Transport Coefficients from PLσM

BLTP Seminar

JINR-Dubna, January 28, 2015, 16:00

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• Sigma model and symmetries
• SU(3) $L_{\sigma}M$ with Polyakov-Loop Potential

• Electrical and Heat Conductivity
• Bulk and Shear Viscosity
Sigma Models

Sigma-Model is a Physical system with the Lagrangian

\[ \mathcal{L}(\phi_1, \phi_2, \ldots, \phi_n) = \sum_{i=1}^{n} \sum_{j=1}^{n} g_{ij} \, d\phi_i \wedge *d\phi_j \]

The fields \( \phi_i \) represent a map from a base manifold (spacetime (worldsheet)) to a target (Riemannian) manifold of the scalars linked together by internal symmetries.

The scalars \( g_{ij} \) determines linear and non-linear properties.

It was introduced by Gell-Mann and Levy in 1960. The name \( \sigma \)-model comes from a field corresponding to the spinless meson \( \sigma \), scalar introduced earlier by Schwinger.
**Symmetries**

- **LoM** is an effective theory for QCD dof at low-energy and incorporates global $SU(N_f)_r \times SU(N_f)_e \times U(1)_A$ symmetry (not local $SU(3)_c$).

- For $N_f=2$ massless quarks, the phase transition can be of:
  - $2^{nd}$-order, if $U(1)_A$ symmetry is explicitly broken by **instantons**
  - $1^{st}$-order (fluctuations), if $U(1)_A$ symmetry is restored at $T_c$

- For $N_f=3$ massless quarks, the transition is always of $1^{st}$-order.

- In last case, the term which breaks $U(1)_A$ symmetry explicitly drives $1^{st}$-order phase-transition.

- In absence of explicit $U(1)_A$ symmetry breaking, the transition is fluctuation-induced of $1^{st}$-order.

• LσM is one of lattice QCD alternatives

• Various symmetry-breaking scenarios can be investigated in a more easy way

• Various properties of strongly interacting matter can be studied

• But, finite-T LσM requires many-body resummation schemes, because the IR divergences cause perturbation theory to break down
• Again, for \( N_f \) massless quarks, QCD Lagrangian has \( \text{SU}(N_f)_r \times \text{SU}(N_f)_\ell \times \text{U}(1)_A \) symmetry

• In vacuum, a non-vanishing expectation value of the quark-antiquark condensate, spontaneously breaks this symmetry to diagonal \( \text{SU}(N_f)_V \) group of vector transformations, \( V = r + \ell \)

• For \( N_f = 3 \), effective low-energy dof of QCD are scalar and pseudoscalar mesons. Since mesons are quark-antiquark states, they fall in singlet and octet representations of \( \text{SU}(3)_V \).

• The \( \text{SU}(N_f)_r \times \text{SU}(N_f)_\ell \times \text{U}(1)_A \) symmetry of QCD Lagrangian is explicitly broken by nonvanishing quark masses

• For \( M \leq N_f \) degenerate quarks, \( \text{SU}(M)_V \) symmetry is preserved

• If \( M > N_f \), mass eigenstates are mixtures of singlet and octet states

Symmetries imply conservation laws: invariance of Lagrangian under translations in space and time $\Rightarrow$ momentum and energy conservation

QCD Lagrangian for massless quarks shows symmetry under vector and axial transformation.

Equally (vector) left- and right-handed parts treated differently (Axial)

For example: symmetry of vector transformations leads to isospin conservation
Chiral symmetry of vector field under unitary transformation

\[ \Phi \mapsto e^{-i \theta^a T^a_{ij}} \Phi \]

\( \theta^a \) corresponding the rotational angle, \( T^a_{ij} \) matrix generates the transformation and \( a \) index indicating several generators associated with the symmetry transformation.

Vector transformation \( \Lambda_V \)

Axil transformation \( \Lambda_A \)

\[ \Psi \mapsto e^{-i \frac{\gamma^5}{2} \theta} \Psi \approx (1 - i \frac{\gamma^5}{2} \theta) \Psi \]

\[ \Psi \mapsto e^{-i \gamma_5 \frac{\gamma^5}{2} \theta} \Psi \approx (1 - i \gamma_5 \frac{\gamma^5}{2} \theta) \Psi \]

Conjugate

Fermions Dirac Lagrangian which describes free Fermion particle of mass \( m \)

\[ \mathcal{L}_D = \bar{\psi} (i \gamma_\mu \partial^\mu - m^2) \psi \]

Under vector transformation \( \Lambda_V \) \( \mathcal{L}_D \) is invariant.

