcal{O}\left((\alpha L)^3\right) \\[2ex]
\text{EW} & \alpha\left(M_Z^2\right) & = & \dfrac{1}{128.9} & \text{up to } \mathcal{O}\left(\alpha^2\right) \\[2ex]
\text{QCD} & \alpha_S\left(M_Z^2\right) & = & 0.119 & \text{up to } \mathcal{O}\left(\alpha_S^3\right)
\end{array}
$$

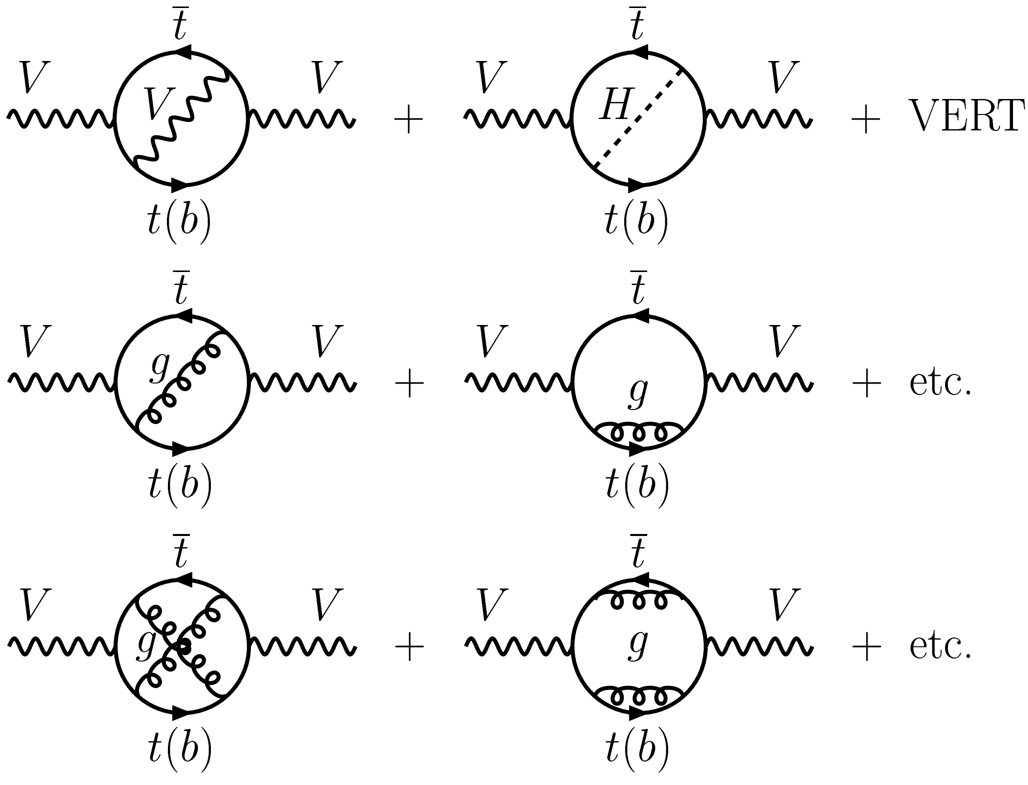
where

$$L = \ln\frac{s}{m_e^2} - 1 = 23 \qquad \text{at } s = M_Z^2, \qquad \alpha L = 0.169$$

Mixed corrections are also needed

$$\mathcal{O}\left(\alpha\alpha_S\right), \qquad \mathcal{O}\left(\alpha\alpha_S^2\right)$$

Some examples of higher order diagrams:



Parameters Δr and $\delta\rho$

Sirlin's (1980) parameter Δr originates from calculation of one-loop EWRC for the μ lifetime. For one-loop QED corrections in Fermi theory

$$\begin{aligned}\tau_\mu^{-1} &= \frac{G_F^2 m_\mu^5}{192\,\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right) \left[1 + \frac{\alpha}{2\,\pi}\left(\frac{25}{4} - \pi^2\right)\right] \\ F(r) &= 1 - 8\,r + 8\,r^3 - r^4 - 12\,r^2 \ln r\end{aligned}$$

$G_F = (1.6639 \pm 0.00001) \times 10^{-5}\,\mathrm{GeV}^{-2}$ and on top

$$G_F = \frac{\alpha\pi}{\sqrt{2} M_W^2 (1 - M_W^2/M_Z^2)} \frac{1}{1 - \Delta r}$$

Δr is a *gauge invariant* (observable) quantity.

Recently the complete two-loop QED corrections were computed (*Ritbergen and Stuart, 1998*). The new value with better error

$$G_F = (1.16637 \pm 0.00001) \times 10^{-5}\,\mathrm{GeV}^{-2}$$

Veltman's (1975) parameter $\Delta\rho$ is closely related to the ratio of the neutral to the charged current effective couplings for low q^2:

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W^0}$$

where $\cos^2\theta_W^0$ is a weak mixing angle measured at low q^2. One may show that

$$\delta\rho_0 = \frac{g^2}{16\pi^2 M_W^2}\Big[\Sigma_{WW}(0) - \Sigma_{ZZ}(0)\Big]$$

For Z resonance observables more relevant is the quantity defined as the difference of two self-energies taken at their physical masses,

$$\delta\rho^\alpha = \frac{\alpha}{4\pi s_W^2}\frac{1}{M_W^2}\Big[\Sigma_{WW}(M_W^2) - \Sigma_{ZZ}(M_Z^2)\Big]$$

which, is gauge invariant, although non-observable, even divergent!

$\delta\rho$ terms known in the literature

$$\delta\rho = \delta\rho^{\alpha} + \delta\rho^{\alpha\alpha_S} + \delta\rho^{\alpha^2} + \delta\rho_L^{\alpha\alpha_S^2}$$

$\mathcal{O}\left(\alpha\alpha_S\right)$ – complete – *Djouadi, 1988; Kniehl, 1990*
$\mathcal{O}\left(\alpha^2\right)$ – two terms: *leading* and *next-to-leading* in m_t^2
Degrassi et al. 1996–1997

$$\begin{array}{ccccc} \delta\rho^{\alpha^2} & = & \delta\rho_L^{\alpha^2} & + & \delta\rho_{NL}^{\alpha^2} \\ & & \mathcal{O}\left(\alpha^2 m_t^4/M_W^4\right) & & \mathcal{O}\left(\alpha^2 m_t^2/M_W^2\right) \end{array}$$

For the three-loop correction $\delta\rho^{\alpha\alpha_S^2}$, only the leading in m_t^2 term is available *Avdeev et al, 1994; Chetyrkin, Kühn, Steinhauser, 1995*.

Fanchiotti and Sirlin, 1991 introduced

$$\delta\hat{\rho}^{\alpha} \equiv \frac{\alpha}{4\pi s_W^2}\frac{1}{M_W^2}\Big[\Sigma_{WW}(M_W^2) - \Sigma_{ZZ}(M_Z^2)\Big]\Big|_{\overline{MS},\, \mu=M_Z}$$

understood as regularized in the $\overline{MS}$ sence,

The leading in m_t^2 terms should be considered *Consoli, Hollik, Jegerlehner, 1989*, with the coupling constant G_F i.e. one should perform a *conversion* of couplings:

$$\alpha \to G_F$$

This may be achieved with the factor

$$f = \frac{\sqrt{2}G_F M_Z^2 s_W^2 c_W^2}{\pi\alpha}$$

The superscript G will emphasises the use of the scale G_F.

The leading QCD terms $\delta\rho_L^{G\alpha_S} + \delta\rho_L^{G\alpha_S^2}$ should be treated with the scale G_F, *Halzen and Kniehl, 1991*.

Δr at one loop

$$\Delta r = \Delta\alpha(M_Z^2) + \frac{c_W^2}{s_W^2}\Delta\hat{\rho}^{\alpha} + \Delta\hat{r}_{\mathrm{rem}}$$

contains two *large* terms: 1) the running coupling $\Delta\alpha^{(5)}(M_Z^2)$, *logarithmically* enhanced by fermionic mass singularities,
2) the parameter $\Delta\rho$ enhanced *quadratically* by m_t^2.
Notion of *leading* and *reminder* terms an of *leading-reminder splitting*.

