A Determination of the Running b-quark Mass $\overline{m}_b(M_Z)$

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hep-ex/9905017 [illegible]
final version

Quarkmasses in Q D

Quark masses cannot be directly measured due to confinement.

Indirect determination:

1.) Compute influence of quark masses on properties of hadrons

2.) Measure these properties and compare to predictions

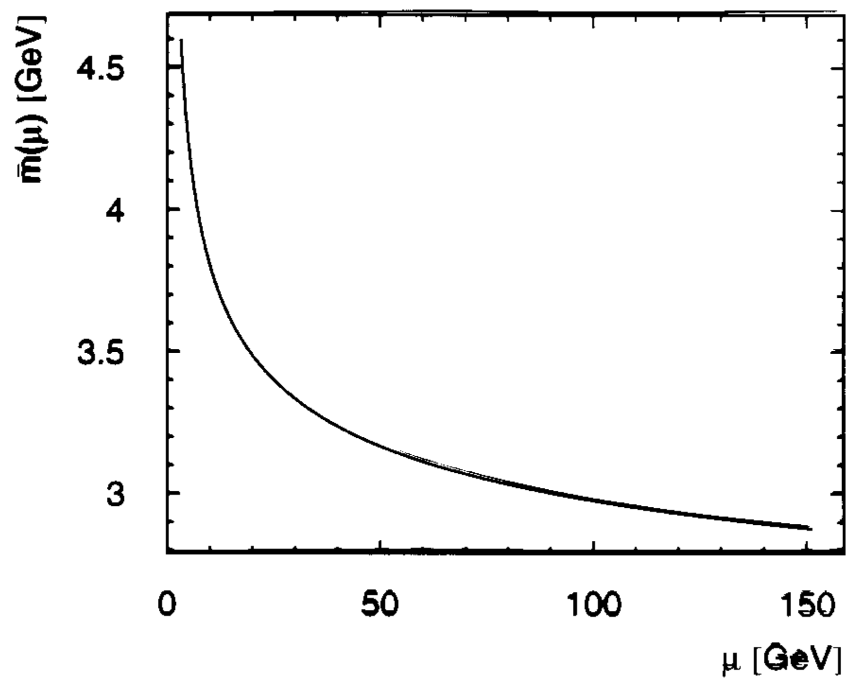
Important consequence:

Quark mass values depend on computational scheme that one uses to make predictions.

Two of the most common quark mass definitions:

- Pole mass m^{pole}
- $\overline{MS}$ mass (running mass) $\overline{m}(\mu)$

Quark masses in Q D



1.) $\overline{\mathrm{m}}_\mathrm{b}(\mu)$ depends on the renormalization scale!

$$\mu^2 \frac{\partial \overline{\mathrm{m}}(\mu)}{\partial \mu^2} = \gamma_\mathrm{m}(\alpha_\mathrm{s}) \overline{\mathrm{m}}(\mu)$$

$$\rightsquigarrow \overline{\mathrm{m}}(\mu) = \overline{\mathrm{m}}(\mu_0) \left[\frac{\alpha_\mathrm{s}(\mu)}{\alpha_\mathrm{s}(\mu_0)} \right]^{\frac{12}{33-2n_\mathrm{f}}} \{1 + \mathrm{O}(\alpha_\mathrm{s})\}$$

(solution of the renormalization group equations)

2.) $\mathrm{m}_\mathrm{b}^\mathrm{pole}$ and $\overline{\mathrm{m}}_\mathrm{b}(\mu)$ differ only in order α_s:

$$\mathrm{m}^\mathrm{pole} = \overline{\mathrm{m}}(\mu) \left[1 + \frac{\alpha_\mathrm{s}(\mu)}{\pi} \left(\frac{4}{3} - \ln \frac{\overline{\mathrm{m}}(\mu)^2}{\mu^2} \right) + \ldots \right]$$

$\rightsquigarrow$ we need a NLO calculation to fix the renormalization scheme of the mass parameter

$\rightsquigarrow$ study mass effects in jetrates at NLO

theoretical predictions from 3 different groups:

- [illegible]
- [illegible]
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Why a determination of the b mass at high scale?

at low scale (~ 10 GeV):

+ $m_{pol} \leftrightarrow \overline{m}(\mu)$ well behaved, no large log's

+ small errors of the exp. results

– large higher order corrections if m_{pol} is used (can be avoided by PS mass)

$\leadsto$ precise determination of the b mass possible

$$\overline{m}(\mu = \overline{m}) = 4.25 \pm 0.08 \text{ GeV}$$

at high scale ($\sim m_Z$):

+ pQCD works fine

+ high statistic

+ measurement of $\overline{m}(\mu)$ at high scale μ

– mass effects are in general small, but...