BUT axial-vector transformation \( \Lambda_A \) reads

\[ \Lambda_A : \quad m \bar{\psi} \psi \mapsto e^{-i \gamma^5 \frac{\gamma^5}{2} \theta} m \bar{\psi} \psi \approx (1 - i \gamma_5 \frac{\gamma^5}{2} \theta) m \bar{\psi} \psi, \]

\[ = m \bar{\psi} \psi - 2im \bar{\psi} (\gamma_5 \frac{\gamma^5}{2} \psi) \]

\( \phi \) are component fields such as \( \pi \)'s
Combination of quarks (q# of mesons), a meson-like state

Sigma fields

\[ \sigma = \bar{\psi} \psi \]

\[ \pi = i\bar{\psi} \gamma_5 \psi \]

Gell-Mann & Levy obtained an invariant form if squares of the two states are summed

\( \Lambda_V: \pi^2 \rightarrow \pi^2 \)
\( \sigma^2 \rightarrow \sigma^2 \)

\( \Lambda_A: \pi^2 \rightarrow \pi^2 + 2\sigma\theta\pi \)
\( \sigma^2 \rightarrow \sigma^2 - 2\sigma\theta\pi \)

\( (\pi^2 + \sigma^2) \overset{\Lambda_V, \Lambda_A}{\leftrightarrow} (\pi^2 + \sigma^2) \)
Vector transformation

\[ \pi_i : \quad i\bar{\psi}\bar{\tau}\gamma_5\psi \longrightarrow i\bar{\psi}\bar{\tau}\gamma_5\psi + \theta_j \left[ \bar{\psi}\tau_i\gamma_5\frac{\tau_j}{2}\psi - \bar{\psi}\frac{\tau_j}{2}\tau_i\gamma_5\psi \right] \]

\[ = i\bar{\psi}\bar{\tau}\gamma_5\psi + i\theta\epsilon_{ijk}\bar{\psi}\gamma_5\tau_k\psi, \]

Vector transformation

\[ \left[ \tau_i, \tau_j \right] = 2i\epsilon_{ijk}\tau_k \]

Levi-Civita Symbols

\[ \epsilon_{ijk} = \begin{cases} 
+1 & \text{for even permutation } 123, \\
-1 & \text{for odd permutation } 123, \\
0 & \text{Otherwise} 
\end{cases} \]

\[ \bar{\pi} \longrightarrow \bar{\pi} + \epsilon_{ijk}\bar{\theta}\bar{\pi}_k \]
Axial-Vector transformation

\[ \pi_i : \quad i\bar{\psi}\tau_i\gamma_5\psi \rightarrow i\bar{\psi}\tau_i\gamma_5\psi + \theta_j \left[ \bar{\psi}\tau_i\gamma_5\gamma_5\frac{\tau_j}{2}\psi - \bar{\psi}\gamma_5\frac{\tau_j}{2}\gamma_5\tau_i\psi \right] \]

\[ = \quad i\bar{\psi}\tau_i\gamma_5\psi + \theta_j\bar{\psi}\psi\delta_{ij}, \]

\[ \gamma_5\gamma_5 = 1 \text{ and the commutation relation between matrices} \]

\[ [\tau_i, \tau_j] = 2\delta_{ij} \]

\[ \delta_{ij} = \begin{cases} +1 & \text{for } i = j, \\ 0 & \text{for } i \neq j \end{cases} \]

\[ \bar{\pi} \rightarrow \bar{\pi} + \theta\bar{\pi} \]
The chiral part of LoM-Lagrangian has $SU(3)_R \times SU(3)_L$ symmetry

where fermionic part

$$\mathcal{L}_q = \sum_f \overline{\psi}_f (i \gamma^\mu D_\mu - g T_a (\sigma_a + i \gamma_5 \pi_a)) \psi_f$$

and mesonic part

$$\mathcal{L}_m = \text{Tr}(\partial_\mu \Phi^\dagger \partial^\mu \Phi - m^2 \Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 + c [\text{Det}(\Phi) + \text{Det}(\Phi^\dagger)] + \text{Tr}[H(\Phi + \Phi^\dagger)],$$

