Progress of Few-Body Calculational Methods Stimulated by $\mu$CF

— Stau-catalyzed nuclear fusion —

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1. Gaussian Expansion Method (GEM) for various 3-body problems in $\mu$CF

2. Examples of applications of GEM to various 3-, 4- and 5-body problems in physics

3. Stau-catalyzed d-d fusion and d-$\alpha$ fusion

$\text{stau } (\tilde{\tau}) = \text{supersymmetry (SUSY) partner of } \text{tau lepton } (\tau)$
Section 1
Gaussian Expansion Method (GEM) for various 3-body problems in μCF
My colleagues, Kino and Hiyama, and myself have been developing a few-body calculational method, called Gaussian Expansion Method (GEM), which is reviewed in

"Gaussian Expansion Method for Few-Body Systems"
E. Hiyama, Y. Kino and M. Kamimura,

This method was first proposed by myself at \( \mu \text{CF-1987} \) held in Leningrad.
As is explained later on, this method, GEM, is accurately applicable both
  i) to bound-state calculations such as for 
    \[(d \, t\mu)_{11}\]
  ii) to reaction calculations such as for
    \[(d\mu)_{1s} + t \rightarrow d + (t\mu)_{1s} + 48\,\text{eV}\]

\textbf{\textmu CF-1987} in Leningrad:
  I reported calculated results for the two cases.

\textbf{\textmu CF-1988} in Florida:
  I applied the method to the calculations of
  i) fusion rate in \((d \, t\mu)\)
  ii) probability of muon sticking to \(\alpha\) after the fusion.
μCF-1992 in Uppsala:

Kino and myself applied GEM to (d Heμ) molecules.

We discussed competition between the particle decay and radiative decay from the excited J=1 states of the molecule.

This calculation nicely explained the isotope dependence of the radiative decay seen in (d $^3$Heμ), (d $^4$Heμ) and (p $^4$Heμ).

Experimental data for the isotope dependence was just reported in the same conference by Nagamine group.
Section 1.1
Brief survey of GEM
3-body Gaussian basis functions

On 3-sets of Jacobi coordinates

\[
\begin{bmatrix}
\phi_{nclc}^G(r_e) \\
\psi_{NcLc}^G(R_c)
\end{bmatrix}_{JM} (c = 1 \text{ to } 3)
\]

\[
\phi_{nlm}^G(r) = \phi_{nl}^G(r) Y_{lm}(\hat{r}),
\]
\[
\phi_{nl}^G(r) = r^l e^{-\nu_n r^2},
\]
\[
\psi_{NLM}^G(R) = \psi_{NL}^G(R) Y_{LM}(\hat{R}),
\]
\[
\psi_{NL}^G(R) = R^L e^{-\lambda_N R^2}.
\]
Gaussian ranges:

\[ \phi_{nl}^G(r) = r^l e^{-\nu_n r^2}, \quad \psi_{NL}^G(R) = R^l e^{-\lambda_N R^2} \]

\[ \nu_n = 1/r_n^2, \quad r_n = r_1 a^{n-1} \quad (n = 1 - n_{\text{max}}), \]

\[ \lambda_N = 1/R_N^2, \quad R_N = R_1 A^{N-1} \quad (N = 1 - N_{\text{max}}). \]

Geometric progression

To take geometric progression is very suited for describing
Simultaneously both long-range asymptotic behavior
and short-range correlations:

Precisely reviewed in
"Gaussian Expansion Method for Few-Body Systems"
E. Hiyama, Y. Kino and M. Kamimura,
Using the 3-body basis functions,

\[ \left[ \phi_{nclc}(r_c) \psi_{NcLc}(R_c) \right]_{JM} \quad (c = 1 - 3) \]

we

1) diagonalize the 3-body Hamiltonian,

\[ \langle \Phi_{JM,\nu} | H | \Phi_{JM,\nu'} \rangle = E_{J\nu} \delta_{\nu\nu'}, \]

\( (\nu, \nu' = 1 - \nu_{\text{max}}) \)

2) obtain 3-body eigenstates:

\[ \{ \Phi_{JM,\nu}; \nu = 1 - \nu_{\text{max}} \}, \]

\( \nu_{\text{max}} \sim 10^3 \) for 3-body problem

\( \sim 10^4 \) for 4-body problem

Function space of the 3-body Gaussian basis functions spanned over the 3 sets of Jacobi coordinates is very wide.

