On August 21, 1979 Academician Nikolai Nikolaevich Bogolyubov celebrates his 70th birthday. The Editorial Board and authors of Teoreticheskaya i Matematicheskaya Fizika offer sincere greetings to the Principal Editor of the Journal on his birthday and dedicate this issue to him.

# NIKOLAI NIKOLAEVICH BOGOLYUBOV

## (ON HIS 70TH BIRTHDAY)

Academician A. A. Logunov and
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A great modern scientist – Academician Nikolai Nikolaevich Bogolyubov – was born on August 21, 1909 in Nizhnii Novgorod (today Gor'kii). Bogolyubov began his scientific work in Kiev, where from his 13th year he began to work in the seminar of Academician N. M. Krylov; in 1924, he already wrote his first scientific paper.

The initial period of Bogolyubov's scientific work was devoted to a number of mathematical questions – direct methods of variational calculus, the theory of almost periodic functions, methods of approximate solutions of differential equations, and dynamical systems.

Already the young scientist's early investigations into the development of direct methods of solution of extremal problems made him widely known. One of the papers from this cycle was awarded a prize by the Bolon' Academy of Sciences.

In the same years, Bogolyubov gave a new construction of the theory of uniform almost periodic functions and found a deep connection between this theory and the general theorem on the behavior of linear combinations of an arbitrary bounded function.

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In 1932, in collaboration with his teacher Krylov, Bogolyubov turned to the development of a completely new field of mathematical physics – the theory of nonlinear vibrations, which they called nonlinear mechanics. The investigations were aimed at the development of new methods of asymptotic integration of nonlinear equations describing vibrational processes. Bogolyubov created a new mathematical formalism for studying general nonconservative systems with a small parameter. He investigated the nature of an exact stationary solution near an approximate solution for sufficiently small value of the parameter and proved a number of existence and stability theorems for quasiperiodic solutions.

Among the methods formulated and developed by Bogolyubov in nonlinear mechanics the method of averaging and the method of integral manifolds, which are now classic, are of particular importance.

The pioneering ideas and fundamental results of Bogolyubov in nonlinear mechanics provide the basis of many modern investigations in general mechanics, the mechanics of the continuous medium, celestial mechanics, the mechanics of rigid bodies and gyroscopic systems, the theory of the stability of motion, control theory, control and stabilization of the mechanics of space flight, mathematical ecology, and other directions in natural science and technology.

Of great importance for the subsequent development of not only nonlinear mechanics but also the general theory of dynamical systems were Bogolyubov's papers on the qualitative investigation of the equations of nonlinear mechanics, which, essentially, led to a new construction of the theory of invariant measures. The basis of this theory was the concept of an ergodic set and a number of subtle theorems on the possibility of decomposition of an invariant measure into nondecomposable invariant measures localized in ergodic sets. All these concepts have long become classic in the modern theory of random processes.

The mathematical methods developed by Bogolyubov for investigating dynamical systems enabled him to approach in an essentially new manner the problems of the mechanics of systems consisting of many particles. In early papers of this cycle (the first of them in 1939) he considered the manifestation of stochastic features traditionally described by the Fokker–Planck equation in dynamical systems subject to the random influence of a thermal bath. First, for the example of an exactly solvable model, he established general features of the evolution of a statistical system and the establishment in it of the state of equilibrium. It was found that the random process which describes the behavior of the system can, depending on the choice of the time scale, be regarded as a dynamical, Markov, and, in the general case, non-Markov process. Thus was introduced for the first time the concept of a hierarchy of times in nonequilibrium statistical physics, which was the key to all the subsequent development of the statistical theory of irreversible processes.

A major contribution of Bogolyubov to the statistical mechanics of nonideal classical systems were the papers in his monograph Problems of a Dynamical Theory in Statistical Physics (1946), which has become well known throughout the world; in it, he developed the method of hierarchies of equations for equilibrium and nonequilibrium many-particle distribution functions.

The new physical concepts established in these papers amounted to a new stage in the development of statistical mechanics following on after the work of Gibbs and Boltzmann. At the present time, no serious investigation into the theory of nonideal systems, both classical and quantum, can dispense with some modification of the formalism of correlation functions developed by Bogolyubov and the Bogolyubov (BBGKY) hierarchy of equations for them.

