Induced charges and fields in QGP and dense fermion media in magnetic fields at finite temperature

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Abstract

In QCD, the deconfinement phase transition is accompanied by the creation of the $A_0 = \text{const}$ condensate and strong temperature dependent chromomagnetic $H^3, H^8$ and usual magnetic $H^{em}$ fields.

A gauge invariance of the $A_0$ condensation is proven within the Nielsen identity method. It is shown that the effective action accounting for the one-loop, two-loop and plasmon diagram contributions satisfies the Nielsen identity. At this background, the color charges $Q_{\text{ind}}^3$ and $Q_{\text{ind}}^8$ are generated. They are temperature dependent and produce related color electric fields $E_{\text{color}}^3$ and $E_{\text{color}}^8$.

Similar phenomenon - generation of induced electric charge $Q_{\text{ind}}^{el}$ and electric field $E_{\text{ind}}$ - happen in dense fermionic media with non-zero chemical potential $\mu$. We investigate this in the presence of finite temperature and external magnetic field.

All these may serve as signals of the phase transitions - creation of either quark-gluon plasma or fermionic media. The role of temperature and magnetic fields and possible applications are discussed.
Outline

• New signals of Deconfinement PT
• \(QGP, \ A_0\) condensation
• \(QGP\), spontaneous magnetization
• Violation of Furry’s theorem in \(QGP\)
• Induced charges \(Q_{\text{ind.}}^3, Q_{\text{ind}}^8\) and potentials \(\bar{\phi}^3, \bar{\phi}^8\)
• Photon dispersion equation
• Effective \(\gamma\gamma g\) and \(g^3\) vertexes
• Inelastic scattering of photons in \(QGP\)
• Fermionic media at nonzero \(\mu, T, H\)
• Conclusion
• Appendixes
1 Deconfinement phase transition (DPT)

Investigations of deconfinement phase of $QCD$ is a hot topic nowadays. Due to asymptotic freedom of non-Abelian gauge field interactions at high temperature $T \geq 150 \text{ MeV}$ quarks are deliberated from hadrons and new matter state - quark-gluon plasma (QGP) - is formed. At lower temperatures quarks are confined inside hadrons. The order parameter of the $DPT$ is the Polyakov loop (PL)

$$P(\vec{x}) = T \exp[i g \int dx_4 A_0(\vec{x}, x_4)].$$ \hspace{1cm} (1)

It equal 0 at low temperature and $P \neq 0$ at $T > T_d$.

If $A_0(x_4) = \text{const}$

$A_0 \neq 0$ is also the order parameter of the $DPT$. The condensation of the $A_0$ was demonstrated in either lattice simulations or in analytic calculations. $A_0 \neq 0$ violates the $Z(3)$ and gauge symmetries.

Review paper O.A. Borisenko, J. Bohacik, V.V. Skalozub, $A_0$ condensate in QCD, Fortschr. Phys. v. 43 (1995) 301.
Other important order parameter is the temperature dependent chromo (magnetic) fields $H(T) \neq 0$ spontaneously created in the volume of the $QGP$. This point will not be discussed in this talk. In the literature, numerous applications of the $PL$ in the $QGP$ have been discussed. The combinations of both $A_0 \neq 0$ and $H(T) \neq 0$ were also investigated.

In particular, it was observed that the $A_0$ is dominant at temperatures not much greater $T_d$. So, in what follows we consider this case.

**We describe some new phenomena and effects taking place due to the $A_0$ presence.**
Spontaneous vacuum magnetization at LHC

Recently (Skalozub, Minaiev (2018)) it was obtained that at LHC experiment energies the QGP should be spontaneously magnetized.

The strengths of the large scale temperature dependent chromomagnetic, $B_3(T)$, $B_8(T)$, and usual magnetic, $H(T)$, fields spontaneously generated after the $DPT$, were estimated.

