Topology of strong interactions, between the QCD and the EW transition.

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INFN Firenze

Preamble
The two faces of QCD topology

Window to Dark Matter

Strong interactions dynamics
We will concentrate on the topology of gauge fields in this range of temperatures, and on their observable properties.
Topology: geometric properties which do not change under continuous deformations.

how do we ‘measure’ them?
Topological charge $Q$: adding up a local - butterfly-like - operator

$$Q = \sum Q(x) = \sum F^a_{\mu \nu} \tilde{F}_a^{\mu \nu}(x)$$

Topological fluctuations measured by the susceptibility

$$\langle Q^2 \rangle - \langle Q \rangle^2$$
History of the Universe

Today $t_0$
- Life on earth
- Solar system
- Quasars

Recombination
- Relic radiation decouples [CBR]

Matter domination
- Onset of gravitational instability

Nucleosynthesis
- Light elements created - D, He, Li

Quark-hadron transition
- Hadrons form - protons & neutrons

QCD transition

Electroweak phase transition
- Electromagnetic & weak nuclear forces become differentiated:
  $SU(3) \times SU(2) \times U(1) \rightarrow SU(3) \times SU(2) \times U(1)$

EW transition

Grand unification transition
- $G \rightarrow H \rightarrow SU(3) \times SU(2) \times U(1)$
- Inflation, baryogenesis, monopoles, cosmic strings, etc.

The Planck epoch
- The quantum gravity barrier

$T = 3\, K$ (1 meV)

$T = 1\, GeV$

$T = 10^3\, GeV$

$T = 10^4\, GeV$

$T = 10^5\, GeV$

$T = 10^6\, GeV$

$T = 10^7\, GeV$

$T = 10^8\, GeV$

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$T = 10^{10}\, GeV$

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$T = 10^{140}\, GeV$

$T = 10^{141}\, GeV$

$T = 10^{142}\, GeV$
QCD Lagrangian symmetries:

Breaking/restoration at $T_c$ studied a lot on the lattice

Always exact

Always broken if topological charge fluctuates!

BUT:
the ‘amount’ of breaking, may depend on temperature!

HOW ARE THESE RELATED??

DOES IT?

IMPLICATIONS?
A mystery of QCD…
Pseudoscalar light spectrum: eight pseudoGoldstones

\[ SU(3)_L \times SU(3)_R \rightarrow SU(3)_V \]

\[ \chi PT \text{ predicts} \]

\[ m_{\pi}^2 \propto (m_u + m_d) \Lambda_{QCD} \]

\[ m_K^2 \propto (m_s + m_{u,d}) \Lambda_{QCD} \]

\[ m_\eta^2 \propto \frac{1}{3} (m_u + m_d + 4m_s) \Lambda_{QCD} \]

<table>
<thead>
<tr>
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\[ U(1)_A \]

should be broken as well producing a 9th Goldstone BUT:

\[ \eta' \]

is too heavy

Exception!
A mystery of QCD: $\eta'$ too heavy

can be solved by topological charge fluctuations!

Crucial ingredient: $\langle Q^2 \rangle \neq 0$

and $\langle Q(0)Q(t) \rangle$

(more later. ...)
It is possible to couple QCD to topological charge

\[ \mathcal{L} = \mathcal{L}_{QCD} + \theta \frac{g^2}{32\pi^2} F_{\mu\nu}^a \tilde{F}^{\mu\nu}_a \]

Q — topological charge
CP-violating term

but: phenomenology tells us that $\theta$ must be unnaturally small

This is the strong CP problem of QCD!
A second mystery of QCD…

the strong CP problem

..can be solved by introducing the AXION

a new particle which is a viable dark matter candidate

Crucial ingredient: $\langle Q^2(T) \rangle$

(more later..)
The experimental side

$T_c$:

<table>
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<th>340 – 380 MeV</th>
<th>420 – 480 MeV</th>
<th>500 – 600 MeV</th>
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<tbody>
<tr>
<td>RHIC AuAu</td>
<td>LHC</td>
<td>LHC hot spots</td>
</tr>
<tr>
<td>200 GeV</td>
<td>2.76 TeV</td>
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</table>