Investigating the hierarchy of equations for the distribution functions in the case of statistical equilibrium, Bogolyubov proposed regular methods for the most important physical cases – short-range (low-density gas) and long-range interaction (system with Coulomb interaction) – in the form of an expansion in a small parameter: either in inverse powers of the ratio of the specific volume to the cube of the interaction range, or in powers of the specific volume divided by the cube of the Debye radius.

In nonequilibrium systems, these expansions of the distribution functions are, on account of the appearance of circular terms, valid only for very short time intervals. The resolution of this difficulty and the associated possibility of developing regular methods of perturbation theory in nonequilibrium statistical mechanics were made possible by the use of a particular variant of the methods of nonlinear mechanics developed earlier and the important physical concept of the existence of different time scales established by Bogolyubov.

A physically meaningful concept of a single-particle distribution function appears only in a certain approximation on the transition to time scales long compared with the time of correlation weakening. For such a distribution function, one can obtain a closed kinetic equation of, for example, Boltzmann type.

To derive the kinetic equation itself, Bogolyubov proposed, in place of Boltzmann's hypothesis of molecular chaos, which involves ignoring the correlations between the dynamical states of the colliding particles, a new physical approach, in which the conditions of correlation weakening are used as boundary conditions imposed on every initial condition compatible with the considered special case, this having the consequence that the explicit structure of the collision integral is already obtained at a dynamical level. This method makes it possible to derive not only the principal Boltzmann term but also to investigate the higher approximations.

Continuing his investigation into the evolution of the system, Bogolyubov showed that the next stage, the hydrodynamic, is associated with the transition to an even coarser time scale, which is appreciably longer than the mean free time of the particles (i.e., the time of formation of the local thermodynamic characteristics). In this stage of the evolution, the single-particle distribution function itself depends on the time only through the functional dependence on the hydrodynamics parameters of the system (local velocity, density, and specific internal energy). For these last quantities, Bogolyubov constructed a closed system of hydrodynamic equations directly on the basis of Liouville's equation and without recourse to a kinetic equation (1948). This idea, which provided the logical completion to the description of the evolution of many-particle systems, had a great influence on the subsequent development of the theory of nonequilibrium processes.

Bogolyubov obtained just as fundamental results in quantum statistics. Generalizing the method of classical correlation functions to the case of quantum statistical systems, he constructed hierarchies of equations for the equilibrium and nonequilibrium density matrices and proposed a method for constructing kinetic equations in the quantum case (1947). The idea of different scales of microscopic processes in statistical systems was used by him subsequently in the construction of the equations of the hydrodynamics of a superfluid liquid (1963).

In recent years, Bogolyubov has returned to considering general questions of the evolution of statistical systems. In his papers in the period 1975-1978 he has significantly deepened understanding of transient processes in nonequilibrium systems and found the microscopic structure of the Boltzmann approximation in kinetics. His scheme proposed earlier for the development of stochastic processes for a small system (consisting, possibly, of even a single particle) interacting weakly with a large system enabled him, in conjunction with a new, effective, and, as always with Bogolyubov, powerful technique, to approach from a unified point of view the problem of describing all phases of evolution, including the kinetic phase, with allowance for all higher correlations of the particles, and the hydrodynamic stage.

Bogolyubov's name is inseparably linked with the creation of the modern theory of nonideal quantum macroscopic systems.