The critical temperature for the magnetized plasma is found to be $T_d(H) \sim 110 - 120$ MeV. This is essentially lower compared to the zero field value $T_d(H = 0) \sim 160 - 180$ MeV usually discussed in the literature. Due to contribution of quarks, the color magnetic fields act as the sources generating $H$. The strengths of the fields are $B_3(T)$, $B_8(T) \sim 10^{18} - 10^{19}G$, $H(T) \sim 10^{16} - 10^{17}G$ for temperatures $T \sim 160 - 220$ MeV.

The presence of strong large scale (color) magnetic fields modifies the spectrum of the (color) charged particles that influence various processes of interest.
2 QGP, $A_0$ condensate

Quarks interact with electromagnetic field and gluons according the form

$$L^{int.} = \bar{\psi}^a \left[ \gamma_\mu (\partial_\mu \delta^{ab} - ie_f A_\mu \delta^{ab} - ig (Q_\mu \frac{\lambda}{2})^{ab} - m_f \delta^{ab}) \right] \psi^b,$$

where $A_\mu$ is potential of electromagnetic fields, $Q_\mu$ is potential of gluon field, $e_f$ is electric charge of quark with flavor $f$, $m_f$ is quark mass, $g$ is charge of strong interactions, $a$, $b$ are color indexes.

Since quarks carry both electric and strong charges in the QGP the effective interactions of color and white objects are possible due to the quark virtual loops.

The $A_0$ is an element of the center $Z(3)$ of the $SU(3)$ group. When it is non zero,

both of these symmetries are broken.

The $A_0$ is a specific classical external fields. It can be introduced by splitting

$$Q_\mu^a = (A_0)^a_\mu + (Q^a_\mu)^{rad}.$$ of the gluon field potential. In what follows we consider the case $(A_0)^a_\mu = (A_0)_\mu \delta^{a3}$. This is for short.
3 Violation of Furry’s theorem in $QGP$

In the vacuum, Furry’s theorem holds:

The amplitudes having odd number of photon(gluon) lines, generated by the fermion loops, equal zero.

It is the consequence of $C$-parity invariance. The contribution of particles cancels the contribution of antiparticles.

The presence of the $A_0$ condensate violates this symmetry. So that new type processes are permissible.

In particular, the diagram with one gluon external line results in an induced color charge in the plasma. This may result in the scattering of quarks on this external charge.
Other interesting object is

**Three line vertex - photon-photon-gluon** - relates colored and white states. This is new type effective vertex which generates new observable processes - inelastic scattering of photons, splitting (dissociation) of gluons in two photons in the *QGP*. One of our goals is to calculate this vertex and investigate these processes in the plasma.

**These can be signals of the creation of QGP.**
4 Gluon and photon spectra in QGP

Before doing that we have to detect the normal photon and gluon modes presented in the QGP with $A_0$. This can be done by solving the dispersion equations for these fields.

M. Bordag, V. Skalozub (2019)

Basically, in the plasma the spectra of the excitations can be obtained from the dispersion relations of the type

$$\omega^2 - \vec{k}^2 = \text{Re}\Pi(\omega, \vec{k}),$$

where $\omega$ and $\vec{k}$ are the frequency and the momentum of the modes.

In the QGP the transverse and the longitudinal excitations present. They are derived from relevant polarization tensors $\Pi(\omega, \vec{k})_T$ and $\Pi(\omega, \vec{k})_L$. 
The expression for the photon polarization tensor reads

$$\Pi_{\mu\nu}(k) = -e^2 \sum_{p_4} \int \frac{d^3p}{(2\pi)^3} \beta Tr [\gamma_{\mu}^{(p + k)\sigma}\gamma_{\sigma} + m \gamma_{\nu}^{(p + k)^2 + m^2} \gamma_{\rho}^{p_\rho\gamma_\rho + m}] (4)$$

Here, imaginary time formalism is used. $\gamma_{\mu}$, ... are the Dirac matrixes, $p_4 = 2\pi T(l + \frac{1}{2}) + A_0$, $k\mu = (k_4 = 2\pi T(n), k)$, and $l, n = 0, \pm 1, \pm 2, ...$

Such type objects must be calculated in the gluon sector of the model.

