Confinement viewed with caloron, dyon and dimeron ensembles

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Outline:

1. Introduction, motivation
2. Instantons at $T > 0$: calorons with non-trivial holonomy
3. Simulating caloron ensembles
4. Dyon gas ensembles and confinement
5. $T = 0$: Simulating dimeron ensembles
6. Summary
1. Introduction, motivation

Reinvestigation of an old issue: semiclassical approach to QCD path integral

[’t Hooft, ’76; Callan, Dashen, Gross, ’78; Gross, Pisarski, Yaffe, ’81;
Ilgenfritz, M.-P., ’81; Shuryak, ’82; Diakonov†, Petrov, ’84]

\( T = 0: \) Belavin-Polyakov-Shvarts-Tyupkin (BPST) instantons
\( T > 0: \) Harrington-Shepard (HS) calorons

\[ \text{BPST: } A_{a,\mu}^{\text{inst}}(x) = R_{a\alpha} \bar{\eta}_{\mu \nu} \frac{x_\nu}{(x^2 + \rho^2)x^2}, \quad A_{a,\mu}^{\text{antiinst}} : \bar{\eta} \leftrightarrow \eta \]

superpositions of “pseudo-particles”

\[ A_{\text{class}}(z = \{\rho^{(i)}, x^{(i)}, R^{(i)}\}) = \sum_{i=1}^{N_+ + N_-} A_{a,\mu}^{(i)}(x - x^{(i)}, \rho^{(i)}, R^{(i)}), \]

in order to approximate the functional integral by \( A = A_{\text{class}}(z) + \varphi \)

\[
\int DA \exp(-S[A]) \simeq \sum_{\text{class}} \int [dz] \exp(-S[A_{\text{class}}]) \int D\varphi \exp \left( -\frac{1}{2!} \int \frac{\delta^2 S}{\delta A^2} \big|_{A_{\text{class}}\varphi} \right) + \cdots
\]

\( \int [dz] \) — modular space integration (“collective coordinates”).

Turns path integral into stastistical mechanics partition function.
• Powerful approach for non-perturbative phenomena like chiral symmetry breaking and $U_A(1)$ problem $\iff$ confinement hard to explain
  [Reviews by Schäfer, Shuryak, ’98; Forkel, ’00; Diakonov†, ’03; ...]

• (Anti)selfdual solutions seen on the lattice with “cooling”, “smoothing”,...
  [Teper, ’86; Ilgenfritz, Laursen, M.-P., Schierholz, Schiller, ’86; Polikarpov, Veselov†, ’88; ...]

Old idea to implement confinement $\implies$ increase entropy by “dissociation” of instantons into constituents (“instanton quarks”).

• $T = 0$: “meron” mechanism [Callan, Dashen, Gross, ’77 - ’79].
  [Lenz, Negele, Thies, ’03-’04; M. Wagner, ’06]

• $T > 0$: KvBLL (multi-) calorons with non-trivial holonomy – “dyons”

Here simulate caloron, dyon ensembles – for $0 < T < T_c$,
  meron pair (“dimeron”) ensembles – for $T = 0$. 
2. Instantons at $T > 0$: calorons with non-trivial holonomy

Partition function

$$Z_{YM}(T, V) \equiv \text{Tr} \, e^{-\frac{\hat{H}}{T}} \propto \int DA \, e^{-S_{YM}[A]} \quad \text{with} \quad A(\vec{x}, x_4+b) = A(\vec{x}, x_4), \quad b = 1/T.$$ 

Old treatment with HS caloron solutions

$$\equiv \text{$x_4$-periodic instanton chains} \quad \text{Gross, Pisarski, Yaffe, '81}$$

$$A_{\mu}^{a\text{HS}} = \tilde{\eta}_a^{\mu\nu} \partial_\nu \log(\Phi(x))$$

$$\Phi(x) = 1 + \sum_{k \in \mathbb{Z}} \frac{\rho^2}{(\vec{x} - \vec{z})^2 + (x_4 - z_4 - kb)^2}$$

$$= 1 + \frac{\pi \rho^2}{b |\vec{x} - \vec{z}|} \frac{\sinh \left( \frac{2\pi b |\vec{x} - \vec{z}|}{|\vec{x} - \vec{z}|} \right)}{\cosh \left( \frac{2\pi b |\vec{x} - \vec{z}|}{|\vec{x} - \vec{z}|} \right) - \cos \left( \frac{2\pi b (x_4 - z_4)}{|\vec{x} - \vec{z}|} \right)}$$

