In the present paper we discuss the interrelation of the advanced interdisciplinary concepts of modern physics such as symmetry breaking, quantum protectorate, emergence and the Bogoliubov’s concept of quasiaverages in the context of modern theoretical physics, and, in particular, quantum and statistical physics. The main aim of this analysis was to demonstrate the connection and interrelation of these conceptual advances of the many-particle physics and to try to show explicitly that those concepts, though different in details, have certain common features. Some problems in the field of statistical physics of complex materials and systems e.g. foundation of the microscopic theory of magnetism and superconductivity were pointed in relation to these ideas. The main suggestion is that the emphasis of symmetry breaking concept is on the symmetry itself, whereas the method of quasiaverages emphasizes the degeneracy of a system. The concept of quantum protectorate reveals essential difference in the behavior of the complex many-body systems at the low-energy and high-energy scales. Thus the notion of quantum protectorate might provide distinctive signatures and good criteria for a hierarchy of energy scales and the appropriate emergent behavior.

Key words and phrases: theoretical physics, quantum physics, quantum statistical mechanics, symmetry, broken symmetry, Bogoliubov’s quasiaverages, quantum protectorate, emergence, quantum theory of magnetism, microscopic theory of superconductivity.

1. Introduction

The development of experimental techniques over the last decades opened the possibility for studies and investigations of the wide class of extremely complicated and multidisciplinary problems in physics, astrophysics, biology, material science, etc. In this regard theoretical physics is a kind of science which forms and elaborates the appropriate language for treating these problems on the firm ground [1]. This idea was expressed in the statement of F. Wilczek [2]: “primary goal of fundamental physics is to discover profound concepts that illuminate our understanding of nature”. For example, the theory of symmetry is a basic tool for understanding and formulating the fundamental notions of physics [3–11].It is well known that symmetry principles play a crucial role in physics. Many fundamental laws of physics in addition to their detailed features possess various symmetry properties [6,7]. These symmetry properties lead to certain constraints and regularities on the possible properties of matter. “Symmetry pervades the inner world of the structure of matter, the outer world of the cosmos, and the abstract world of mathematics itself. The basic laws of physics, the most fundamental statements we can make about nature, are founded upon symmetry” [6]. Thus the principles of symmetry belongs to the underlying principles of physics. Moreover, the idea of symmetry is a useful and workable tool for many areas of quantum field theory [7], physics of elementary particles [6,10,11], statistical physics and condensed matter physics [12–15]. Symmetry considerations show that symmetry arguments are very powerful tool for bringing order into the very complicated picture of the real world.

It is known that when the Hamiltonian of a system is invariant under a symmetry operation, but the ground state is not, the symmetry of the system can be spontaneously broken [16]. Symmetry breaking is termed spontaneous when there is no explicit term in a Lagrangian which manifestly breaks the symmetry. Symmetries and breaking of symmetries play an important role in statistical physics, quantum field theory, physics of elementary particles, etc. [17–19]
In physics, spontaneous symmetry breaking occurs when a system that is symmetric with respect to some symmetry group goes into a vacuum state that is not symmetric. When that happens, the system no longer appears to behave in a symmetric manner. It is a phenomenon that naturally occurs in many situations. The mechanism of spontaneous symmetry breaking is usually understood as the mechanism responsible for the occurrence of asymmetric states in quantum systems in the thermodynamic limit and is used in various fields of quantum physics [16]. The intriguing mechanism of spontaneous symmetry breaking is a unifying concept that lies at the basis of most of the recent developments in theoretical physics, from statistical mechanics to many-body theory and to elementary particles theory [17–21].

It should be stressed that symmetry implies degeneracy. The greater the symmetry, the greater the degeneracy. The study of the degeneracy of the energy levels plays a very important role in quantum physics. There is an important aspect of the degeneracy problem in quantum mechanics when a system possesses more subtle symmetries. This is the case when degeneracy of the levels arises from the invariance of the Hamiltonian \( H \) under groups involving simultaneous transformation of coordinates and momenta that contain as subgroups the usual geometrical groups based on point transformations of the coordinates. For these groups the free part of \( H \) is not invariant, so that the symmetry is established only for interacting systems. For this reason they are usually called dynamical groups.

It is of importance to emphasize that when spontaneous symmetry breaking takes place, the ground state of the system is degenerate. Substantial progress in the understanding of the broken symmetry concept was connected with Bogoliubov’s fundamental ideas on quasiaverages [22–32]. Studies of degenerate systems led Bogoliubov in 1960–61 to the formulation of the method of quasiaverages. This method has proved to be a universal tool for systems whose ground states become unstable under small perturbations. Thus the role of symmetry (and the breaking of symmetries) in combination with the degeneracy of the system was reanalyzed and essentially clarified by N. N. Bogoliubov in 1960–1961. He invented and formulated a powerful innovative idea of quasiaverages in statistical mechanics [22–25, 28, 31]. The very elegant work of N. N. Bogoliubov [23] has been of great importance for a deeper understanding of phase transitions, superfluidity and superconductivity [22–31, 33, 34], quantum theory of magnetism [32] and other fields of equilibrium and nonequilibrium statistical mechanics [14, 15, 22–32, 35, 36]. The concept of quasiaverages is indirectly related to the theory of phase transition. The instability of thermodynamic averages with respect to perturbations of the Hamiltonian by a breaking of the invariance with respect to a certain group of transformations means that in the system transition to an extremal state occurs. The mathematical apparatus of the method of quasiaverages includes the Bogoliubov theorem [14, 23, 25, 28, 31] on singularities of type \( 1/q^2 \) and the Bogoliubov inequality for Green and correlation functions as a direct consequence of the method. It includes algorithms for establishing non-trivial estimates for equilibrium quasiaverages, enabling one to study the problem of ordering in statistical systems and to elucidate the structure of the energy spectrum of the underlying excited states. Thus the Bogoliubov’s idea of quasiaverages is an essential conceptual advance of modern physics.