≈ 200 MeV

1 GeV

LHC

7 TeV

Quark Gluon Plasma @ Colliders

Highest man-made temperature

Who

CERN, LARGE HADRON COLLIDER

What

$5 \times 10^{12}$ DEGREE(S) KELVIN
HI experiments: $T < 500$ MeV

Lattice: $T < 600$-$700$ MeV - sufficient for $T_c$, hadron spectrum in the plasma and QGP dynamics

Lattice + extrap. $T$ about 1000 MeV - and more needed to study axions

Topology plays a major role in all this
Plan

Axions
Topology in QCD

Results:

Topological Susceptibility
Bounds on the QCD axion’s mass
The $\eta'$ and its fate in the plasma
Axions ‘must’ be there (?)

\[ \theta \] term, strong CP problem and topology

\[ \mathcal{L}_{QCD}(\theta) = \mathcal{L}_{QCD} + \frac{g^2}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma}. \]

\[ Q = \int d^4 x \frac{g^2}{32\pi^2} \text{tr} F \tilde{F} \]

\[ |d_n| < 2.9 \times 10^{-26} \text{ e cm} \]

\[ d_n(\theta) \sim e \theta \frac{m_u m_d}{(m_u + m_d) m_n^2} \Rightarrow |\theta| < 10^{-9} \]

\[ Z_{QCD}(\theta, T) = \int [dA][d\psi][d\bar{\psi}] \exp \left( -T \sum_t d^3 x \mathcal{L}_{QCD}(\theta) \right) = \exp[-VF(\theta, T)] \]

\[ \frac{\partial^2 F(\theta, T)}{\partial \theta^2} \bigg|_{\theta=0} \equiv \chi(T) = (\langle Q^2 \rangle - \langle Q \rangle^2)/V \]
Axions ‘must’ be there: solution to the strong CP problem

\[ L_{\text{QCD}}(\theta) = L_{\text{QCD}} + \frac{g^2 \theta}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma} \]

\[ Q = \int d^4x \frac{g^2}{32\pi^2} \text{tr} F \tilde{F} \]

Postulate axions, coupled to Q:

\[ L_{\text{axions}} = \frac{1}{2} (\partial_\mu a)^2 + \left( \frac{a}{f_a} + \theta \right) \frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \]

\[ Z_{\text{QCD}}(\theta, T) = \int [dA][d\psi][d\bar{\psi}] \exp \left( -T \sum_t d^3x \ L_{\text{QCD}}(\theta) \right) = \exp[-VF(\theta, T)] \]

Axion potential

Axion mass

\[ m_a^2(T) f_a^2 = \left. \frac{\partial^2 F(\theta, T)}{\partial \theta^2} \right|_{\theta=0} \equiv \chi(T), \quad f_A \gtrsim 4 \times 10^8 \text{ GeV} \]

weakly coupled
After freezout
constant

Wantz, Shellard 2010

Time from Big Bang
Axions’s freezout

\[ 3H(T) = m_a(T) \]

Axions’ mass and density today

After freezout
\[ \frac{n_a}{s} \] constant

\[ \rho_{a,0} = \frac{n_a}{s} m_a s_0 \]

Wantz, Shellard 2010

Hubble parameter
\[ H(T) \sim T^2/M_P \]

Quark Gluon Plasma: Topology

\[ m_a(T) = \sqrt{\chi(T)/f_a} \]
Cold Dark Matter candidates might have been created after the inflation.

Several CDM candidates are highly speculative - but one, the axion, is the

Theoretically well motivated in QCD
Amenable to quantitative estimates once QCD topological properties are known:

\[ m_a(T) = \sqrt{\chi(T)/f_a} \]
Almost all hadrons can be described taking into account chiral symmetry breaking and confining potential.

Chiral perturbation theory + Potential models = Hadron spectrum
Almost all hadrons can be described taking into account chiral symmetry breaking and confining potential with an important exception.

