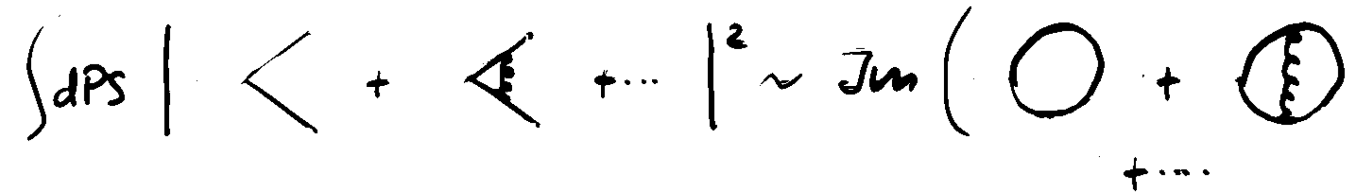
Quark mass effects to $\sigma(e^+e^- \to \text{hadrons})$ at $\mathcal{O}(\alpha_s^3)$

[Univ. Karlsruhe]

- Introduction + Motivation
- Method: OPE + RG
- Results
- Conclusions

$$\boxed{R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}}$$

- optical theorem : $R(s) = 12\pi \, \mathrm{Im}\, \Pi(s + i\epsilon)$



- 2 scales: m^2, s

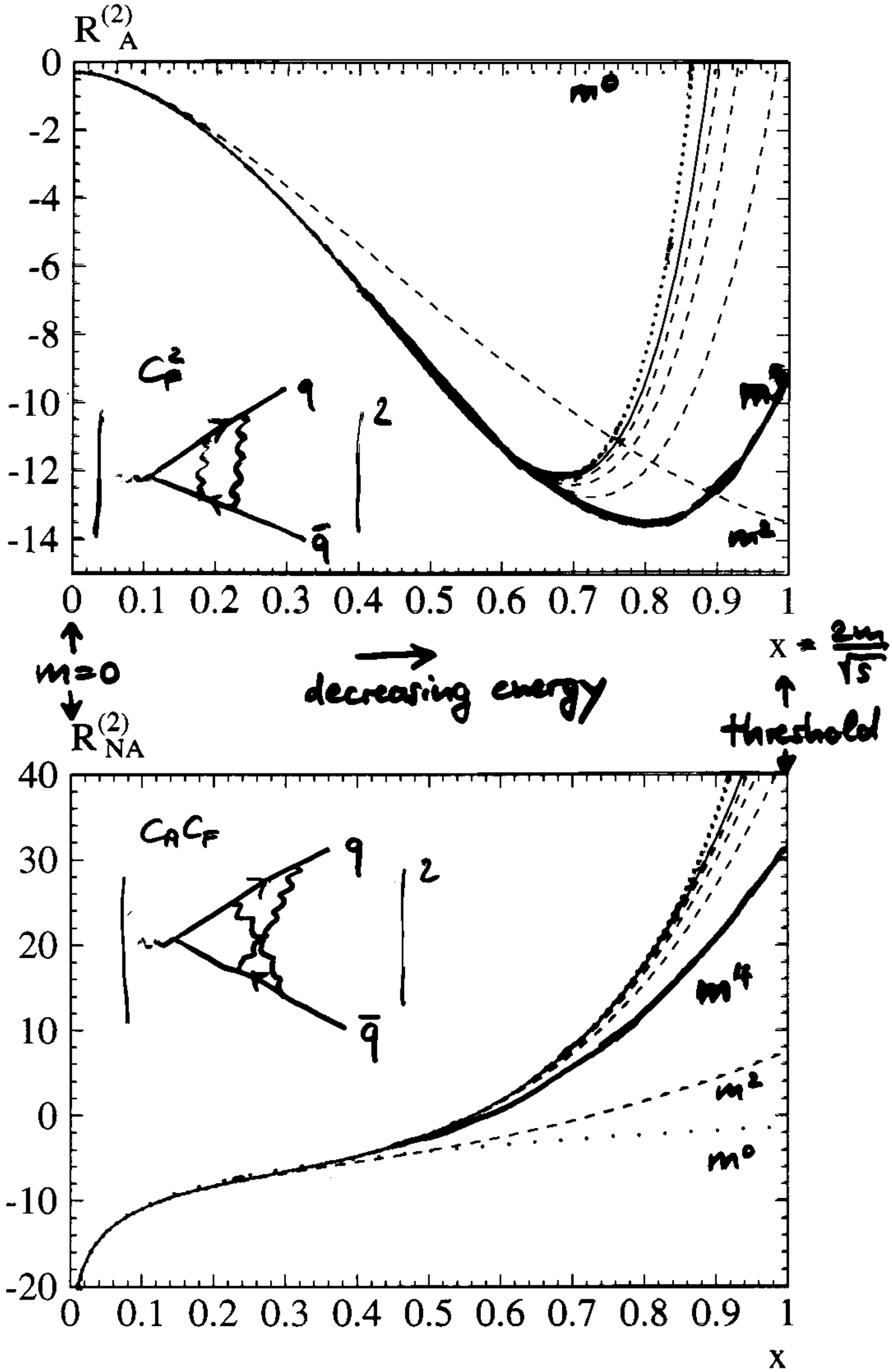
- <u>single scale</u> up to 3 loops by computer algebra

