Next to next to leading orde
analysis of xF_3 S.F.
(3 - loops)

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Presented by: Gonzalo Parente, Univ. of Santiag
Work in collaboration with: A.L. Kataev, INR,
A.V. Sidorov, Dubna

THEORY : pQCD + Parton Model

$$F(x,Q^2) = \int_x^1 \frac{dy}{y}\, q\left(\frac{x}{y}, Q^2\right) C\,(y, Q^2)$$

DGLAP
evolution
equation

$$\frac{dq}{d\ln Q^2} = \int_x^1 \frac{dy}{y}\, q\left(\frac{x}{y}, Q^2\right) P(y, Q^2)$$

Coefficients

$$C = \underbrace{\delta(1-x)}_{\text{LO}} + \underbrace{C^{(1)}(x)\,\alpha_s(Q^2)}_{\text{NLO}} + \underbrace{C^{(2)}(x)\,\alpha_s^2(Q^2)}_{\text{N-NLO}} + \ldots$$

Splitting Functions

$$P = \underbrace{\alpha_s(Q^2)\,P^{(0)}(x)}_{\text{LO}} + \underbrace{P^{(1)}(x)\,\alpha_s^2(Q^2)}_{\text{NLO}} + \underbrace{P^{(2)}(x)\,\alpha_s^3(Q^2)}_{\text{N-NLO}} + \ldots$$

$C^{(2)}(x)$ [illegible]

$P^{(2)}(x)$ NOT YET CALCULATED !

but ! moments [illegible]

Anomalous dimensions at 3 loops

$$\gamma_n^{(2)} = \int_0^1 dx\, x^{n-2}\, P^{(2)}(x)$$

$n = 2, 4, 6, 8, 10$

drive the evolution of moments of structure functions

$$M_n(Q^2) = \int_0^1 dx\, x^{n-2}\, F(x, Q^2)$$

O.P.E. + R.G.E.

$$M_n(Q^2) = M_n(Q_0^2) \exp\left[\int_{\alpha_s(Q_0^2)}^{\alpha_s(Q^2)} d\alpha' \frac{\gamma_n(\alpha')}{\beta(\alpha')} \right] \frac{C_n(\alpha_s(Q^2))}{C_n(\alpha_s(Q_0^2))}$$

(non-singlet)

	LO	NLO	N-NLO	N3LO
$C_n =$	$1 +$	$C_n^{(1)}\alpha_s +$	$C_n^{(2)}\alpha_s^2$	
$\gamma_n =$	$\gamma_n^{(0)}\alpha_s +$	$\gamma_n^{(1)}\alpha_s^2 +$	$\gamma_n^{(2)}\alpha_s^3$	
$\beta =$	$\beta_0\alpha_s^2 +$	$\beta_1\alpha_s^3 +$	$\beta_2\alpha_s^4 +$	$\beta_3\alpha_s^5$

RECONSTRUCTION OF STRUC. FUN FROM MOMENTS BY USING JACOBI POLYNOMIALS

$$F(x,Q^2) = x^{\alpha}(1-x)^{\beta} \sum_{n=0}^{N} \Theta_n^{\alpha\beta}(x) \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta) M_{j+2}(Q^2)$$

N-N

"Effective method for analyzing scaling violations" at LO

NLO

At N-NLO it is essential !!

*

Analysis of xF_3 CCFR '97 data

(ν_μ Fe)

- $x > 0.01$, $Q^2 > 5\ \mathrm{GeV}^2$

- Up to N-NLO in pQCD ($\ln Q^2$)

- Target Mass Corrections

$$M_n^{TMC} = \frac{n(n+1)}{n+2}\,\frac{M^2}{Q^2}\,M_{n+2}^{xF_3}(Q^2)$$

$$\downarrow$$

$N_{max} = 6 \Rightarrow \gamma_{16}^{(2)}$ needed!