Chiral symm. breaking
Confinement:
Chiral perturbation theory + Potential models
= Hadron spectrum
Pseudoscalar light spectrum:
eight pseudoGoldstones

\[ SU(3)_L \times SU(3)_R \rightarrow SU(3)_V \]

\[ \chi PT \text{ predicts} \]
\[ m^2_\pi \propto (m_u + m_d) \Lambda_{QCD} \]
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\[ U(1)_A \]
should be broken as well
producing a 9th Goldstone BUT:

\[ \eta' \]
is too heavy

Exception!
Topology, $\eta'$ and the $U_A(1)$ problem:

The $U_A(1)$ symmetry

$$ q \rightarrow e^{i\alpha \gamma_5} q $$

would be broken by the (spontaneously generated)

$q q$  

the candidate Goldstone is the $\eta'$

too heavy!! (900 MeV)

BUT:

the divergence of the current

$$ j_5^\mu = \bar{q} \gamma_5 \gamma_\mu q, $$

contains a mass independent term

$$ \partial_\mu j_5^\mu = m\bar{q} \gamma_5 q + \frac{1}{32\pi^2} F F. $$

IF

$$ \frac{1}{32\pi^2} \int d^4 x F F \neq 0 $$

The $U_A(1)$ symmetry is **explicitly** broken.
It can be proven that

\[ \frac{1}{32\pi^2} \int d^4xF\tilde{F} = Q \]  

Gluonic definition

and

\[ Q = n_+ - n_- \]  

Fermionic definition
Topology, $\eta'$ and the $U_A(1)$ problem:

It can be proven that

$$\frac{1}{32\pi^2} \int d^4x F \tilde{F} = Q$$  
Gluonic definition

and

$$Q = n_+ - n_-$$  
Fermionic definition

The $\eta'$ mass may now be computed from the decay of the correlation

$$\langle \partial_\mu j_5^\mu(x) \partial_\mu j_5^\mu(y) \rangle \propto \frac{1}{N^2} \langle F(x) \tilde{F}(x) F(y) \tilde{F}(y) \rangle$$

which at leading order gives the Witten-Veneziano formula

$$m_{\eta'}^2 = \frac{2N_f}{F_\pi^2} \chi_t^{qu}$$
It can be proven that

\[
\frac{1}{32\pi^2} \int d^4 x F \tilde{F} = Q \quad \text{Gluonic definition}
\]

and

\[ Q = n_+ - n_- \quad \text{Fermionic definition} \]

The \( \eta' \) mass may now be computed from the decay of the correlation

\[
\langle \partial_\mu j_5^\mu (x) \partial_\mu j_5^\mu (y) \rangle \propto \frac{1}{N^2} \langle F(x) \tilde{F}(x) F(y) \tilde{F}(y) \rangle
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which at leading order gives the Witten-Veneziano formula

\[
m_{\eta'}^2 = \frac{2N_f}{F^2} \chi^{\text{qu}}
\]

Successful at \( T=0 \)
Topology, $\eta'$ and the $U_A(1)$ problem...
Results

Twisted mass Wilson Fermions, $N_f=2+1+1$
Wilson fermions with a twisted mass term

Frezzotti Rossi 2003

A twisted mass term in flavor space:

\( i \mu \tau_3 \gamma_5 \) for two degenerate light flavors

is added to the standard mass term in the Wilson Lagrangian

Consequences:
- simplified renormalization properties
- automatic O(a) improvement
- control on unphysical zero modes

Successful phenomenology at T=0

ETMC collaboration 2003—
Why $N_f = 2 + 1 + 1$?

- $T_c$: RHIC AuAu 340–380 MeV, LHC 420-480 MeV, 500–600 MeV, LHC hot spots 2.76 TeV, 1 GeV, LHC 7 TeV
- $\approx 200\text{MeV}$

Quark Gluon Plasma @ Colliders

Analytic studies suggest that a dynamical charm becomes relevant above 400 MeV, well within the reach of LHC

Laine Schroeder 2006
Trace anomaly: effects of a dynamical charm

$T_{mft}$

$Wuppertal$-$Budapest$

$$\frac{\epsilon - 3p}{T^4}$$

$N_f = 2 + 1 + 1, m_\pi \simeq 370$:

- $\beta = 1.90$
- $\beta = 1.95$
- $\beta = 2.10$

$N_f = 2 + 1, m_\pi \simeq 160$

$N_f = 2, m_\pi \simeq 360$

Staggered
For each lattice spacing we explore a range of temperatures 150MeV — 500 MeV by varying Nt. We repeat this for three different lattice spacings following ETMC T=0 simulations.