→ massless to $\mathcal{O}(\alpha_s^2)$

<u>mass effects:</u> at $\mathcal{O}(\alpha_s^2)$

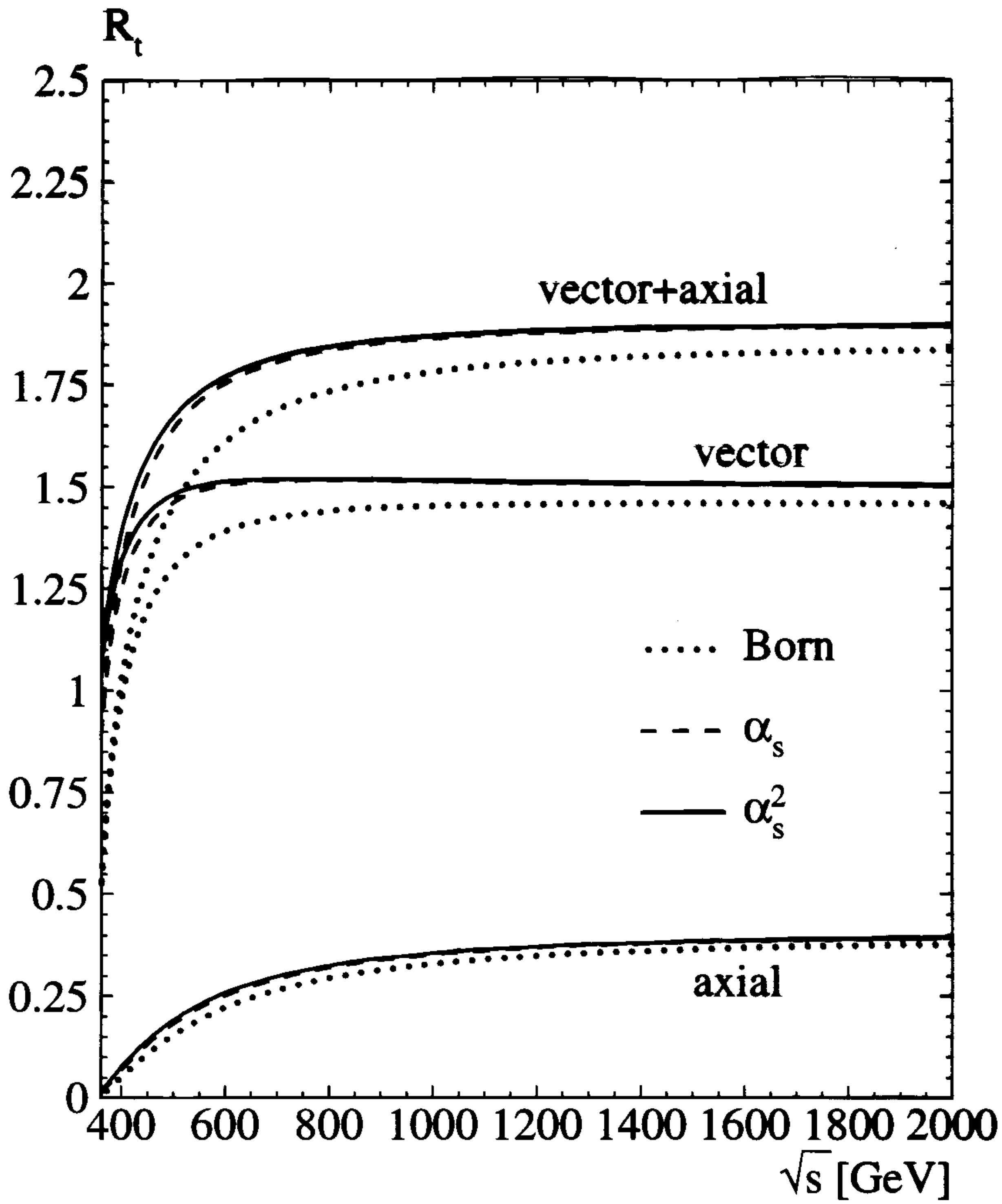
- asymptotic expansions + Padé approximation