- Higher twist (HT)

pheno: $xF_3 = \dfrac{h(x)}{Q^2}$

IRR: $M_n^{HT} = \tilde{C}(n)\,\dfrac{A_2'}{Q^2}\,M_n^{xF_3}(Q^2)$

(Dasgupta + Webber 1996)

- $M_n(Q_0^2) = \int_0^1 dx\, x^{n-2} A x^a (1-x)^b (1+\gamma x)$

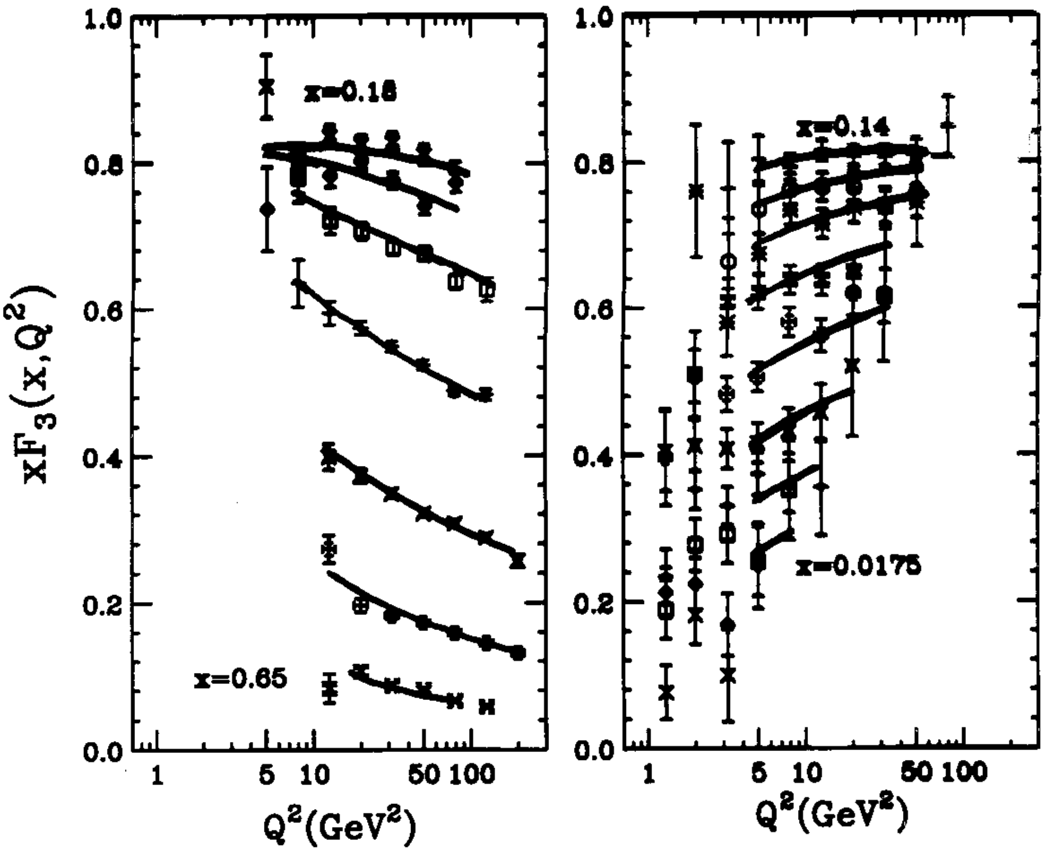
- Free param.

$A, a, b, \gamma, \Lambda_{\overline{MS}}(4), \overbrace{A_2', h(x_i)}^{\text{when H.T. included}}$

experimental x bins

CCFR '97 data

QCD fit at N-NLO



RESULTS OF FITS TO CCFR'97 data

Table 2
The results of the fits of CCFR'97 data with the cut $Q^2 > 5\ GeV^2$, obtained in the case $Q_0^2 = 20\ GeV^2$.