Advantages: we rely on the setup of ETMC T=0 simulations. Scale is set once for all.

Disadvantages: mismatch of temperatures - need interpolation before taking the continuum limit.

<table>
<thead>
<tr>
<th>Number of flavours</th>
<th>m_{\pi}\pm</th>
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<tbody>
<tr>
<td>N_f = 2 + 1 + 1</td>
<td>210, 260, 370, 470</td>
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<tr>
<td>N_f = 2</td>
<td>360, 430</td>
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<table>
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<tr>
<th>Nf = 2 + 1 + 1 Setup</th>
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<tr>
<td>( T = 0 ) (ETMC) nomenclature</td>
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Overview of Chiral observables

Nf 2 + 1 +1

Outcome: twisted mass ok; and the results confirm that a dynamical charm does not contribute around Tc

Spacing effects below statistical errors
Topology
Topological and chiral susceptibility

\[ \chi_{top} = \langle Q_{top}^2 \rangle / V = m_l^2 \chi_{5,disc} \]

From:

\[ m(T) \int d^4 x \bar{\psi} \gamma_5 \psi = Q_{top} \]

Kogut, Lagae, Sinclair 1999
HotQCD, 2012
Chiral susceptibility

Within errors, no discernable spacing dependence
Results for physical pion mass

Rescaled according to

\[ \chi_{\text{top}} = m_l^2 \chi_{\psi\psi}^{\text{disc}} = \sum_{n=0} a_n m_\pi^{4(n+1)}. \]
Power-law decay?

\[ \chi^{0.25}(T) = \alpha T^{-d(T)} \]

For instanton gas

\[ d(T) = -T \frac{d}{dT} \ln \chi^{0.25}(T) \]

Possibly consistent with instant-dyon?

Shuryak 2017

Faster decrease before DIGA sets in
Effective exponent $d(T)$:

\[ \frac{1}{4} \chi_{top} = a T^{-d(T)} \]
QCD axion
From exponent $d$ to axion mass in three steps

1. Time from Big Bang
   Axions' freezeout
   \[ 3H(T) = m_a(T) \]

2. After freezeout \( \frac{n_a}{s} \) constant

3. \( \rho_{a,0} = \frac{n_a}{s} m_a s_0 \)

\( \rho_a(m_a) \propto \frac{3.053 + d/2}{2.027 + d/2} m_a \)

\[ \chi_{\text{top}} \approx A T^{-d} \]

\[ d = (6.26, 6.88, 7.52, 7.48) \]

\[ m_\pi = (470, 370, 260, 210) \text{ MeV} \]

\[ m_a = \sqrt{\chi(T)/f_a} \]

\[ \begin{align*}
   &\begin{array}{c}
   \text{Temperature}
   \\
   \text{Hubble parameter}
   \\
   H(T) \approx T^2/M_P
   \\
   m_a(T) = \sqrt{\chi(T)/f_a}
   \\
   \text{Quark Gluon Plasma: Topology}
   \\
   \end{array}
   \\
   &\begin{array}{c}
   \text{Expansion and cooling}
   \\
   \text{Compression and heating}
   \\
   \text{Spectroscopy and density}
   \\
   \end{array}
   \\
   &\begin{array}{c}
   \text{470 MeV}
   \\
   \text{370 MeV}
   \\
   \text{260 MeV}
   \\
   \text{210 MeV}
   \\
   \end{array}
\end{align*} \]
\[ \Omega_a = \frac{\rho_{a,0}}{\rho_c}, \]

\[ \Omega_{a/\Omega_{DM}} \]

Axion mass [\mu eV]