Δr beyond one loop

With the two-loop EW terms $\mathcal{O}\left(G_F^2 m_t^4\right)$, $\mathcal{O}\left(G_F^2 m_t^2 M_Z^2\right)$ under controll, the Δr must be written as follows

$$\frac{1}{1-\Delta r} = \frac{1}{\left[1-\Delta\alpha(M_Z^2) - \Delta r^G_{\mathrm{rem}}\right]\left[1 - \frac{c_W^2}{s_W^2}\delta\hat{\rho}^G\right]}$$

(*Degrassi, Gambino, Vicini, Sirlin, 1996, 1997*)

Irreducible higher order corrections are applied by means of simple modifications of the leading and remainder terms

$$\delta\hat{\rho}^G \rightarrow \delta\hat{\rho}^{(G)}$$
$$\Delta r^G_{\mathrm{rem}} \rightarrow \Delta r^{(G)}_{\mathrm{rem}}$$

where (G) denotes sum of all available corrections and their scale

$$\delta\hat{\rho}^{(G)} = \delta\hat{\rho}^G + \delta\hat{\rho}^{G\alpha_S} + \delta\hat{\rho}^{G\alpha_S^2} + \delta\hat{\rho}^{G^2}$$
$$\Delta r^{(G)}_{\mathrm{rem}} = \Delta r^G_{\mathrm{rem}} + \Delta r^{G\alpha_S}_{\mathrm{rem}} + \Delta r^{G^2}_{\mathrm{rem}}$$

There is available yet qudratically enchanced two loop Higgs contribution *van der Bij, Veltman, 1984*

$$\Delta r_L^{H,\alpha^2} = -0.005832\frac{\alpha^2}{\pi^2 s_\theta^4}\frac{M_H^2}{M_W^2}$$

which is not accounted for by $\mathcal{O}(G_F^2 m_t^4)$ and $\mathcal{O}(G_F^2 m_t^2 M_Z^2)$ terms.

Z-decay

The amplitude of the decay $Z \to f\overline{f}$

$$\sim \sqrt{\rho_Z^f} I_f^{(3)} \gamma_\mu \left[(1+\gamma_5) - 4|Q_f|\kappa_Z^f s_W^2\right]$$

ρ_Z^f and κ_Z^f – *effective Z-decay couplings*, some complex-valued constants; an example of *amplitude form factors*

$$\rho_Z^f \equiv \rho_Z^{f,\alpha}, \qquad \kappa_Z^f \equiv \kappa_Z^{f,\alpha}$$

Two cases:

1) Z decays into a pair of leptons $l = e, \mu, \tau$:

$$\Gamma_l \equiv \Gamma\left(Z \to l\overline{l}\right) \;=\; \Gamma_0 |\rho_Z^f| \sqrt{1 - \frac{4m_l^2}{M_Z^2}} \left[\left(1 + \frac{2m_l^2}{M_Z^2}\right)(|g_Z^l|^2 + 1) - \frac{6m_l^2}{M_Z^2}\right] \times \left[1 + \frac{3}{4}\frac{\alpha(M_Z^2)}{\pi} Q_f^2\right]$$

Here

$$\Gamma_0 = \frac{G_F M_Z^3}{24\sqrt{2}\,\pi} = 82.945(7)\,\mathrm{MeV}$$

is the ‘standard’ partial width; the complex-valued variable

$$g_Z^f = \frac{v_f}{a_f} = 1 - 4|Q_f|(\kappa_Z^f s_W^2 + I_f^2)$$

is the ratio of effective vector over axial couplings,

$$v_f \;=\; I_f^{(3)} - 2Q_f(\kappa_Z^f s_W^2 + I_f^2), \qquad a_f \;=\; I_f^{(3)}$$

The combinations,

$$\sin^2\theta_{\mathrm{eff}}^f = \mathrm{Re}(\kappa_Z^f) s_W^2 + I_f^2$$

define the flavour dependent *effective weak mixing angles*.

$I_f^{(3)}, Q_f, c_f$ – third projection of weak isospin, fraction of electric charge and color factor
I_f^2 is an $\mathcal{O}(\alpha^2)$ real-valued term originating from the product of two imaginary parts

$$I_f^2 = \left(\frac{\alpha(s)}{4\,\pi}\right)^2 \mathrm{Im}\Pi_{\gamma\gamma}(s)\;\mathrm{Im}\Pi_{ZA}(s) = \alpha^2(s)\frac{35}{18}\left[1 - \frac{8}{3}\mathrm{Re}(\kappa_Z^f) s_W^2\right]$$

giving a non-negligible contribution (π^2 enchanced)

2) $\underline{Z \text{ decays into a pair of quarks}}$ $f = q = u, d, c, s, b$:

$$\Gamma_f \equiv \Gamma\left(Z \to f\overline{f}\right) = \Gamma_0\, c_f\, |\rho^f_Z| [|g^f_Z|^2\, R^f_V(M^2_Z) + R^f_A(M^2_Z)] + \Delta_{\mathrm{EW/QCD}}$$

Radiator factors, $R^f_{V,A}(M^2_Z)$
describe corrections from an interaction in the final state, FSR,
(*Chetyrkin, Gorishny, Kataev, Kühn, Larin, et al. 1979-1997*)
QED, QCD, mixed QED $\times$ QCD

$$\begin{aligned}
R^q_V(s) &= 1 + \frac{3}{4}Q^2_q\frac{\alpha(s)}{\pi} + \frac{\alpha_S(s)}{\pi} - \frac{1}{4}Q^2_q\frac{\alpha(s)}{\pi}\frac{\alpha_S(s)}{\pi} \\
&\quad + C_{02}\left(\frac{\alpha_S(s)}{\pi}\right)^2 + C_{03}\left(\frac{\alpha_S(s)}{\pi}\right)^3 \\
&\quad + \frac{m^2_c(s)}{s}C^{c,V}_2 + \frac{m^2_b(s)}{s}C^{b,V}_2 + \frac{m^4_c(s)}{s^2}C^{c,V}_4 + \frac{m^4_b(s)}{s^2}C^{b,V}_4 \\
&\quad + \text{vector singlet contribution},
\end{aligned}$$

$$\begin{aligned}
R^q_A(s) &= 1 + \frac{3}{4}Q^2_q\frac{\alpha(s)}{\pi} + \frac{\alpha_S(s)}{\pi} - \frac{1}{4}Q^2_q\frac{\alpha(s)}{\pi}\frac{\alpha_S(s)}{\pi} \\
&\quad + C_{02}\left(\frac{\alpha_S(s)}{\pi}\right)^2 + C_{03}\left(\frac{\alpha_S(s)}{\pi}\right)^3 \\
&\quad + \frac{m^2_c(s)}{s}C^{c,A}_2 + \frac{m^2_b(s)}{s}C^{b,A}_2 + \frac{m^4_c(s)}{s^2}C^{c,A}_4 + \frac{m^4_b(s)}{s^2}C^{b,A}_4 \\
&\quad + \text{axial singlet contribution}
\end{aligned}$$

All $C^{q,\{V,A\}}_{2,4}$ are presently known up to $\mathcal{O}\left(\alpha^3_S\right)$, very important for energies not too far from quark threshold $4m^2_q$.
For $C^{q,\{V,A\}}_4$ owing to the work by *K.G. Chetyrkin, R. Harlander, J.H. Kühn: Abstract # 1_444 to EPS-HEP99*

K.G. Chetyrkin, J.H. Kühn, A. Kwiatkowski, Rhys. Rep. 277(1996)189.

$\Delta_{\mathrm{EW/QCD}}$, is *non-factorizable* EW⊗QCD correction (*Czarnecki, Kühn, 1996; Harlander, Seidensticker, Steinhauser, 1998; Fleischer, Jegerlehner, Tentyukov, Veretin, 1999; York-Peng E. Yao: Abstract #* 5_376 *to EPS-HEP99*):

$$\Delta_{\mathrm{EW/QCD}} = \begin{cases} -0.113 \;\; \mathrm{MeV}, & \text{for } u \text{ and } c \text{ quarks} \\ -0.160 \;\; \mathrm{MeV}, & \text{for } d \text{ and } s \text{ quarks} \\ -0.040 \;\; \mathrm{MeV}, & \text{for } b \text{ quark} \end{cases}$$

The form factors comprise one-loop and known two-loop electroweak and internal QCD corrections $\mathcal{O}\left(\alpha\alpha_{S}\right)$. Their implementation relies on the notion of leading and reminder terms.