'\text{t Hooft symbols:} \quad \eta_{a \mu\nu} = \varepsilon_{a \mu\nu}, \quad \eta_{a \mu4} = - \eta_{a4\mu} = \delta_{a\mu} \quad \text{for} \quad \mu, \nu = 1, 2, 3, \quad \eta_{a44} = 0;$$

$$\tilde{\eta}_{a \mu\nu} = (-1)^{\delta_{a4\mu} + \delta_{4\nu4}} \eta_{a \mu\nu}.$$ 

HS (anti)caloron exhibits trivial holonomy, i.e. Polyakov loop behaves as:

$$\frac{1}{2} \text{tr} \mathbf{P} \exp \left( i \int_0^{b=1/T} A_4(\vec{x}, t) \, dt \right) \bigg|_{|\vec{x}| \to \infty} \xrightarrow{\text{as}} \pm 1.$$
Kraan - van Baal - Lee - Lu solutions (KvBLL)
= (multi-) calorons with non-trivial asymptotic holonomy ($SU(2)$)

\[ P(\vec{x}) = \mathcal{P} \exp \left( i \int_{0}^{b=1/T} A_{4}(\vec{x},t) \, dt \right) \xrightarrow{\text{as } |\vec{x}| \to \infty} P_{\infty} = e^{2\pi i \omega \tau_{3}} \notin \mathbb{Z}(2) \]

Holonomy parameter: \(0 \leq \omega \leq \frac{1}{2}, \quad \omega = \frac{1}{4}\) – maximally non-trivial holonomy.

Action density of an $SU(3)$ caloron (van Baal, ’99)

\[ \implies \text{not a simple } SU(2) \text{ embedding into } SU(3) !! \]
**SU(2) calorons with non-trivial holonomy**

[K. Lee, Lu, '98, Kraan, van Baal, '98 - '99, Garcia-Perez et al. '99]

- $x_4$-periodic, (anti)selfdual solutions from ADHM formalism,
- generalize Harrington-Shepard calorons (i.e. $x_4$ periodic BPST instantons).

Holonomy parameter $\bar{\omega} = 1/2 - \omega$, $0 \leq \omega \leq 1/2$

\[
A^C_\mu = \frac{1}{2} \tilde{\eta}_{\mu\nu} \tau_3 \partial_\nu \log \phi + \frac{1}{2} \phi \text{ Re} \left( (\tilde{\eta}_{\mu\nu}^1 - i \tilde{\eta}_{\mu\nu}^2) (\tau_1 + i \tau_2) (\partial_\nu + 4\pi i \omega \delta_\nu, 4) \tilde{\chi} \right) + \delta_{\mu, 4} 2\pi \omega \tau_3 ,
\]

\[
\phi(x) = \frac{\psi(x)}{\tilde{\psi}(x)}, \quad x = (\vec{x}, x_4 \equiv t), \quad r = |\vec{x} - \vec{x}_1|, \quad s = |\vec{x} - \vec{x}_2| ,
\]

\[
\psi(x) = -\cos(2\pi t) + \cosh(4\pi r \bar{\omega}) \cosh(4\pi s \omega) + \frac{r^2 + s^2 + \pi^2 \rho^4}{2rs} \sinh(4\pi r \bar{\omega}) \sinh(4\pi s \omega) + \frac{\pi \rho^2}{s} \sinh(4\pi s \omega) \cosh(4\pi r \bar{\omega}) + \frac{\pi \rho^2}{r} \sinh(4\pi r \bar{\omega}) \cosh(4\pi s \omega),
\]

\[
\tilde{\psi}(x) = -\cos(2\pi t) + \cosh(4\pi r \bar{\omega}) \cosh(4\pi s \omega) + \frac{r^2 + s^2 - \pi^2 \rho^4}{2rs} \sinh(4\pi r \bar{\omega}) \sinh(4\pi s \omega) ,
\]