- 370 MeV
- 260 MeV
- 210 MeV
- \( d = 8 \) (DIGA)
- \( d = 4 \)
- \( A \times 10^4 \)
- \( A / 10^4 \)
Example: if axions constitute 80% DM, our results give a lower bound for the axion mass of $\approx 30 \mu\text{eV}$.
Adapted from MpL, Nature N&V 2016
What happens to topology in the Quark Gluon Plasma?
So far, only results from model's studies in the QGP

Horvatic et al. 2018
Different mechanisms leading to \( \eta' \) (900 MeV) mass reduction

Adopting the basis

\[
\begin{align*}
I & \equiv \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \\
S & \equiv s\bar{s}
\end{align*}
\]

The mass matrix of the \( \eta \) complex is:

\[
\begin{pmatrix}
m^2_\pi + m^2_A \\
\frac{m^2_A}{\sqrt{2}} & \frac{m^2_A}{\sqrt{2}}
\end{pmatrix}
\begin{pmatrix}
m^2_\pi + m^2_A \\
2m^2_K - m^2_\pi + \frac{m^2_A}{2}
\end{pmatrix}
\]

Veneziano, 1981

Non anomalous: \( \eta' \simeq 700 \text{ MeV} \)

(strange only)

However: also sensitive to SU(2)XSU(2)
Indication of topology suppression in PHENIX

Effects of chain decays, radial flow and $U_A(1)$ restoration on the low-mass dilepton enhancement in $\sqrt{s_{NN}}=200$ GeV Au+Au reactions

Márton Vargyas$^{a,b,1}$, Tamás Csörgő$^{b,2}$, Róbert Vértesi$^{b,c,3}$

This is at finite density!
\( \eta' \) mass from topological charge correlators

\[
G(\tau) = \int d^3 \bar{x} q(0)q(\tau, \bar{x}) = \int d^3 \bar{x} \frac{1}{32\pi^2} F_{\mu\nu} \tilde{F}_{\mu\nu}(0) \times \\
\times \frac{1}{32\pi^2} F_{\mu\nu} \tilde{F}_{\mu\nu}(\tau, \bar{x}) \quad \simeq e^{-m_{\eta'} \tau}
\]

Pion mass 210 MeV

Pion mass 370 MeV
Pion mass = 370 MeV

- $a = 0.0646$ fm, Fit, $\tau > 1.6\sqrt{8t}$
- $a = 0.0646$ fm, Fit, $\tau > 1.8\sqrt{8t}$
- $a = 0.0646$ fm, Fit, $\tau > 2.0\sqrt{8t}$
- $a = 0.0646$ fm, Plateau
- $a = 0.0823$ fm, Fit, $\tau > 1.8\sqrt{8t}$
- $a = 0.0823$ fm, Plateau

Non anomalous
Pion mass 210 MeV

Fit, $\tau > 1.6\sqrt{8t}$
Fit, $\tau > 1.8\sqrt{8t}$
Fit, $\tau > 2.0\sqrt{8t}$
Plateau
ETMC

$m_\pi = 370$ MeV
$m_\pi = 210$ MeV

$m_{\pi} = 370$ MeV
$m_{\pi} = 210$ MeV

$\Sigma = 370$ MeV
$\Sigma = 210$ MeV
Correlations?

$m_{\eta'}(T)/m_{\eta'}(0)$:
- 210 MeV
- 370 MeV

$R\langle \bar{\psi} \psi \rangle$:
- 210 MeV
- 370 MeV

Small dip for pion mass 210 MeV
Small dip for pion mass 370 MeV

$T$ [MeV]
Minimum of the $\eta'$

Approx. correlated with $T_\chi$

$T_{\eta'}$ [MeV] 
\[ \simeq 150 \]

$T_{\eta'}$ [MeV] 
\[ \simeq 170 \]

Consistent with suppression of the anomalous contribution
Summary

Axions are attractive dark matter candidates

The QCD topological susceptibility at high temperature gives a strict lower bound on the axion mass. Some of the planned experiments do not seem to be able to explore this region.

The $\eta'$ meson is an important probe of axial symmetry and of its interplay, or lack thereof, with chiral symmetry.

The correlators of the QCD topological charge afford an estimate of the $\eta'$ mass, which appears to be correlated with signals of chiral symmetry restoration.
Thank You!