	$\Lambda_{\overline{MS}}^{(4)}$ [MeV]	A	b	c	γ	$A_2'[GeV^2]$	χ^2/n.e.p.
LO	264 ± 36	4.98 ± 0.23	0.68 ± 0.02	4.05 ± 0.05	0.96 ± 0.18	—	113.1/86
	433 ± 51	4.69 ± 0.13	0.64 ± 0.01	4.03 ± 0.04	1.16 ± 0.12	-0.33 ± 0.12	83.1/86
NLO	339 ± 35	4.67 ± 0.11	0.65 ± 0.01	3.96 ± 0.04	0.95 ± 0.09	—	87.6/86
	369 ± 37	4.62 ± 0.16	0.64 ± 0.01	3.95 ± 0.05	0.98 ± 0.17	-0.12 ± 0.06	82.3/86
NNLO	326 ± 35	4.70 ± 0.34	0.65 ± 0.03	3.88 ± 0.08	0.80 ± 0.28	—	77.0/86
	327 ± 35	4.70 ± 0.34	0.65 ± 0.03	3.88 ± 0.08	0.80 ± 0.29	-0.01 ± 0.05	76.9/86

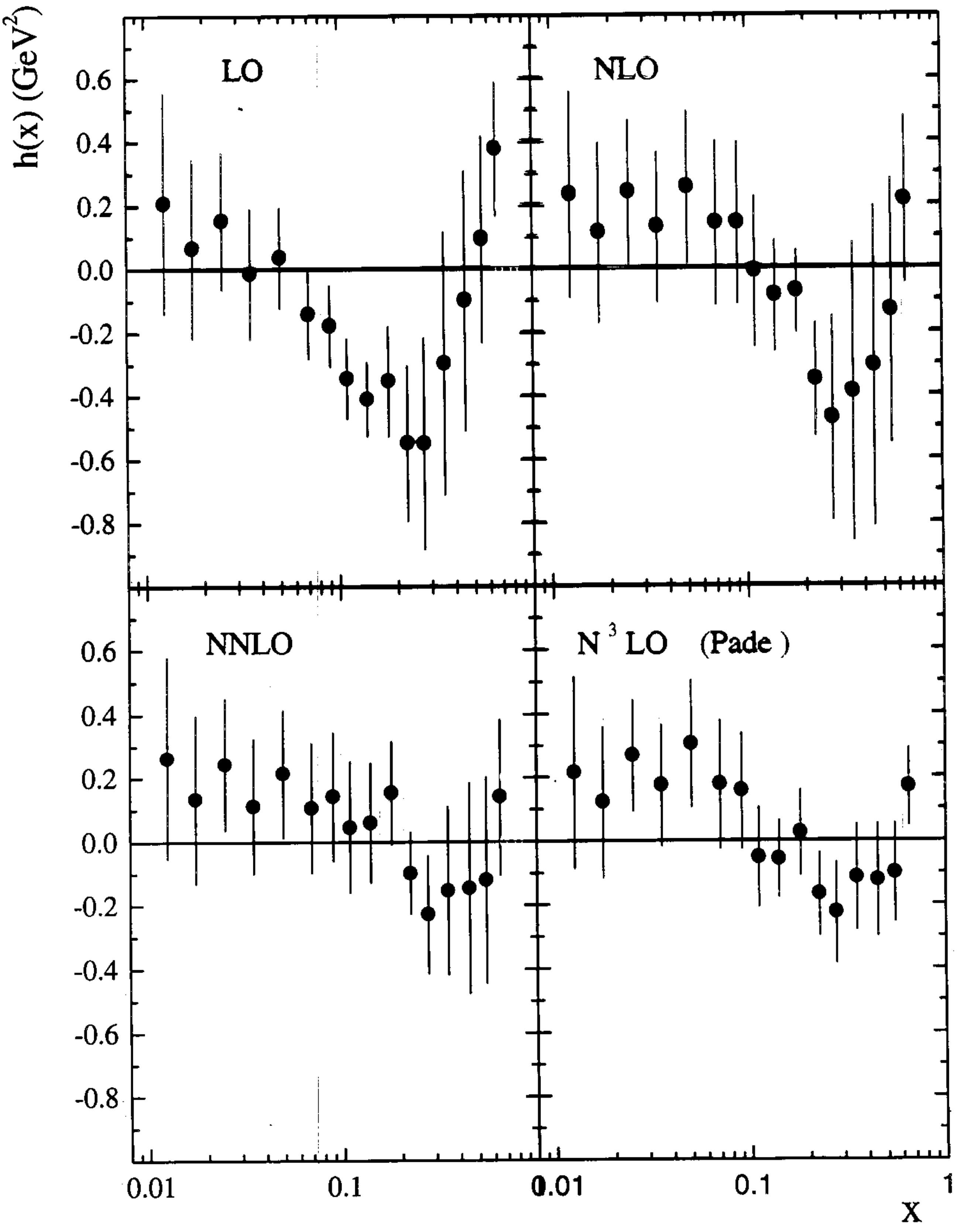
LO $\boxed{\alpha_s(\mathrm{NLO})}$: $\Lambda_{\overline{MS}}^{(4)} = 382 \pm 38$ MeV