→ reduction to 3-loop massless

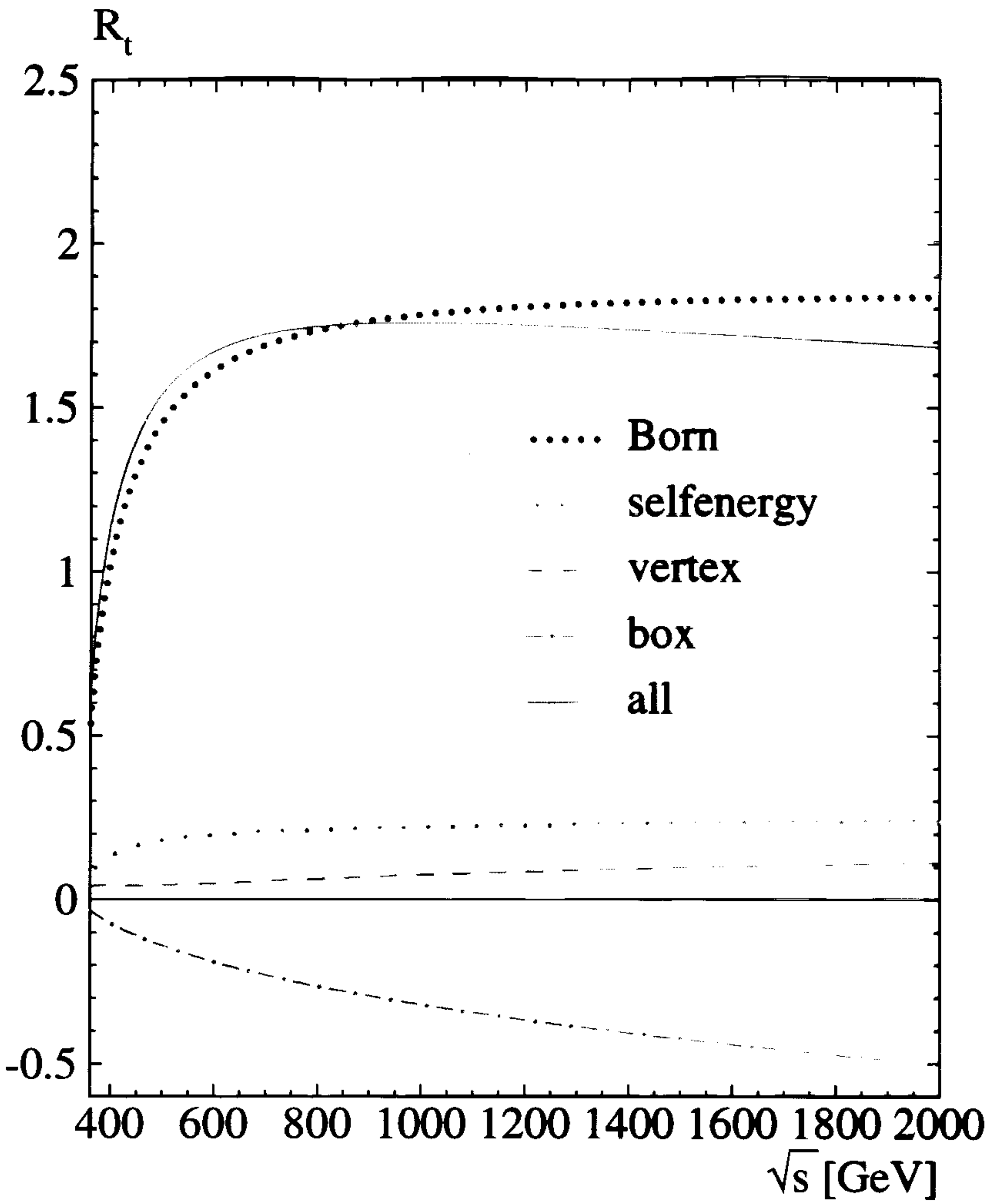


—— semi-analytical [Chetyrkin, Kühn, Steinhauser]

