NIKOLAI NIKOLAEVICH BOGOLYUBOV

(FIFTY YEARS OF SCIENTIFIC WORK)

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The great mathematician and theoretical physicist Academician Nikolai Nikolaevich Bogolyubov began his scientific work in Kiev, where at the age of 13 he began to work in the Seminar directed by Academician N. M. Krylov, and already in 1924 wrote his first scientific paper.

The initial period of Bogolyubov's scientific work was devoted to a number of mathematical questions – direct methods of variational calculus, the theory of almost periodic functions, methods of approximate solution of differential equations.

The early investigations of the young scientist into the development of direct methods of solution of extremal problems already made him widely known. One of the original papers of this cycle was awarded a prize of the Bolonskii Academy of Sciences.

During these years, Bogolyubov developed a new construction of the theory of uniform almost periodic functions. It was shown that the main results of this theory are a consequence of a general theory according to which certain linear combinations of an arbitrary bounded function behave as trigonometric sums and on the average have the property of almost periodicity.

In the theory of boundary-value problems for ordinary differential equations, Bogolyubov obtained a number of interesting results related directly to the application of a difference method to variational calculus.

In 1932, in conjunction with Academician N. M. Krylov, Bogolyubov turned to the development of a completely new branch of mathematical physics – the theory of nonlinear oscillations, which they called nonlinear mechanics. The primary aim of the investigations was to perfect the methods of asymptotic integration of the nonlinear equations that describe oscillatory processes. Aspects of the asymptotic integration of differential equations with a "small" parameter had been studied earlier, but exclusively in connection with conservative systems. Having overcome fundamental difficulties, Bogolyubov created a formalism capable of just an effective description of the behavior of general nonconservative systems, and he constructed new asymptotic methods of nonlinear mechanics, giving them a rigorous mathematical foundation. In papers devoted to this question, he investigated the nature of an exact stationary solution near an approximate solution for a sufficiently small value of the parameter and proved theorems on the existence and stability of quasiperiodic solutions.

Of great importance for the subsequent development of not only nonlinear mechanics but also the general theory of dynamical systems were Bogolyubov's papers on the qualitative investigation of the equations of nonlinear mechanics; these led essentially to a new construction of the theory of invariant measure. This theory was based on the concept of an ergodic set and a number of subtle theorems and the possibility of decomposing an invariant measure into nondecomposable invariant measures localized in ergodic sets. All these concepts have long become classic in the modern theory of random processes.

The new methods of investigation of dynamical systems made possible a new approach to the problems of classical statistical physics. In 1945, Bogolyubov considered the influence of a random force on an harmonic oscillator and investigated the establishment of statistical equilibrium in a system coupled to a thermal bath. An harmonic oscillator was taken as the system, and a large number of harmonic oscillators as the thermal bath. He showed that, depending on the choice of the time scale and appropriate

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approximations, one and the same random process could be regarded as either dynamical or Markov or as a certain non-Markov process. This was thus the first enunciation and application of the idea of a hierachy of times in statistical physics; it was to be decisive in all the subsequent development of the statistical theory of irreversible processes.

Bogolyubov's investigations into the statistical mechanics of classical systems are summarized in his monograph Problems of a Dynamical Theory in Statistical Physics (1946). He developed the chain method for the distribution functions of complexes of one, two, etc., particles. This method is now the most effective in the statistical mechanics of equilibrium and nonequilibrium processes.

Investigating statistical equilibrium, Bogolyubov proposed regular methods of solution of the chains of equations for the distribution functions; for example, in the case of a low density or a Coulomb interaction in the form of expansions in powers of small parameters: the density or the reciprocal Debye radius.

In the study of nonequilibrium systems, the solution of the equations for the distribution functions as series in powers of a small parameter leads to expansions that are applicable for only a very short interval of time because of the presence of secular terms, as in nonlinear mechanics and astronomy. Bogolyubov succeeded in overcoming this difficulty. For nonequilibrium processes he developed regular methods of constructing kinetic equations for the one-particle distribution functions of a system of interacting particles. From a unified point of view he succeeded in obtaining different types of kinetic equations; for example, for systems with short- or long-range but not weak forces. Application of regular methods of perturbation theory in statistical mechanics was found to be possible because of the existence of two different time scales in the relaxation process of the distribution functions. After a very short time interval – of the order of the collision time – has elapsed, all the higher distribution functions are completely determined by the one-particle distribution function, which itself varies comparatively slowly and on a different time scale (kinetic stage).

Bogolyubov showed that kinetic equations can be constructed for the kinetic state, i.e., for time scales appreciably longer than the "synchronization" time of the distribution functions, and also for systems in which one can separate a small parameter.

In deriving a kinetic equation, Bogolyubov replaced Boltzmann's hypothesis of molecular chaos by the boundary conditions of correlation weakening, imposing them on every initial condition that is compatible with the special case when all the distribution functions are determined by the one-particle distribution function.

Whereas Boltzmann's kinetic method was based on the complete neglect of correlations between the dynamical states of colliding molecules (hypothesis of molecular chaos), Bogolyubov's method enables one to take into account successively the higher terms of the expansion in powers of the density.