- $m^2$ is tree-level mass of the fields in the absence of symmetry breaking
- $\lambda_1$ and $\lambda_2$ are the two possible quartic coupling constants,
- $c$ is the cubic coupling constant,
- $g$ flavor-blind Yukawa coupling of quarks to mesons and of quarks to background gauge field $A_\mu = \delta_{\mu 0} A_0$

$$c = 4.80; g = 6.5; \lambda_1 = 5.90; \lambda_2 = 46.48; m^2 = (0.495)^2;$$
\( \phi \) is a complex 3 \( \times \) 3 matrix and parameterizing scalar \( \sigma_a \) and pseudoscalar \( \pi_a \) (nonets) mesons

\[
\Phi = T_a \phi_a = T_a (\sigma_a + i\pi_a)
\]

where \( \sigma_a \) are the scalar fields and \( \pi_a \) are the pseudoscalar fields. The 3 \( \times \) 3 matrix \( H \) breaks the symmetry explicitly and is chosen as

\[
H = T_a h_a
\]

where \( h_a \) are nine external fields and \( T_a = \hat{\lambda}_a / 2 \) are generators of U(3) with \( \hat{\lambda}_a \) are Gell-Mann matrices \( \hat{\lambda}_0 = \sqrt{2} / 3 \) 1

The \( T_a \) are normalized such that \( \text{Tr}(T_a T_b) = \delta_{ab} / 2 \) and obey the U(3)

\[
[T_a , T_b] = i f_{abc} T_c ,
\]

\[
\{T_a , T_b\} = d_{abc} T_c ,
\]

where \( f_{abc} \) and \( d_{abc} \) for \( a, b, c = 1, \ldots, 8 \) are the standard antisymmetric and symmetric structure constants of \( SU(3) \) and

\[
f_{ab0} \equiv 0 , \quad d_{ab0} \equiv \sqrt{2 / 3} \delta_{ab}
\]
Gell-Mann matrices with \( \lambda_0 = \sqrt{\frac{2}{3}} I \)

\[
\begin{align*}
\hat{\lambda}_1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
\hat{\lambda}_2 &= \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
\hat{\lambda}_3 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
\hat{\lambda}_4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \\
\hat{\lambda}_5 &= \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \\
\hat{\lambda}_6 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \\
\hat{\lambda}_7 &= \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ 0 & i & 0 \end{pmatrix}, \\
\hat{\lambda}_8 &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}
\end{align*}
\]

as required \( \lambda_a \) span all traceless Hermitian matrices, then the generators follow

\[
[T_a, T_b] = i \sum_{c=1}^{8} f_{abc} T_c \{T_a, T_b\} = \frac{1}{3} \delta_{ab} + \sum_{c=1}^{8} d_{abc} T_c
\]

where \( f \) are structure constant given by

\[
\begin{align*}
f_{123} &= f_{147} = -f_{156} = f_{246} = f_{257} = f_{345} = -f_{367} = \frac{1}{2} f_{458} = f_{678} = \frac{\sqrt{3}}{2}, \\
d_{118} &= d_{228} = d_{338} = -d_{888} = \frac{1}{\sqrt{3}} d_{448} = d_{558} = d_{668} = d_{778} = -\frac{1}{2\sqrt{3}}
\end{align*}
\]
\[ H = \left( \begin{array}{cccc} \sqrt{\frac{2}{3}} h_0 + h_3 + \frac{h_8}{\sqrt{3}} & h_1 - ih_2 & h_4 - ih_5 \\ h_1 + ih_2 & \sqrt{\frac{2}{3}} h_0 - h_3 + \frac{h_8}{\sqrt{3}} & h_6 - ih_7 \\ h_4 + ih_5 & h_6 + ih_7 & \sqrt{\frac{2}{3}} h_0 - 2 \frac{h_8}{\sqrt{3}} \end{array} \right) ; \]