$\leadsto$ less precise, but direct observation of the running

Mass effects in Jet physics

what size one would expect?

naively: mass effects of order $\left(\frac{m_b^2}{s}\right) \sim 0.1\%$

this naive expectation is not necessarily true for observables with an additional scale

in jetrates effects may be enhanced due to additional scale $s\, y_{cut}$:

$$\left(\frac{m_b^2}{s\, y_{cut}}\right) \sim 1\%$$

qualitative understanding of mass effects: diminished phase space for gluon emission due to quark mass

kinematic effects in the definition of jet clustering schemes

mass enters differently in different jet algorithms
$\rightsquigarrow$ effects depend on the algorithm

Jetrates

Algorithm	Resolution y_{ij}	Recombination
DURHAM	$\frac{2\min(E_i^2, E_j^2)}{s}(1 - \cos\vartheta_{ij})$	$p_k = p_i + p_j$
GENEVA	$\frac{8E_iE_j(1 - \cos\vartheta_{ij})}{9(E_i + E_j)^2}$	$p_k = p_i + p_j$
E	$\frac{(p_i + p_j)^2}{s}$	$p_k = p_i + p_j$
E0	$\frac{(p_i + p_j)^2}{s}$	$E_k = E_i + E_j$, $\vec{p}_k = \frac{E_k}{\lvert\vec{p}_i + \vec{p}_j\rvert}(\vec{p}_i + \vec{p}_j)$
P	$\frac{(p_i + p_j)^2}{s}$	$\vec{p}_k = \vec{p}_i + \vec{p}_j$, $E_k = \lvert\vec{p}_k\rvert$
P0	$\frac{(p_i + p_j)^2}{\hat{s}}$	same as P

Experimental Results

SLD analyses, flavor independence of α_s

(comparison with theoretical prediction for fixed value of m_b)

Alg.	y_c	r^b	stat.	exp. syst.	had.
D	0.010	0.964	0.023	$^{+0.038}_{-0.041}$	$^{+0.001}_{-0.006}$
G	0.080	0.995	0.032	$^{+0.035}_{-0.036}$	$^{+0.020}_{-0.008}$
E	0.040	1.050	0.026	$^{+0.038}_{-0.042}$	$^{+0.011}_{-0.046}$
E0	0.020	1.054	0.019	$^{+0.030}_{-0.037}$	$^{+0.007}_{-0.045}$
P	0.020	1.048	0.019	$^{+0.027}_{-0.037}$	$^{+0.002}_{-0.026}$
P0	0.015	1.055	0.017	$^{+0.028}_{-0.035}$	$^{+0.007}_{-0.037}$

Definition of the Observable

define R_3^i to be the flavour specific fraction of events containing 3 or more jets
consider as observable:

$$r^b(y_c) \equiv R_3^b(y_c)/R_3^{uds}(y_c),$$

expanded in α_s

$$r^b = \frac{A^b}{A^{uds}} + \frac{\alpha_s}{2\pi}\left(\frac{B^b + C^b}{A^{uds}} - \frac{B^{uds} + C^{uds}}{A^{uds}}\frac{A^b}{A^{uds}}\right) + O\left(\alpha_s^2\right)$$

with the coefficients A, B, C defined by

$$R_3^q(y_c) = \frac{\alpha_s}{2\pi}A^q(y_c) + \left(\frac{\alpha_s}{2\pi}\right)^2 \left(B^q(y_c) + C^q(y_c)\right) + O\left(\alpha_s^3\right),$$

A $\sim$ LO 3-Jet contribution, B $\sim$ NLO 3-Jet contribution, C $\sim$ LO 4-Jet contribution.

