RESUMMATION AND POWER CORRECTIONS FOR FACTORIZED CROSS SECTIONS

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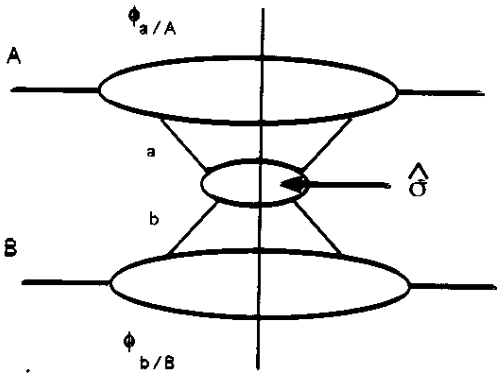
Threshold Corrections

- Factorized cross sections ($\overline{\text{MS}}$)

$$\sigma_{AB\to F}(Q) = \int_0^1 dx_a dx_b\, \phi_{a/A}^{(\overline{\text{MS}})}(x_a,\mu^2)\phi_{a/A}^{(\overline{\text{MS}})}(x_a,\mu^2) \times \hat{\sigma}_{ab\to F}\left(z \equiv Q^2/x_a x_b S, Q, \mu, \alpha_s(\mu^2)\right)$$

- Moment-space Drell-Yan

$$\int_0^1 d\tau \tau^{N-1} \frac{1}{\sigma_0}\frac{d\sigma_{AB\to V}}{dQ^2}(\tau \equiv Q^2/S, Q^2) = \sum_{a,b=q,\bar{q},g} e_f^2\, \tilde{\phi}_{a/A}^{(\overline{\text{MS}})}(N,\mu^2)\, \tilde{\phi}_{b/B}^{(\overline{\text{MS}})}(N,\mu^2) \times \tilde{\omega}_{ab}^{\text{DY}}\left(N, Q, \mu, \alpha_s(\mu^2)\right)$$



- $\hat{\sigma}_{ij}$, $\tilde{\omega}_{ab}$ process dependent but *universal among external hadrons*
- Singular corrections and their moments

$$\hat{\sigma}_{ii\to V}^{(k)} \sim \left(\frac{\alpha_s}{\pi}\right)^k \left[\frac{\ln^{2k-1}(1-z)}{1-z}\right]_+ + \dots$$

$$\int_0^1 dz\, z^N \left[\frac{\ln^m(1-z)}{1-z}\right]_+ = \frac{(-1)^{m+1}}{m+1}\ln^{m+1} N + \dots$$

- *Threshold Resummation*: Singular corrections to all orders

Example: Resummed Drell Yan

- $\overline{\rm MS}$ scheme

$$\begin{aligned}\tilde{\omega}_{ab}^{\rm DY}(N) &= \delta_{b\bar{a}} F_a(\alpha_s(Q^2)) \, \exp\Bigg[-\int_0^1 dz \frac{z^{N-1}-1}{1-z} \\ &\times \left(\int_{(1-z)^2Q^2}^{Q^2} \frac{d\xi^2}{\xi^2} A^{(a\bar{a})}[\alpha_s(\xi^2)] + B^{(a\bar{a})}[\alpha_s(Q^2)] \right) \Bigg] \\ &+ \mathcal{O}\left(\frac{1}{N}\right)\end{aligned}$$

$$\begin{aligned}A^{(ab)} &= (C_a + C_b)\left(\frac{\alpha_s}{\pi} + \frac{1}{2} K \left(\frac{\alpha_s}{\pi}\right)^2 \right) \\ K &= C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} n_f \,, \quad B^{a\bar{a}} = \mathcal{O}(\alpha_s^2)\end{aligned}$$

- Structure valid to $\rm N^nL$ in exponent – known explicitly to NNL
- Extension to QCD hard scattering, 1PI kinematics known