Therefore, eigenfunctions obtained above

i) are found to form a complete set in the finite spatial region,

ii) are useful to expand any asymptotically-vanishing function.
3-body reaction calculations

\[(d\mu)_{1s} + t \to d + (t\mu)_{1s} + 48 \text{ eV}\]

Total wave function

\[
\Psi_{JM} = \phi_{1s}^{(d\mu)}(r_1) \chi_{JM}^{(d\mu-t)}(R_1) + \phi_{1s}^{(t\mu)}(r_2) \chi_{JM}^{(t\mu-d)}(R_2) + \Psi_{JM}^{(closed)}. 
\]

\[
\Psi_{JM}^{(closed)} = \sum_{\nu=1}^{\nu_{\text{max}}} b_{JM,\nu} \Phi_{JM,\nu} \quad \text{to be solved}
\]

The third term, \[\Psi_{JM}^{(closed)}\], stands for all the closed (virtually-excited) channels in the energy range of this reaction;

This term is responsible for all the asymptotically-vanishing 3-body amplitudes that are not included in the first two scattering terms.

The term is then expanded in terms of the complete set in the finite region.
Section 2
Examples of applications of GEM to various 3-, 4- and 5-body problems in physics
Examples of application and development of GEM

1) Antiprotonic helium atom and mass of antiproton (3-body) (Kino)
2) 3-cluster structure of light nuclei (Kamimura)
3) 3- and 4-body structure of light hypernuclei (strangeness= -1, -2) (Hiyama)
4) 4-nucleon ground state and excited states (realistic NN force) (Hiyama)
5) Resonance and scattering states of 5-quark states (Hiyama)
6) 4-body breakup reactions induced by unstable halo nuclei (Hiyama, Matsumoto)
7) Stau-catalyzed nuclear fusion (3-body) (Kino, Kamimura) --- in PLB(2007)

As is recognized from this list, developments to 4- and 5-body problems has been accomplished by Hiyama.
She proposed a new type of Gaussian basis functions, called **infinitesimally-shifted Gaussian-Lobe** basis functions.

\[
\phi_{nm}^{G}(\mathbf{r}) = r^l e^{-\nu r^2} Y_{lm}(\hat{\mathbf{r}}) = \lim_{\varepsilon \to 0} \frac{1}{(\varepsilon \nu)^l} \sum_{k=1}^{k_{\text{max}}} \mathcal{C}_{lm,k} e^{-\nu(r-\varepsilon D_{lm,k})^2}
\]

and a skilful method to take the limiting analytically after analytical calculation of few-body Hamiltonian matrix elements.

The new type of Gaussian basis functions makes 3-, 4- and 5-body calculations extremely easier than before.

Due to this new method and many applications mentioned above, **the 2006 Yukawa Memorial Award**, a very honorable award in theoretical physics in Japan, was given to Hiyama.

It is my pleasure to see this and to note that such a development of the method started with solving difficult 3-body problems in \(\mu\)CF.
Section 3

Stau-catalyzed $d$-$d$ fusion and $d$-$\alpha$ fusion
**stau (scalar tau) particle**

1) Supersymmetry (SUSY) particle (lepton) beyond the standard model (not discovered yet)

2) the scalar partner (boson) of the tau lepton (fermion)

3) the lightest SUSY particle = gravitino
   (candidate of the dark matter, the fermion partner of the graviton),
   the next lightest SUSY particle (NLSP) = stau

4) stau mass \( \sim \) a few 100 GeV
   lifetime \( \sim \) seconds to years

5) charged lepton (usually written as \( X^- , X^+ , X^0 \) )

6) Coulombic interaction and weak interaction
The \textbf{stau} particle is expected to be discovered at \textbf{LHC} (Linear Hadron Colider) in CERN at the early stage after the first beam (2007) (before Higgs particle?). Therefore, many theorists in the elementary particle physics are eagerly making many predictions about the \textbf{stau} particle.

Six months ago, Kino and myself were asked to help three of them who are studying \textbf{stau}-catalyzed nuclear fusion.

K. Hamaguchi, T. Hatsuda and T. Yanagida (University of Tokyo)

Here, I introduce you two examples of their study.
Section 3.1

stau-catalyzed d-d fusion
stau ($X^-$) particle

- Long-lived, negatively-charged, heavy lepton
- Coulombic interaction (and weak interaction)

muon-catalyzed d-d fusion

stau-catalyzed d-d fusion

extremely exotic atom!
They discussed about

i) feasibility of stau-catalyzed d-d fusion

ii) possible production of stau particles.