As an example, we show the high temperature dispersion equation for the transversal plasma oscillations generated by the gluons

\[(ik_4)^2 = g^2 T^2 \left[ B_2\left(\frac{x}{2}\right) + B_2(0) \right] \xi^2 \left( \frac{\xi^2}{\xi^2 - 1} - \frac{\xi}{2} \log \frac{\xi + 1}{\xi - 1} \right) + i\Gamma. \] (5)

In this formula, \(B_2(z) = z^2 - |z| + 1/6\) is the Bernoulli polynomial, \(x = A_0/\pi T\), \(\xi = (ik_4 + A_0)/|\vec{k}|\) and \(\Gamma\) is an imaginary part of the expression. It describes the damping of the plasma oscillations.

The similar expression have been obtained for longitudinal oscillations (plasmons) in the high temperature limit \(T \to \infty\).

To find Dispersion relations we have to replace \(ik_4 \to \omega\). In such a way all the quasi particle states of photons and gluons have been derived.

The \(A_0\) condensate stabilizes the infrared behavior of the plasma and has a lower energy as compared to the empty vacuum case.
5 Induced charge in $QGP$

Generation of the strong charge due to one-line non-zero diagram.

I. Baranov, V. Skalozub (2018)

Its quark loop contribution can be calculate from the expression

$$Q_{\text{induced}}^{\text{quark}} = -g \sum_{p_4} \int \frac{d^3 p}{(2\pi)^3 \beta} Tr \gamma^4 \left[ \frac{\lambda^3 (p + k)_\sigma \gamma_\sigma + m_f}{2 (p + k)^2 + m^2_f} \right].$$  \hspace{1cm} (6)

Here, the momentum $p = (p_4 = p_4 \pm A_0, \vec{p})$, $p_4 = 2\pi T (l + 1/2), l = 0, \pm 1, ..., \beta = 1/T$.

Similar expressions can be calculated from tadpole gluon diagram having charged gluon loop.

These also hold for the color charge $Q_8$.

The resulting induced charge changes the coupling constant of gluons in the QGP.
We obtain in the high temperature limit ($\beta \to 0, (T \to \infty)$)

$$Q_{3\text{ind.}} = gA_0 \left( \frac{T^2}{3} - \frac{m^3}{T} + O(1/T^3) \right).$$

In the presence of the induced charge the Slavnov-Taylor identity reads

$$\hat{p}_\mu \Pi_{\mu\nu}^\perp(\hat{p}_4, \vec{p}) = gJ_\nu^3.$$  

The induced current is

$$J_\nu^3 = 2igQ_{3\text{ind.}}u_\nu,$$  

$u_\nu$ is plasma velocity.
6 Potentials of classical color fields

V. Skalozub (2019)

The induced color charges in the plasma result in the generation of classical gluon potentials. We introduce a simple model motivated by heavy-ion collisions.

We consider the QGP confined in the plate of the size $L$ in $z$-axis direction and infinite in $x$-, $y$- directions. For this geometry, we calculate the classical potentials $\bar{\phi}^3 = G_4^3, \bar{\phi}^8 = G_4^8$ by solving the classical field equations for the gluon fields $G_4^3, G_4^8$ generated by the induced charges $Q_{ind}^3, Q_{ind}^8$. In doing so we take into consideration the results by Kalashnikov (1994, 96) who calculated the gluon modes at the $A_0$ background. Either transversal or longitudinal modes were derived. For our problem, we are interested in the latter ones. The longitudinal modes of fields $G_4^3, G_4^8$ have temperature masses $\sim g^2T^2$. They are not affected by the background fields.