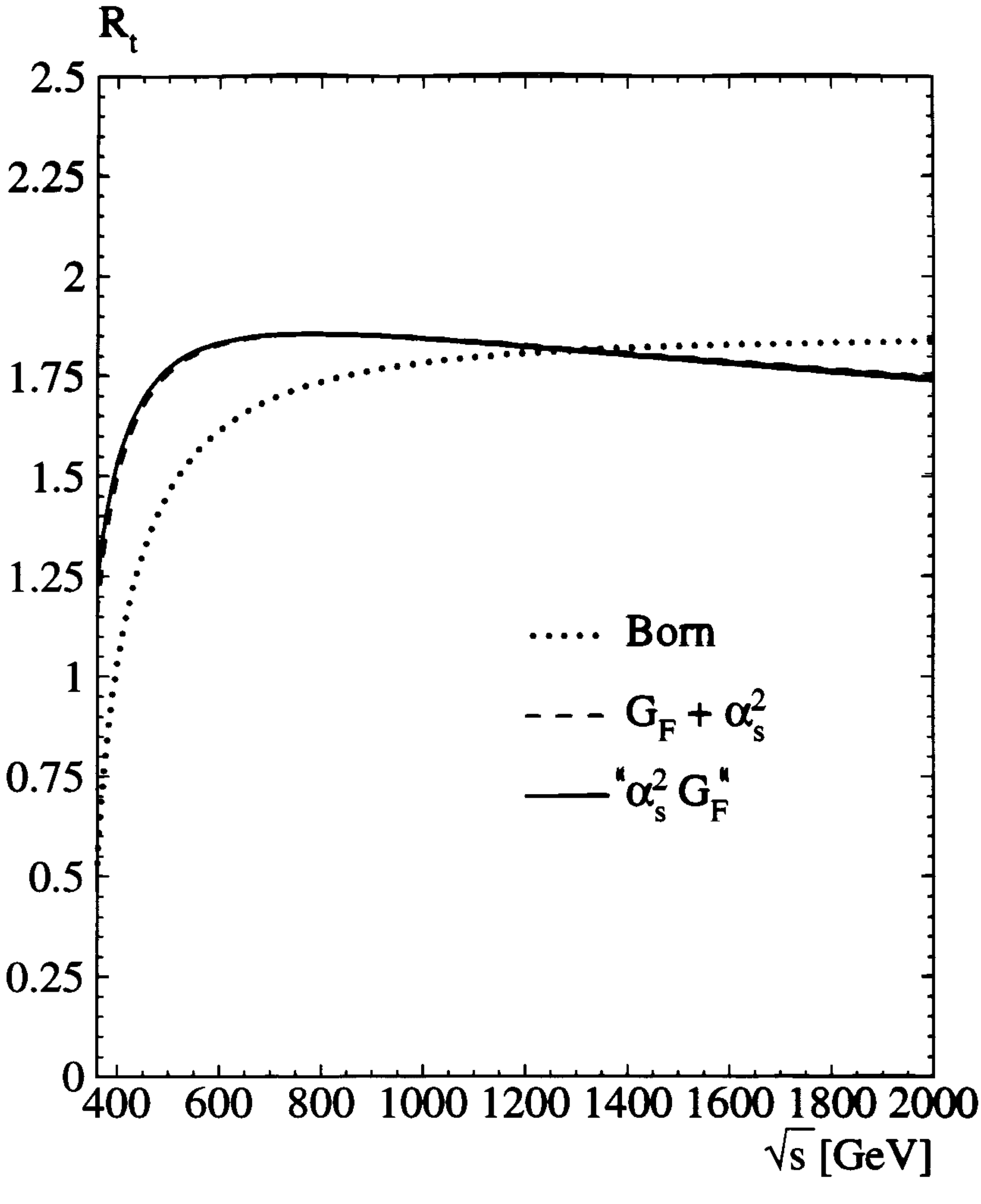
- - - - asympt. exp. [Chetyrkin, RH, Kühn, Steinhauser]