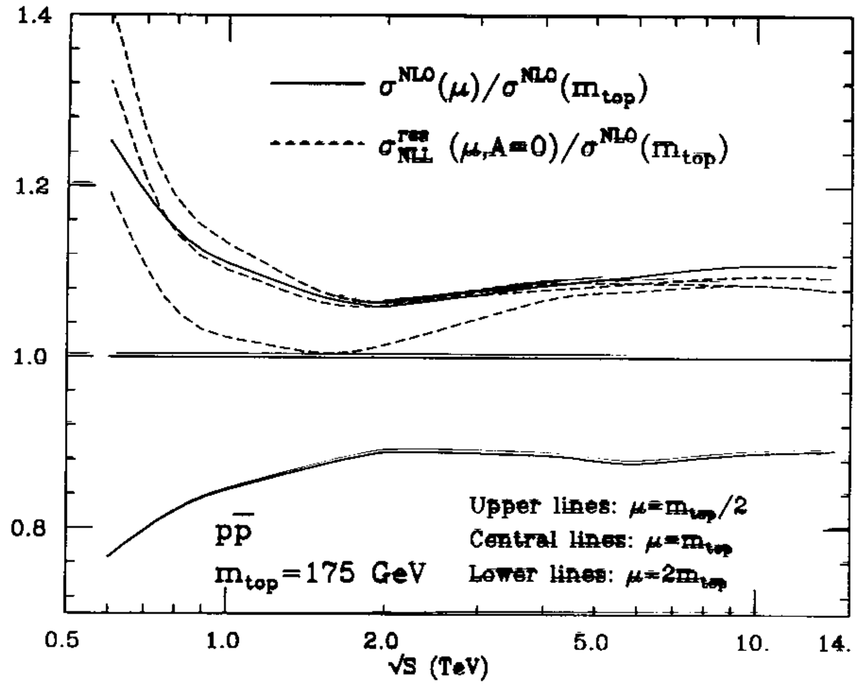
Status of the Method

⋆ Inversion of Moments ($\tilde{\omega}(N) \rightarrow \hat{\sigma}(z)$)

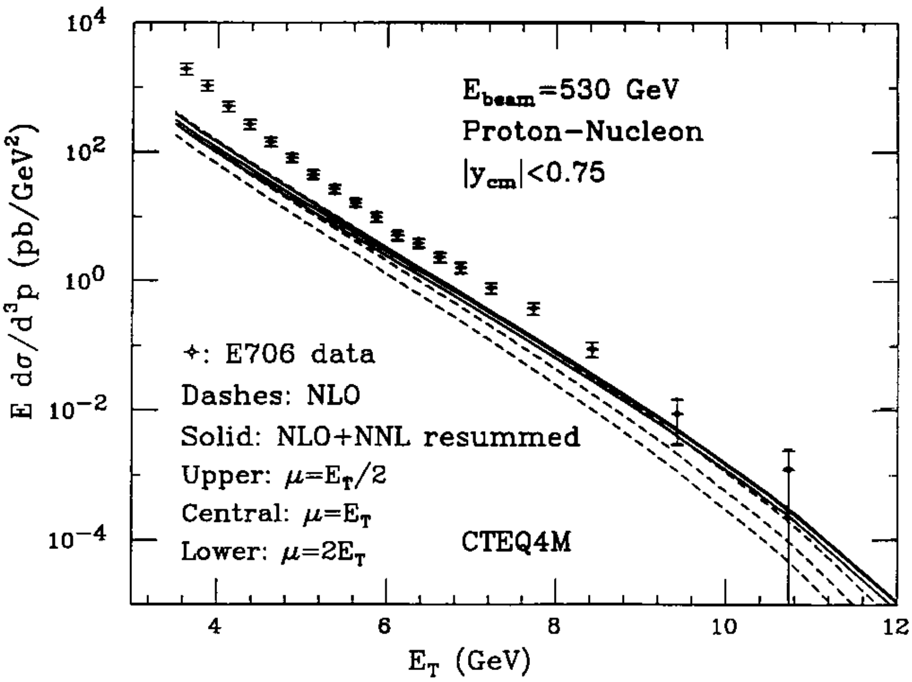
- NLL in moment space – numerical inversion
- NLL in momentum space ($z = Q^2/x_a x_b S$) – analytic inversion

⋆ Motivation & Applications

- Justification of NLO (HO corrections under control)
- Reduction of scale dependence
- Guide to power corrections
- Extension to k_T-resummation?



$\sigma^{NLO}(\mu)/\sigma^{NLO}(m_{top})$
$\sigma^{res}_{NLL}(\mu, A=0)/\sigma^{NLO}(m_{top})$
$p\bar{p}$
m_{top}=175 GeV
Upper lines: $\mu=m_{top}/2$
Central lines: $\mu=m_{top}$
Lower lines: $\mu=2m_{top}$
1.4
1.2
1.0
0.8
0.5
1.0
2.0
5.0
10.
14.
$\sqrt{S}$ (TeV)