The classical potential $\bar{\phi}^3$ is calculated from the equation

$$\left[\frac{\partial^2}{\partial x_\mu^2} - m_D^2\right]\bar{\phi}^3 = -Q_{ind}^3.$$ (10)

Similar equation is for $\bar{\phi}^8$. 
Making Fourier’s transformation to momentum $k$-space we derive the spectrum of modes, $-k_4^2 = k_x^2 + k_y^2 + k_z^2 + m_D^2$, where $k_z^2 = (\frac{2\pi}{L})^2 l^2$ and $l = 0, \pm 1, \pm 2, \ldots$. The discreteness of $k_z$ is due to the periodic boundary condition for the plane: $\bar{\phi}^3(z) = \bar{\phi}^3(z + L)$. The general solution to Eq.(10) is

$$\bar{\phi}^3(x_4, \vec{x}) = d + ae^{-i(k_4 x_4 - \vec{k} \cdot \vec{x})} + be^{i(k_4 x_4 - \vec{k} \cdot \vec{x})}. \tag{11}$$

At zero induced charge, $d = 0$, and we have two well known plasmon modes. In case of $Q_{ind.}^3 \neq 0$, the values $a, b, d$ calculated from the confinement boundary condition

$$\bar{\phi}^3(z = -\frac{L}{2}) = \bar{\phi}^3(z = \frac{L}{2}) = 0 \tag{12}$$

result in the expression

$$\bar{\phi}^3(z) = \frac{Q_{ind.}^3}{m_D^2} \left[ 1 - \frac{\cos(k_z z)}{\cos(k_z L/2)} \right]. \tag{13}$$

There are no dynamical plasmon states at all. This is the main observation. In the presence of the induced charges, the static classical color potentials (and, hence, fields) have to realize in the plasma.

By dimension analysis we have $\frac{Q_{ind.}^3}{m_D^2} \sim \frac{g A_0 T^2}{g^2 T^2}$ and $g A_0 \sim g^2 T$.

**Hence,** $\bar{\phi}^3(z) \sim c T$,

where $c \geq 0$ is a positive number!
For applications it is also necessary to get the Fourier’s transform \( \bar{\phi}^3(k) \) of the potential (13) to momentum space \( k \). Fulfilling that for the interval of \( z \left[ -\frac{L}{2}, \frac{L}{2} \right] \) we obtain

\[
\bar{\phi}^3(k) = \frac{Q^3_{\text{ind}}}{m_D^2} \sin(kL/2) \frac{k_z^2}{(kL/2) k_z^2 - k^2},
\]

where the values of \( k_z \) are given after Eq.(10).

The energy for a one mode with momentum \( k_z \) is positive and equals to

\[
E_l = \frac{(Q^3_{\text{ind}})^2 k_z^2}{m_D^4} \frac{2}{L} = \frac{(Q^3_{\text{ind}})^2 2\pi^2}{m_D^4 L} l^2.
\]

The total energy is given by the sum over \( l \) of energies (15).

In the presence of the induced charges the static gluon potentials with positive energy should be generated. Dynamical longitudinal modes do not exist. This is the consequence of the condition Eq.(12).

Obvious that such a situation is independent of the form of the bag where the plasma is confined. In general, we have to expect that the color static potentials \( \bar{\phi}^3, \bar{\phi}^8 \) have to exist in the \( QGP \) and produce specific processes.
7 Effective $\gamma\gamma G$ vertexes in QGP

Explicit form for the photon-photon-gluon vertex, its dominant terms are

M. Bordag, V. Skalozub (2019)

\[
\Pi_{\mu\nu\lambda}(k^1, k^2, k^3) = \delta(k^1 + k^2 + k^3)(-e^2 g \Lambda) \sum_{p_4} \int \frac{d^3 p}{(2\pi)^3} \beta (\Gamma_{\mu\nu\lambda}^{(1)} + \Gamma_{\mu\nu\lambda}^{(2)}),
\]

