Finite Size Correction to the $dd\mu$ and $dt\mu$ Molecules

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Formation of a muonic molecule in a loosely bound excited state with $J = v = 1$ is an essentially important process in the muon catalyzed fusion cycle. In order to analyze an experimental data, an accuracy of 0.1 meV is required for the energy of the bound state. Various corrections to the energy should be estimated carefully as far as they were at rest 0.1 meV. The finite size correction to the $dd\mu$ and $dt\mu$ molecules is one of the most important corrections. Many calculations have been done for $dt\mu$-e and $dd\mu$-e four-body systems and $dt\mu$-dee and $dd\mu$-dee six-body systems. The calculations [1–5] are based on the second order perturbation theory. Since the size of the muonic molecule is two orders of magnitude smaller than the outer electron orbit, this approximation would seem to be reasonable. However, the first order and the second order contributions were almost the same amplitude and opposite sign.

$$E_{\rm FS}^{dt\mu e} = 0.50\text{meV}, \qquad E^{\rm 1st} + E^{\rm 2nd} = 18.253 + (-17.752)\text{meV}$$
$$E_{\rm FS}^{dd\mu e} = 1.46\text{meV}, \qquad E^{\rm 1st} + E^{\rm 2nd} = 11.577 + (-10.113)\text{meV}$$

In the present paper, we directly solve the four-body problem without the perturbation theory. The four-body system is not a true bound state, but a Feshbach-type resonance state. The four-body state Auger transfers to the lower three-body bound states emitting an Auger electron. We use the Gaussian expansion method [6] and the complex coordinate rotation method [7] to calculate the energy shift and Auger rate. Table 1 shows the energy shift due to the finite size of the $dt\mu$ molecule in the $dt\mu$-e four-body system together with literature values. The higher-oder contribution in the perturbation theory is not negligible.

Ref. [1]	Ref. [2]	Ref. [3]	Ref. [4]	Ref. [5]	This work
8	1.2	0.54	0.58	0.50	2.31

Table 1: Energy shift due to the finite size of the $dt\mu$ molecule in meV.

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