In i), since the lifetime of $X^-$ is sufficiently long (seconds to years), essential issue is the probability of ($^3\text{He} - X^-$) sticking after fusion.
The authors estimated as

probability of $x^-$ sticking to $^3\text{He} = 2 \times 10^{-6}$ (cf. 0.12 in $d\text{d}$)$\mu$)

Therefore

Energy production \[= \frac{4 \text{ MeV}}{(2 \times 10^{-6})} = \frac{2000 \text{ GeV}}{X^-} \]

per one fusion

\[ (d\text{d}$\mu$: $\sim 2 \text{ GeV} / \mu) \]

Amazing!
I found that the authors did not calculate the $ddX^-$ 3-body wave function, but assumed too naive wave function $\Phi(R)$ between $d$-$d$ pair and $X^-$ when $d$-$d$ fusion takes place.

Kino and myself solved the 3-body problem of $d + d + X^-$ system accurately and calculated the sticking probability. We obtained: $X^-$ sticking to $^3$He = 0.023 (not $2 \times 10^{-6}$)

It is pity to conclude that the stau-catalyzed $d$-$d$ fusion in the $(ddX^-)$ atom is not feasible.
Section 3.2

stau-catalyzed $d$-$\alpha$ fusion

in Big-Bang nucleosynthesis
In the Big-Bang nucleosynthesis, 

\[ ^{6}\text{Li} \text{ nucleus is produced mainly by} \]

\[ \text{d} + \text{\(\alpha\)} \rightarrow ^{6}\text{Li} + \gamma \]

Since, in this capture-\(\gamma\)-reaction, E1 transition is heavily suppressed, 

\[ \sigma( \text{d} + \text{\(\alpha\)} \rightarrow ^{6}\text{Li} + \gamma ) \sim 10^{-5} \times \sigma( \text{t} + \text{\(\alpha\)} \rightarrow ^{7}\text{Li} + \gamma ) \]

Producton of \(^{6}\text{Li}\) is much smaller than that of \(^{7}\text{Li}\).

This is one of the key points of

the beautiful success of the Big-Bang nucleosynthesis scenario.

( H, D, T, \(^{3}\text{He}\), \(^{4}\text{He}\), \(^{6}\text{Li}\), \(^{7}\text{Li}\) )

But, an exciting idea by Pospelov (2006): 

Stau-catalyzed nuclear fusion might destroy the success.
A very exciting paper:


“Particle-physics catalysis of thermal Big-Bang nucleosynthesis“
M. Pospelov

(According to the SUSY physics, $X^-$ is generated just after the inflation of the universe)

If the lifetime of $X^-$ is long enough ($10^3$–$10^4$ sec)

- $X^-$ survives until the Big Bang time
- $X^-$ forms a bound state such as $(\alpha X^-)$
- $X^-$ causes a dangerous reaction such as $(X^-$ catalyzed fusion in flight)

$$d + (\alpha X^-) \rightarrow ^6\text{Li} + X^- + 1.1 \text{ MeV}$$

1.1 MeV

$(X^-\alpha)_{1s}$ atom

$10$ keV

$6$ fm

$^6\text{Li}$
M. Pospelov (2006) : stau-catalyzed d-α fusion

\[ d + (\alpha X^-) \rightarrow ^6\text{Li} + X^- + 1.1 \text{ MeV} \]

Serious problem: production of too much $^6\text{Li}$

This destroys the success of the Big-Bang-nucleosynthesis scenario. If this estimation is correct, we are enforced to assume

i) very short life time of $X^-$ to disappear before the nucleosynthesis time,

ii) very small density of $X^-$ at the nucleosynthesis time,

which strongly confines the property of $X^-$. Using a very naive model, he gave

\[ \sigma( \ d + (X^-\alpha) \rightarrow ^6\text{Li} + X^- \ ) \sim 10^8 \times \sigma( \ d + \alpha \rightarrow ^6\text{Li} + \gamma \ ) \]
Therefore, this is one of fashionable subjects in an overlap region of cosmology, elementary particle physics and nuclear astrophysics.

Many people wanted to know whether the Pospelov’s naive estimation
\[ \sigma(d + (X^-\alpha) \rightarrow ^6\text{Li} + X^-) \sim 10^8 \times \sigma(d + \alpha \rightarrow ^6\text{Li} + \gamma) \]
is valid or not.

I was asked, 6 month ago, by these 3 authors, to examine this estimation.