(16)

\[
\Lambda = -16 A_0 m_f^2,
\]

(17)

\[
\Gamma_{\mu\nu\lambda}^{(1)} = \frac{\delta_{\mu\nu} \delta_{\lambda4} + \delta_{\mu\lambda} \delta_{\nu4} + \delta_{\lambda\nu} \delta_{\mu4}}{d^2(p)d^2(p, k^1)d^2(p, k^3)},
\]
and

$$\Gamma_{\mu\nu\lambda}^{(2)} = \frac{-2S_{\mu\nu\lambda}}{d^2(p)d^2(p, k^1)d^2(p, k^3)} \left( \frac{(p + k^3)^4}{d^2(p, k^3)} + \frac{(p - k^1)^4}{d^2(p, k^1)} + \frac{p_4}{d^2(p)} \right), \quad (18)$$

where $d^2(p) = p^2 + m^2_f$, $d^2(p, k^1) = (p - k^1)^2 + m^2_f$, $d^2(p, k^3) = (p + k^3)^2 + m^2_f$,

$$S_{\mu\nu\lambda} = \delta_{\mu\nu}(p + k^1 + k^3)_{\lambda} + \delta_{\lambda\nu}(p - k^1 - k^3)_{\mu} + \delta_{\mu\lambda}(p - k^1 + k^3)_{\nu}. \quad (19)$$

In the above formulas, $k^1$, $k^3$ are momenta of ingoing photons and $k^2 = -(k^1 + k^2)$ is momentum of ingoing color neutral gluon $Q^{a=3}$.

All the other three-vertexes composing photons and gluons are zero. So, we have a possibility for direct interaction of color and white world.
The most important points:

1. The vertex is not transversal

2. It relates transversal and longitudinal modes of photons and gluons

   In particular, new phenomena such as scattering of photons on the QGP as an effective vertex become possible. All the necessary ingredients to investigate these are calculated. These are the spectra of photons and gluons in the QGP, the effective charges.

   There are two sorts of the processes of interest:

   1) Scattering of photons on the plasma as on the external field generated due to quark current and induced color charge. Radiation of photon pairs from plasma.

   2) Scattering on the real gluon excitations in the plasma.

   In these processes the plasma exhibits itself via the effective vertex and therefore the inelastic (or even elastic) scattering may be realized. Specific values for these cases depend on the characteristics of $QGP$. 
Scattering of photons in the $QGP$ has to be estimated by two parameters - induced charge and deviation of of the photon beams from an initial direction.

Other important expected process is splitting of the gluon field $G^3, G^8$ generated by the induced charge $Q_{ind.}^3, Q_{ind.}^8$ in two photons which have to move along the direction of the plasma motion.

These processes are basically different from the scattering of photons on chaotically moving particles of usual plasma.
8 Fermionic media at nonzero $\mu, T, H$

Similar processes may take place in dense nuclear matter and electron-positron plasma. Here the role of the $A_0$ plays the chemical potential $\mu$ related with the matter formation.

To get the corresponding formulas from the above ones, we have to substitute $A_0 \rightarrow i\mu$

The role of gluon fields $Q_3$ play neutral $\rho^0$ meson and photon fields. In matter, the induced electric charge has to appear. This may serve as a signal for the nuclear matter creation. As a result, inelastic scattering of photons on the matter is expected and strong temperature dependent electric fields have to appear. Below, for simplicity, we consider the QED with finite $\mu, T, H$.