\[
\tilde{\chi}(x) = \frac{1}{\psi} \left\{ e^{-2\pi it} \frac{\pi \rho^2}{s} \sinh(4\pi s \omega) + \frac{\pi \rho^2}{r} \sinh(4\pi r \bar{\omega}) \right\} .
\]
Properties:
- (anti)selfdual with topological charge \( Q_t = \pm 1 \),
- has two centers at \( \vec{x}_1, \vec{x}_2 \) \( \Rightarrow \) “instanton quarks”,
  carrying opposite magnetic charges (visible in maximally Abelian gauge),
- scale-size \( \rho \) versus distance \( d \):
  \[ \pi \rho^2 T = |\vec{x}_1 - \vec{x}_2| = d, \]
- limiting cases:
  - \( \omega \to 0 \) \( \Rightarrow \) ‘old’ HS caloron,
  - \( |\vec{x}_1 - \vec{x}_2| \) large \( \Rightarrow \) two static BPS monopoles or “dyon pair” (\( DD \))
    with topological charges (\( \sim \) masses)
    \[ q_{t}^{\text{dyon}} = 2\omega, \ 1 - 2\omega, \]
  - \( |\vec{x}_1 - \vec{x}_2| \) small \( \Rightarrow \) non-static single caloron (\( CAL \)).
- \( L(\vec{x}) = \frac{1}{2} \text{tr} P(\vec{x}) \to \pm 1 \) close to \( \vec{x} \simeq \vec{x}_{1,2} \) \( \Rightarrow \) “dipole” structure
SU(2) KvBLL caloron

Action density  Polyakov loop

singly localized caloron (CAL)

caloron dissociated into dyon-dyon pair (DD)

Seen by cooling also on the lattice.

- Localization of the zero-mode of the Dirac operator:

  - **time-antiperiodic b.c.**:
    around the center with \( L(\vec{x}_1) = -1 \),
    \[
    |\psi^-(x)|^2 = - \frac{1}{4\pi} \partial_\mu^2 \left[ \tanh(2\pi r \bar{\omega}) / r \right] \quad \text{for large } d,
    \]

  - **time-periodic b.c.**:
    around the center with \( L(\vec{x}_2) = +1 \),
    \[
    |\psi^+(x)|^2 = - \frac{1}{4\pi} \partial_\mu^2 \left[ \tanh(2\pi s \omega) / s \right] \quad \text{for large } d.
    \]

- Lattice search for KvBLL configurations:

  - **Caloron constituents seen in MC generated lattice gauge fields by cooling, smearing, and filtering with eigenmodes of the (overlap) lattice Dirac operator.**

  - Their occurrence at \( T < T_c \) significantly differs from \( T > T_c \).

[V. Bornyakov, E.-M. Ilgenfritz, B. Martemyanov, M. M.-P., ..., '02 - '09]
3. Simulating caloron ensembles

[HU Berlin master thesis by P. Gerhold, '06; Gerhold, Ilgenfritz, M.-P., NPB 760, 1 (2007)]

Model based on random superpositions of KvBLL calorons.

Superpositions made in the algebraic gauge – $A_4$-components fall off.

Gauge rotation into periodic gauge

$$A_{\mu}^{per}(x) = e^{-2\pi i x_4 \vec{\omega} \cdot \vec{\tau}} \sum_i A_{\mu}^{(i), alg}(x) \cdot e^{+2\pi i x_4 \vec{\omega} \cdot \vec{\tau}} + 2\pi \vec{\omega} \cdot \delta_{\mu,4}.$$ 

First important check: study the influence of the holonomy

- same fixed holonomy for all (anti)calorons: $P_{\infty} = \exp 2\pi i \omega \tau_3$, $\omega = 0$ – trivial versus $\omega = 1/4$ – maximally non-trivial,

- put equal number of calorons and anticalorons randomly in a 3d box with open b.c.’s,

- for measurements use a $32^3 \times 8$ lattice grid and lattice observables,

- fix parameters and lattice scale:
  
  temperature: $T = 1 \text{ fm}^{-1} \simeq T_c$,  
  density: $n = 1 \text{ fm}^{-4}$,  
  scale size: compare fixed $\rho = 0.33 \text{ fm}$  
  with distribution $D(\rho) \propto \rho^{7/3} \exp(-c\rho^2)$, such that $\bar{\rho} = 0.33 \text{ fm}$.  

Polyakov loop correlator $\rightarrow$ quark-antiquark free energy

$$F(R) = -T \log \langle L(\mathbf{x})L(\mathbf{y}) \rangle, \quad R = |\mathbf{x} - \mathbf{y}|$$

with trivial ($\omega = 0$) and maximally non-trivial holonomy ($\omega = 0.25$).