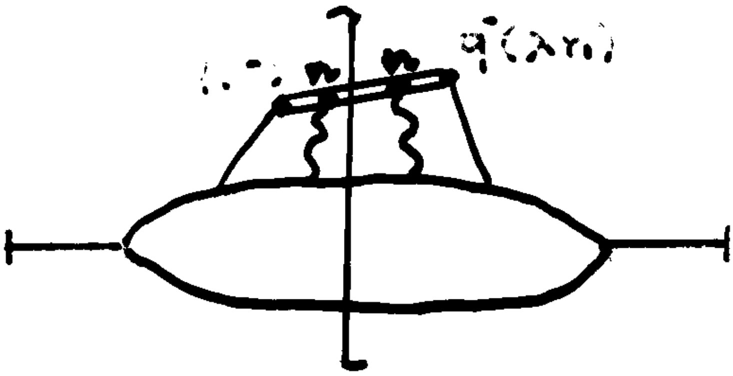
E_{beam}=530 GeV
Proton−Nucleon
$|y_{cm}|$<0.75
✧: E706 data
Dashes: NLO
Solid: NLO+NNL resummed
Upper: $\mu=E_T/2$
Central: $\mu=E_T$
Lower: $\mu=2E_T$
CTEQ4M
$E\, d\sigma/d^3p$ (pb/GeV^2)
10^4
10^2
10^0
10^{-2}
10^{-4}
4
6
8
10
12
E_T (GeV)

Generalized PDFs

- Generalized PDF (gauge-invariant)

$$\phi_{q/h}(x, 2p\cdot n, \mu, \epsilon) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi}\, \mathrm{e}^{-ixp\cdot n\lambda} \langle h(p)|\bar{q}(\lambda, \vec{0})\, \frac{1}{2} v\cdot\gamma\, \Phi_n(\lambda)\; q(0)|h(p)\rangle$$

$$\Phi_n(\lambda) = P\, \exp\left(-ig\int_0^{\lambda} d\lambda'\, n\cdot A(\lambda' n)\right)\,,$$



- Light-cone and energy PDF's:

$$\phi_{i/h}^{(\overline{\mathrm{MS}})} = \phi_{i/h}(n^\mu = \delta^{\mu -} \equiv v)\,, \quad \psi_{i/h} = \phi_{i/h}(n^\mu = \delta^{\mu 0})$$

Refactorization

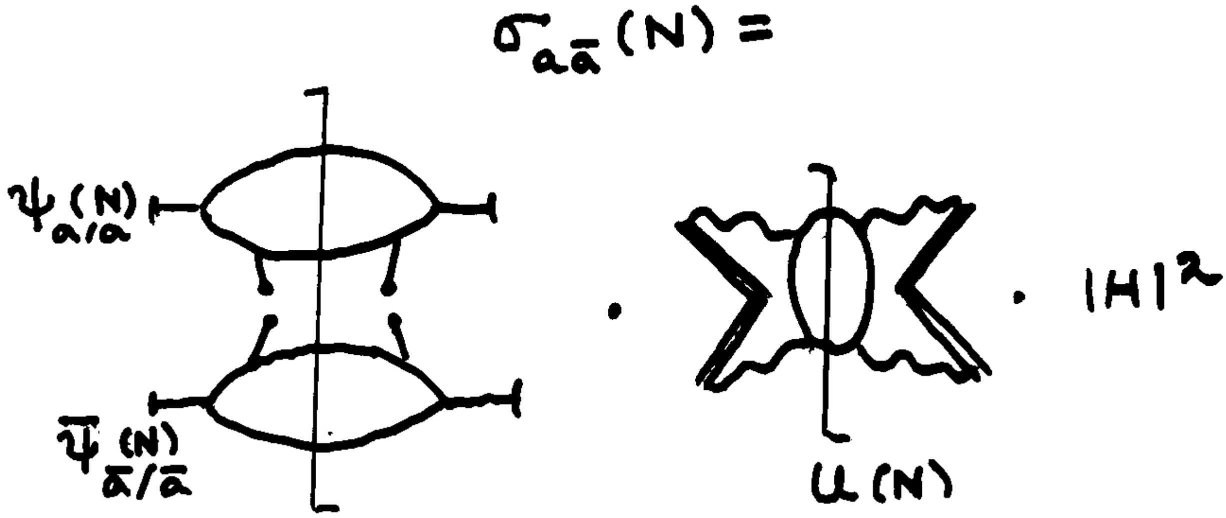
- *External partons $(A = a,\ B = \bar{a})$* $\rightarrow$ same $\hat{\sigma}_{cd}$
- Alternate (re)factorization into *energy* PDFs