Bogolyubov showed that when the distribution functions evolve over time scales appreciably longer than the mean free time they become even smoother (hydrodynamic stage), and the one-particle distribution function begins to depend on the time only through the macroscopic parameters – the mean particle density, the mean velocity, and the mean internal energy. For this stage, the equations of hydrodynamics can be constructed directly from Liouville's equation, without recourse to a kinetic equation (1948). This idea had a profound influence on the further development on the theory of nonequilibrium processes.

Bogolyubov obtained no less important results in quantum statistics. He generalized the method of constructing kinetic equations to quantum systems (1947). He applied the method of constructing hydrodynamic equations to construct the hydrodynamics of a superfluid liquid (1963).

Bogolyubov's name is indissolubly linked to the creation of the modern theory of nonideal quantum macrosystems. His explanation of important physical phenomena such as superfluidity (1946) and superconductivity (1957) was a pioneering contribution to this theory. In a number of papers in the forties devoted to these problems, Bogolyubov developed the method of approximate second quantization, which has now become one of the main tools of quantum statistics. In particular, the new methods made it possible to discover a very important physical phenomenon – stabilization of a condensate in nonideal systems at near-zero temperatures.

The phenomenon of superfluidity was discovered in 1938 by the great Soviet physicist Academician P. L. Kapitsa. It was shown that near the absolute zero of temperature helium-2 acquires a vanishing viscosity. It was obvious that a new type of energy spectrum had been discovered and that its investigation must become the main task in the study of the properties of matter at low temperatures. However, the dynamical nature of the spectrum remained undiscovered for a long time. It was not clear whether it could be explained at all in the framework of the ordinary quantum-mechanical scheme of a binary interaction of individual particles.

In his classical paper of 1946, Bogolyubov gave a brilliantly simple and subtle physical analysis of the phenomenon of superfluidity. He showed that an important role in the formation of the ground state of a Bose system is played by the correlation of pairs of particles with opposite momenta, and that the ordinary binary interaction does not destroy but, on the contrary, stabilizes the ground state of the system. He constructed a mathematical formalism capable of treating the phenomenon, this being based on a singular transformation of the Bose amplitudes, which has now become widely known as the Bogolyubov transformation. As a result of these investigations, a microscopic theory of superfluidity was constructed; it was capable of a consistent description of the energy spectrum of a superfluid system and capable of explaining the relation between the superfluid and the normal state, which only become possible after the fundamental role of condensate stabilization due to the interaction had been elucidated.

Bogolyubov made a large contribution to the development of a microscopic theory of superconductivity. For a long time it had been assumed that only Bose systems can have the ground-state structure characterized by the presence of a condensate leading to superfluidity. Bogolyubov showed that under certain conditions a condensate is formed in a nonideal Fermi gas, and that a fundamental role is here again played by correlation between particles with opposite momenta. It is in fact such a structure of the ground state of a nonideal Fermi gas (conduction electrons in a metal interacting with lattice phonons) that leads to the phenomenon of superconductivity.

The development of the concept of superconductivity as superfluidity of Fermi systems led Bogolyubov to the discovery of a new fundamental effect – the superfluidity of nuclear matter (1958). The concept of superfluidity of nuclear matter is now the basis of the modern theory of the nucleus.

Bogolyubov's subsequent investigations showed that stabilization of a condensate in nonideal systems is a consequence of degeneracy with respect to the particle number – a property characteristic of systems with infinitely many degrees of freedom. The study of the properties of systems with degeneracy led Bogolyubov to formulate the method of quasiaverages (1961), which is now well known. Essentially, this method is a universal means of studying systems whose ground state is unstable against small perturbations (for a superconductor against an infinitesimally small source of pairs, for a ferromagnetic against the switching on of a small magnetic field, etc.).

The greatest achievement of the method of quasiaverages is Bogolyubov's fundamental theorem (1961), which proved that the density of the momentum distribution of the particles in superfluid systems in the neighborhood of zero momentum tends to infinity not slower than the inverse square of the momentum.

The ideas and methods that Bogolyubov developed in the study of nonideal quantum systems had not only a profound influence on the development of modern statistical physics but they also proved extremely fruitful in important questions of quantum field theory related to the problem of degeneracy and stability of the vacuum – the ground state in field theory.

From the beginning of the fifties, Bogolyubov's attention was attracted to quantum field theory. The formulation of field theory that he proposed is based on the S matrix, which is regarded as a functional of the free fields and an arbitrary space–time region.

It was postulated that the S matrix must satisfy fundamental physical principles – unitarity, relativistic covariance, and causality. A fundamental role in the development of the scheme was played by the causality condition, for which Bogolyubov found a new, more general and more convenient formulation. Today, this condition is widely known as Bogolyubov's condition of microcausality.

In the framework of perturbation theory, Bogolyubov showed (1955) that the S matrix in all perturbation orders is completely determined by the adopted system of postulates and that the ordinary Hamiltonian formalism of field theory can be readily obtained from the S matrix formalism. Thus, an axiomatic perturbation theory in quantum field theory was created for the first time. This cycle of papers has influenced the further development of field theory right up to the present time.