$\rho$ fixed $\quad \rho$ sampled with distribution

$\Rightarrow$ Non-trivial (trivial) holonomy creates long-distance coherence (incoherence) and (de)confines for standard instanton or caloron liquid model parameters.
Building a more realistic, $T$-dependent model:

Main ingredients:

- **Holonomy parameter:** $\omega = \omega(T)$
  from lattice results for the (renormalized) average Polyakov loop.
  
  Digal, Fortunato, Petreczky, '03; Kaczmarek, Karsch, Zantow, Petreczky, '04
  
  $\Rightarrow \omega = 1/4$ for $T \leq T_c$, $\omega$ smoothly decreasing for $T > T_c$.

- **Density parameter:** $n = n(T)$ for uncorrelated caloron gas to be identified with top. susceptibility $\chi(T)$ from lattice results
  
  Alles, D’Elia, Di Giacomo, '97

- **$\rho$-distribution:**
  
  $T = 0$: Ilgenfritz, M.-P., '81; Diakonov, Petrov, '84
  
  $T > 0$: Gross, Pisarski, Yaffe, '81

| $T < T_c$ | $D(\rho, T) = A \cdot \rho^{7/3} \cdot \exp(-c\rho^2)$ | $\int D(\rho, T)d\rho = 1$, $\bar{\rho}$ fixed |
| $T > T_c$ | $D(\rho, T) = A \cdot \rho^{7/3} \cdot \exp(-\frac{4}{3}(\pi\rho T)^2)$ | $\int D(\rho, T)d\rho = 1$, $\bar{\rho}$ running |

Distributions sewed together at $T_c \implies$ relates $\bar{\rho}(T = 0)$ to $T_c$,

$\bar{\rho}(T = 0)$ to be fixed from known lattice space-like string tension $T_c/\sqrt{\sigma_s(T = 0)} \simeq 0.71$: $\implies \bar{\rho} = 0.37$ fm.
Color averaged free energy versus distance $R$ at different temperatures from Polyakov loop correlators.

\[ F_{\text{avg}}(R, T) \text{ [MeV]} \]

\[
\begin{array}{c}
T = 0.80 \cdot T_C \\
T = 0.90 \cdot T_C \\
T = 1.00 \cdot T_C \\
T = 1.10 \cdot T_C \\
T = 1.20 \cdot T_C \\
T = 1.32 \cdot T_C \\
\end{array}
\]

-- successful description of the deconfinement transition,
-- but still no realistic description of the deconf. phase.
4. Dyon gas ensembles and confinement

Working hypothesis (cf. Polyakov, ’77):
Confinement evolves from magnetic monopoles effectively in 3D.
Here: monopoles = dyons (KvBLL caloron constituents) for $0 < T < T_c$.

Assume:
integration measure over KvBLL caloron moduli space
rewritten in terms of dyon degrees of freedom,
⇒ difficult task for (still unknown) general multi-caloron solutions.

Diakonov, Petrov, ’07 :
proposed integration measure for all kind dyons
(Abelian fields; no antidyons, i.e. CP is violated).
Dyon ensemble statistics analytically solved ⇒ confinement.

However, observation from numerical simulation:
Moduli space metric satisfies positivity only for a small fraction
of dyon configurations and only for low density ⇒ inconsistent metric.
Simplify the model:

- Far-field limit, i.e. purely Abelian monopole fields, non-trivial holonomy.
- Neglect moduli space metric and describe random monopole gas.
- Compute free energy of a static quark-antiquark pair from Polyakov loop correlators.

[Bruckmann, Dinter, Ilgenfritz, Maier, M.-P., Wagner, PRD 85, 034502 (2012)]

Monopole field:

\[ a_0(r; q) = \frac{q}{r}, \quad a_1(r; q) = -\frac{qy}{r(r - z)}, \quad a_2(r; q) = +\frac{qx}{r(r - z)}, \quad a_3(r; q) = 0, \]

\[ r = (x, y, z), \quad r = |r| - 3d \text{ distance to the dyon center}, \quad q - \text{magnetic charge}. \]

With 't Hooft's symbol write:

\[ a_\mu(r; q) = -q \tilde{\eta}^3_{\mu\nu} \partial_\nu \ln(r - z). \]