It is well known that there are many branches of physics and chemistry where phenomena occur which cannot be described in the framework of interactions amongst a few particles. As a rule, these phenomena arise essentially from the cooperative behavior of a large number of particles. Such many-body problems are of great interest not only because of the nature of phenomena themselves, but also because of the intrinsic difficulties in solving problems which involve interactions of many particles (in terms of known P.W. Anderson’s statement: “more is different” [37]). It is often difficult to formulate a fully consistent and adequate microscopic theory of complex cooperative phenomena. Statistical mechanics relates the behavior of macroscopic objects to the dynamics of their constituent microscopic entities. Primary examples include the entropy increasing evolution of nonequilibrium systems and phase transitions in equilibrium systems. Many aspects of these phenomena can be captured in greatly
simplified models of the microscopic world. They emerge as collective properties of large aggregates, i.e. macroscopic systems, which are independent of many details of the microscopic dynamics. More recently it has been possible to make a step forward in solving of these problems. This step leads to a deeper understanding of the relations between microscopic dynamics and macroscopic behavior on the basis of emergence concept [37–46].

It was shown [46] that emergence phenomena in physics can be understood better in connection with other disciplines. In particular, since emergence is the overriding issue receiving increasing attention in physics and beyond, it is of big value for philosophy also. Different scientific disciplines underlie the different senses of emergence. There are at least three senses of emergence and a suggestive view on the emergence of time and the direction of time have been discussed intensely. The important aspect of emergence concept is different manifestations at different levels of structures, hierarchical in form, and corresponding interactions. It is not easy task to formulate precisely observations pertaining to the concepts, methodology and mechanisms required to understand emergence and describe a platform for its investigation [46].

The “quantum protectorate” concept was formulated in the paper [47]. Its authors, R. Laughlin and D. Pines, discussed the most fundamental principles of matter description in the widest sense of this word. The notion of quantum protectorate [47–49] complements the concepts of broken symmetry and quasiaverages by making emphasis on the hierarchy of the energy scales of many-particle systems [14,50].

The chief purpose of this paper was to formulate the connection and interrelation of the complementary conceptual advances (or “profound concepts”) of the many-body physics, namely the quasiaverages, emergence and quantum protectorate, and to try to show explicitly that those concepts, though different in details, have a certain common features.

2. Bogoliubov’s Quasiaverages

In the work of N. N. Bogoliubov “Quasiaverages in Problems of Statistical Mechanics” the innovative notion of quasiaverage [23] was introduced and applied to various problem of statistical physics. In particular, quasiaverages of Green’s functions constructed from ordinary averages, degeneration of statistical equilibrium states, principle of weakened correlations, and particle pair states were considered. In this framework the $1/q^2$-type properties in the theory of the superfluidity of Bose and Fermi systems, the properties of basic Green functions for a Bose system in the presence of condensate, and a model with separated condensate were analyzed [51].

The method of quasiaverages is a constructive workable scheme for studying systems with spontaneous symmetry breakdown such as superfluidity and superconductivity [14,15,32,51–61]. A quasiaverage is a thermodynamic (in statistical mechanics) or vacuum (in quantum field theory) average of dynamical quantities in a specially modified averaging procedure, enabling one to take into account the effects of the influence of state degeneracy of the system. The method gives the so-called macro-objectivation of the degeneracy in the domain of quantum statistical mechanics and in quantum physics. In statistical mechanics, under spontaneous symmetry breakdown one can, by using the method of quasiaverages, describe macroscopic observable within the framework of the microscopic approach.

In considering problems of findings the eigenfunctions in quantum mechanics it is well known that the theory of perturbations should be modified substantially for the degenerate systems. In the problems of statistical mechanics we have always the degenerate case due to existence of the additive conservation laws. The traditional approach to quantum statistical mechanics [31] is based on the unique canonical quantization of classical Hamiltonians for systems with finitely many degrees of freedom together with the ensemble averaging in terms of traces involving a statistical operator $\rho$. For an operator $A$ corresponding to some physical quantity $A$ the average value
of $A$ will be given as

$$
\langle A \rangle_H = \text{Tr} \rho A; \quad \rho = \exp^{-\beta H} / \text{Tr} \exp^{-\beta H},
$$

where $H$ is the Hamiltonian of the system, $\beta = 1/kT$ is the reciprocal of the temperature.

The core of the problem lies in establishing the existence of a thermodynamic limit (such as $N/V = \text{const, } V \to \infty$, $N = \text{number of degrees of freedom, } V = \text{volume}$) and its evaluation for the quantities of interest. Thus in the statistical mechanics the average $\langle A \rangle$ of any dynamical quantity $A$ is defined in a single-valued way. In the situations with degeneracy the specific problems appear. In quantum mechanics, if two linearly independent state vectors (wave functions in the Schrodinger picture) have the same energy, there is a degeneracy. In this case more than one independent state of the system corresponds to a single energy level. If the statistical equilibrium state of the system possesses lower symmetry than the Hamiltonian of the system (i.e. the situation with the spontaneous symmetry breakdown), then it is necessary to supplement the averaging procedure (1) by a rule forbidding irrelevant averaging over the values of macroscopic quantities considered for which a change is not accompanied by a change in energy. This is achieved by introducing quasiaverages, that is, averages over the Hamiltonian $H_{\nu\vec{e}}$ supplemented by infinitesimally-small terms that violate the additive conservation laws $H_{\nu\vec{e}} = H + \nu(\vec{e} \cdot \vec{M})$, ($\nu \to 0$). Thermodynamic averaging may turn out to be unstable with respect to such a change of the original Hamiltonian, which is another indication of degeneracy of the equilibrium state.