Theoretical prediction

the calculation of A^b, B^b, C^b as a function of m is based on

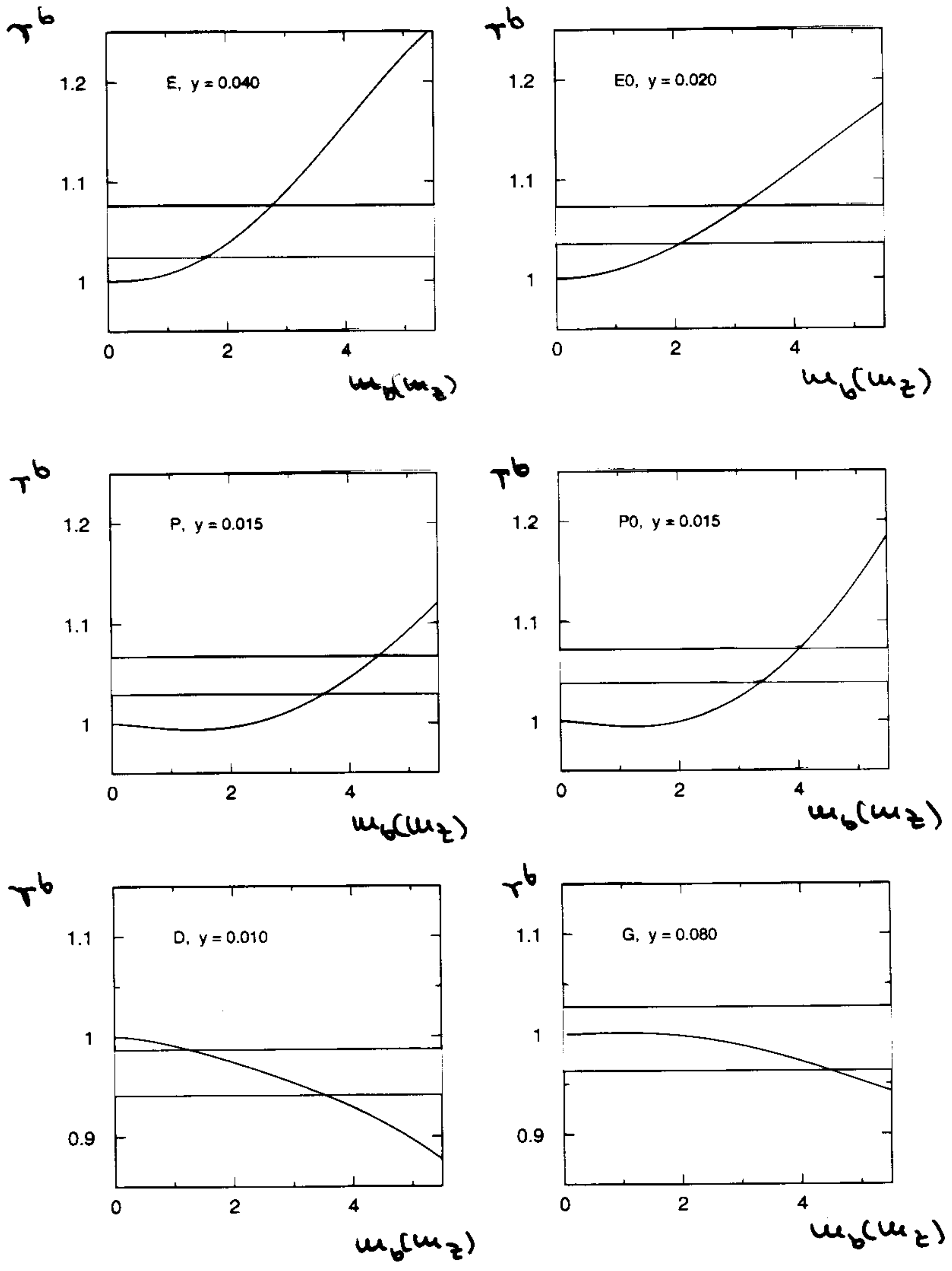
we use a fit of the form

$$r^b_{\text{theo.}} = 1 + \alpha\frac{m^2}{s} + \beta\frac{m^2}{s}\ln(\frac{m^2}{s}) + \gamma\frac{m^4}{s^2}$$

to parmetrize the numerical results

Algorithm	α	β	γ
D	79.3	17.16	−4610.8
G	−89.6	−11.04	3229.9
E	207.6	16.10	−13029.9
E0	42.2	−3.58	−3881.3
P	211.7	28.51	−5060.9
P0	236.8	30.95	−3417.6

Experiment vs Theory



($\mu = m_Z$)

Refined analyses

take correlations into account and fit only one value for the mass:

Algorithm	E	E0	P	P0	D	G
E	1.00	0.70	0.67	0.65	0.61	0.49
E0		1.00	0.84	0.82	0.61	0.49
P			1.00	0.71	0.65	0.56
P0				1.00	0.52	0.41
D					1.00	0.64
G						1.00

E,E0,P,P0 are highly correlated (> 0.65)

try to fit all algorithms with one mass
$\rightsquigarrow \chi^2 = 22/5$

try to omit one algorithm in the fitting procedure
$\rightsquigarrow \chi^2 > 12$

try to omit two algorithms $\rightsquigarrow \chi^2 < 5$ only if two of (E,E0,P,P0) are omitted.