NLO $\boxed{\alpha_s(\mathrm{N\text{-}NLO})}$: $\Lambda_{\overline{MS}}^{(4)} = 322 \pm 29$ MeV

NLO $\alpha_s(M_Z) = \underline{0.120}(\pm 0.003(\mathrm{stat}) \pm 0.005(\mathrm{sys}) \pm 0.004(\mathrm{theo}))$

N-NLO $\alpha_s(M_Z) = \underline{0.118}(\pm 0.002(\mathrm{stat}) \pm 0.005(\mathrm{sys}) \pm 0.003(\mathrm{theo}))$

$$xF_3^{H.T.} = \frac{h(x_i)}{Q^2}$$



Q_0^2 dependence

Table 1
The Q_0^2-dependence of the parameters of xF_3 model, extracted from the fits of CCFR'97 data with the cut $Q^2 > 5\ GeV^2$. The statistical errors are taken into account.

$Q_0^2 = 5\ GeV^2$	$\Lambda^{(4)}_{\overline{MS}}$ [MeV]	A	b	c	γ	χ^2/n.e.p.
LO	266 ± 35	5.13 ± 0.46	0.72 ± 0.03	3.87 ± 0.05	1.42 ± 0.33	113.2/86
NLO	341 ± 30	4.05 ± 0.36	0.65 ± 0.03	3.71 ± 0.06	1.93 ± 0.36	87.1/86
NNLO	$\boxed{292 \pm 29}$	4.25 ± 0.38	0.66 ± 0.03	3.56 ± 0.07	1.33 ± 0.33	78.4/86
$Q_0^2 = 8\ GeV^2$						
LO	266 ± 35	5.10 ± 0.15	0.70 ± 0.01	3.94 ± 0.04	1.23 ± 0.12	113.2/86
NLO	340 ± 40	4.38 ± 0.17	0.66 ± 0.02	3.81 ± 0.07	1.47 ± 0.19	87.4/86
NNLO	312 ± 33	4.42 ± 0.36	0.66 ± 0.03	3.68 ± 0.07	1.13 ± 0.31	76.5/86
$Q_0^2 = 10\ GeV^2$						
LO	265 ± 34	5.07 ± 0.15	0.70 ± 0.01	3.97 ± 0.04	1.16 ± 0.12	113.2/86
NLO	340 ± 35	4.48 ± 0.15	0.66 ± 0.02	3.85 ± 0.04	1.32 ± 0.10	87.5/86
NNLO	318 ± 33	4.50 ± 0.36	0.65 ± 0.03	3.73 ± 0.07	1.05 ± 0.31	76.3/86
$Q_0^2 = 15\ GeV^2$						
LO	265 ± 35	5.02 ± 0.16	0.68 ± 0.01	4.01 ± 0.04	1.04 ± 0.11	113.1/86
NLO	339 ± 37	4.61 ± 0.42	0.65 ± 0.03	3.92 ± 0.11	1.08 ± 0.42	87.6/86
NNLO	324 ± 34	4.60 ± 0.34	0.65 ± 0.03	3.83 ± 0.07	0.92 ± 0.29	76.6/86
$Q_0^2 = 20\ GeV^2$						
LO	264 ± 35	4.98 ± 0.15	0.68 ± 0.01	4.05 ± 0.04	0.96 ± 0.12	113.1/86
NLO	339 ± 36	4.67 ± 0.16	0.65 ± 0.02	3.96 ± 0.05	0.95 ± 0.13	87.6/76
NNLO	326 ± 35	4.70 ± 0.34	0.65 ± 0.03	3.88 ±0.08	0.80 ± 0.30	77.0/86
$Q_0^2 = 30\ GeV^2$						
LO	264 ± 37	4.92 ± 0.17	0.67 ± 0.02	4.09 ± 0.05	0.86 ± 0.09	113.0/86
NLO	338 ± 37	4.72 ± 0.43	0.64 ± 0.03	4.01 ± 0.08	0.80 ± 0.35	87.5/86
NNLO	327 ± 35	4.78 ± 0.32	0.64 ± 0.03	3.96 ± 0.08	0.67 ± 0.27	77.8/86
$Q_0^2 = 50\ GeV^2$						
LO	264 ± 35	4.84 ± 0.15	0.65± 0.01	4.13 ± 0.04	0.75 ± 0.12	112.9/86
NLO	337 ± 34	4.74 ± 0.13	0.64± 0.01	4.06 ± 0.06	0.64 ± 0.14	87.5/86
NNLO	326 ± 36	4.85 ± 0.31	0.64± 0.02	4.03 ± 0.09	0.53 ± 0.27	78.8/86
$Q_0^2 = 100\ GeV^2$						
LO	263 ± 36	4.73 ± 0.23	0.64± 0.02	4.19 ± 0.09	0.62 ± 0.24	112.6/86
NLO	337 ± 37	4.73 ± 0.15	0.62± 0.02	4.12 ± 0.06	0.46 ± 0.12	87.3/86
NNLO	325 ± 36	4.91 ± 0.28	0.63± 0.02	4.11 ± 0.10	0.36 ± 0.25	80.0/86