\[ T_a \sigma_a = \frac{1}{\sqrt{2}} \left( \begin{array}{cccc} \frac{1}{\sqrt{2}} \sigma^0 + \frac{1}{\sqrt{6}} \sigma^8 + \frac{1}{\sqrt{3}} \sigma^0 & \sigma^- & \kappa^- \\ \sigma^0 & -\frac{1}{\sqrt{2}} \sigma^0 + \frac{1}{\sqrt{6}} \sigma^8 + \frac{1}{\sqrt{3}} \sigma^0 & \kappa^0 \\ \kappa^+ & \kappa^0 & -2 \frac{\sigma^8 + \sigma^0}{\sqrt{3}} \end{array} \right) ; \]

\[ T_a \pi_a = \frac{1}{\sqrt{2}} \left( \begin{array}{cccc} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \pi^8 + \frac{1}{\sqrt{3}} \pi^0 & \pi^- & K^- \\ \pi^+ & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \pi^8 + \frac{1}{\sqrt{3}} \pi^0 & \bar{K}^0 \\ K^+ & K^0 & -2 \frac{\pi^8 + \pi^0}{\sqrt{3}} \end{array} \right) . \]
When shifting $\Phi$ field by vacuum expectation value,

$$
\mathcal{L} = \frac{1}{2} \left[ \partial_\mu \sigma_a \partial^\mu \sigma_a + \partial_\mu \pi_a \partial^\mu \pi_a - \sigma_a (m^2_S)_{ab} \sigma_b - \pi_a (m^2_P)_{ab} \pi_b \right] \\
+ \left( G_{abc} - \frac{4}{3} F_{abcd} \bar{\sigma}_d \right) \sigma_a \sigma_b \sigma_c - 3 \left( G_{abc} + \frac{4}{3} H_{abcd} \bar{\sigma}_d \right) \pi_a \pi_b \pi_c \\
- 2 H_{abcd} \sigma_a \sigma_b \pi_c \pi_d - \frac{1}{3} F_{abcd} (\sigma_a \sigma_b \sigma_c \sigma_d + \pi_a \pi_b \pi_c \pi_d) - U(\bar{\sigma}) ,
$$

where the tree-level potential is

$$
U(\bar{\sigma}) = \frac{m^2}{2} \bar{\sigma}_a^2 - G_{abc} \bar{\sigma}_a \bar{\sigma}_b \bar{\sigma}_c + \frac{1}{3} F_{abcd} \bar{\sigma}_a \bar{\sigma}_b \bar{\sigma}_c \bar{\sigma}_d - h_a \bar{\sigma}_a
$$

$\bar{\sigma}_a$ is determined from

$$
\frac{\partial U(\bar{\sigma})}{\partial \bar{\sigma}_a} = m^2 \bar{\sigma}_a - 3 G_{abc} \bar{\sigma}_b \bar{\sigma}_c + \frac{4}{3} F_{abcd} \bar{\sigma}_b \bar{\sigma}_c \bar{\sigma}_d - h_a = 0
$$
coefficients $G_{abc}$, $F_{abcd}$, and $H_{abcd}$ are given by

\[
G_{abc} = \frac{c}{6} \left[ d_{abc} - \frac{3}{2} (\delta_{a0} d_{0bc} + \delta_{b0} d_{a0c} + \delta_{c0} d_{ab0}) + \frac{9}{2} d_{000} \delta_{a0} \delta_{b0} \delta_{c0} \right],
\]

\[
F_{abcd} = \frac{\lambda_1}{4} (\delta_{ab} \delta_{cd} + \delta_{ad} \delta_{bc} + \delta_{ac} \delta_{bd}) + \frac{\lambda_2}{8} (d_{abn} d_{nca} + d_{acn} d_{nbd} + d_{bnc} d_{nbd})
\]

\[
H_{abcd} = \frac{\lambda_1}{4} \delta_{ab} \delta_{cd} + \frac{\lambda_2}{8} (d_{abn} d_{nca} + f_{acn} f_{nbd} + f_{bnc} f_{nbd})
\]

where tree-level masses, $(m_S^2)_{ab}$ and $(m_P^2)_{ab}$ are given by

\[
(m_S^2)_{ab} = m^2 \delta_{ab} - 6 G_{abc} \bar{\sigma}_c + 4 F_{abcd} \bar{\sigma}_c \bar{\sigma}_d
\]

\[
(m_P^2)_{ab} = m^2 \delta_{ab} + 6 G_{abc} \bar{\sigma}_c + 4 H_{abcd} \bar{\sigma}_c \bar{\sigma}_d
\]