Holonomy to be added to superponed monopole fields:

\[ A_0 \rightarrow 2\pi \omega T \tau_3, \]

\[ P(r) \equiv \frac{1}{2} \text{Tr} \left( \exp \left( i \int_0^{1/T} dx_0 \, A_0(x_0, r) \right) \right) \rightarrow \frac{1}{2} \text{Tr} \left( \exp \left( 2\pi i \omega \tau_3 \right) \right) = \cos(2\pi \omega), \]
Superposition of gauge fields of $2K$ dyons:

$$A_\mu (\mathbf{r}) = \left( \delta_{\mu 0} 2\pi \omega T + \frac{1}{2} \sum_{i=1}^{K} \sum_{m=1}^{2} a_\mu (\mathbf{r} - \mathbf{r}_i^m; q_m) \right) \tau_3,$$

$\mathbf{r}_i^m$ and $q_m = (-1)^m$ – positions and magnetic charges of $i$-th dyon ($m = 1$), antidyon ($m = 2$), respectively.

Local Polyakov loop $P(\mathbf{r})$:

$$P(\mathbf{r}) = \cos \left( 2\pi \omega + \frac{1}{2T} \Phi(\mathbf{r}) \right), \quad P(\mathbf{r}) \bigg|_{\omega=1/4} = -\sin \left( \frac{1}{2T} \Phi(\mathbf{r}) \right),$$

where

$$\Phi(\mathbf{r}) \equiv \sum_{i=1}^{K} \sum_{m=1}^{2} \frac{q_m}{|\mathbf{r} - \mathbf{r}_i^m|} = \sum_{i=1}^{K} \left[ \frac{1}{|\mathbf{r} - \mathbf{r}_i^1|} - \frac{1}{|\mathbf{r} - \mathbf{r}_i^2|} \right].$$

Compute free energy of a static $\bar{Q}Q$ pair from Polyakov loop correlator

$$F_{\bar{Q}Q}(d) = -T \ln \left< P(\mathbf{r}) P^\dagger (\mathbf{r}') \right>, \quad d \equiv |\mathbf{r} - \mathbf{r}'|. $$
Expectation value:
\[
\langle O \rangle = \frac{\int \prod_{i=1}^{K} dr^1_i dr^2_i O \left( \{r^1_i, r^2_i\} \right)}{\int \prod_{i=1}^{K} dr^1_i dr^2_i} = \frac{\int \prod_{i=1}^{K} dr^1_i dr^2_i O \left( \{r^1_i, r^2_i\} \right)}{V^{2K}},
\]

\(V\) – spatial volume \(\Rightarrow\) \(\rho = \frac{2K}{V}\) – density of monopole gas. 
\(\omega, T,\) and \(\rho\) – basic parameters of the model.

Expectation values can be computed numerically generating random distributions of the dyon positions in the volume \(V\).

Monopole fields have infinite range \(\Rightarrow\) strong finite size effects, well-known in plasma and statistical physics.
Solution: Ewald’s method  [P. Ewald, Ann. Phys. (1921)]

- use infinite identical replica of a basic 3d cell (“super cell”) with randomly distributed monopoles:
  \[
  \Phi(r) = \sum_{n \in \mathbb{Z}^3} \sum_{j} \frac{q_j}{|r - r_j - nL|},
  \]

  \[j = (i, m)\] running over all dyons and antidyons coming in equal number \((j\) takes \(n_D = 2K\) different values).

- Split phase \(\Phi(r)\) into short-distance and long-distance contributions

  \[
  \Phi(r) = \Phi^{\text{short}}(r) + \Phi^{\text{long}}(r)
  \]

  \[
  \Phi^{\text{short}}(r) \equiv \sum_{n \in \mathbb{Z}^3} \sum_{j} \left(1 - \text{erf}\left(\frac{|r - r_j - nL|}{\sqrt{2}\lambda}\right)\right) \frac{q_j}{|r - r_j - nL|}
  \]

  \[
  \Phi^{\text{long}}(r) \equiv \sum_{n \in \mathbb{Z}^3} \sum_{j} \text{erf}\left(\frac{|r - r_j - nL|}{\sqrt{2}\lambda}\right) \frac{q_j}{|r - r_j - nL|},
  \]

  where \(\text{erf}(\cdot)\) – error function produces smeared charges cancelling with point charges at rising distances, 
  \(\lambda\) – certain cutoff scale.
Short-distance contribution converges exponentially.