According to Bogoliubov [23], the quasiaverage of a dynamical quantity $A$ for the system with the Hamiltonian $H_{\nu\vec{e}}$ is defined as the limit

$$
\langle A \rangle_{\nu\vec{e}} = \lim_{\nu \to 0} \langle A \rangle_{\nu\vec{e}},
$$

where $\langle A \rangle_{\nu\vec{e}}$ denotes the ordinary average taken over the Hamiltonian $H_{\nu\vec{e}}$, containing the small symmetry-breaking terms introduced by the inclusion parameter $\nu$, which vanish as $\nu \to 0$ after passage to the thermodynamic limit $V \to \infty$. Thus the existence of degeneracy is reflected directly in the quasiaverages by their dependence upon the arbitrary unit vector $\vec{e}$. It is also clear that

$$
\langle A \rangle = \int \langle A \rangle_{\nu\vec{e}} \, d\vec{e}.
$$

According to definition (3), the ordinary thermodynamic average is obtained by extra averaging of the quasiaverage over the symmetry-breaking group. Thus to describe the case of a degenerate state of statistical equilibrium quasiaverages are more convenient, more physical, than ordinary averages [31]. The latter are the same quasi-averages only averaged over all the directions $\vec{e}$.

It is necessary to stress, that the starting point for Bogoliubov’s work [23] was an investigation of additive conservation laws and selection rules, continuing and developing the approach by P. Curie for derivation of selection rules for physical effects. Bogoliubov demonstrated that in the cases when the state of statistical equilibrium is degenerate, as in the case of the Heisenberg ferromagnet, one can remove the degeneracy of equilibrium states with respect to the group of spin rotations by including in the Hamiltonian $H$ an additional noninvariant term $\nu M_2 V$ with an infinitely small $\nu$. For the Heisenberg ferromagnet the ordinary averages must be invariant with regard to the spin rotation group. The corresponding quasiaverages possess only the property of covariance. Thus the quasiaverages do not follow the same selection rules as those which govern ordinary averages, due to their invariance with regard to the spin rotation group. It is clear that the unit vector $\vec{e}$, i.e., the direction of the magnetization $\vec{M}$ vector, characterizes the degeneracy of the considered state of statistical equilibrium. In order to remove the degeneracy one should fix the direction of the
unit vector \( \hat{e} \). It can be chosen to be along the \( z \) direction. Then all the quasiaverages will be the definite numbers. This is the kind that one usually deals with in the theory of ferromagnetism.

The question of symmetry breaking within the localized and band models of antiferromagnets was studied by the author of this report in Refs. [14, 32, 50, 62]. It has been found there that the concept of spontaneous symmetry breaking in the band model of magnetism is much more complicated than in the localized model [32]. In the framework of the band model of magnetism one has to additionally consider the so called anomalous propagators of the form [14,32,50,62]

\[
\text{FM : } G_{fm} \sim \langle \langle a_{k\sigma} a_{k-\sigma}^\dagger \rangle \rangle, \\
\text{AFM : } G_{afm} \sim \langle \langle a_{k+Q\sigma} a_{k+Q'-\sigma}^\dagger \rangle \rangle. 
\]

In the case of the band antiferromagnet the ground state of the system corresponds to a spin-density wave (SDW), where a particle scattered on the internal inhomogeneous periodic field gains the momentum \( Q - Q' \) and changes its spin: \( \sigma \rightarrow \sigma' \). The long-range order parameters are defined as follows:

\[
\text{FM : } m = 1/N \sum_{k\sigma} \langle a_{k\sigma}^\dagger a_{k-\sigma} \rangle, \\
\text{AFM : } M_Q = \sum_{kQ} \langle a_{k\sigma}^\dagger a_{k+Q-\sigma} \rangle.
\]

It is important to stress, that the long-range order parameters here are functionals of the internal field, which in turn is a function of the order parameter. Thus, in the cases of Hamiltonians of band ferro- and antiferromagnetics one has to add the following infinitesimal sources removing the degeneracy:

\[
\text{FM : } \nu \mu_B H_x \sum_{k\sigma} a_{k\sigma}^\dagger a_{k-\sigma}, \\
\text{AFM : } \nu \mu_B H \sum_{kQ} a_{k\sigma}^\dagger a_{k+Q-\sigma}.
\]

Here, \( \nu \rightarrow 0 \) after the usual in statistical mechanics infinite-volume limit \( V \rightarrow \infty \). The ground state in the form of a spin-density wave was obtained for the first time by Overhauser. There, the vector \( \vec{Q} \) is a measure of inhomogeneity or translation symmetry breaking in the system.

The analysis performed by various authors showed that the antiferromagnetic and more complicated states (for instance, ferrimagnetic) can be described in the framework of a generalized mean-field approximation. In doing that we have to take into account both the normal averages \( \langle a_{i\sigma}^\dagger a_{i\sigma} \rangle \) and the anomalous averages \( \langle a_{i\sigma}^\dagger a_{i-\sigma} \rangle \). It is clear that the anomalous terms break the original rotational symmetry of the Hubbard Hamiltonian. Thus, the generalized mean-field’s approximation has the following form

\[
\begin{align*}
\langle a_{k+p\sigma} a_{k+q-\sigma} a_{q-\sigma} a_{k+\sigma} \rangle \omega &= \langle a_{k+p\sigma} a_{k+q-\sigma} a_{q-\sigma} a_{k+\sigma} \rangle \omega - \\
&- \delta_{p,0} \langle n_{q-\sigma} \rangle G_{k\sigma} - \langle a_{k+p\sigma} a_{k+q-\sigma} \rangle \langle a_{q-\sigma} a_{k+\sigma} \rangle \omega.
\end{align*}
\]