$$\int_0^1 d\tau \tau^{N-1} \frac{1}{\sigma_0} \frac{d\sigma_{a\bar{a}\rightarrow V}}{dQ^2}(\tau, Q^2, \epsilon) = \tilde{\phi}^{(\overline{\rm MS})}_{a/a}(N, \mu^2)\, \tilde{\phi}^{(\overline{\rm MS})}_{\bar{a}/\bar{a}}(N, \mu^2)$$

$$\times\ \tilde{\omega}^{\rm DY}_{ab}\left(N, Q, \mu, \alpha_s(\mu^2)\right) + \mathcal{O}(1/N)$$

$$= \tilde{\psi}_{a/a}(N, Q, \epsilon)\ \tilde{\psi}_{\bar{a}/\bar{a}}(N, Q, \epsilon)$$

$$\times \left| H\left(Q, \alpha_s(Q^2)\right) \right|^2 \tilde{U}^{(i)}\Big(N, \alpha_s(Q^2)\Big)\ + \mathcal{O}(1/N)$$



- U – factorized eikonal scattering
- Equate factorized and refactorized partonic cross sections

$$\Downarrow$$

$$\tilde{\omega}^{\rm DY}_{a\bar{a}}(N, Q, \mu) = \left[\frac{\tilde{\psi}_{a/a}(N, Q)}{\tilde{\phi}^{(\overline{\rm MS})}_{a/a}(N, \mu)}\right]^2\ U^{(a)}(N, Q)\ |\, H_{a\bar{a}}\,|^2 + \mathcal{O}(1/N)$$

- ψ/ϕ Ratios: exponentiation of N-dependence
- *All factorization scale dependence is in* $\phi^{(\overline{\rm MS})}$

Resummation and Scale Dependence

- Scale dependence in the hadronic cross section

$$\begin{aligned}
\sigma^{\rm DY}_{AB}(N) &= \sum_f \phi_{f/A}(N,\mu)\,\tilde{\omega}^{\rm DY}_{f\bar{f}}(N,Q,\mu)\,\phi_{\bar{f}/B}(N,\mu) \\
&= \sum_f \left[\frac{\tilde{\psi}_{f/f}(N,Q)}{\tilde{\phi}^{(\overline{\rm MS})}_{f/f}(N,\mu)}\right]^2 U^{(f)}(N,Q)\,|\,H_{f\bar{f}}\,|^2\,\phi_{\bar{f}/B}(N,\mu)+\mathcal{O}(1/N)
\end{aligned}$$

- But scale dependence of ϕ's is *universal*

$$\Downarrow$$

$$\mu\frac{d}{d\mu}\left[\frac{\phi_{f/A}(N,\mu)}{\phi^{(\overline{\rm MS})}_{f/f}(N,\mu)}\right]=0 + \mathcal{O}(1/N)$$

- μ_F dependence disappears to $\mathcal{O}(1/N)$ (!)
- Seen in phenomenological applications

Resummation Beyond Logs

- *But:* $\mathcal{O}(1/N)$ *singlet evolution dominates the gluon distribution:*

$$G(N,\mu) \to \frac{\Sigma(N,\mu)}{N \ln N}$$

- Need to extend formalism to $\mathcal{O}(1/N)$
- Example: $F_L(N,Q) \sim 1/N$
- Example: Importance of $1/N$ terms in NLO Higgs production

⋆ Generalized Refactorization

$$\tilde{\omega}^{\rm DY}_{ab}(N,Q,\mu) = \sum_{e,f} \left[\tilde{\psi}_{c/e}(N,Q)\tilde{\phi}^{(\overline{\rm MS})-1}_{e/a}(N,\mu)\right]\ \rho_{cd}(N,Q)\ \left[\tilde{\psi}_{d/f}(N,Q)\tilde{\phi}^{(\overline{\rm MS})-1}_{f/b}(N,\mu)\right]$$