Long-distance contribution converges in its Fourier transform (for charge-neutral system only $\implies$ for dyon gas satisfied).

$$\Phi_{\text{long}}(r) = \frac{4\pi}{L^3} \sum_{n \in \mathbb{Z}^3 \setminus \{0\}} e^{+ik(n)r} \left( e^{-\lambda^2 k(n)^2/2} \frac{k(n)^2}{k(n)^2} \left( \sum_{j=1}^{N} q_j e^{-ik(n)r_j} \right) \right),$$

$$k(n) = \frac{2\pi}{L} n.$$

Cutoff scale $\lambda$ chosen such that CPU time is optimized.
Model can be solved also analytically:

Infinite-volume limit and for large distance $d = |\mathbf{r} - \mathbf{r}'|$

$$\left\langle P(\mathbf{r})P(\mathbf{r}') \right\rangle = \frac{1}{2} \exp \left( -\frac{\pi d \rho}{2T^2} + \text{const.} \right)$$

$$\Rightarrow$$ free energy $F_{\bar{Q}Q}(d) = \sigma d + \text{const.}$

$$\Rightarrow$$ string tension $\sigma = \frac{\pi}{2} \frac{\rho}{T}$.

At arbitrary finite distance $d$:

Polyakov loop correlator can be obtained by numerical integrations.
Compare Ewald’s method with analytical results:

\[
\frac{F_{\bar{Q}Q}(d)}{T} \text{ vs. } d \quad (\rho/T^3 = 1.0)
\]
Ewald’s result extrapolated to $V \to \infty$  \[ F_{Q\bar{Q}}(d)/T \text{ vs. } d \text{ for } V \to \infty \]

$\implies$ excellent agreement !!

$\implies$ simplest dyon gas model provides confinement (in terms of Polyakov loop correlator).
5. $T = 0$: Simulating dimeron ensembles

[F. Zimmermann, H. Forkel, M. M.-P., PRD 86, 094005 (2012)]

– KvBLL-like solutions in Euclid. 4d Yang-Mills theory unknown.
– Possibility: Dimeron (DM) configurations

[De Alfaro, Fubini, Furlan, ’76 - ’77]

single (anti)merons: $Q_t = \pm 1/2, \ S \rightarrow \infty$.

(anti)dimeron = (anti)meron pair ($r \equiv$ regular gauge, $SU(2)$): $Q_t = \pm 1$

\[
A_{\mu}^{(DM,r)}(x; \{x_0, a, u\}) = \left[ \frac{(x - x_0 + a)_{\nu}}{(x - x_0 + a)^2} + \frac{(x - x_0 - a)_{\nu}}{(x - x_0 - a)^2} \right] u^\dagger \sigma_{\mu\nu} u,
\]

$\sigma_{\mu\nu} := \eta_{a\mu\nu} \tau_a/2$, for $DM \leftrightarrow \overline{DM}$ replace $\eta \leftrightarrow \bar{\eta}$, $u$ – colour rotations.

Limiting cases:

- $a \rightarrow 0 \quad \Rightarrow \quad$ regular gauge instanton,
- $a \rightarrow \infty \quad \Rightarrow \quad$ well-separated merons with ‘locked’ colour orientation.

DM has 11 collective coordinates $z \equiv \{x_0, a, u\}$

(instead of 8 for one instanton, 14 for two single, colour-unlocked merons).

$\Rightarrow$ corresponding increase of entropy in the path integral.
To superpone (anti)dimerons put them into singular gauge ("s")

$$\Rightarrow \quad A_{\mu}^{(DM,s)} \sim \frac{1}{x^3} \text{ for } |x| \gg |a|, \quad \text{i.e. better localized.}$$

For numerical integrations (action, top. charge, parallel transporters etc.) meron and gauge singularities have to be regularized.

