Annihilating States in Close-Coupling Method for Collisions between Hadronic and Ordinary Atoms

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Traditional close-coupling method for atomic and nuclear collisions supposes an expansion of the total wave function in terms of inner stationary states of colliding subsystems. In the case of hadronic atoms, a similar expansion has to involve *inter alia* the low angular-momentum states \((ns, np)\) with large annihilation (nuclear absorption) widths \(\Gamma_{nl}\) and small life times \(\tau_{nl} = \hbar/\Gamma_{nl}\). These states can disappear during collision. The parameter \(\nu_{nl} = \Gamma_{nl}R_i/\hbar v\) (the ratio of the collision time \(\tau_{\text{coll}} = R_i/v\) to the life time) gives a criterion of the state non-stationarity during collisions: at \(\nu \ll 1\) the annihilation during collision is negligible, whereas at \(\nu \gtrsim 1\) it is appreciable. The annihilation during collisions was considered previously within semiclassical approximations (see, e.g., [1, 2]). As far as we know, the single attempt to consider this effect within the quantum close-coupling method was done in the paper [3] for the collisions of \((\pi^-p)_{nl}\) with H atom. However the authors of [3] use an artificial assumption that the \(\Gamma_{nl}\) is turned off at distances between two atoms \(r > R_0 = 5a_0\) that contradicts to the physical reality.

In order to consider close-coupling equations with account for annihilating states correctly, we divide the total space of \(N\) channels into the subspace \(\alpha\) of the stationary states and the subspace \(\beta\) of the annihilating states, and construct two types of the \((N \times N)\) matrix solutions \(X(r)\) and \(Y(r)\), which are defined by the asymptotic forms at \(r \to \infty\): \(X_{ij}, Y_{ij} \to 0\) at \(i \neq j\), \(X_{ii}\) and \(Y_{ii}\) at \(i \in \alpha\) tend to ordinary incoming and outgoing waves, whereas at \(i \in \beta\) they tend to damping incoming and outgoing waves, respectively. The damping of the both latter waves is provided by the complex conjugation of the complex wave numbers in two waves, \(k_{\beta}^{-}\) and \(k_{\beta}^{+}\), and \(\text{Im}k_{\beta}^{+} > 0\). Total matrix of solutions with the correct asymptotic behaviour is \(F(r) = [X(r) - Y(r)]C\). At a small \(r = r_s\) it has to be sewed with the \((N \times N)\) matrix of regular solutions \(U(r)\) obtained by a standard way with account for the complex energy shifts in the annihilating channels. This procedure yields the \((N \times N)\) matrix \(C\). The submatrix \(C_{\alpha\alpha} = S\) is the S-matrix of the transitions between the states in the subspace \(\alpha\), whereas other elements of the \(C\) don’t have a real physical meaning. The S-matrix is not unitary, because the hamiltonian of the problem is non-hermitian. The ‘unitary defect’ \((1 - \sum_{j \in \alpha} |S_{ji}|^2)\) gives the cross section of induced annihilation for the initial state \(i \in \alpha\).