The masses are not diagonal, thus $\sigma_a$ and $\pi_a$ fields are not mass generators in standard basis of SU(3). As, the mass matrices are symmetric and real, diagonalization is achieved by an orthogonal transformation

\[
\tilde{\sigma}_i = O^{(S)}_{ia} \sigma_a,
\]

\[
\tilde{\pi}_i = O^{(P)}_{ia} \pi_a,
\]

\[
(m_{S,P}^2)_{ab} = O^{(S,P)}_{ai} (m_{S,P}^2)_{ab} O^{(S,P)}_{bi}
\]
The expectation values \( \langle \Phi \rangle = T_0 \bar{\sigma}_0 + T_8 \bar{\sigma}_8 \)

where

\[
h_0 = \left[ m^2 - \frac{c}{\sqrt{6}} \bar{\sigma}_0 + \left( \lambda_1 + \frac{\lambda_2}{3} \right) \bar{\sigma}_0^2 \right] \bar{\sigma}_0 + \left[ \frac{c}{2\sqrt{6}} + (\lambda_1 + \lambda_2) \bar{\sigma}_0 - \frac{\lambda_2}{3\sqrt{2}} \bar{\sigma}_8 \right] \bar{\sigma}_8^2
\]

\[
h_8 = \left[ m^2 + \frac{c}{\sqrt{6}} \bar{\sigma}_0 + \frac{c}{2\sqrt{3}} \bar{\sigma}_8 + (\lambda_1 + \lambda_2) \bar{\sigma}_0^2 - \frac{\lambda_2}{\sqrt{2}} \bar{\sigma}_0 \bar{\sigma}_8 + \left( \lambda_1 + \frac{\lambda_2}{2} \right) \bar{\sigma}_8^2 \right] \bar{\sigma}_8
\]

From PCAC relations

\[
\bar{\sigma}_0 = \frac{f_\pi + 2f_K}{\sqrt{6}},
\]

\[
\bar{\sigma}_8 = \frac{2}{\sqrt{3}} (f_\pi - f_K)
\]

\[
f_\pi = 92.4 \text{ MeV}, \quad f_K = 113 \text{ MeV}
\]
Why Polyakov loop?

- the chiral model does NOT describe effects of QCD gluonic dof
- absence of confinement results in a non-zero quark number density even in confined phase

- The functional form of the potential is motivated by the QCD symmetries of in the pure gauge limit

\[
\frac{U(\phi, \phi^*, T)}{T^4} = -\frac{b_2(T)}{2}|\phi|^2 - \frac{b_3}{6}(\phi^3 + \phi^*^3) + \frac{b_4}{4}(|\phi|^2)^2,
\]

\[
b_2(T) = a_0 + a_1 \left(\frac{T_0}{T}\right) + a_2 \left(\frac{T_0}{T}\right)^2 + a_3 \left(\frac{T_0}{T}\right)^3.
\]

\[
\begin{align*}
a_0 &= 6.75, & a_1 &= -1.95, & a_2 &= 2.625, & a_3 &= -7.44 \\
b_3 &= 0.75 & b_4 &= 7.5
\end{align*}
\]
The thermal expectation value of color traced Wilson loop in the temporal direction determines Polyakov-loop potential

\[ \Phi(\vec{x}) = \frac{1}{N_c} \langle \mathcal{P}(\vec{x}) \rangle, \]

Polyakov-loop potential and its conjugate

\[ \phi = \frac{(\text{Tr}_c \mathcal{P})}{N_c}, \]

\[ \phi^* = \frac{(\text{Tr}_c \mathcal{P}^\dagger)}{N_c}, \]