In the lowest order:

$$\begin{aligned} \rho_{Z}^{f} &= 1 - \delta\hat{\rho}^{\alpha} + \delta\rho^{f,\alpha}_{\mathrm{rem}} \\ \kappa_{Z}^{f} &= 1 - \frac{c^2_W}{s^2_W}\delta\hat{\rho}^{\alpha} + \delta\kappa^{f,\alpha}_{\mathrm{rem}} \end{aligned}$$

With higher orders, resummed:

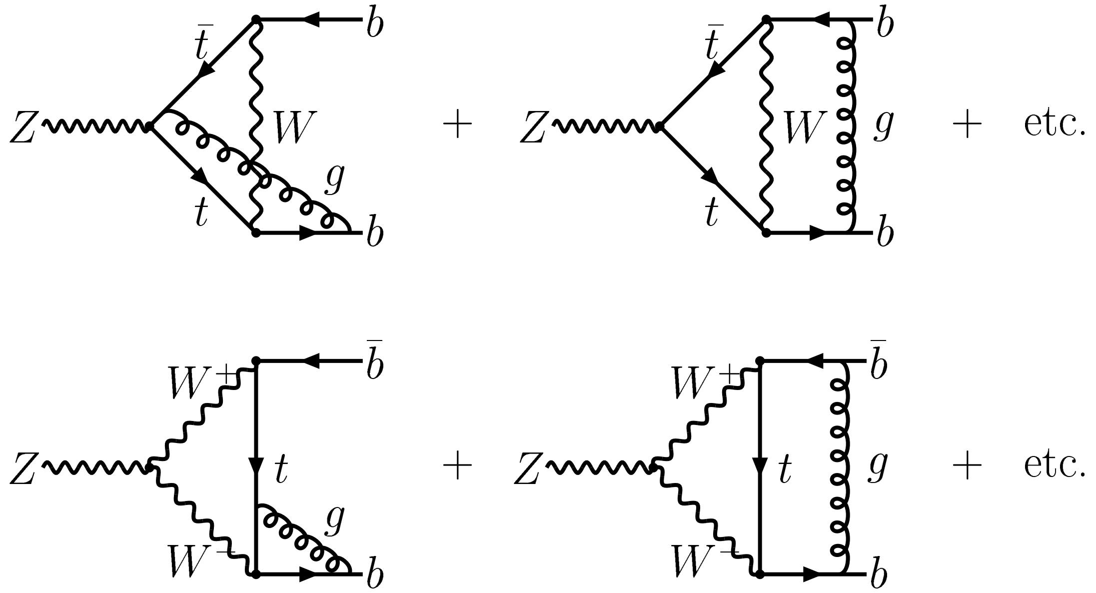
$$\begin{aligned} \rho_{Z}^{f} &= \frac{1 + \delta\rho^{f,[G]}_{\mathrm{rem}} + \delta\rho^{f,G^2}_{\mathrm{rem}}}{1 + \delta\hat{\rho}^{(G)}\left(1 - \Delta\bar{r}^{[G]}_{\mathrm{rem}}\right)} \\ \kappa_{Z}^{f} &= \left(1 + \delta\kappa^{f,[G]}_{\mathrm{rem}} + \delta\kappa^{f,G^2}_{\mathrm{rem}}\right)\left[1 - \frac{c^2_W}{s^2_W}\delta\hat{\rho}^{(G)}\left(1 - \Delta\bar{r}^{[G]}_{\mathrm{rem}}\right)\right] \end{aligned}$$

(G) stands for inclusion of all known terms, $[G]$ denotes inclusion of one-loop EW corrections together with all known orders in α_{S}, i.e. $[G] = G + G\alpha_{S} + \ldots$

$$\Delta\bar{r}^{[G]}_{\mathrm{rem}} = \Delta\bar{r}^{G}_{\mathrm{rem}} + \Delta r^{G\alpha_S}_{\mathrm{rem}},$$

where $\Delta\bar{r}^{G}_{\mathrm{rem}}$ is a part of one-loop remainder with extracted $\Pi_{\gamma\gamma}$ contributions, (*Degrassi, Gambino, 1999*)

Non-universal corrections for the $Z \to b\bar{b}$ contribute additional m_t-dependent terms that are absent in the cases of light quarks (*Akhundov, Bardin, Riemann, 1986; Beenakker, Hollik, 1988; Bernabeu, Pich, Santamaria, 1988, 1991*), and also higher order vertex corrections



They are into account with the correction τ_b:

$$\tau_b = -2x_t \left[1 - \frac{\pi}{3}\alpha_S(m_t^2) + x_t \tau^{(2)} \left(\frac{m_t^2}{M_H^2} \right) \right]$$

second term – `FTJR` correction – (*Fleischer et al., Buchalla, Buras, 1993; Degrassi, 1993 Chetyrkin, Kwiatkowski, Steinhauser, 1993*), the last term (*Barbieri et al., 1992*).

τ_b is acconted for by means of the following replacement:

$$\rho_b \to \rho_b \left(1 + \tau_b\right)^2, \qquad \kappa_b \to \frac{\kappa_b}{(1 + \tau_b)}$$

as was proved in *Barbieri et al., 1992.*

A compact form of the function $\tau^{(2)}$ was found by *Fleischer, Tarasov, Jegerlehner, 1993.*

$\mathcal{O}\left(G_F^2 m_t^2 M_Z^2\right)$ **corrections are not known!!!**

Pseudo-Observables, PO's

Examples:

- Δr and related quantity M_W or $s^2_W = 1 - M^2_W/M^2_Z$;
- all partial widths, Γ_f, in particular:
  - invisible widths: $\Gamma_{\rm inv} = 3\Gamma_\nu$,
  - total hadronic widths: $\Gamma_h = \sum_q \Gamma_q$, the sum of all quarkonic widths,
  - the total width: $\Gamma_Z = \sum_f \Gamma_f$;
- quantities strightforwardly derived from widths:
  - ratios $R_l = \dfrac{\Gamma_h}{\Gamma_e}, \qquad R^0_f = \dfrac{\Gamma_f}{\Gamma_h}$
  - the hadronic and leptonic pole cross-sections

$$\sigma^0_h = 12\pi \frac{\Gamma_e \Gamma_h}{M^2_Z \Gamma^2_Z}, \qquad \sigma^0_\ell = 12\pi \frac{\Gamma_e \Gamma_l}{M^2_Z \Gamma^2_Z}$$

- effective weak mixing angles $\sin^2\theta^f_{\rm eff}$;
- ρ^f_Z parameters;
- *coupling factors* trivially made of **real parts** of ratios of effective couplings (aka *left-right flavor asymmetries*):

$$\mathcal{A}_f = \frac{2{\rm Re}g^f_Z}{({\rm Re}g^f_Z)^2 + 1}$$

- *forward-backward peak asymmetries* are mere combinations of coupling factors:

$$A^{\rm 0f}_{\rm FB} = \frac{3}{4}\mathcal{A}_e \mathcal{A}_f$$

Definition: In a very general sence, the pseudo-observable is a construction made of gauge invariant form factors of an amplitude of a process, for instance Z decay.

Observable	TOPAZ0	ZFITTER	$10^3 \times \frac{T-Z}{T}$
$1/\alpha^{(5)}(M_Z)$	128.877	128.877	
$1/\alpha(M_Z)$	128.887	128.887	
M_W [GeV]	80.3731	80.3738	-0.009
σ_h^0 [nb]	41.4761	41.4777	-0.04
σ_l^0 [nb]	1.9995	1.9997	-0.12
Γ_h [GeV]	1.74211	1.74223	-0.07
Γ_Z [GeV]	2.49549	2.49573	-0.10
Γ_ν [MeV]	167.207	167.234	-0.16
Γ_e [MeV]	83.983	83.995	-0.14
Γ_μ [MeV]	83.983	83.995	-0.14
Γ_τ [MeV]	83.793	83.805	-0.14
Γ_u [MeV]	300.129	300.154	-0.08
Γ_d [MeV]	382.961	382.996	-0.09
Γ_c [MeV]	300.069	300.092	-0.08
Γ_b [MeV]	375.997	375.993	0.01
$\Gamma_{\rm inv}$ [GeV]	0.50162	0.50170	-0.16
R_l	20.7435	20.7420	0.07
R_b^0	0.215829	0.215811	0.08
R_c^0	0.172245	0.172246	-0.01
$\sin^2\theta_{\rm eff}^{\rm lept}$	0.231596	0.231601	-0.02
$\sin^2\theta_{\rm eff}^{b}$	0.232864	0.232950	-0.37
$\sin^2\theta_{\rm eff}^{c}$	0.231491	0.231495	-0.02
ρ_e	1.00513	1.00528	-0.15
ρ_b	0.99413	0.99424	-0.11
ρ_c	1.00582	1.00598	-0.16
Observable	TOPAZ0	ZFITTER	$10^3 \times (T - Z)$
$A_{\rm FB}^{0,l}$	0.016084	0.016074	0.01
$A_{\rm FB}^{0,b}$	0.102594	0.102617	-0.02
$A_{\rm FB}^{0,c}$	0.073324	0.073300	0.02
$\mathcal{A}_e$	0.146440	0.146396	0.04
$\mathcal{A}_b$	0.934654	0.934607	0.05
$\mathcal{A}_c$	0.667609	0.667595	0.01

Table 1: Complete table of POs, from TOPAZ0 and ZFITTER.