In his lecture to the Section of Physical and Mathematical Sciences of the USSR Academy of Sciences in 1946, Bogolyubov explained the phenomenon of superfluidity, the explanation being remarkable for its simplicity and subtlety of physical analysis. He showed that the phenomenon of superfluidity in Bose systems with a weak interaction is due to the occurrence in the system of a condensate, and, if the condensate is thermodynamically stable, interaction does not disrupt but, in contrast, stabilizes this state. Bogolyubov constructed a mathematical formalism adequate for the description of this phenomenon. The formation of the condensate was taken into account by separating out a classical component from the operator wave function, and the quantum part of the Hamiltonian was diagonalized by a canonical transformation which is now widely known as the Bogolyubov transformation. This transformation, which defines quasiparticles that do not interact in the first approximation, made it possible to establish that the correlation of pairs of particles with opposite momenta play the principal part in the stabilization of the condensate. These investigations made it possible to construct a microscopic theory of superfluidity, in which it was possible to describe consistently the energy spectrum of a superfluid system and elucidate the relationship between the superfluid and the normal state. Modern investigations into the behavior of a nonideal Bose gas have as their point of departure the Bogolyubov spectrum and the Bogolyubov structure of the ground state, from which further investigations of the system commence.

The generality of the new concepts defined in the lecture, in particular the connection between the stability of the classical state and the positive definiteness of the quantum part of the Hamiltonian went far beyond the needs of the problem under consideration, and one can now say that they are numbered among the classical concepts of statistical physics and quantum field theory.

In September 1957, Bogolyubov applied his canonical transformation, generalized to fermion operators, to the systematic construction of the theory of superconductivity on the basis of the Fröhlich model, taking into account electron–phonon interaction in metals. The vacuum of quasiparticles defined by this transformation is a state with an indefinite number of particles – a kind of condensate in which the correlation of particles with oppositely directed momenta and spins plays the principal part. It is the stability of this condensate which determines the singular properties of the superconducting state. The method of the canonical transformation, which is the most adequate formalism for taking into account the existence of Cooper pairs near the Fermi surface, proved to be a powerful tool for investigating the energy spectrum of superconductors. As a result, it was established that the system contains, in addition to excitations of fermion type associated with the breaking of the Cooper pairs and characterized by a definite value of the energy gap, collective gapless Bose excitations, the presence of which is of fundamental importance (1958).

Thus, in his papers of 1957 Bogolyubov created independently and almost simultaneously with Bardeen, Cooper, and Schrieffer the microscopic theory of superconductivity.

Development of the concept of superconductivity as the superfluidity of Fermi systems led Bogolyubov to the discovery of the new fundamental effect of superfluidity of nuclear matter (1958). The concept of superfluidity of nuclear matter has since become fundamental in modern nuclear theory.

Further investigations of Bogolyubov showed that the stabilization of the condensate in nonideal systems is a consequence of degeneracy with respect to the particle number, a property characteristic of systems with infinitely many degrees of freedom. Study of the properties of systems with degeneracy led Bogolyubov to the formulation of the now widely known method of quasi-averages (1961). In conjunction with his own method of two-time thermal Green's functions (1959) and the technique of spectral expansion, this method is essentially a universal method for studying systems whose ground state is unstable against small perturbations (for a superconductor against the source of pairs, and for a ferromagnet against the switching on of a small magnetic field).

A major achievement of the method of quasi-averages was Bogolyubov's fundamental theorem (1961) which proved that a long-range interaction always results from spontaneous breaking of symmetry in a system. This theorem made it possible to solve the fundamental question of the structure of the energy spectrum of low-lying elementary excitations in nonideal Bose and Fermi systems, relating it to the requirement of gauge invariance of these models.

The ideas and methods developed by Bogolyubov in the study of nonideal quantum systems had not only a tremendous influence on the development of modern statistical physics but also proved to be extremely fruitful in the study of an important problem in quantum field theory relating to the problem of degeneracy and stability of the vacuum. Indeed, the very idea of vacuum instability in quantum field theory arose through his investigations.

At the beginning of the fifties, Bogolyubov's attention was drawn to quantum field theory. His work in this field led above all to the formulation of new concepts. At that time, quantum field theory had only one effective formalism – perturbation theory – and the main shortcoming of this formalism, the ultraviolet divergences, was eliminated by heuristic arguments about the possibility of renormalizing the mass and the charge. Bogolyubov's papers emphasized that the interpretation of the divergences as a shortcoming of theory is due essentially to the direct transfer to quantum field theory of ordinary concepts of macroscopic physics. The origin of the divergences is contained in the basic concept of microscopic physics, in which an elementary particle is understood as a quantum of a local wave field. Therefore, an adequate mathematical formalism must include organically generalized functions.