This can be represented by a matrix in the color space

\[ \mathcal{P}(\vec{x}) = \mathcal{P}_{\exp} \left[ i \int_0^\beta d\tau A_4(\vec{x}, \tau) \right], \]

\[ \beta = \frac{1}{T} \quad \text{Temperature} \]

\[ A_4 = i A^0 \quad \text{Polyakov gauge} \]
The coupling between Polyakov loop and quarks is given by the covariant derivative

$$D_\mu = \partial_\mu - iA_\mu \text{ in PLSM Lagrangian}$$

$$A_\mu = \delta_{\mu0}A_0 \text{ in the chiral limit}$$

$$\mathcal{L}_{\text{PLSM}} = \mathcal{L}_{\text{meson}} + \mathcal{L}_{\text{quark}} + \overline{q}\gamma_0A_0 - \mathcal{U}(\phi, \phi^*, T),$$

invariant under chiral flavor group (like QCD Lagrangian)

$$\mathcal{L}_{\text{chiral}} = \mathcal{L}_{\text{meson}} + \mathcal{L}_{\text{quark}} + \overline{q}\gamma_0A_0$$

$$\mathcal{U}(\phi, \phi^*, T) \text{ is T-dependent Polyakov Potential}$$

In case of no quarks, then $\phi = \phi^*$ and the Polyakov loop is considered as an order parameter for the deconfinement phase-transition
In thermal equilibrium, the grand partition function can be defined by using a path integral over quark, antiquark and meson fields

\[ Z = \text{Tr} \exp[-(\hat{\mathcal{H}} - \sum_{f=u,d,s} \mu_f \hat{N}_f)/T] \]

\[ = \int \prod_a D\sigma_a D\pi_a \int D\psi D\bar{\psi} \exp \left[ \int_x (\mathcal{L} + \sum_{f=u,d,s} \mu_f \bar{\psi}_f \gamma^0 \psi_f) \right], \]

where \( \int_x \equiv i \int_0^{1/T} dt \int_V d^3x \) and \( \mu_f \) chemical potential

Thermodynamic potential density

\[ \Omega(T, \mu) = \frac{-T \ln Z}{V} = U(\sigma_x, \sigma_y) + \mathcal{U}(\phi, \phi^*, T) + \Omega_{\bar{\psi}\psi}. \]
The quarks and antiquarks Potential contribution

\[
\Omega_{\bar{\psi}\psi} = \frac{g^2}{2} \int_0^\infty \frac{d^3\vec{p}}{(2\pi)^3} \left\{ \ln \left[ 1 + 3(\phi + \phi^* e^{-E-T} \times e^{-(E-T)/T} + e^{-3(E-T)/T}) \right] \\
+ \ln \left[ 1 + 3(\phi^* + \phi e^{-(E+T)} \times e^{-(E+T)/T} + e^{-3(E+T)/T}) \right] \right\},
\]

where \( N \) gives the number of quark flavors,

\[
E = \sqrt{p^2 + m^2}
\]

\[
m_q = g \frac{\sigma_x}{2}, \\
m_s = g \frac{\sigma_y}{\sqrt{2}}.
\]

Mesonic potential

\[
U(\sigma_x, \sigma_y) = \frac{m^2}{2} (\sigma_x^2 + \sigma_y^2) - h_x \sigma_x - h_y \sigma_y - \frac{c}{2\sqrt{2}} \sigma_x^2 \sigma_y
\]

\[
+ \frac{\lambda_1}{2} \sigma_x^2 \sigma_y^2 + \frac{1}{8} (2\lambda_1 + \lambda_2) \sigma_x^4 + \frac{1}{4} (\lambda_1 + \lambda_2) \sigma_y^4.
\]

Vandermonde determinant is found negligibly small
The thermodynamic potential

\[ \Omega(T, \mu) = -\frac{T \ln Z}{V} = U(\sigma_x, \sigma_y) + U(\phi, \phi^*, T) + \Omega_{\bar{\psi}\psi}. \]

has the parameters

\[ m^2, h_x, h_y, \lambda_1, \lambda_2, c \text{ and } g \]
\[ \sigma_x \text{ and } \sigma_y \]
\[ \phi \text{ and } \phi^* \]

Condensates (chiral order parameters)
(deconfinement order parameters)

\[ \frac{\partial \Omega}{\partial \sigma_x} = \frac{\partial \Omega}{\partial \sigma_y} = \frac{\partial \Omega}{\partial \phi} = \frac{\partial \Omega}{\partial \phi^*} \bigg|_{\text{min}} = 0, \]

\[ \sigma_x = \bar{\sigma}_x, \sigma_y = \bar{\sigma}_y, \phi = \bar{\phi} \text{ and } \phi^* = \bar{\phi}^* \text{ are the global minimum} \]

can be fixed, experimentally minimizing the potential

refined by lattice QCD,
**Transport Coefficients from PLSM**

**Electrical and Heat Conductivity**

\[ \sigma_e(T, \mu) = \sum_k \frac{4\pi^2}{137} q_k^2 n_k(T, \mu) \tau_k(T, \mu) \]

\[ \kappa(T, \mu) = \frac{1}{3} \psi_{rel} c_V(T, \mu) \sum_k \tau_k(T, \mu) \]