`TOPAZO/ZFITTER` Comparison for Pseudo-Observables

IPS:

$$\begin{aligned} M_Z &= 91.1867\,\mathrm{GeV}, \quad m_t = 173.8\,\mathrm{GeV}, \quad M_H = 100\,\mathrm{GeV}, \\ \alpha_S(M_Z) &= 0.119, \quad G_F = 1.16639 \times 10^{-5}\,\mathrm{GeV}^{-2} \end{aligned}$$

PO's and relative and absolute difference between `TOPAZO` and `ZFITTER` in units of per-mill:

$$\delta = 10^3 \times \frac{\mathrm{T} - \mathrm{Z}}{\mathrm{T}} \qquad \Delta = 10^3 \times \; (\mathrm{T} - \mathrm{Z})$$

Some conclusions:

- deviations are typically below 0.15 per-mill:
- with an exception of $\sin^2\theta^b_{\mathrm{eff}}$ with a difference of 0.4 per-mill
- wide M_H range $M_H = 10, 30, 100, 300, 1000\,\mathrm{GeV}$, was investigated:
  - largest deviation for $\mathcal{A}_e$ at $M_H = 10\,\mathrm{GeV}$ – 0.14×10^{-3};
  - for $M_H = 100\,\mathrm{GeV}$ all deviations are below 0.08×10^{-3}
- CERN Yellow Report 95–03 versus 1999,
  - for $\sin^2\theta^{\mathrm{lept}}_{\mathrm{eff}}$ was 0.21 now 0.004 per-mill at $M_H = 300\,\mathrm{GeV}$ and it is now everywhere below 0.06 per-mill,
  - for M_W was 0.087 now 0.015 per-mill at $M_H = 300\,\mathrm{GeV}$ with maximum of 0.029 for very large values of M_H
  - for $\mathcal{A}_e$ was 0.38×10^{-3} now 0.05×10^{-3} at $M_H = 300\,\mathrm{GeV}$ with a maximum of 0.14×10^{-3} at low values of M_H

Theoretical Uncertainties from TOPAZO

TOPAZO has been run over all the options remaining after implementation of NLO corrections.

- *central* values are evaluated with the *preferred setup*;
- minus error for $\mathrm{PO}_{central} - \min_{\mathrm{opt}}\ \mathrm{PO}$;
- plus error for $\max_{\mathrm{opt}}\ \mathrm{PO} - \mathrm{PO}_{central}$.

Observable	central	minus error	plus error	total exp. error
$1/\alpha^{(5)}(M_Z)$	128.877	-	-	
$1/\alpha(M_Z)$	128.887	-	-	
M_W [GeV]	80.3731	5.8 MeV	0.3 MeV	64 MeV
σ_h^0 [nb]	41.4761	1.0 pb	1.6 pb	58 pb
σ_ℓ^0 [nb]	1.9995	0.17 pb	0.26 pb	3.5 pb
Γ_ν [MeV]	167.207	0.017	0.001	
Γ_e [MeV]	83.983	0.010	0.0005	0.10^*
Γ_μ [MeV]	83.983	0.010	0.0005	
Γ_τ [MeV]	83.793	0.010	0.0005	
Γ_u [MeV]	300.129	0.047	0.013	
Γ_d [MeV]	382.961	0.054	0.010	
Γ_c [MeV]	300.069	0.047	0.013	
Γ_b [MeV]	375.997	0.208	0.077	
Γ_{had} [GeV]	1.74211	0.26 MeV	0.11 MeV	2.3 MeV*
Γ_{inv} [GeV]	0.50162	0.05 MeV	0.002 MeV	1.9 MeV*
Γ_Z [GeV]	2.49549	0.34 MeV	0.11 MeV	2.4 MeV

Table 2: Theoretical uncertainties on POs from TOPAZO.
The experimental errors – summer 1998.
$*$) assumes lepton universality.

Observable	central	minus error	plus error	total exp. error
R_l	20.7435	0.0020	0.0013	0.026*
R_b^0	0.215829	0.000100	0.000031	0.00074
R_c^0	0.172245	0.000005	0.000024	0.0044
$\sin^2\theta_{\rm eff}^{\rm lept}$	0.231596	0.000035	0.000033	0.00018*
$\sin^2\theta_{\rm eff}^{b}$	0.232864	0.000002	0.000048	
$\sin^2\theta_{\rm eff}^{c}$	0.231491	0.000029	0.000033	
$A_{\rm FB}^{0,l}$	0.016084	0.000057	0.000060	0.00096*
$A_{\rm FB}^{0,b}$	0.102594	0.000184	0.000195	0.0021
$A_{\rm FB}^{0,c}$	0.073324	0.000142	0.000149	0.0044
$\mathcal{A}_e$	0.146440	0.000259	0.000275	0.0017*
$\mathcal{A}_b$	0.934654	0.000032	0.000005	0.035
$\mathcal{A}_c$	0.667609	0.000114	0.000103	0.040
ρ_e	1.00513	0.00010	0.000005	0.0012*
ρ_b	0.99413	0.00048	0.000001	
ρ_c	1.00582	0.00010	0.000005	

Table 3: Theoretical uncertainties on POs from `TOPAZ0`.
The experimental errors – summer 1998.
$*$) assumes lepton universality.

One can see from Tables that there is a sizeable reduction of the theoretical uncertainties of POs as compared to CERN Yellow Report 95–03.

This is mainly due to the implementation of:
– next-to-leading two loop EWRC corrections $\mathcal{O}\left(G_F^2 m_t^2 M_Z^2\right)$
– and non-factorizable EW⊗QCD corrections
in `TOPAZ0` and `ZFITTER`.

Caution:
Complete two-loop EW calculations are still lacking!

LEPTOP fit to electroweak observables

Observable	Exp. data	Fit one-loop	Pull	Fit two-loops	Pull
Γ_Z [GeV]	2.4939(24)	2.4959(18)	-0.8	2.4960(18)	-0.9
σ_h [nb]	41.491(58)	41.472(16)	0.3	41.472(16)	0.3
R_l	20.765(26)	20.747(20)	0.7	20.746(20)	0.7
A^l_{FB}	0.0168(10)	0.0161(3)	0.8	0.0161(4)	0.7
A_τ	0.1431(45)	0.1465(14)	-0.8	0.1467(16)	-0.8
A_e	0.1479(51)	0.1465(14)	0.3	0.1467(16)	0.2
R_b	0.2166(7)	0.2158(2)	1	0.2158(2)	1.0
R_c	0.1735(44)	0.1723(1)	0.3	0.1723(1)	0.3
A^b_{FB}	0.0990(21)	0.1027(10)	-1.8	0.1028(12)	-1.8
A^c_{FB}	0.0709(44)	0.0734(8)	-0.6	0.0734(9)	-0.6
$s^2_l(Q_{FB})$	0.2321(10)	0.2316(2)	0.5	0.2316(2)	0.5
$s^2_l\ (A_{LR})$	*0.2311(3)*	*0.2316(2)*	*-1.7*	*0.2316(2)*	-1.6
A_b	0.8670(350)	0.9348(1)	-1.9	0.9348(2)	-1.9
A_c	0.6470(400)	0.6676(6)	-0.5	0.6677(7)	-0.5
m_W [GeV]	80.3902(64)	80.36(3)	0.5	80.37(3)	0.4
$s^2_W\ (\nu N)$	0.2254(21)	0.2234(6)	0.9	0.2233(7)	1.0
m_t [GeV]	173.8(5.0)	171.6(4.9)	0.4	170.8(4.9)	0.6
m_H [GeV]		139^{+134}_{-76}		71^{+82}_{-43}	
$\hat{\alpha}_s$		0.1195(30)		0.1194(29)	
$\chi^2/n_{d.o.f.}$		15.1/14		15.0/14	