R_t
2.5
2.25
2
1.75
1.5
1.25
1
0.75
0.5
0.25
0
vector+axial
vector
axial
Born
α_s
α_s^2
400
600
800
1000
1200
1400
1600
1800
2000
$\sqrt{s}$ [GeV]



R_t
2.5
2
1.5
1
0.5
0
-0.5
400
600
800
1000
1200
1400
1600
1800
2000
$\sqrt{s}$ [GeV]
Born
selfenergy
vertex
box
all

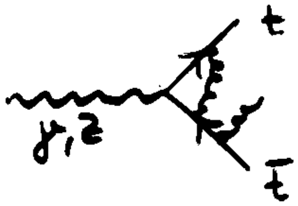


R_t
2.5
2.25
2
1.75
1.5
1.25
1
0.75
0.5
0.25
0
400
600
800
1000
1200
1400
1600
1800
2000
√s [GeV]
Born
G_F + α_s^2
"α_s^2 G_F"

Application: $\sigma(e^+e^- \to t\bar{t})$

- <u>QCD:</u> $\sigma(e^+e^- \to t\bar{t}) = \frac{4\pi\alpha^2}{3s}\left(R_V + R_A\right)$

  full m_t-dep. known to $\mathcal{O}(\alpha_s^2)$



singlet

- <u>EW:</u> $\mathcal{O}(\alpha_{QED})$

- <u>QCD + EW:</u>

$$\left(|\mathcal{M}_B^{v/a}|^2 + 2\,\mathrm{Re}\,\mathcal{M}_B^{v/a}\mathcal{M}_{1l}^{EW*}\right)\cdot R_{V/A}$$

$$\sim 1 + \mathcal{O}(\alpha) + \mathcal{O}(\alpha_s^n) + \text{"}\mathcal{O}(\alpha\alpha_s^n)\text{"}$$

- $R_{V/A}$ up to $\mathcal{O}(\alpha_s^3 m^4)$

Mass effects at $\mathcal{O}(\alpha_s^3)$

- $\underline{m=0:}$ $R(s)$ to $\mathcal{O}(\alpha_s^3)$:

  3-loop massless + IR rearrangement

- $\underline{\alpha_s^3 m^2:}$ RG transformations

  $\rightarrow$ no "hard" calculation necessary

- $\underline{\alpha_s^3 m^4:}$ OPE + RG

  3-loop massless + massive

Operator Product Expansion

- $R(s) = 12\pi \, \mathrm{Im}\, \Pi(s+i\epsilon)$
- $\Pi(q^2) = \langle 0 | T(q^2) | 0 \rangle$
- $T(q^2) = i \int d^4x \, e^{iqx} \, T \, j(x) \, j(0)$, $j = \bar{\psi} \Gamma \psi$

- OPE $\boxed{T(q^2) \xrightarrow{q^2 \to \infty} \sum_i C_i(q^2)\, \mathcal{O}_i(0)}$

- dim 0: $\mathbb{1}$ dim 2: m^2

- dim 4:

$$\mathcal{O}_1 = G_{\mu\nu}^2 , \quad \mathcal{O}_2 = m\bar{\psi}\psi , \quad \mathcal{O}_3 = m^4$$

Calculation of $\langle O_n \rangle$ and C_n

• $\langle O_n \rangle$:

O_1

O_2

$O_3 \equiv m^4$

3-loop $\hat{=}$ $O(\alpha_s^2)$

• C_n $\boxed{\text{Method of projectors}}$

C_1: $\langle g | T(q^2) | g \rangle \Big|_{p=m=0} =$  α_s $\Big|_{p=m=0}$ $+ \cdots +$  α_s^3

C_2: $\langle f | T(q^2) | f \rangle \Big|_{p=m=0} =$  α_s $\Big|_{p=m=0}$ $+ \cdots +$  α_s^3

C_3: $\langle 0 | T(q^2) | 0 \rangle \Big|_{m=0} =$  α_s^0 $\Big|_{m=0}$ $+ \cdots +$  α_s^2

$+$  α_s^3 !

$$\boxed{\text{Solution: Renormalization group}}$$

$$\mu^2 \frac{d}{d\mu^2} \left(C_1 O_1 + C_2 O_2 + C_3 O_3 \right) = 0$$

$$\Rightarrow \frac{\partial}{\partial L} C_3 = 4\gamma^0 C_2 - 4 C_1 \alpha_s \frac{\partial}{\partial \alpha_s} \gamma^0$$
$$- \beta \cdot \alpha_s \frac{\partial}{\partial \alpha_s} C_3 - 4 \gamma^m C_3$$