**Bulk and Shear Viscosity**

\[ \xi = \frac{1}{9T} \left[ T^5 \frac{\partial}{\partial T} \left( \frac{\epsilon - 3p}{T^4} \right) + 16 |\epsilon_v| \right] = \frac{1}{9T} \left[ -16\epsilon + 9TS + Tc_V + 16|\epsilon_v| \right] \]

\[ \eta \sim \frac{\xi}{-0.45(c_s^2 - \frac{1}{3})} \]

- Number density
- Quark mass
- Decay time
- Specific heat
- Vacuum energy density
- Pressure, energy density, entropy, speed of sound
Based on parton-hadron-string dynamics transport approach

\[ \frac{d}{dt} p^j_z = q_j e E_z \]

an additional force causes the propagation of charge.

The electrical current density

\[ j_z(t) = \frac{1}{V} \sum_j q_j e \left( \frac{p^j_z(t)}{M_j(t)} \right) \]

In natural units, the ratio of current density and electric field strength \( \frac{\sigma_0}{T} \) is proportional to the electric conductivity

\[ \frac{\sigma_0}{T} = \frac{j_{eq}}{E_z T} \]

in relaxation time approximation, \( \sigma \) is described in Gases, Liquids and Solid State,

\[
\sigma_0 = \frac{e^2 n_e \tau}{m_e^*}
\]

for partonic degrees of freedom within the dynamical quasiparticle model (DQPM), the thermal dependence reads:

\[
\frac{\sigma_0(T)}{T} \approx \frac{2}{9} \frac{e^2 n_q(T)}{M_q(T)} \frac{1}{\Gamma_q(T) T}
\]

- \( n \) density of nonlocalized charges
- \( \tau \) relaxation time of charge carriers
- \( m_e^* \) effective masses
- \( \Gamma_q \) width of quasiparticle spectral function
- \( M_q \) pole mass = spectral dist. of quark-mass
- Flavor averaged fractional quark charge squared

In PHSD: DQPM matches quasiparticles properties to lattice QCD results in equilibrium for EOS, electromagnetic correlator, among others.
Electrical Conductivity

**Durde-Lorentz conductivity**

\[
\sigma_e(T, \mu) = \sum_k \frac{4\pi}{137} q_k^2 \frac{n_k(T, \mu)}{m_k(T, \mu)} \tau_k(T, \mu)
\]

- Fine structure etc.
- Number density
- Quark mass
- Decay time
- Quarks flavors

σ is related to flow of charges in presence of an electric field (decay constant & relaxation time)

response of the strongly interacting system in equilibrium to an external e-field

- external e-field is applied on flowing charges, the induced electric current \( J \) is related to the e-field. \( \sigma \) is the proportionally constant.

- self-interaction between quarks and gluons, Green-Kubo corrector
Normalized Electrical Conductivity

μ = 0.0 MeV

Non-Normalized Electrical Conductivity

\[ \sigma_e \text{ [GeV]} \]

\[ T/T_c \]

PLSM  
NJL [24]  
DQPM [24]

Heat Conductivity

From relativistic Navier-Stokes ansatz, heat flow is proportional to the gradient of thermal potential

\[ q^\mu = -\kappa \frac{nT^2}{\epsilon + p} \nabla^\mu \alpha = \kappa \left( \nabla^\mu T - \frac{T}{\epsilon + p} \nabla^\mu p \right) \]

\[ q^x = \kappa (\nabla^x T) = -\kappa \partial_x T(x) \]

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Alternatively, linearizing Boltzmann Eq.

\[ f_i = f_i^{le} + \frac{\partial f_i^0}{\partial \epsilon_i} \frac{\nabla T}{T} \]

Temperature profile

PRD48, 2916 (1993)