A self-consistent theory of band antiferromagnetism was developed by the author of this report using the method of the irreducible Green functions [14,32,50,62]. The following definition of the irreducible Green functions was used:

\[
ir \langle \langle a_{k+p\sigma} a_{k+q-\sigma} a_{q-\sigma} a_{k+\sigma} \rangle \rangle \omega = \langle a_{k+p\sigma} a_{k+q-\sigma} a_{q-\sigma} a_{k+\sigma} \rangle \omega - \\
- \delta_{p,0} \langle n_{q-\sigma} \rangle G_{k\sigma} - \langle a_{k+p\sigma} a_{k+q-\sigma} \rangle \langle a_{q-\sigma} a_{k+\sigma} \rangle \omega.
\]
The algebra of relevant operators must be chosen as follows \((a_{i\sigma}, a_{i\sigma}^\dagger, n_{i\sigma}, a_{i\sigma}^\dagger a_{i-\sigma})\). The corresponding initial GF will have the following matrix structure:

\[
\mathcal{G}_{AFM} = \begin{pmatrix}
\langle\langle a_{i\sigma}|a_{j\sigma}^\dagger \rangle\rangle & \langle\langle a_{i\sigma}|a_{j-\sigma}^\dagger \rangle\rangle \\
\langle\langle a_{i-\sigma}|a_{j\sigma}^\dagger \rangle\rangle & \langle\langle a_{i-\sigma}|a_{j-\sigma}^\dagger \rangle\rangle
\end{pmatrix}.
\]

The off-diagonal terms select the vacuum state of the band’s antiferromagnet in the form of a spin-density wave. It is necessary to stress that the problem of the band’s antiferromagnetism is quite involved, and the construction of a consistent microscopic theory of this phenomenon remains a topical problem.

D.N. Zubarev showed [35] that the concepts of symmetry breaking perturbations and quasiaverages play an important role in the theory of irreversible processes as well. The method of the construction of the nonequilibrium statistical operator [32,35,36] becomes especially deep and transparent when it is applied in the framework of the quasiaverage concept. The main idea of this approach was to consider infinitesimally small sources breaking the time-reversal symmetry of the Liouville equation, which become vanishingly small after a thermodynamic limiting transition.

To summarize, the Bogoliubov’s method of quasiaverages gives the deep foundation and clarification of the concept of broken symmetry. It makes the emphasis on the notion of a degeneracy and plays an important role in equilibrium statistical mechanics of many-particle systems. According to that concept, infinitely small perturbations can trigger macroscopic responses in the system if they break some symmetry and remove the related degeneracy (or quasidegeneracy) of the equilibrium state. As a result, they can produce macroscopic effects even when the perturbation magnitude is tend to zero, provided that happens after passing to the thermodynamic limit. Therefore the method of quasiaverages plays a fundamental role in equilibrium and nonequilibrium statistical mechanics and is one of the pillars of modern physics.

3. Emergent Phenomena

Emergence and complexity refer to the appearance of higher-level properties and behaviors of a system that obviously comes from the collective dynamics of that system’s components [37–49]. These properties are not directly deducible from the lower-level motion of that system. Emergent properties are properties of the “whole” that are not possessed by any of the individual parts making up that whole. Such phenomena exist in various domains and can be described, using complexity concepts and thematic knowledges. Thus this problematic is highly pluridisciplinary. Emergence — macro-level effect from micro-level causes - is an important and profound interdisciplinary notion of modern science [37–49].

Emergence is a key notion when discussing various aspects of what are termed self-organizing systems, spontaneous orders, chaotic systems, system complexity, and so on. This variety of the problems reflects the multidisciplinary nature of the emergence concept, because the concept has appeared relatively independently in various contexts within philosophy [46], the social and the natural sciences [41,42]. Emergence unites these disciplines in the sense that it emphasizes their common focus on phenomena where orders arises from elements within a system acting independently from one another. In addition, emergence stresses that such an action is realized within a framework of procedural rules or laws that generate positive and negative feedback such that independent behavior takes the actions of others into consideration without intending to do so. Moreover, the impact of that behavior tends to facilitate more complex relationships of mutual assistance than could ever be deliberately created. Such systems may generate order spontaneously. In doing so they can act in unanticipated ways because there is no overarching goal, necessity, or plan that orders the actions of their components or the responses they make to feedback generated within the system.
Indeed, self-organization, fractals, chaos, and many other interesting dynamical phenomena can be understood better with the help of the emergence concept [39, 41, 42]. For example, a system with positive and negative feedback loops is modeled with nonlinear equations. Self-organization may occur when feedback loops exist among component parts and between the parts and the structures that emerge at higher hierarchical levels. In chemistry, when an enzyme catalyzes reactions that encourage the production of more of itself, it is called auto-catalysis. It was suggested that auto-catalysis played an important role in the origins of life [39]. Thus the essence of self-organization lies in the connections, interactions, and feedback loops between the parts of the system [39, 41, 42]. It is clear then that system must have a large number of parts. Cells, living tissue, the immune system, brains, populations, communities, economies, and climates all contain huge number of parts. These parts are often called agents because they have the basic properties of information transfer, storage and processing. An agent could be a ferromagnetic particle in a spin glass, a neuron in a brain, or a firm in an economy. Models that assign agency at this level are known as individual-based models. It is possible to say that emergence is a kind of observation, when the observer’s attention shifts from the micro-level of the agents to the macro-level of the system. Emergence fits well into hierarchy theory as a way of describing how each hierarchical level in a system can follow discrete rule sets.

Emergence also points to the multiscale interactions [39, 41, 42, 47–49] and effects in self-organized systems. The small-scale interactions produce large-scale structures, which then modify the activity at the small scales. For instance, specific chemicals and neurons in the immune system can create organism-wide bodily sensations which might then have a huge effect on the chemicals and neurons. Some authors has argued that macro-scale emergent order is a way for a system to dissipate micro-scale entropy creation caused by energy flux, but this is still a hypothesis which must be verified.