**Superpositions:** (anti)dimerons superponed (neglect meron-antimeron pairs)

$$A_{\mu}(x, \{z_I, \bar{z}_{\bar{I}}\}) = \sum_{I}^{N_{DM}} A_{\mu}^{(DM,s)}(x, \{z_I\}) + \sum_{\bar{I}}^{N_{DM}} \bar{A}_{\mu}^{(DM,s)}(x, \{\bar{z}_{\bar{I}}\}).$$

**Partition function:**

$$Z = \int \prod_{I, \bar{I}}^{N_{DM}, N_{DM}} dz_I d\bar{z}_{\bar{I}} \exp\{-S[A_{\mu}(x, \{z_I, \bar{z}_{\bar{I}}\})]\},$$

with

$$S[A] = \frac{1}{2 g^2} \int d^4 x \text{ tr } \{F_{\mu\nu} F_{\mu\nu}\} =: \int d^4 x \ s(x),$$

$$F_{\mu\nu}(x) = \partial_{\mu} A_{\nu}(x) - \partial_{\nu} A_{\mu}(x) - i [A_{\mu}(x), A_{\nu}(x)].$$

In the following study $g^2$ dependence ("temperature").
Simulating the dimeron gas:

- Take statistical weight of dimeron confs. $\exp -S[A]$ into account.

Multi-layered multigrid to control boundary effects.

*Inner measurement volume* $\Leftrightarrow$ approximate constant action density.
• MC method: Metropolis.
• For approaching to equilibrium adapt step-sizes and grid resolution.
• Ensemble: $N_{DM} = 243$, $N_{DM} = 244$ in whole volume.
• Bare coupling ("temperature"): $g^2 \in \{1, 25, 100, 1000, \infty\}$.

Measurements:
• Ensemble parameters: intermeron distances, neighbour densities, colour correlations,...
• Topology: spatial top. charge distribution, topological susceptibility.
• Wilson loops: static $\bar{Q}Q$-potential, string tension.
Results:
Left: Average inter-meron distance $2\langle |a| \rangle$ versus $g^2$.
Right: Probability for the nearest neighbour of a dimeron to have opposite topological charge $\langle f_{DM\overline{DM}} \rangle$ versus $g^2$.

$\Rightarrow$ Dimerons dissociate into their constituents with increasing $g^2$.

$\Rightarrow$ Topological order at $g^2 = 1 \iff$ disorder for $g^2 > 10$. 
Spatial topological order / disorder:
average radial density of neighbour pseudoparticles vs. distance $d$
equal (black), opposite (red) sign topological charges

g^2 = 1

g^2 = 100
Nearest neighbour dimeron colour-angle distribution $\langle f_\alpha \rangle$
black squares: equal top. charge,
red bullets: opposite top. charge,
blue crosses: random distribution ($g^2 = \infty$).

\[ g^2 = 1 \]

$\Rightarrow \alpha \simeq \pi/2$ dominant for $DM - DM$ pairs,

$DM - \overline{DM}$ pairs randomly mutually orientated.

$\Rightarrow$ qualitatively understood from two-dimeron ($\simeq$ two-instanton) interactions.
Wilson loops $\log < W >$ vs. area $A$

$g^2 = 1, 25, 100, 1000, \infty$

static potential $V(R)$:

$g^2 = 1, 25, 100$
Dimensionless ratio:
string tension / top. susceptibility \( \sigma^{1/2}/\chi^{1/4} \) vs. \( g^2 \).

\[ \chi^{1/4}/\sigma^{1/2} \simeq 0.30, \ldots, 0.55. \]

\( \Rightarrow \) Compatible with lattice \((SU(2))\): \( \chi^{1/4}/\sigma^{1/2} = 0.483 \pm 0.006 \) [Lucini, Teper, 01]

\( \Rightarrow \) Compatible also with simulations of single meron and regular instanton ensembles [Lenz, Negele, Thies, 08]
7. Summary

- Topological aspects in QCD occur naturally and have phenomenological impact.
  Standard instanton gas/liquid remains phenomenologically important: chiral symmetry breaking, solution of $U_A(1)$, ..., but fails to explain confinement.
- $0 < T < T_c$: KvBLL caloron gas model with non-trivial holonomy very encouraging for description of confinement. Model can be improved.
- Non-interacting Abelian dyon gas model analytically solvable $\Rightarrow$ confinement.
  Ewald’s method allows to keep finite-size effects under control and provides same infinite volume result.
  Full modular space metric (?) should be taken into account.
- $T = 0$: Dimerons play similar role as KvBLL calorons for $0 < T < T_c$.
  Shows Callen-Dashen-Gross mechanism of meron disorder at strong coupling.
  Reasonable results for topological susceptibility in units of the string tension obtained.
Thank you for your attention
Bol’shoye spasibo za vnimaniye