“Stau-catalyzed Nuclear Fusion“
K. Hamaguchi, T. Hatsuda and T. Yanagida
(University of Tokyo)
Enhance factor = \[ \frac{\text{CBBN}}{\text{SBBN}} \approx \left( \frac{130 \text{ fm}}{3.6 \text{ fm}} \right)^5 \approx 5 \times 10^7 \]

(No consideration on nuclear \(\alpha\)-d potential and on angular momentum between \(\alpha\) and \(X\))
Kino and myself did precise 3-body reaction calculation of
\[ d + (\alpha X^-) \rightarrow ^6\text{Li} + X^- + 1.1 \text{ MeV} \]
and published it together with the 3 authors (including an additional discussions from the Particle Physics models):

“Stau catalyzed $^6\text{Li}$ production in Big-Bang nucleosynthesis “
K. Hamaguchi, T. Hatsuda, M. Kamimura, Y. Kino and T.T Yanagida

We simply applied the same calculational method of our muon-transfer-reaction calculation to this stau-catalyzed nuclear fusion reaction because the two types of the reactions have the same structure as is seen in the next figure:
\[ d + (\alpha X^-) \rightarrow ^6\text{Li} + X^- + 1.1 \text{ MeV} \]

\[ t + (d\mu^-) \rightarrow (t\mu^-) + d + 48 \text{ eV} \]
However, large difference between two types of reactions:

1) **Stau-catalyzed nuclear fusion** takes place much below the Coulomb barrier.

   \[ d + (\alpha X^-) \rightarrow ^6\text{Li} + X^- + 1.1 \text{ MeV} \]

   incoming energy = \(10 - 100 \text{ keV}\) \((\text{Temperature}=10 \text{ keV})\)

   Coulomb barrier = \(500 \text{ keV}\)

2) We have to treat simultaneously both the long-range Coulomb potential and the short-range nuclear potential which is the driving force of the fusion reaction.

   \[ t + (d\mu^-) \rightarrow (t\mu^-) + d + 48 \text{ eV} \]

This 3-body reaction calculation is much more tedious than in the muon transfer reaction.
Therefore, for safety, Kino and myself solved the same 3-body Schroedinger equation, using quite different 2 methods to each other and compared the calculated S-matrix elements.

We were so careful about this exciting problem.

We found
1) Kino's result with the direct numerical (finite-difference) method
2) Kamimura's result with the Kohn-type variational method
   agree very well to each other.
Calculated astrophysical S-factor

\[ d + (\alpha X^-) \rightarrow ^6\text{Li} + X^- + 1.1 \text{ MeV} \]

Gamov peak at Temperature=10 keV

Reaction rate

\[ N_A \langle \sigma v \rangle = 2.37 \times 10^8 (1 - 0.34 T_9) T_9^{-2/3} \exp(-5.33 T_9^{-1/3}) \text{ cm}^3 \text{ s}^{-1} \text{ mol}^{-1}. \]  

PLB (in press, 2007)
Comparison in reaction cross section

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Pospelov (2006)</th>
<th>Kino &amp; Kamimura</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>too naive model</td>
<td>3-body calculation</td>
</tr>
<tr>
<td>d + (X^-α) → 6Li + X^-</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>t + (X^-α) → 7Li + X^-</td>
<td>1000</td>
<td>1</td>
</tr>
</tbody>
</table>

Reason of the difference:

Pospelov’s virtual photon model pays no attention
i) to the angular momenta between particle pairs
ii) to the nuclear potential
    (so simple model).
But, the Pospelov’s idea (stau-catalyzed nuclear fusion) itself is very much interesting and appreciated.

His estimation of the $^6\text{Li}$ production at the Big Bang time

$$\sigma(\, d + (X^-\alpha) \rightarrow ^6\text{Li} + X^- \, ) \sim 10^8 \times \sigma(\, d + \alpha \rightarrow ^6\text{Li} + \gamma \, )$$

is reduced to

$$\sigma(\, d + (X^-\alpha) \rightarrow ^6\text{Li} + X^- \, ) \sim 10^7 \times \sigma(\, d + \alpha \rightarrow ^6\text{Li} + \gamma \, )$$

by our calculation,

but still enough large to destroy the success of the standard Big-Bang scenario.

By the way, I was surprised to see the following thing:
within only 2 weeks after our preprint was posted on the arXive, 5 new preprints appeared in the arXive citing our result.

So busy the community is.
σ( d + (X^-α) → ^6Li + X^- ) \sim 10^7 \times \sigma( d + α → ^6Li + γ )

One of those 5 arXive preprints says:

“this factor of $10^7$ is very severe from the viewpoint of the compatibility between particle physics models and Big-Bang nucleosynthesis”.

We are now calculating all the possible cases of the stau-catalyzed nuclear reactions in Big-Bang nucleosynthesis and studying its influence on particle physics models.

It is my pleasure to see that developments of calculational methods which have been stimulated by $\mu$CF are now very useful to studies in other fields.

Let me skip the summary.

Thank you.