Then, the thermal current reads

\[ \frac{1}{\kappa} = \frac{24}{\pi^3} \alpha_s^2 T^{-2} I_\kappa(T/q_D) \]

\[ I_\kappa(T/q_D) = \begin{cases} \frac{1}{3} \ln(T/q_D) + 0.30, & T \gg q_D, \\ 2\zeta(3) \left( \frac{T}{q_D} \right)^2, & T \ll q_D. \end{cases} \]

Equilibrium distribution function

Non-Equilibrium distribution function

Equilibrium distribution function

\[ f_i^{le} = \left\{ \exp\left[ (\epsilon_i - \mu)/T(z) \right] + 1 \right\}^{-1} \]

\[ J_T = v_q \sum_p (\epsilon_p - \mu_q) v_z \frac{\partial f^0}{\partial \epsilon_p} \Psi_p = \frac{1}{3} \mu_q^2 T^2 \]

\[ \kappa = q^x \frac{(ax + b)^2}{ap} \]

\[ \alpha_s \text{ running strong coupling} \]

\[ q_D \text{ Debye wave number} \]

\[ g^2 N_q \mu^2 / (2\pi^2) \]
Heat Conductivity

\[ \kappa = \frac{1}{3} v_F^2 C_v T \kappa \]

Relaxation time, specific heat are T- and mu-dependent

Relative velocity

\[ \nu_{rel} = \sqrt{(p_1 p_2)^2 - (m_1 m_2)^2 / E_1 E_2} \]

\[ \kappa(T, \mu) = \frac{1}{3} \nu_{rel} c_v(T, \mu) \sum_{k} \tau_k(T, \mu) \]

\( \kappa \) is related to heat flow of relativistic fluid (rate of energy change)

\( \kappa \) can be estimated through irradiation caused by energetic ions
Normalized Heat Conductivity

\[ \frac{\kappa}{T^2} \text{ vs } \frac{T}{T_c} \]

- \( \mu = 0.0 \text{ MeV} \)
- PLSM
- NJL [24]
- DQPM [24]

Non-Normalized Heat Conductivity

PLSM
NJL [24]
DQPM [24]

\[ \kappa \text{ [GeV}^2] \]

\[ \frac{T}{T_c} \]

Kubo’s formula: shear $\eta$ and bulk $\zeta$ viscosities are related to the correlation function of stress tensor

$$\zeta = \frac{1}{9} \lim_{\omega \to 0} \frac{1}{\omega} \int_{0}^{\infty} dt \int d^{3}r \ e^{i\omega t} \langle [\theta_{ii}(x), \theta_{kk}(0)] \rangle$$

$$\zeta = \frac{1}{9} \lim_{\omega \to 0} \frac{1}{\omega} \int_{0}^{\infty} dt \int d^{3}r \ e^{i\omega t} \langle [\theta_{\mu}^{\mu}(x), \theta_{\mu}^{\mu}(0)] \rangle$$

In low energy theorems: bulk viscosity is a measure for violation of conformal invariance

$$\langle m\bar{q}q \rangle^* = \langle m\bar{q}q \rangle^*_0 - 4|\epsilon_v|$$

$$9 \omega_0 \zeta = Ts \left( \frac{1}{c_s^2} - 3 \right) - 4(E - 3P) + \left( T \frac{\partial}{\partial T} - 2 \right) \langle m\bar{q}q \rangle^* + 16|\epsilon_v| + 6(M_\pi^2 f_\pi^2 + M_K^2 f_K^2)$$
Bulk Viscosity

\[ \xi = \frac{1}{9T} \left[ T^5 \frac{\partial}{\partial T} \left( \frac{\epsilon - 3p}{T^4} \right) + 16 \left| \epsilon \right| \right] = \frac{1}{9T} \left[ -16 \epsilon + 9T S + TcV + 16 \left| \epsilon \right| \right] \]

Vacuum energy density

\( \mu = 0.0 \text{ MeV} \)

\( \xi/s \)

\( T/T_c \)

Shear Viscosity

(b) $\mu = 0.0 \text{ MeV}$

Summary

• PLSM seems to be able to generate lattice QCD transport confidents
• Approaches a wide horizon in understanding QGP properties at finite T and mu

شكراً!
Спасибо!
Thanks!
Danke!

http://atawfik.net/