ONE-PHOTON VERTEX IN DENSE FERMION MEDIUM

E. Reznikov, V. Skalozub (2019)

The one-photon-line tensor is defined as

$$\Pi_\nu = \frac{e}{(2\pi)^3} Tr \int d^4p \gamma_\nu G(p), \quad (20)$$

$\nu$ runs from one to four, $G(p)$ is the electron Green function with the presence of magnetic field

$$G(p) = \frac{-i\hat{p} + m}{p^2 + m^2}, \quad (21)$$
The Euclidean metric is used,

\[
p = \begin{cases} 
  p_\rho + eA_\rho, & \rho = 1, 2, 3, \\
  p_4 + i\mu, & \rho = 4 
\end{cases}.
\]  

(22)

\(\mu\) - chemical potential, \(\gamma_\nu\) - Dirac matrices and \(p_4 = i p_0\). For homogeneous field, the vertex function is simplified, and the tensor components are:

\[
\Pi_4 = \frac{e}{(2\pi)^3} \int dp_3 dp_4 \times 
\]

\[
\times \sum_{n=1}^{p_4 - i\mu} \left( p_4 - i\mu \right)^2 + (2n + 1)eH - \sigma eH + p_3^2 + m^2, 
\]

\(\Pi_i = 0\).

(23)

spin variable \(\sigma = \pm 1\). By summing up over \(\sigma\), then over \(n\), and integration, we obtain

\[
\Pi_4 = \frac{e}{2\pi} \theta(\mu^2 - (eH + m^2))(\mu^2 - (eH + m^2)),
\]  

(24)

were \(\theta(\mu^2 - (eH + m^2))\) is Heaviside’s step function.

In contrast to the case of a ”pure” medium, in function’s argument the square of the mass is replaced by the sum of the magnetic field strength and the mass squared. Moreover, in this case not only the threshold of the function is shifted but also its value in the allowed regions changed. At zero temperature and in the field, the generation of the induced charge is partially or completely suppressed.
One-photon tensor at finite temperature

Summation over $p_4$ is realized according to the formula

$$
\frac{1}{\beta} \sum_{p_4} F(\ldots p_4) = \frac{1}{\beta} \sum_n F(\ldots \frac{(2n+1)\pi}{\beta}) = -\frac{1}{\beta} \sum_k f(z_k) \text{Res} (F(\ldots z_k)),
$$

where $p_4 = \frac{(2n+1)\pi}{\beta}$ and $p_4 = z_k$ is the position of the poles of the function $F$. The function $f(z)$ for points $p_4 = z_k = \frac{(2n+1)\pi}{\beta}$ is $f(z) = \frac{-i\beta}{1+e^{i\beta z}}$.

We get

$$
\Pi_4 = \frac{e}{\beta(2\pi)^2} \int dp_3 \left[ \frac{(\frac{\pi}{\beta} + i\mu)}{((\frac{\pi}{\beta} + i\mu)^2 + eH + p_3^2 + m^2)} - \frac{i\beta \sinh(\beta\mu)}{2(\cosh(\beta\mu) + \cosh \sqrt{eH + p_3^2 + m^2})} \right].
$$

This integral can be calculated only partially in the form

$$
\Pi_4 = \frac{e}{2\pi} \frac{(\frac{\pi}{\beta} + i\mu) \theta(\lambda)}{\beta \sqrt{(\frac{\pi}{\beta} + i\mu)^2 + eH + m^2}} - \frac{e}{4\pi^2} \int \frac{i \sinh(\beta\mu) dp_3}{2 \left( \cosh(\beta\mu) + \cosh \beta \sqrt{eH + p_3^2 + m^2} \right)},
$$

where $\lambda$ is

$$
\lambda = \mu^2 - \frac{\pi^2}{\beta^2} - m^2 - eH + \sqrt{\frac{4\pi^2 \mu^2}{\beta^2} + \left( \mu^2 - m^2 - eH - \frac{\pi^2}{\beta^2} \right)^2}.
$$
At low-temperature, it tends to $\lambda = 2(\mu^2 - (eH + m^2))$ and $\lambda = 0$. Step-function comes from strict inequality $\text{Im} \sqrt{\frac{\pi}{\beta} + i\mu)^2 + eH + m^2} > 0$, so it is defined as $\theta(\lambda) = \begin{cases} 0, & \lambda \leq 0, \\ 1, & \lambda > 0. \end{cases}$ Therefore, in case $\lambda = 0$ Heaviside’s function equals zero.