- Mass factorization universal among generalized PDFs
- Ratios $\longrightarrow$ matrices for singlet sector
- $\rho(N,Q)$ – final state dependence

$$\tilde{\psi}_{c/e}(N,Q)\tilde{\phi}^{(\overline{\rm MS})-1}_{e/a}(N,\mu) = \bar{\mathcal{P}}_\xi \exp\left\{\int_{Q^2/N^2}^{Q^2}\frac{d\xi^2}{\xi^2}\left[\bar{\Gamma}_1(N,\xi^2) - A(\xi^2)\ln(\frac{N^2\xi^2}{Q^2})\right]\right\}_{ca}$$

$$\bar{\Gamma}_1(N,\xi^2) \; = \; \frac{\alpha_s(\xi^2)}{2\pi}\begin{pmatrix} -C_F\left(\frac{1}{N}+\frac{1}{N+1}\right) & 2T_f\left(\frac{1}{N}-\frac{2}{N+1}+\frac{2}{N+2}\right) \\ C_F\left(\frac{2}{N-1}-\frac{2}{N}+\frac{1}{N+1}\right) & 2C_A\left(\frac{1}{N-1}-\frac{2}{N}+\frac{1}{N+1}-\frac{1}{N+2}\right)\end{pmatrix} + \mathcal{O}(

Power Corrections from Resummation

⋆ Power Corrections in DIS, e^+e^-

- $\mathcal{O}(1/N) \Rightarrow \mathcal{O}(N)^a$ (flavor diagonal, "higher twist") $1/Q^b$
- Generic power correction from resummation:

$$\int_0^{\mu_1} dk\ \alpha_s(k^2)\ k^p \rightarrow \mu_1^{p+1}\ \alpha_p(\mu_1)$$

- α_p phenomenological; dependent on new "factorization" scale μ_1 separating PT from NP
- DIS: $(N/Q^2)^a$ (OPE)
- e^+e^- event shapes (thrust ...): $(N/Q)^a$ (renormalon, resum)
- $\mathcal{O}(N/Q)$: shift in $t \equiv 1 - T$:

$$\frac{d\sigma_{\rm PT}(t)}{dt} \rightarrow \frac{d\sigma_{\rm PT}(t - \lambda/Q)}{dt} + \mathcal{O}\left(\frac{1}{(tQ)^2}\right)$$

- $\mathcal{O}(N/Q)^a$: convolution

$$\frac{d\sigma_{\rm PT}(t)}{dt} \rightarrow \int_0^{tQ} d\epsilon\ U(\epsilon)\ \frac{d\sigma_{\rm PT}(t - \epsilon/Q)}{dt} + \mathcal{O}\left(\frac{1}{tQ^2}\right)$$

- $U(\epsilon) \leftrightarrow$ eikonal cross section in DY
- $(N/Q)^a \leftrightarrow$ "wide angle soft gluon emission"
  (linear dependence of thrust on energy)

⋆ Drell-Yan Power Corrections from Resummatiion

- $1/Q$ *vs* $1/Q^2$ in Drell-Yan
- What about $N \leftrightarrow 1-z$ dependence?
- Can resummation predict power correction dependence uniquely?
  Beyond large-n_f approximation? At all?
- Example: ratio at order $N^0 \times \ln^m N$ (Lowest order in exponent)

$$\ln\left[\frac{\psi_{i/i}(N,Q)}{\phi_{i/i}^{\overline{\rm MS}}(N,Q)}\right] \sim \int_0^1 \frac{dz}{1-z}\int_0^{(1-z)^2Q^2}\frac{d\mu^2}{\mu^2}$$
$$\times\left[{\rm e}^{-N[(1-z)+\mu^2/(1-z)Q^2]} - {\rm e}^{-N(1-z)}\right] C_i\frac{\alpha_s(\mu^2)}{2\pi}$$
$$-\int_0^1 \frac{dz}{1-z}\left({\rm e}^{-N(1-z)}-1\right)\int_{(1-z)^2Q^2}^{Q^2}\frac{d\mu^2}{\mu^2}C_i\frac{\alpha_s(\mu^2)}{2\pi}$$

- First line: no logs, but N^0
- Second line: all logs of N
- First line was abosored into overall "constant" in
  explicit resummation formulas (kept $\ln^m N$, $m>0$).
  The information was there all along.
- But: all odd powers of (N/Q) cancel: remaining
  power corrections are

$$\left(\frac{N^2}{Q^2}\right)^a \quad\leftrightarrow\quad \left[\frac{1}{(1-x)^2Q^2}\right]^a$$

- $1/(1-x)Q$ still natural NP scale
- Generalizes to all orders in α_s at $\mathcal{O}(N^0)$

Outlook

- Justification of NLO ⋆⋆ but no guarantee of agreement with experiment!
- Reduction of scale dependence ⋆⋆ extension to mixing underway; control at $1/N$
- Guide to power corrections ⋆ (shift, convolution ...) Insight into extensions of "minimal" resum and of relations between different approches to moment inversion
- k_T Many lessons; subject of active research, but that's another talk!

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