Thus emergent entities (properties or substances) arise out of more fundamental entities and yet are novel or irreducible with respect to them. Each of these terms are uncertain in its own right, and their specifications yield the varied notions of emergence that have been discussed in literature [37–49]. There has been renewed interest in emergence within discussions of the behavior of complex systems [41–43] and debates over the reconcilability of mental causation, intentionality, or consciousness with physicalism. This concept is also at the heart of the numerous discussions on the interrelation of the reductionism and functionalism [37–39, 46–49].

A vast amount of current researches focuses on the search for the organizing principles responsible for emergent behavior in matter [47, 63], with particular attention to correlated matter, the study of materials in which unexpectedly new classes of behavior emerge in response to the strong and competing interactions among their elementary constituents. As it was formulated at Ref. [63], “we call emergent behavior … the phenomena that owe their existence to interactions between many subunits, but whose existence cannot be deduced from a detailed knowledge of those subunits alone”.

Models and simulations of collective behaviors are often based on considering them as interactive particle systems [42]. The focus is then on behavioral and interaction rules of particles by using approaches based on artificial agents designed to reproduce swarm-like behaviors in a virtual world by using symbolic, sub-symbolic and agent-based models. New approaches have been considered in the literature [41–43] based, for instance, on topological rather than metric distances and on fuzzy systems. Recently a new research approach [42] was proposed allowing generalization possibly suitable for a general theory of emergence. The coherence of collective behaviors, i.e., their identity detected by the observer, as given by meta-structures, properties of meta-elements, i.e., sets of values adopted by mesoscopic state variables describing collective, structural aspects of the collective phenomenon under study and related to a higher level of description (meta-description) suitable for dealing with coherence, was considered. Mesoscopic state variables were abductively identified by the observer detecting emergent properties, such as sets of suitably clustered distances, speed, directions, their ratios and ergodic properties of sets. This research approach
is under implementation and validation and may be considered to model general processes of collective behavior and establish an possible initial basis for a general theory of emergence.

Statistical physics and condensed matter physics supply us with many examples of emergent phenomena [14, 15]. For example, taking a macroscopic approach to the problem, and identifying the right degrees of freedom of a many-particle system, the equations of motion of interacting particles forming a fluid can be described by the Navier-Stokes equations for fluid dynamics from which complex new behaviors arise such as turbulence. This is the clear example of an emergent phenomenon in classical physics.

Including quantum mechanics into the consideration leads to even more complicated situation. In 1972 P. W. Anderson published his essay “More is Different” which describes how new concepts, not applicable in ordinary classical or quantum mechanics, can arise from the consideration of aggregates of large numbers of particles [37]. Quantum mechanics is a basis of macrophysics. However macroscopic systems have the properties that are radically different from those of their constituent particles. Thus, unlike systems of few particles, they exhibit irreversible dynamics, phase transitions and various ordered structures, including those characteristic of life [37–49]. These and other macroscopic phenomena signify that complex systems, that is, ones consisting of huge numbers of interacting particles, are qualitatively different from the sums of their constituent parts [37].

Many-particle systems where the interaction is strong have often complicated behavior, and require nonperturbative approaches to treat their properties. Such situations are often arise in condensed matter systems. Electrical, magnetic and mechanical properties of materials are emergent collective behaviors of the underlying quantum mechanics of their electrons and constituent atoms. A principal aim of solid state physics and materials science is to elucidate this emergence. A full achievement of this goal would imply the ability to engineer a material that is optimum for any particular application. The current understanding of electrons in solids uses simplified but workable picture known as the Fermi liquid theory. This theory explains why electrons in solids can often be described in a simplified manner which appears to ignore the large repulsive forces that electrons are known to exert on one another. There is a growing appreciation that this theory probably fails for entire classes of possibly useful materials and there is the suspicion that the failure has to do with unresolved competition between different possible emergent behaviors.

In Ref. [40] Levine and Wen proposed to consider photons and electrons as emergent phenomena. Their arguments are based on recent advances in condensed-matter theory which have revealed that new and exotic phases of matter can exist in spin models (or more precisely, local bosonic models) via a simple physical mechanism, known as “string-net condensation”. These new phases of matter have the unusual property that their collective excitations are gauge bosons and fermions. In some cases, the collective excitations can behave just like the photons, electrons, gluons, and quarks in the relevant vacuum. This suggests that photons, electrons, and other elementary particles may have a unified origin-string-net condensation in that vacuum. In addition, the string-net picture indicates how to make artificial photons, artificial electrons, and artificial quarks and gluons in condensed-matter systems.

4. Quantum Mechanics and its Emergent Macrophysics

The notion of emergence in quantum physics has attracted recently big attention [64–72]. Although quantum mechanics (QM) has been the generally accepted basis for most of progress in fundamental physics during the last 100 years, the extension of the current theoretical frontier to Planck’s scale physics, and recent enlargements of our experimental capabilities, may make our time the period in which possible limits of quantum theory will be subjected to a thorough scrutiny. Some authors speculate that QM may be actually not a complete ontological system, but in fact it represents a very accurate low-energy approximation to a deeper level of dynamics (hierarchy of
energy scales). But what is exactly the “deeper level dynamics” is not clear at all. There is a growing interest in these problems which was partially supported by the belief that to make a convincing synthesis of QM and general relativity [44] new conceptual paradigms should be formulated to describe physics at very small space-time scales.

The interrelation of notion of emergence and QM was considered by Sewell in his book “Quantum Mechanics And Its Emergent Macrophysics” [66]. According to his point of view, the quantum theory of macroscopic systems is a vast, ever-developing area of science that serves to relate the properties of complex physical objects to those of their constituent particles. Its essential challenge is that of finding the conceptual structures needed for the description of the various states of organization of many-particle quantum systems. In that book, Sewell proposes a new approach to the subject, based on a “macrostatistical mechanics”, which contrasts sharply with the standard microscopic treatments of many-body problems.