The axiomatic construction of perturbation theory showed that the physicists must, above all, be clear as to the nature of the mathematical entities with which they deal. Although the physicists laid the foundation of the theory of generalized functions, these were long treated from the point of view of classical analysis. Therefore, in the framework of the Hamiltonian formalism the divergent integrals were regarded as a serious defect of the theory to be eliminated only by additional physical arguments. Bogolyubov was the first to point out that, since quantum theory deals with entities whose adequate description requires the consistent application of generalized functions, it is necessary to define correctly the concept of the product of these functions in order to give a definite meaning to the chronological product of generalized functions present in the expansion of the S matrix. Thus, the source of the divergences is to be found in bad definitions rather than in physics. Bogolyubov pointed out a method of correct definition of chronological products and a consistent procedure for eliminating divergences (1955), which is known today as the R-operation.

Although the creation of an axiomatic perturbation theory was itself an important contribution to field theory, it is only a small part of the whole of the axiomatic method. Analysis of the conditions of causality and locality led Bogolyubov to conclude that these conditions can, essentially, be formulated as conditions on the choice of the class of the generalized functions allowed in quantum field theory. This circumstance made it possible to improve the axiomatics of field theory and prove dispersion relations in field theory (1956).

The proof of dispersion relations required the development of a special method of analytic continuation of generalized functions. Among the purely mathematical results in this direction, one must mention the so-called "edge of the wedge" theorem, which was first discovered and proved by Bogolyubov. This theorem is a generalization of the principal of analytic continuation of analytic functions of many complex variables. It has found many nontrivial applications in modern theoretical physics and mathematics. At the present time there are ~10 proofs of it, and it has been the subject of numerous papers and monographs, in which it is extended to entities more general than generalized functions – hyperfunctions. The significance of this formalism extends well beyond the direct requirements of physics. However, the important thing about the papers on the fundamentals of dispersion relations was their influence on the further development of field theory. For the first time an axiomatic physical theory had been constructed, and this led to a change of the very style of physical thought.

It became obvious that further progress in quantum field theory necessarily entailed a new standard of mathematical tools and more stringent requirements on the proofs.

The proof of dispersion relations opened up a new period in the theory of strong interactions. The important point was not only that a consistent mathematical formalism had been constructed independently of the assumption of weak interaction of the elementary particles, although in the low-energy region the dispersion method in conjunction with the unitarity condition made it possible to obtain a complete picture of physical phenomena. The ideas introduced into physics in the proof of dispersion relations became the basis of a new language of the theory of strong interactions. Physics obtained the concept of the scattering amplitude as a single analytic function of the scattering variables; this concept became decisive in the subsequent development of the theory of strong interactions. The apparently purely mathematical concept was a reflection of the deep relations in physics between apparently distinct physical processes. It became obvious that even if one could not find the scattering amplitude of a given process, one could find how it is related to the amplitudes of other processes. The idea of coupling of the different channels of a reaction is the point of departure for numerous heuristic arguments concerning the structure of a scattering amplitude.

Bogolyubov's studies laid the foundation of a new direction in the physics of strong interactions, in which the interaction of hadrons at asymptotically high energies is studied. The value of this direction is due to the fact that it is based solely on the general axioms of local field theory, i.e., they can serve as a criterion of the validity of the modern theory of elementary particles.

Besides his scientific activity, Bogolyubov always devoted very serious attention to the training of young scientists. As head of departments in Kiev and then at the Moscow University, he regularly gave lectures, which aroused great interest among his hearers. Through his lectures and also as the leader of seminars, Bogolyubov had a great influence on the training of many Soviet scientists – mathematicians, engineers, physicists.

He was frequently invited to give lectures and talks on his investigations in foreign universities and institutes and at international congresses and conferences.

Another achievement of his has been the creation of several successful scientific schools. He created the School of Mathematical Physics and Nonlinear Mechanics at Kiev and of Theoretical Physics at Moscow and Dubna.

He is also an eminent organizer in science. He is at present a member of the Presidium of the Academy of Sciences of the USSR and Academician-Secretary of the Mathematics Branch of the Academy of Sciences. He is the director of an important international center – The Joint Institute of Nuclear Research at Dubna.

He devotes much time and attention to public activity, being a deputy of the Supreme Soviet of the USSR and a member of the Pugwash Movement.

The scientific and public work of Bogolyubov has been acclaimed by Party and Government. He is the winner of a Lenin Prize, twice winner of a State Prize of the USSR, has been awarded four Lenin Orders and a number of other orders and medals. In 1969 his eminent services were acknowledged by the award of a Gold Star of Hero of Socialist Labor.

Bogolyubov's studies have been awarded many prizes in both the Soviet Union and abroad. He is an honorary member of many foreign academies and scientific societies and an honorary doctor of a number of foreign universities.

After 50 years of scientific endeavor Nikolai Nikolaevich is still at the height of his creative powers. For his numerous students and followers this is a happy event.

We wish him long and happy years of creative inspiration and new discoveries.