LEPTOP Tampere July 1999

Novikov, Okun, Rozanov, Vysotsky, 1999

The Improved Born Approximation, IBA

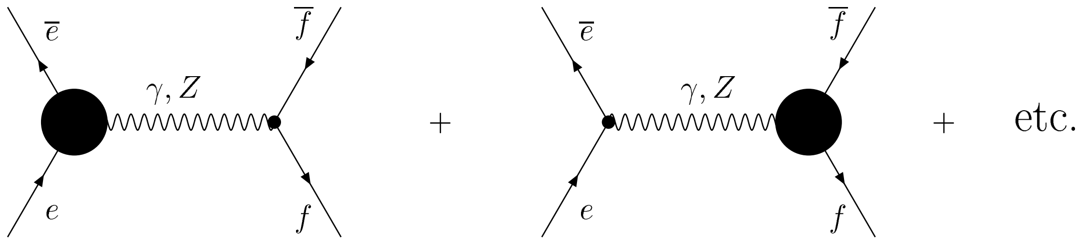


Figure 1: Process $e\overline{e} \to (Z, \gamma) \to f\overline{f}$ in the IBA

Improved γ exchange:

$$A_{\gamma}^{\mathrm{IBA}} = \chi_{\gamma}(s)\alpha(s)\gamma_{\mu} \otimes \gamma_{\mu}$$

with running electromagnetic coupling $\alpha(s)$

Improved Z exchange:

$$\alpha(s) = \frac{\alpha}{1 - \dfrac{\alpha}{4\pi}\Big[\Pi_{\gamma\gamma}^{\mathrm{fer}}(s) - \Pi_{\gamma\gamma}^{\mathrm{fer}}(0)\Big]}$$

$$\begin{aligned}
\mathcal{A}_{Z}^{\mathrm{IBA}}(s,t) \;=\; & \sqrt{2}G_{\mu}I_e^{(3)}I_f^{(3)}M_{Z}^2\chi_Z(s)\rho_{ef}(s,t)\Big\{\gamma_{\mu}(1+\gamma_5)\otimes\gamma_{\mu}(1+\gamma_5) \\
& -4|Q_e|s_{W}^2\kappa_e(s,t)\gamma_{\mu}\otimes\gamma_{\mu}(1+\gamma_5) \\
& -4|Q_f|s_{W}^2\kappa_f(s,t)\gamma_{\mu}(1+\gamma_5)\otimes\gamma_{\mu} \\
& +16|Q_eQ_f|s_{W}^4\kappa_{e,f}(s,t)\gamma_{\mu}\otimes\gamma_{\mu}\Big\}
\end{aligned}$$

with **four** complex-valued electroweak form factors – **functions** of (s,t) (t due to weak boxes)

$$\rho_{ef}(s,t), \quad \kappa_e(s,t), \quad \kappa_f(s,t), \quad \kappa_{e,f}(s,t)$$

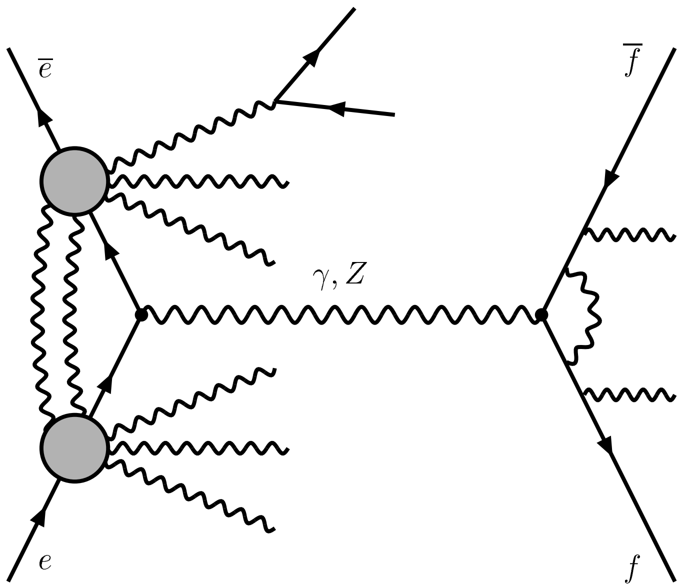
Propagator factors:

$$\chi_{\gamma}(s) = 4\pi\frac{Q_eQ_f}{s}, \qquad \chi_Z(s) = \frac{1}{s - M_{Z}^2 + is\dfrac{\Gamma_{Z}}{M_{Z}}}$$

Chain of calculations

IBA amplitudes $\rightarrow$ IBA cross-sections $\rightarrow$
$\rightarrow$ IBA cross-sections $\oplus$ FSR corrections

$$\rightarrow \sigma^{\mathbf{dec}} = \hat{\sigma}(\mathbf{s}', \mathrm{cuts})$$



ISR$\oplus$FSR QED corrections for $e\overline{e} \rightarrow (Z, \gamma) \rightarrow f\overline{f}$

$$\rightarrow \sigma^{\mathbf{con}} = \sigma(\mathbf{s}) = \int \mathbf{dz}\ \mathbf{H}_{\mathrm{in}}(\mathbf{z}, \mathbf{s})\, \hat{\sigma}(\mathbf{zs})$$

Pure QED ISR corrections

- From complete calculations up to $\mathcal{O}(\alpha^2)$ (*Berends, Burgers, van Neerven, 1988*)

- From structure function method in the leading logarithmic approximation $\mathcal{O}(\alpha^3 L^3)$ (*Kuraev and Fadin, 1988; Jadach et al, 1990–1992; Montagna et al, 1996*)

Semi-Model Independent approach

Short-hand notation for relevant combinations

$$\begin{aligned}
\rho_{ef} &= \rho_{ef}(s,t) \\
G_e &= 1 - 4|Q_e|[\kappa_e(s,t)s_W^2 + I_e^2] \\
G_f &= 1 - 4|Q_f|[\kappa_f(s,t)s_W^2 + I_f^2] \\
G_{ef} &= -1 + G_e + G_f + 16|Q_e||Q_f|\{s_W^4\kappa_{ef}(s,t) \\
&\quad + s_W^2[\kappa_e(s,t)I_f^2 + \kappa_f(s,t)I_e^2]\}
\end{aligned}$$

At the Z resonance, some approximations and factorizations are fulfilled:

$$\begin{aligned}
\rho_{ef}^2(M_Z^2,t) &\approx \rho_Z^e\rho_Z^f\,, \qquad t \text{ irrelevant} \\
\kappa_{ef}(M_Z^2,t) &\approx \kappa_Z^e\kappa_Z^f \\
\kappa_e(M_Z^2,t) &\approx \kappa_Z^e \\
\kappa_f(M_Z^2,t) &\approx \kappa_Z^f
\end{aligned}$$

If factorization property were fulfilled, then we would have

$$G_{ef} = G_e G_f$$

Short-hand notations for the real parts of on-resonance couplings

$$\rho_f = \mathrm{Re}\,\rho_Z^f\,, \qquad g_f = \mathrm{Re}\,g_Z^f$$

Let $\bar{\rho}_e,\ \bar{\rho}_f,\ \bar{g}_e,\ \bar{g}_f$ denote constant, real-valued coupling constants to be fitted to the experimental data.

Define the *SM trajectory*

$$\begin{aligned}
\bar{\rho}_e &= \rho_e\,, \qquad & \bar{\rho}_f &= \rho_f \\
\bar{g}_e &= g_e\,, \qquad & \bar{g}_f &= g_f
\end{aligned}$$

In the vicinity of Z resonance the following approximations hold:

$$\begin{aligned}
\rho_{ef} &\approx \sqrt{\rho_e\rho_f} \\
G_e &\approx g_e \\
G_f &\approx g_f \\
G_{ef} &\approx g_e g_f
\end{aligned}$$

The makes meaningful to define new set of couplings

$$\begin{aligned}
\hat{\rho}_{ef} &= \sqrt{\bar{\rho}_e\bar{\rho}_f} - \sqrt{\rho_e\rho_f} + \rho_{ef} \\
\hat{G}_e &= \bar{g}_e - g_e + G_e \\
\hat{G}_f &= \bar{g}_f - g_f + G_e \\
\hat{G}_{ef} &= \bar{g}_e\bar{g}_f - g_e g_f + G_{ef}
\end{aligned}$$

At the SM trajectory,

$$\hat{\rho}_{ef} = \rho_{ef}\,, \quad \text{etc.}$$

The quantities

$$-\sqrt{\rho_e\rho_f} + \rho_{ef}\,, \quad \text{etc.}$$

represent an example of the so-called **SM remnants**.