According to Sewell, quantum theory began with Planck’s derivation of the thermodynamics of black body radiation from the hypothesis that the action of his oscillator model of matter was quantized in integral multiples of a fundamental constant, $\hbar$. This result provided a microscopic theory of a macroscopic phenomenon that was incompatible with the assumption of underlying classical laws. In the century following Planck’s discovery, it became abundantly clear that quantum theory is essential to natural phenomena on both the microscopic and macroscopic scales.

As a first step towards contemplating the quantum mechanical basis of macrophysics, Sewell notes the empirical fact that macroscopic systems enjoy properties that are radically different from those of their constituent particles. Thus, unlike systems of few particles, they exhibit irreversible dynamics, phase transitions and various ordered structures, including those characteristic of life. These and other macroscopic phenomena signify that complex systems, that is, ones consisting of enormous numbers of interacting particles, are qualitatively different from the sums of their constituent parts (this point of view was also stressed by Anderson [37]).

Sewell proceeds by presenting the operator algebraic framework for the theory. He then undertakes a macrostatistical treatment of both equilibrium and nonequilibrium thermodynamics, which yields a major new characterization of a complete set of thermodynamic variables and a nonlinear generalization of the Onsager theory. He focuses especially on ordered and chaotic structures that arise in some key areas of condensed matter physics. This includes a general derivation of superconductive electrodynamics from the assumptions of off-diagonal long-range order, gauge covariance, and thermodynamic stability, which avoids the enormous complications of the microscopic treatments. Sewell also re-analyzes a theoretical framework for phase transitions far from thermal equilibrium. It gives a coherent approach to the complicated problem of the emergence of macroscopic phenomena from quantum mechanics and clarifies the problem of how macroscopic phenomena can be interpreted from the laws and structures of microphysics.

Correspondingly, theories of such phenomena must be based not only on the quantum mechanics, but also on conceptual structures that serve to represent the characteristic features of highly complex systems [47–49]. Among the main concepts involved here are ones representing various types of order, or organization, disorder, or chaos, and different levels of macroscopicality. Moreover, the particular concepts required to describe the ordered structures of superfluids and laser light are represented by macroscopic wave functions that are strictly quantum mechanical, although radically different from the Schrodinger wave functions of microphysics.

Thus, according to Sewell, to provide a mathematical framework for the conceptual structures required for quantum macrophysics, it is clear that one needs to go beyond the traditional form of quantum mechanics, since that does not discriminate qualitatively between microscopic and macroscopic systems. This may be seen from the fact that the traditional theory serves to represent a system of $N$ particles within the standard Hilbert space scheme, which takes the same form regardless of whether $N$ is “small” or “large”.

Sewell’s approach to the basic problem of how macrophysics emerges from quantum mechanics is centered on macroscopic observables. The main objective of his approach is to obtain the properties imposed on them by general demands of quantum theory and many-particle statistics. This approach resembles in a certain sense the Onsager’s irreversible thermodynamics, which bases also on macroscopic observables and certain general structures of complex systems [35, 36].

The conceptual basis of quantum mechanics which go far beyond its traditional form was formulated by S. L. Adler [67]. According to his view, quantum mechanics is not a complete theory, but rather is an emergent phenomenon arising from the statistical mechanics of matrix models that have a global unitary invariance. The mathematical presentation of these ideas is based on dynamical variables that are matrices in complex Hilbert space, but many of the ideas carry over to a statistical dynamics of matrix models in real or quaternionic Hilbert space. Adler starts from a classical dynamics in which the dynamical variables are non-commutative matrices or operators. Despite the non-commutativity, a sensible Lagrangian and Hamiltonian dynamics was obtained by forming the Lagrangian and Hamiltonian as traces of polynomials in the dynamical variables, and repeatedly using cyclic permutation under the trace. It was assumed that the Lagrangian and Hamiltonian are constructed without use of non-dynamical matrix coefficients, so that there is an invariance under simultaneous, identical unitary transformations of all the dynamical variables, that is, there is a global unitary invariance. The author supposed that the complicated dynamical equations resulting from this system rapidly reach statistical equilibrium, and then shown that with suitable approximations, the statistical thermodynamics of the canonical ensemble for this system takes the form of quantum field theory. The requirements for the underlying trace dynamics to yield quantum theory at the level of thermodynamics are stringent, and include both the generation of a mass hierarchy and the existence of boson-fermion balance. From the equilibrium statistical mechanics of trace dynamics, the rules of quantum mechanics emerge as an approximate thermodynamic description of the behavior of low energy phenomena. “Low energy” here means small relative to the natural energy scale implicit in the canonical ensemble for trace dynamics, which author identify with the Planck scale, and by “equilibrium” he means local equilibrium, permitting spatial variations associated with dynamics on the low energy scale. Brownian motion corrections to the thermodynamics of trace dynamics then lead to fluctuation corrections to quantum mechanics which take the form of stochastic modifications of the Schrodinger equation, that can account in a mathematically precise way for state vector reduction with Born rule probabilities [67].

Adler emphasizes [67] that he have not identified a candidate for the specific matrix model that realizes his assumptions; there may be only one, which could then provide the underlying unified theory of physical phenomena that is the goal of current researches in high-energy physics and cosmology. He admits the possibility also that the underlying dynamics may be discrete, and this could naturally be implemented within his framework of basing an underlying dynamics on trace class matrices. The ideas of the Adler’s book suggest, that one should seek a common origin for both gravitation and quantum field theory at the deeper level of physical phenomena from which quantum field theory emerges [67]. Recently Adler discussed his ideas further [68]. He reviewed the proposal made in his 2004 book [67], that quantum theory is an emergent theory arising from a deeper level of dynamics. The dynamics at this deeper level is taken to be an extension of classical dynamics to non-commuting matrix variables, with cyclic permutation inside a trace used as the basic calculational tool. With plausible assumptions, quantum theory was shown to emerge as the statistical thermodynamics of this underlying theory, with the canonical commutation-anticommutation relations derived from a generalized equipartition theorem. Brownian motion corrections to this thermodynamics were argued to lead to state vector reduction and to the probabilistic interpretation of quantum theory, making contact with phenomenological proposals for stochastic modifications to Schrodinger dynamics.