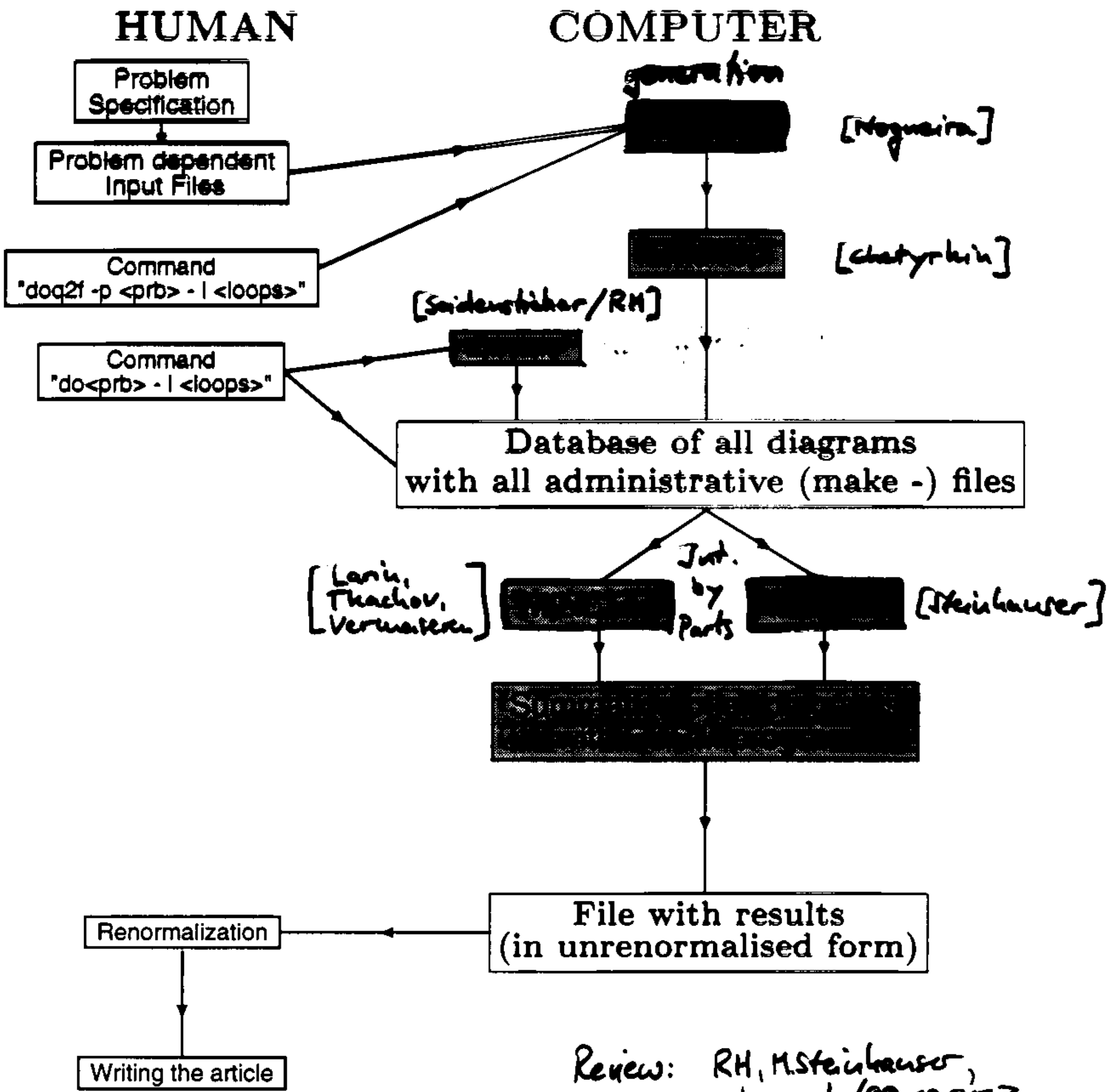
$$L = \ln\left(-\frac{\mu^2}{q^2}\right) \xrightarrow{q^2 \to s + i\epsilon} i\pi + \ln \frac{\mu^2}{s}$$

$$R(s)\Big|_{m^4} = 12\pi \, \mathrm{Im} \sum_{n=1}^{3} C_n \langle O_n \rangle$$