This is the essence of a parameter release with exact s-dependent SM remnants.

Procedure is not unique,
<u>multiplicative variant</u>:

$$\begin{aligned}
\hat{\rho}_{ef} &= \frac{\sqrt{\bar{\rho}_e\bar{\rho}_f}}{\sqrt{\rho_e\rho_f}}\rho_{ef} \\
\hat{G}_e &= \frac{\bar{g}_e}{g_e}G_e \\
\hat{G}_f &= \frac{\bar{g}_f}{g_f}G_e \\
\hat{G}_{ef} &= \frac{\bar{g}_e\bar{g}_f}{g_e g_f}G_{ef}
\end{aligned}$$

<u>SM-remnants fixed at M_Z</u>, `ZFITTER option MISD=0.`

Glossary

- ISR, FSR;
- IFI – Initial Final Interference, a.k.a. ISR/FSR-interference
  - $\mathcal{O}(\alpha)$ – in the lowest QED order ;
  - $\mathcal{O}(\alpha)_{\mathrm{CEEX}}$ – exponentiation at the amplitude level, that is *Jadach, Ward*;
- DD – Doubly Deconvoluted observable (cross-section, asymmetry) that is, without ISR and FSR QED corrections (IFI);
- SD – Single-de-convolution, the kernel cross-sections without initial-state QED radiation, but including all final-state correction factors (without IFI);
- CA3 – Complete RO, with QED ISR implemented through an additive $\mathcal{O}(\alpha^3)$ radiator, that is *Kuraev, Fadin; Berends, Burgers, van Neerven* type;
- CF3 – Complete RO, with QED initial state radiation implemented through a factorized $\mathcal{O}(\alpha^3)$ radiator, that is *Jadach, Skrzypek* type;
- ISPP – Initial State Pair Production corrections:
  - KKKS – complete lowest QED order $\mathcal{O}(\alpha^2)$, that is *Kniehl, Krawczyk, Kühn, and Stuart, 1988*;
  - JSM – third order exponentiated, that is *Jadach, Skrzypek, Martinez, 1992*;
- Kinemetical cuts:
  - s' – invariant mass of the propagator after ISR;
  - $M^2_{f\bar{f}}$ – invariant mass of the final state fermion pair;

– Realistic cuts – $E_\pm > E_{\rm th}$, $\theta_{\rm acol} < \bar{\theta}$;

- Options to simulate theoretical uncertainties:
  - set of working options;
  - preferred set of options;
  - asymmertric theoretical errors.
- Setup for RO's

INPUT	$M_Z = 91.1867, m_t = 173.8, M_H = 100, \alpha_S(M_Z) = 0.119$
$\sqrt{s}$	$M_Z - 3,\ M_Z - 1.8,\ M_Z,\ M_Z + 1.8,\ M_Z + 3$
s_0/s	0.01
WEAK BOXES	YES
IFI	YES/NO
ISPP	YES/NO

Table 4: Extrapolated setup for the calculations of ROs. Masses and $\sqrt{s}$ are in GeV.

σ_μ [nb] with $\theta_{\rm acol} < 10^\circ$					
$\theta_{\rm acc} = 0$	$M_Z - 3$	$M_Z - 1.8$	M_Z	$M_Z + 1.8$	$M_Z + 3$
TOPAZ0	0.21932	0.46287	1.44795	0.67725	0.39366
	0.21776	0.46083	1.44785	0.67894	0.39491
	–7.16	**–4.43**	**–0.07**	**+2.49**	**+3.17**
ZFITTER	0.21928	0.46284	1.44780	0.67721	0.39360
	0.21772	0.46082	1.44776	0.67898	0.39489
	–7.16	**–4.40**	**–0.03**	**+2.60**	**+3.27**

Table 1: *A comparison of cross-section predictions around the Z peak*

$A^\mu_{\rm FB}$ with $\theta_{\rm acol} < 10^\circ$					
$\theta_{\rm acc} = 0$	$M_Z - 3$	$M_Z - 1.8$	M_Z	$M_Z + 1.8$	$M_Z + 3$
TOPAZ0	–0.28450	–0.16914	0.00033	0.11512	0.16107
	–0.28158	–0.16665	0.00088	0.11385	0.15936
	+2.92	**+2.49**	**+0.55**	**–1.27**	**–1.71**
ZFITTER	–0.28497	–0.16936	0.00024	0.11496	0.16083
	–0.28222	–0.16710	0.00083	0.11392	0.15926
	+2.75	**+2.27**	**+0.60**	**–1.03**	**–1.56**

Table 2: *A comparison of predictions for forward-backward asymmetries around the Z peak*

TOPAZ0: from Tables 37 and 38 of: D.Bardin et al., hep-ph/9902452
ZFITTER v.6.11: see DESY 99-070;
new QED calculation for accolinearity cut: P. Christova et al., Phys. Lett. B456 (1999) 264
First row: without IFI, second row: with IFI, third row: relative effect of IFI in per mil
IFI = initial-final-state interference

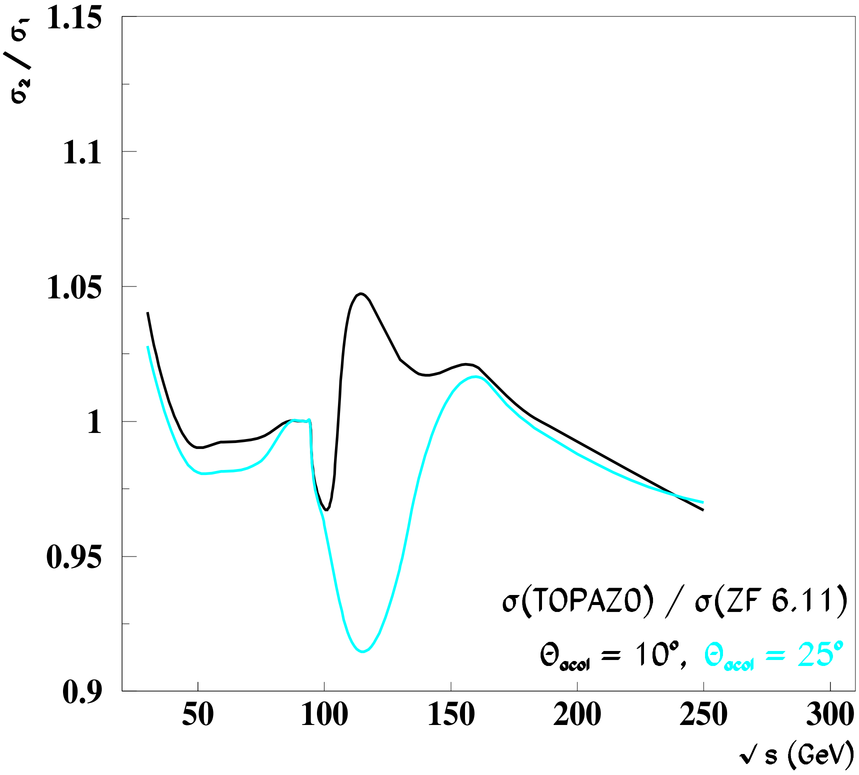


Figure 2: *Cross-section ratio* TOPAZO *v.4 (1999) versus* ZFITTER *v.6.11. (1999)*

An acceptance cut and two different acollinearity cuts are applied

TOPAZO: G. Montagna et al., Comput. Phys. Commun. 117 (1999) 278

Part of TOPAZO numbers: G. Passarino, priv. communication

ZFITTER: D. Bardin et al., DESY 99-070

Figure from: P. Christova et al., DESY 99-037

new QED calculation for acollinearity cut: P. Christova et al., Phys. Lett. B456 (1999) 264