$\rightsquigarrow$ there is some inconsistency

Two ways out

1.) throw away the algorithms of the JADE family because of bad soft gluon behavior
$\rightsquigarrow$ will yield low χ^2 because mass from D, G are very close to each other

2.) introduce an additional uncertainty due to higher order effects which may affect the different algorithms differently

we choose the second way

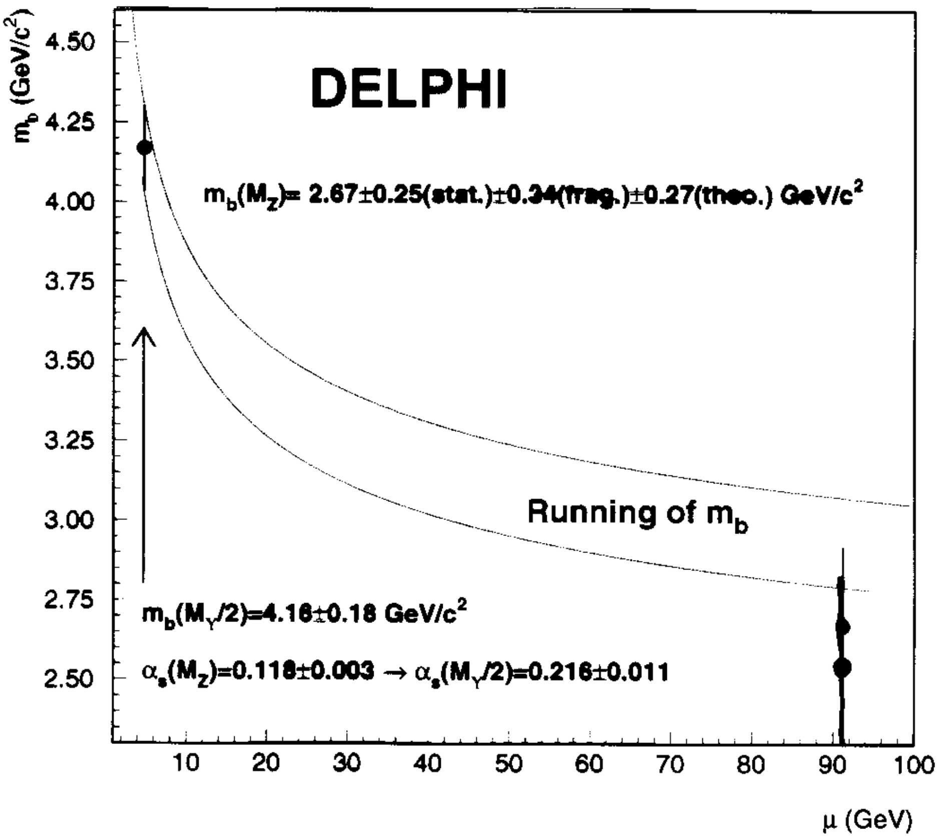
under the assumption of an additional uncertainty $\epsilon = 0.02$ we obtain

$$\overline{\mathrm{m}}(\mu = \mathrm{M_Z})$$
$$= 2.52 \pm 0.27(\mathrm{stat.})^{+0.33}_{-0.47}(\mathrm{syst.})^{+0.54}_{-1.46}(\mathrm{theor.})\ \frac{\mathrm{GeV}}{\mathrm{c}^2}$$

the theoretical uncertainty includes the uncertainty from hadronization, and the introduced uncertainty ϵ

the value for $\overline{\mathrm{m}}(\mu = \mathrm{M_Z})$ remains stable under a further increment of ϵ

Comparison with the DELPHI result



Conclusion

- mass effects well established in jet physics

- high precision reached at e^+e^- colliders makes it possible to extract the mass of the b quark

- direct observation of the running of $\overline{m}$

$\leadsto$ Q D works well