In his investigations in quantum field theory, Bogolyubov dispensed with the usual Hamiltonian formalism and based his theory on the S matrix introduced by Heisenberg. In his investigations in the beginning of the fifties he showed that the S matrix can be reconstructed in all orders of perturbation theory from the interaction Lagrangian by requiring only fulfillment of the fundamental physical principles of the theory – relativistic invariance, the spectral condition, unitarity, and causality. An important part in these and subsequent papers was played by the development of a new principle of causality, which is today widely known as "Bogolyubov's condition of microcausality".

Bogolyubov's theorem that the S matrix in all perturbation orders can be determined systematically up to quasilocal operators (with the origin of the nonuniqueness identified in the singular nature of the coefficient functions of the S matrix, revealed the nature of the ultraviolet divergences and made it possible to develop a consistent scheme for their elimination – the Bogolyubov R operation (1955). Perturbation theory constructed in this manner is essentially purely axiomatic; it is the first systematic axiomatic scheme in quantum field theory.

Bogolyubov's lecture at the conference at Seattle (1956) heralded a new stage in the development of not only the axiomatic method but also in the physics of strong interactions generally. In this lecture (so characteristic of Hilbert with his "we must know, we shall know") Bogolyubov, having established the causal structure of the pion–nucleon scattering amplitude on the basis of his principle of microcausality, directly proved the possibility of analytic continuation of the amplitude to complex values of the energy. The proof is associated with the discovery of a new principle of analytic continuation of generalized functions of many variables, and the "edge-of-the-wedge" theorem thereby proved (today, it carries Bogolyubov's name) became the basis of a new direction in mathematics.

Bogolyubov's papers on the foundation of dispersion relations opened up a new stage in the theory of strong interactions. It was not simply that a systematic formalism had been constructed that did not depend on assumed weakness of the interaction of elementary particles. The ideas introduced into physics by the proof of the dispersion relations became the basis of a new language of the theory of strong interactions. Physicists were given a new concept of the scattering amplitude as a single analytic function of the scattering variables; this concept was to be decisive for the further development of theory. At the first glance, the purely mathematical concept was a reflection of the deep connections existing in physics between apparently different processes. It became obvious that even if one could not find the scattering amplitude of a given process one could find its connection to the amplitudes of other processes. The idea of a connection between different reaction channels was the point of departure for numerous heuristic constructions of the scattering amplitude.

The papers of Bogolyubov and his students outlined wide and varied applications of the asymptotic method, such as asymptotic estimates at high energies, description of low-energy regions using the unitarity condition, the problem of scaling at high energies, and so forth.

Numerous other ideas and investigations in different fields of the relativistic dynamics of particles are due to Bogolyubov.

In the years 1965-1966 there are his investigations into the theory of the symmetry of elementary particles. With these investigations there is in particular associated the introduction of a new quantum number, today called color, which plays a fundamental part in modern schemes of the theory of weak and strong interactions.

The directions listed here by no means exhaust the scientific work of N. N. Bogolyubov. He also made a number of fundamental investigations into plasma theory and kinetic equations that are of practical importance.

Bogolyubov's investigations have been made in many branches of mathematics, mechanics, and physics. In each of these fields he has made a number of fundamental scientific discoveries, and he has written more than 200 papers and monographs.

The main feature of his scientific style is his ability to grasp the key nature of the problem and its essential solvability and then, undaunted by the difficulties, create an adequate formalism for the solution of this problem. The organic fusion of mathematics and physics recalls to the reader of Bogolyubov's papers the times when the representatives of the exact sciences were simply called natural philosophers. This gift enabled Bogolyubov to make a decisive contribution to the development of theoretical physics during the last forty years and in fact create a new modern mathematical physics. All this has long made Bogolyubov one of the great scientists of the world and has left his individual mark on the complete development of theoretical physics in the second half of our century.

Bogolyubov devotes much attention to the education of young scientists. To him is due the honor of having created a number of scientific schools, such as the school of mathematical physics and nonlinear mechanics at Kiev and of theoretical and mathematical physics at Moscow and Dubna. Many well-known scientists are proud to call Bogolyubov their teacher.