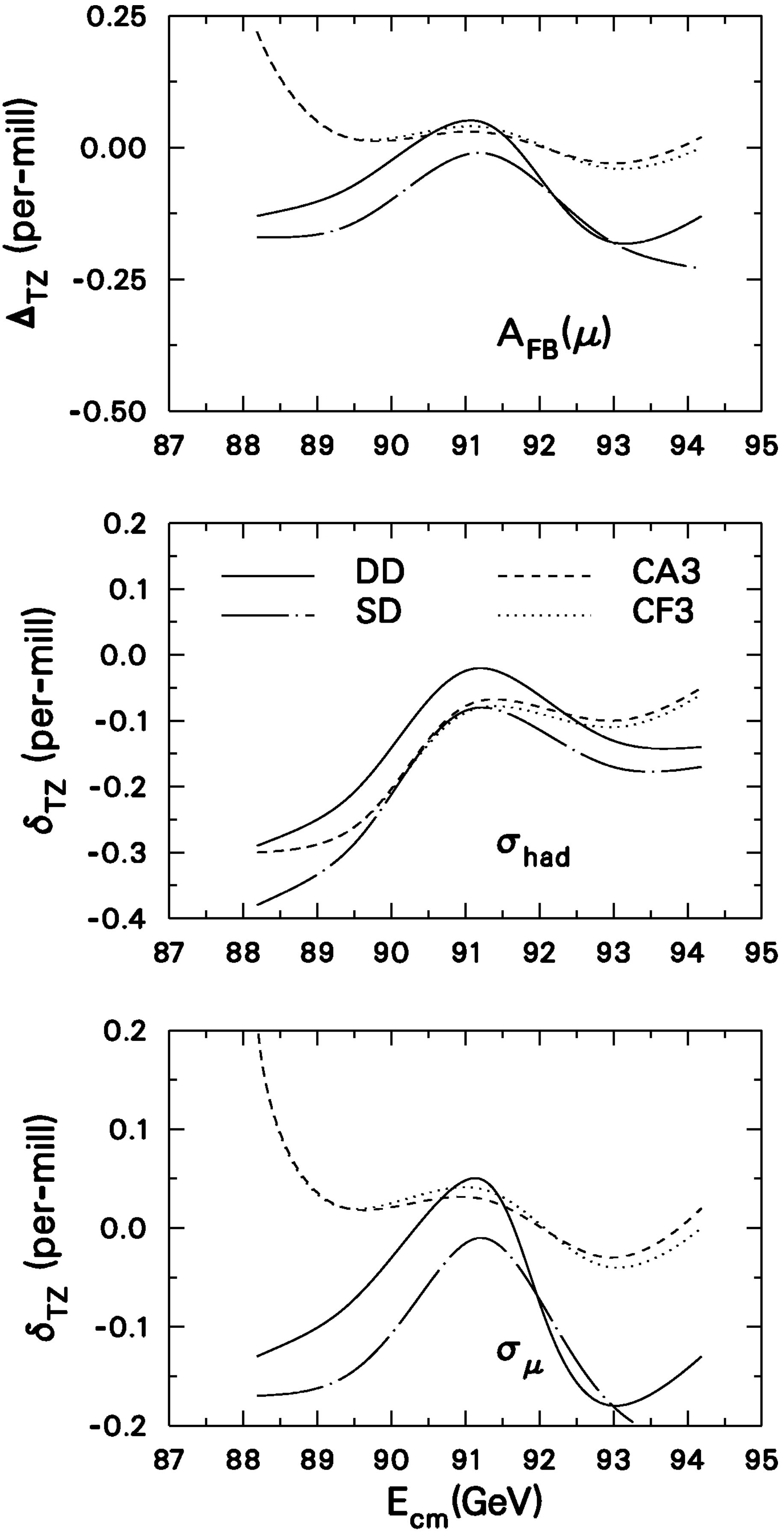


Figure 2: Relative deviations between `TOPAZ0` and `ZFITTER` for muonic and hadronic cross-section in DD, SD, CA3 and CF3 modes. The last three are corresponding to a cut $M^2(f\overline{f})$ of $0.01\,s$.

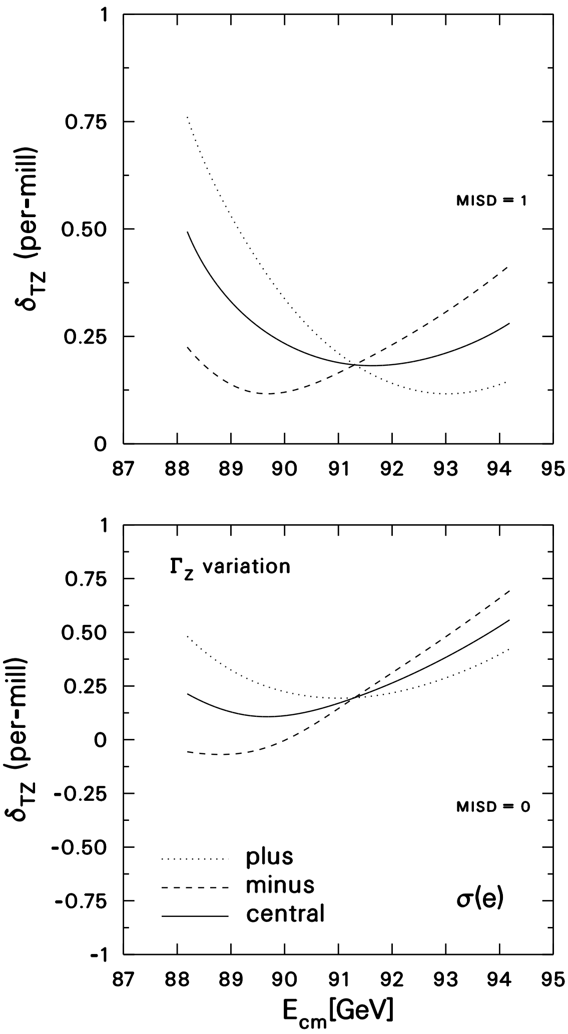
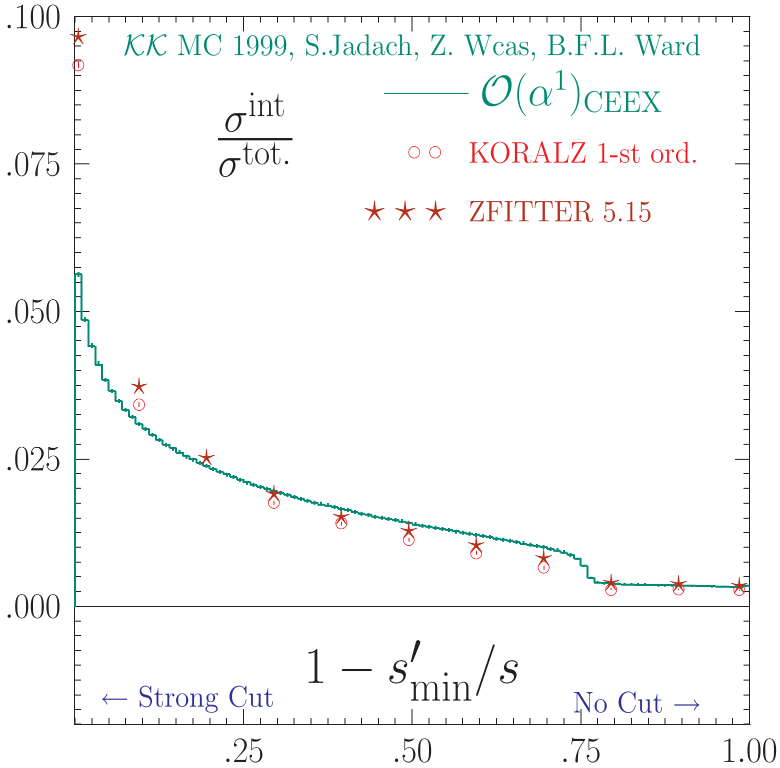


Figure 3: Relative deviations between `TOPAZ0` and `ZFITTER` for muonic cross-section in CA3 mode and for $s' > 0.01\, s$ and for two values of `MISD` option. The solid curve gives deviations corresponding to the current experimental value of Γ_Z, the dotted (dashed) curve corresponds to $\mathrm{PO} \pm 2 \times$ the experimental error.