The analytical term then tends to

$$\frac{e^{i\mu\theta(2(\mu^2 - (eH + m^2)))}}{2\pi \beta \sqrt{(eH + m^2 - \mu)^2}}.$$  

(27)

The integral can be calculated as asymptotic at $\beta \to \infty$. After integration we get

$$\frac{e^{i\mu\theta(2(\mu^2 - (eH + m^2)))}}{4\pi^2} \int \frac{i \sinh(\beta \mu) dp_3}{2 \left( \cosh(\beta \mu) + \cosh \beta \sqrt{eH + p_3^2 + m^2} \right)} =$$

$$= \frac{e^{i\mu\theta(2(\mu^2 - (eH + m^2)))}}{2\pi} \left( \mu^2 - (eH + m^2) \right) \left( \mu^2 - (eH + m^2) \right),$$

(28)

that coincides with eq. (24).

Eq. (26) indicates that the effect of the temperature on the processes of the induced charge generation is significant and cannot be reduced to correction factors.
One-photon tensor at high temperature

We calculate the asymptotic for (26). In this case $\beta$ approaches to zero, so hyperbolic sinuses can be replaced by their argument’s and cosines tend to one. Then integration yields

$$\int dp_3 F = \frac{i\mu \pi \theta (\lambda)}{\sqrt{4 + (\beta)^2(eH + m^2)}}. \quad (29)$$

Combining this and the analytical part of (26) we obtain

$$\Pi_4 = \frac{e}{2\pi} \frac{(\frac{\pi}{\beta} + i\mu) \theta (\lambda)}{\beta \sqrt{(\frac{\pi}{\beta} + i\mu)^2 + eH + m^2}} - \frac{e}{4\pi} \frac{i\mu \theta (\lambda)}{\sqrt{4 + (\beta)^2(eH + m^2)}}. \quad (30)$$

In case of $T >> \mu$ this is simplified to

$$\Pi_4 = \frac{e}{2\pi} \theta (\lambda) \left( \frac{1}{\beta} - \frac{i\mu}{4} \right). \quad (31)$$

For high $T$, $\lambda$ tends to $\lambda = 2\mu^2 \left( 1 - \frac{(eH + m^2)\beta^2}{\pi^2} \right)$. Therefore, the allowed region is any non-zero $\mu$.

Thus, we observe a linear dependence on the temperature. The high temperature ensures the generation of the induced charge even for small values of the chemical potential.
Behavior of induced potential

In the low temperature approximation for \( \rho(\mu, m, \beta) \), we get

\[
\rho(\mu, m, \beta) = 2e\theta(\lambda) \text{Re} \left( \frac{\left( \frac{\pi}{\beta} + i\mu \right)}{\beta \sqrt{\left( \frac{\pi}{\beta} + i\mu \right)^2 + eH + m^2}} + \right.
\]

\[
+ \frac{\mu^2 - m^2 - eH}{1 + \cosh \pi \beta \sqrt{eH - \mu^2 + m^2}},
\]

\[
\lambda = \mu^2 - m^2 - eH + \sqrt{(\mu^2 - m^2 - eH)^2} + \]

\[
+ \frac{2\mu^2 \pi^2}{(\mu^2 - m^2 - eH) \beta^2};
\]

and in high temperature approximation, we obtain

\[
\rho(\mu, m, T) = 2e\theta(\lambda) \text{Re} \left( T - \frac{i\mu}{4} \right),
\]

\[
\lambda = 2\mu^2 \left( 1 - \frac{(eH + m^2)}{\pi^2 T^2} \right).
\] (33)