G. ’t Hooft considered various aspects of quantum mechanics in the context of emergence [69–71]. According to his view, quantum mechanics is emergent if a statis
tical treatment of large scale phenomena in a locally deterministic theory requires the use of quantum operators. These quantum operators may allow for symmetry transformations that are not present in the underlying deterministic system. Such theories allow for a natural explanation of the existence of gauge equivalence classes (gauge orbits), including the equivalence classes generated by general coordinate transformations. Thus, local gauge symmetries and general coordinate invariance could be emergent symmetries, and this might lead to new avenues towards understanding the flatness problem of the Universe. G. ’t Hooft demonstrated also that “For any quantum system there exists at least one deterministic model that reproduces all its dynamics after prequantization”. H.T. Elze elaborated an extension [72] which covers quantum systems that are characterized by a complete set of mutually commuting Hermitian operators (”beables”). He introduced the symmetry of beables: any complete set of beables is as good as any other one which is obtained through a real general linear group transformation. The quantum numbers of a specific set are related to symmetry breaking initial and boundary conditions in a deterministic model. The Hamiltonian, in particular, can be taken as the emergent beable which provides the best resolution of the evolution of the model Universe.

5. Quantum Protectorate

R. Laughlin and D. Pines invented an idea of a quantum protectorate, “a stable state of matter, whose generic low-energy properties are determined by a higher-organizing principle and nothing else” [47]. This idea brings into physics the concept that emphasize the crucial role of low-energy and high-energy scales for treating the propertied of the substance. It is known that a many-particle system (e.g. electron gas) in the low-energy limit can be characterized by a small set of collective (or hydrodynamic) variables and equations of motion corresponding to these variables. Going beyond the framework of the low-energy region would require the consideration of plasmon excitations, effects of electron shell reconstructing, etc. The existence of two scales, low-energy and high-energy, in the description of physical phenomena is used in physics, explicitly or implicitly.

According to R. Laughlin and D. Pines, “The emergent physical phenomena regulated by higher organizing principles have a property, namely their insensitivity to microscopics, that is directly relevant to the broad question of what is knowable in the deepest sense of the term. The low energy excitation spectrum of a conventional superconductor, for example, is completely generic and is characterized by a handful of parameters that may be determined experimentally but cannot, in general, be computed from first principles. An even more trivial example is the low-energy excitation spectrum of a conventional crystalline insulator, which consists of transverse and longitudinal sound and nothing else, regardless of details. It is rather obvious that one does not need to prove the existence of sound in a solid, for it follows from the existence of elastic moduli at long length scales, which in turn follows from the spontaneous breaking of translational and rotational symmetry characteristic of the crystalline state. Conversely, one therefore learns little about the atomic structure of a crystalline solid by measuring its acoustics. The crystalline state is the simplest known example of a quantum protectorate, a stable state of matter whose generic low-energy properties are determined by a higher organizing principle and nothing else . . . .

Other important quantum protectorates include superfluidity in Bose liquids such as $^4\text{He}$ and the newly discovered atomic condensates, superconductivity, band insulation, ferromagnetism, antiferromagnetism, and the quantum Hall states. The low-energy excited quantum states of these systems are particles in exactly the same sense that the electron in the vacuum of quantum electrodynamics is a particle . . . Yet they are not elementary, and, as in the case of sound, simply do not exist outside the context of the stable state of matter in which they live. These quantum protectorates, with their associated emergent behavior, provide us with explicit demonstrations that the underlying microscopic theory can easily have no measurable consequences whatsoever at low energies. The nature of the underlying theory is unknowable until one
raises the energy scale sufficiently to escape protection”. The notion of quantum protectorate was introduced to unify some generic features of complex physical systems on different energy scales, and is a complimentary unifying idea resembling the symmetry breaking concept in a certain sense.

In the search for a “theory of everything” [47], scientists scrutinize ever-smaller components of the universe. String theory postulates units so minuscule that researchers would not have the technology to detect them for decades. R.B. Laughlin [47–49, 63], argued that smaller is not necessarily better. He proposes turning our attention instead to emerging properties of large agglomerations of matter. For instance, chaos theory has been all the rage of late with its speculations about the “butterfly effect”, but understanding how individual streams of air combine to form a turbulent flow is almost impossible [73]. It may be easier and more efficient, says Laughlin, to study the turbulent flow. Laws and theories follow from collective behavior, not the other way around, and if one will try to analyze things too closely, he may not understand how they work on a macro level. In many cases, the whole exhibits properties that can not be explained by the behavior of its parts. As Laughlin points out, mankind use computers and internal combustion engines every day, but scientists do not totally understand why all of their parts work the way they do.