ISR*FSR in σ, cut-off dependence

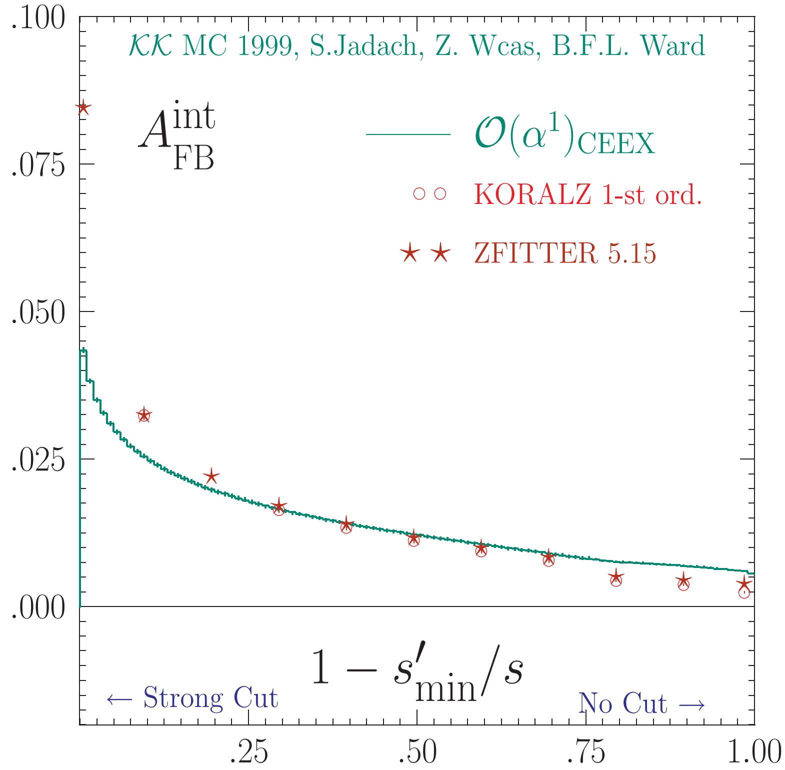
The ISR*FSR interference correction to $\sigma(s'_{\rm min})$ at 189GeV. No cut in $\cos\theta^\bullet$. From $\mathcal{KK}$ M.C. with $\mathcal{O}(\alpha^1)_{\rm CEEX}$ exponentiation at the amplitude level.



DESY

ISR*FSR in A_{FB}, energy cut-off study

The ISR*FSR interference correction to A_{FB} at 189GeV. No cut in $\cos\theta^{\bullet}$. From $\mathcal{KK}$ M.C. with $\mathcal{O}(\alpha^1)_{\rm CEEX}$ exponentiation at the amplitude level.



DESY

Pair corrections to e^+e^- annihilation at LEP

- The complete $O(\alpha^2)$ result for leptonic pairs:
  [1] F.A. Berends *et al.*, Nucl. Phys. B 88'.
- The complete $O(\alpha^2)$ result for hadronic pairs:
  [2] B.A. Kniehl *et al.*, Phys. Lett. B 88'.

The ISR pair radiation is accounted as:

$$\mathrm{d}\sigma^{\mathrm{pair}} = \int\limits_{z_{\min}}^{1} \mathrm{d}z \tilde{\sigma}(zs) H(z).$$

† The convergence of perturbative series for pairs is worse than for photons. Higher orders are strictly required.

† The LLA approximation is not good enough at Z-peak: the particular structure of sub-leading RC gives about 1/2 part at $O(\alpha^2)$ level.

- Simultaneous exponentiation of photonic and pair radiation (*phenomenological ansatz*):
  [3] S. Jadach, *et al.*, Phys. Lett. B 92'.
- The third order corrections as convolution of the pair radiator $H(z)$ with the photonic radiator, the fourth order is estimated in LLA:
  [4] A.B. Arbuzov in preparation.

Results for LEP1

$$\delta = 10^3 \, \frac{\sigma_{\rm RC}^{\rm pairs,\gamma} - \sigma_{\rm RC}^{\gamma}}{\sigma_{\rm RC}^{\gamma}} \,, \qquad z_{\rm min} = 0.04$$

$\boldsymbol{E}_{\rm cm}$	ZF(−1)	ZF(1)	[1] + [2]	[3] + 0.7 · [2]	[4]
88.1867	−2.23	−2.38	−3.22	−2.28	−2.23
89.3867	−2.51	−2.58	−3.61	−2.46	−2.50
91.1867	−2.49	−2.52	−3.44	−2.52	−2.47
92.9867	−0.69	−0.45	−0.43	−0.93	−0.99
94.1867	+0.78	+1.77	+1.36	+0.27	+0.08

Results for LEP2

$\boldsymbol{E}_{\rm cm}$	ZF(−1)	ZF(1)	[1] + [2]	[3] + 0.7 · [2]	[4]
189	11.26	62.58	34.70	13.09	34.90
without singlet pairs in [1] and [4]					
189	11.26	62.58	14.12	13.09	15.51

Note, we obtain 15.19 instead of 13.09 for [3], if we treat the hadronic pairs there in the same way as in [4].

Look at the LLA formula for singlet pair contribution to H [1]:

$$H^S(z) = \left(\frac{\alpha}{\pi}\right)^2 \ln\frac{s}{m_e^2}\left[\frac{1}{2}(1+z)\ln z + \frac{1}{3z} + \frac{1}{4} - \frac{z}{4} - \frac{z^2}{3}\right]$$

The term with z in denominator is responsible for the large correction due to singlet pairs.

$E_{\rm mc} = 189$ GeV, different values for $z_{\rm min}$

$z_{\rm min}$	F(−1)	ZF(1)	[1] + [2]	[3] + 0.7 · [2]	[4]
0.01	11.35	63.29	44.30	13.16	44.42
0.1	11.21	62.08	33.08	13.04	33.31
0.3	2.64	3.77	4.26	2.60	4.03
0.7	−0.31	−0.56	−0.60	−0.80	−0.69
0.9	−2.28	−2.05	−2.74	−2.37	−2.27

Simlilar numbers for $z = 0.1$ and $z = 0.01$ probably mean the large contribution from the radiative return to the Z-peak, via radiation of a pair in the singlet channel.

Conclusion for LEP1:

- TOPAZ0 and ZFITTER were permanently updated to accumulate all world experience in the field and ensure the precision of theoretical computations which meets the ultimate precision of combined LEP1 data;
- A lot of bug fixes affecting the numbers at the level from several units $\times 10^{-3}$ to several units $\times 10^{-4}$;
- Level of agreement at LEP1:
  - $1 \div 2 \times 10^{-4}$ at resonance;
  - $3 \div 5 \times 10^{-4}$ on wings, $Z \pm 3$ GeV;
- Examples of estimates of theoretical uncertainties:
  - for the complete hadronic cross-section:

$$\begin{array}{llllll} 2.73\,\mathrm{pb}, & 4.76\,\mathrm{pb}, & 3.51\,\mathrm{pb}, & 3.23\,\mathrm{pb}, & 1.71\,\mathrm{pb} & \text{or} \\ 0.061\%, & 0.050\%, & 0.012\%, & 0.023\%, & 0.021 & \end{array}$$

  - for the muonic cross-section:

$$\begin{array}{llllll} 0.07\,\mathrm{pb}, & 0.06\,\mathrm{pb}, & 0.19\,\mathrm{pb}, & 0.11\,\mathrm{pb}, & 0.08\,\mathrm{pb} & \text{or} \\ 0.030\%, & 0.014\%, & 0.013\%, & 0.016\%, & 0.021 & \end{array}$$

  In both cases, the uncertainty arising due to the uncertainty on the ISPP contribution has to be added.
- TOPAZ0 and ZFITTER after their recent update are used for final analysis of LEP1 data.

Status for LEP2:

- Both `TOPAZ0` and `ZFITTER` continue update of the codes for LEP2 energies, started at the time of LEP2 Workshop (1995) then interrupted and recently renewed.
- Topics of particular interest:
  - IFI beyond $\mathcal{O}(\alpha)$;
  - critical revision of implementation of ISPP (*Andrej Arbuzov, INFN, Torino*);
  - implementation of 4f processes of simple topology, `NC08`, to allow cutting of secondary pairs (an example of $2f-4f$ interplay);
  - critical revision of the theoretical precision at LEP2 energies;
- some comparison between `KK`, `TOPAZ0` and `ZFITTER` was done within LEP2 difermion group;
- this triange comparison must be continued!

More work is needed!!!