Thus, by changing the magnetic field, we can control at what temperature the induced charge generation begins.
We consider medium confined in the plate of the size $L$ in z-axis direction and infinite in x and y directions. The classical potential $\varphi$ is calculated from the equation

$$\left[ \frac{\partial^2}{\partial x^2} - m_D^2 \right] \varphi = -\rho(\mu, m, \beta). \quad (34)$$

Making the Fourier transformation to momentum $k$-space, we derive the spectrum of modes $-k_4^2 = k_x^2 + k_y^2 + k_z^2 + m_D^2$, where $k_z^2 = \left( \frac{2\pi}{L} \right)^2 l^2$, $l = 0, \pm 1, \pm 2, \ldots$, and $m_D^2$ is the Debye plasmon mass. The discreteness of $k_z$ is due to the periodic boundary condition for the plate: $\varphi(z) = \varphi(z + L)$. The general solution to Eq. (34) is

$$\varphi(x_4, \vec{x}) = d + a e^{-i(k_4 x_4 - \vec{k} \vec{x})} + b e^{i(k_4 x_4 - \vec{k} \vec{x})}. \quad (35)$$

In the case of the zero induced charge $d=0$ and we have two plasmon modes. In presence of induced charge, we use the boundary condition

$$\varphi\left(\frac{L}{2}\right) = \varphi\left(-\frac{L}{2}\right) = -\frac{\rho(\mu, m, \beta) L^2}{2}, \quad (36)$$

that results in the expression

$$\varphi(z) = \frac{\rho(\mu, m, \beta)}{m_D^2} \left[ 1 - \frac{1 - L^2 m_D^2/2}{\cos(k_z L/2) \cos(k_z)} \right]. \quad (37)$$

The generated potential depends on z-coordinate, only.
At high temperature, \( m_D^2 \approx e^2 T^2 \).

We use this approximation and the explicit form of \( \rho (\mu, m, \beta) \) and obtain

\[
\varphi (z) \approx e \left[ \frac{2}{e^2 T^2} \left( 1 - \frac{\cos (k_z z)}{\cos (k_z L/2)} \right) + \frac{L^2 \cos (k_z z)}{\cos (k_z L/2)} T \right] \times \\
\times \Theta \left( 2\mu^2 \left( 1 - \frac{(eH + m^2)}{\pi^2 T^2} \right) \right). \tag{38}
\]

At sufficiently high temperature the charging starts at not dense media, \( \mu \approx 0 \). In this case, the magnetic field strength determines the required temperature at which the induced charge (and potential) arises.
The dependence of $\varphi(z)$ on the z-distance for several values of temperature.

We observe the growth of the induced potential in each point of space at temperature increase. The temperature step also exists. Below it the induced potential equals zero.
9 Conclusion

According to basic principles of QCD, the QGP has to be either magnetized with strong long range temperature dependent magnetic fields $B^3(T), B^8(T), H(T)$ (that lowers the deconfinement transition temperature $T_d$) or charged with color induced charges $Q_{ind}^3, Q_{ind}^8$.

Due to violation of Furry’s theorem, in the QGP new type phenomena have to be generated. Among them the deviation of the photon beam from its initial direction and the change of the frequency. Generation of induced color charges, gluon splitting in two photons. These are the distinguishable signals of the QGP creation.

In dense fermionic matter, the role of $A_0$ plays chemical potential $\mu$. The role of gluon fields $Q_3$ play neutral $\rho$ meson and photon fields. The induced electric charge $Q_{ind}^e$ should appear. This could serve as signal for the nuclear matter creation. As a result, inelastic scattering of photons on the matter is expected and strong temperature and magnetic field dependent electric fields have to be observed. The charging of dense medium is an important phenomenon affecting different processes. In particular, the induced charge will be the explicit signal of the dense medium creation. Magnetic field strength sets the temperature of the medium, at which charging takes place.

The strong spatially alternating potential is generated in the plate and outspace at sufficiently high temperature. Obviously, this conclusion independes of the specific configuration of the medium!