THE EFFECT OF 3-LOOP CORRECTIONS TO THE MOMENTS

$$C_n = 1 + C_n^{(1)} A_s + C_n^{(2)} A_s^2$$

n	$C_n^{(1)}$	$C_n^{(2)}$
2	-1.778	-47.472
3	1.667	-12.715
4	4.867	37.117
5	7.748	95.409
⋮	⋮	⋮

$$\exp\left[\int d\alpha' \frac{\gamma_n(\alpha')}{\beta(\alpha')}\right] \simeq \left[\alpha_s(Q^2)\right]^{\frac{\gamma_n^{(0)}}{2\beta_0}}\left(1 + p_n A_s + q_n A_s^2\right.$$

n	p_n	q_n
2	1.646	4.232
3	1.941	4.774
4	2.051	5.546
5	2.115	6.134
⋮	⋮	⋮

$A_s = \frac{\alpha_s}{4\pi}$ $\quad n_f = 4$

$$M_n(Q^2) \sim \left[\alpha_s(Q^2)\right]^{\frac{\gamma^{(n)}}{2\beta_0}} \left[\overset{\text{LO}}{1} + \overset{\text{NLO}}{K_n^{(1)} A_s} + \overset{\text{N-NLO}}{K_n^{(2)} A_s^2} \ldots\right.$$

n	$K_n^{(1)}$	$K_n^{(2)}$
2	-0.132	-46.16
3	3.608	-4.71
4	6.918	52.64
$\vdots$	$\vdots$	$\vdots$

$Q^2 = 5\,\text{GeV}^2 \longrightarrow A_s = 3 \times 10^{-2}$

	LO	NLO	N-NLO
$n=2$	1	$-\ 0.004$	$-\ 0.041$
$n=3$	1	$+\ 0.108$	$-\ 0.004$
$n=4$	1	$+\ 0.210$	$+\ 0.050$
$\vdots$		$\vdots$	

Conclusions

- Analyzing Structure Functions at N-N
  is already available (in the
  moment space - Jacobi, Bernstein, ...

- $\alpha_s(M_Z)$ from DIS xF_3 at N-NLO

- In xF_3, $\frac{1}{Q^2}$ effects dissapear! (N-N

- Stability of fits in $\Lambda_{\overline{MS}}$ breaks at N-NL
  when $Q_0^2 < 10\ GeV^2$

- anomalous behavior of perturb. series
  for $n = 2$ (low x)