The authors formulate their main thesis: emergent physical phenomena, which are regulated by higher physical principles, have a certain property, typical for these phenomena only. This property is their insensitivity to microscopic description. For instance, the crystalline state is the simplest known example of a quantum protectorate, a stable state of matter whose generic low-energy properties are determined by a higher organizing principle and nothing else. There are many other examples [47]. These quantum protectorates, with their associated emergent behavior, provide us with explicit demonstrations that the underlying microscopic theory can easily have no measurable consequences whatsoever at low energies. The nature of the underlying theory is unknowable until one raises the energy scale sufficiently to escape protection. The existence of two scales, the low-energy and high-energy scales, relevant to the description of magnetic phenomena was stressed by the author of this report in the papers [32, 50] devoted to comparative analysis of localized and band models of quantum theory of magnetism. It was shown there, that the low-energy spectrum of magnetic excitations in the magnetically-ordered solid bodies corresponds to a hydrodynamic pole ($\vec{k}, \omega \to 0$) in the generalized spin susceptibility $\chi$, which is present in the Heisenberg, Hubbard, and the combined $s-d$ model. In the Stoner band model the hydrodynamic pole is absent, there are no spin waves there. At the same time, the Stoner single-particle’s excitations are absent in the Heisenberg model’s spectrum. The Hubbard model with narrow energy bands contains both types of excitations: the collective spin waves (the low-energy spectrum) and Stoner single-particle’s excitations (the high-energy spectrum). This is a big advantage and flexibility of the Hubbard model in comparison to the Heisenberg model. The latter, nevertheless, is a very good approximation to the realistic behavior in the limit $\vec{k}, \omega \to 0$, the domain where the hydrodynamic description is applicable, that is, for long wavelengths and low energies. The quantum protectorate concept was applied to the quantum theory of magnetism by the author of this report in the paper [50], where a criterion of applicability of models of the quantum theory of magnetism to description of concrete substances was formulated. The criterion is based on the analysis of the model’s low-energy and high-energy spectra.

6. Conclusions and Discussion

In our interdisciplinary review [14] we analyzed the applications of the symmetry principles to quantum and statistical physics in connection with some other branches of science. The profound and innovative idea of quasiaverages formulated by N.N. Bogoliubov, gives the so-called macro-objectivation of the degeneracy in domain of quantum statistical mechanics, quantum field theory and in the quantum physics
in general. We discussed also the complementary unifying ideas of modern physics, namely: spontaneous symmetry breaking, quantum protectorate and emergence. The interrelation of the concepts of symmetry breaking, quasiaverages and quantum protectorate was analyzed in the context of quantum theory and statistical physics. The main aim of that paper were to demonstrate the connection and interrelation of these conceptual advances of the many-body physics and to try to show explicitly that those concepts, though different in details, have a certain common features. Many problems in the field of statistical physics of complex materials and systems (e.g. the chirality of molecules) and the foundation of the microscopic theory of magnetism [50] and superconductivity [32] were discussed in relation to these ideas. It is worth to emphasize once again that the notion of quantum protectorate complements the concepts of broken symmetry and quasiaverages by making emphasis on the hierarchy of the energy scales of many-particle systems. In an indirect way these aspects of hierarchical structure arose already when considering the scale invariance and spontaneous symmetry breaking in many problems of classical and quantum physics.

It was shown also in papers [14, 15, 32] that the concepts of symmetry breaking perturbations and quasiaverages play an important role in the theory of irreversible processes as well. The method of the construction of the nonequilibrium statistical operator [36] becomes especially deep and transparent when it is applied in the framework of the Bogoliubov’s quasiaverage concept. For detailed discussion of the Bogoliubov’s ideas and methods in the fields of nonlinear oscillations and nonequilibrium statistical mechanics see Ref. [36]. Thus, it was demonstrated in Ref. [14] that the connection and interrelation of the conceptual advances of the many-body physics discussed above show that those concepts, though different in details, have complementary character.

To summarize, the ideas of symmetry breaking, quasiaverages, emergence and quantum protectorate play constructive unifying role in modern theoretical physics. The main suggestion is that the emphasis of symmetry breaking concept is on the symmetry itself, whereas the method of quasiaverages emphasizes the degeneracy of a system. The idea of quantum protectorate reveals the essential difference in the behavior of the complex many-body systems at the low-energy and high-energy scales. Thus the role of symmetry (and the breaking of symmetries) in combination with the degeneracy of the system was reanalyzed and essentially clarified within the framework of the method of quasiaverages. The complementary notion of quantum protectorate might provide distinctive signatures and good criteria for a hierarchy of energy scales and the appropriate emergent behavior. It was demonstrated also that the Bogoliubov’s method of quasiaverages plays a fundamental role in equilibrium and nonequilibrium statistical mechanics and quantum field theory and is one of the pillars of modern physics.

We believe that all these concepts will serve for the future development of physics [74] as useful practical tools. Additional material and discussion of these problems can be found in recent publications [75, 76].

References

УДК 530.1; 530.145
Фундаментальные принципы теоретической физики и концепции квазисредних, квантового протектората и эмергенции

А. Л. Куземский
Лаборатория теоретической физики им. Н.Н. Боголюбова
Объединённый институт ядерных исследований
Россия, 141980, Дубна, Моск. обл.

В настоящей работе обсуждаются новейшие глубокие концепции современной теоретической физики, имеющие междисциплинарный характер: симметрия и нарушенная симметрия, квазисредние Н.Н. Боголюбова, квантовый протекторат и эмергенция (возникающие явления). Данные концепции обсуждаются в контексте квантовой и статистической физики и квантовой теории твердого тела. Главная цель настоящего анализа...
состояла в том, чтобы показать связь и взаимоотношение обсуждаемых концепций. При этом были проанализированы сходство и различие, а также пределы применимости изучаемых новых понятий на основе рассмотрения ряда задач микроскопической теории магнетизма и сверхпроводимости и коллективного поведения других сложных систем. Можно утверждать (с известной долей условности), что концепция нарушенной симметрии делает упор на симметрию системы в целом, в то время как концепция квазисредних Н.Н. Боголюбова подчеркивает роль вырождения в системе. Концепция квантового протектората подчеркивает различие в поведении сложных систем при низких и высоких энергиях. Иерархия энергетических шкал в сложных системах позволяет глубже понять возникающие явления и их специфические черты благодаря различию в спектрах возбуждений.

**Ключевые слова:** теоретическая физика, квантовая физика, квантовая статистическая физика, симметрия, нарушенная симметрия, квазисредние Н.Н. Боголюбова, квантовый протекторат, эмергенция (возникающие явления), квантовая теория магнетизма, микроскопическая теория сверхпроводимости.