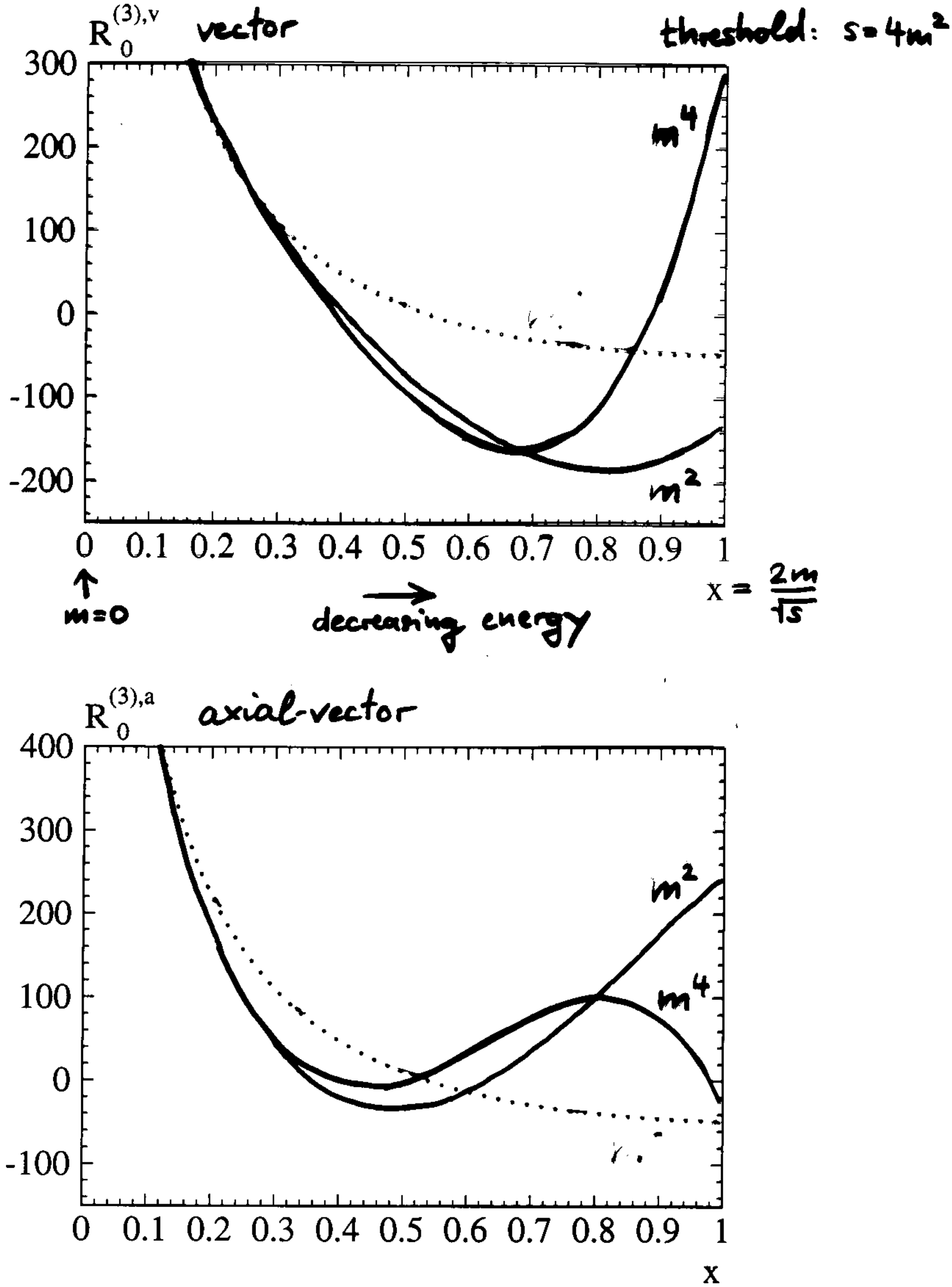
known to $\mathcal{O}(\alpha_s^3)$

GEFICOM [Chetyrkin, Steinhauser]

automatic GEneration, FInding topologies and COMputataion of Feynman diagrams



Review: RH, M.Steinhauser, hep-ph/9812357 (Prog. Part. Nucl. Phys., in print)



$R_0^{(3),v}$ vector
threshold: $s=4m^2$
300
200
100
0
-100
-200
m^4
m^2
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
$m=0$
decreasing energy
$x = \frac{2m}{\sqrt{s}}$
$R_0^{(3),a}$ axial-vector
400
300
200
100
0
-100
m^2
m^4
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
x

Conclusions

- knowledge of R_{had} at $\mathcal{O}(\alpha_s^3)$ extended to energies (not too) close to threshold
- interplay between
  - automated multi-loop calculation
  - field theoretic concepts
- outlook: extension of method to m^{2n}, $n = 3, 4, \ldots$