N. N. Bogolyubov also devotes much attention to the organization of science. He is currently a member of the Presidium of the USSR Academy of Sciences, Director of the Joint Institute for Nuclear Research, and Principal Editor of our journal Theoretical and Mathematical Physics, which he created.

Bogolyubov also devotes much time and interest to general work and is a deputy of the Supreme Soviet of the USSR.

His country has highly valued his scientific and general work. He has been awarded the Lenin Prize, the USSR State Prize twice, the Lomonosov Prize of the USSR Academy of Sciences, five Lenin Orders, and a number of other orders and medals. In 1969, Bogolyubov was made a Gold Star of a Hero of Socialist Labor for his eminent services.

In recognition of Bogolyubov's personal contribution to the development of science and his high scientific and public authority he has been made a Foreign Member of many foreign academies. He has been awarded honorary doctorates by a number of distinguished universities throughout the world and many international prizes and medals.

Nikolai Nikolaevich Bogolyubov celebrates his 70th birthday in the full flower of his creative abilities. We wish him long and happy years of scientific inspiration and new discoveries bringing honor to science in our country.

# TAUBERIAN THEOREMS IN QUANTUM FIELD THEORY

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The use of Tauberian theorems in quantum field theory is studied in particular connection with the scaling of the form factors of deep inelastic scattering and the asymptotic behavior of the Wightman two-point function.

## 1. Introduction

Tauberian theorems are theorems that relate the asymptotic behavior of a (generalized) function in the neighborhood of the origin to the asymptotic behavior of its Fourier transform (Laplace transform or other integral transforms) at infinity. Theorems that are the converses of Tauberian theorems are called Abelian theorems.

In the case of one independent variable, Tauberian theory has been developed fairly far and has numerous applications in the theory of numbers, the theory of differential equations, in harmonic analysis, and in mathematical physics.

For the case of many variables, the problems of Tauberian theory are much more complicated. Only individual results have been achieved in this direction. One of them relates to the multidimensional generalization of the Tauberian theorem of Hardy and Littlewood (see Sec. 10).

A number of important results in multidimensional Tauberian theory has been obtained in connection with the theoretical explanation of the experimentally observed scaling at high energies and large momentum transfers of quantities in quantum field theory, in particular, the form factors of deep inelastic lepton–hadron scattering. One of the main problems of theory is to establish whether these phenomena are consistent with the general principles of local quantum field theory and establish what restrictions these principles impose on the possible asymptotic behaviors. This problem is intimately related to the singular structure of the Fourier transforms of the form factors in the neighborhood of the light cone. These problems have been discussed intensively in the physics literature since Bjorken's paper in 1967 [1] and in the mathematical literature since the paper of Bogolyubov, Vladimirov, and Tavkhelidze in 1972 [2].

These problems are intimately related to the dimensional analysis of deep inelastic scattering processes [3] and the study of the asymptotic behavior of inclusive processes [4-5]. A detailed bibliography on this question can be found in the review [6]. Although the explicit form of the form factors is unknown, some information about them follows from the general principles of local quantum field theory: Lorentz covariance, the spectral condition, locality, unitarity, and various symmetries [7].

Let $F(q)$, $q = (q_0, \mathbf{q})$, $\mathbf{q} = (q_1, q_2, q_3)$ be the form factor in the coordinate system associated with the hadron (we assume that the hadron has mass equal to 1). The generalized function $F(q)$ has the following properties: 1) $F \in \mathcal{S}'(\mathbf{R}^4)$, 2) $F(q) = -F(-q)$, 3) $F(q)=0$, if $-q^2/2|q_0|>1$, 4) $\tilde{F}(x)=0$, if $x^2<0$, 5) $F(q_0, \mathbf{q}) = F(q_0, S\mathbf{q})$, $\forall S \in SO(3)$. Here $q^2 = q_0^2 - |\mathbf{q}|^2 = q_0^2 - q_1^2 - q_2^2 - q_3^2$ is the Lorentz square and $\tilde{F}(x)$ is the

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