Non-perturbative study of the viscosity in $SU(2)$ lattice gluodynamics.

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Outline

- Introduction
- Transport coefficients in lattice calculations
- Improving statistical accuracy of the results
- Analytical continuation problem
- Numerical setup
- Results and discussion
One heavy ion collision produces a huge number of final particles

Large number of particles $\Rightarrow$ hydrodynamical description can be used

In hydrodynamics transport coefficients control flow of energy, momentum, electrical charge and other quantities
Shear viscosity. Value and bounds.


- Experimentally preferred value: \( \frac{\eta}{s} \sim (1 \leftrightarrow 3) \frac{1}{4\pi} \)
- Experimental bound: \( \frac{\eta}{s} < 5 \frac{1}{4\pi} \)
- KSS-bound: \( \frac{\eta}{s} \geq \frac{1}{4\pi} \)

**Comparison of different liquids**

**QGP the most superfluid liquid**

**The aim: first principle calculation of transport coefficients**
Lattice simulations of QCD.

- Allows to study strongly interacting systems
- Based on the first principles of quantum field theory
- Acknowledged approach to study QCD
- Very powerful due to the development of computer systems
Previous lattice calculations ($SU(3)$ gluodynamics).

Viscosity in lattice calculations.

Green-Kubo relation:

\[ \eta = \pi \lim_{\omega \to 0} \frac{\rho_{12,12}(\omega, q = 0)}{\omega} \]

Green function measured on the lattice (Euclidean):

\[ C_{12,12}(x_0, \mathbf{p}) = \beta^5 \int d^3x e^{i\mathbf{p} \cdot \mathbf{x}} \langle T_{12}(0) T_{12}(x_0, \mathbf{x}) \rangle \]

Spectral function and correlator of stress-energy tensor:

\[ C_{12,12}(x_0, \mathbf{p}) = \beta^5 \int_0^\infty \rho_{12,12}(\omega, \mathbf{p}) \frac{\cosh \omega \left( \frac{1}{2} L_0 - x_0 \right)}{\sinh \frac{\omega L_0}{2}} d\omega \]

Stress-energy tensor for gluodynamics:

\[ T_{\mu\nu} = 2 \text{tr}(F_{\mu\sigma} F_{\nu\sigma} - \frac{1}{4} \delta_{\mu\nu} F_{\rho\sigma} F_{\rho\sigma}) \]

Asymptotic behaviour - perturbation theory: \[ \rho(\omega) = \frac{1}{10} \left( \frac{3}{4\pi} \right)^2 \omega^4, \omega \to \infty \]
Main difficulties.

- Large statistical errors in measuring correlator $C_{12,12}(x_0, 0)$
  - Improved action
  - Multilevel algorithm
- Extracting spectral function $\rho_{12,12}$ from

$$C_{12,12}(x_0, p) = \beta^5 \int_0^\infty \rho_{12,12}(\omega, p) \frac{\cosh \omega (\frac{1}{2}L_0 - x_0)}{\sinh \frac{\omega L_0}{2}} d\omega$$

- Fit by model function
- Maximum entropy method
- Linear method
- ...
Statistical error of the correlator $C_{12,12}$. Improved action.

\[
S_{unimpr} = \beta \sum_{pl} S_{pl}
\]

\[
S_{impr} = \beta_{impr} \sum_{pl} S_{pl} - \frac{\beta_{impr}}{20u_0^2} \sum_{rt} S_{rt}
\]

\[
S_{pl,rt} = \frac{1}{2} \text{tr}(1 - U_{pl,rt})
\]

Increases accuracy but is not enough.
Statistical error of the correlator $C_{12,12}$. Multilevel algorithm.

For $t_1$ and $t_2$ in different areas

$$\langle O(t_1)O(t_2) \rangle = \frac{1}{N_{bc}} \sum_{b,c} \langle O(t_1) \rangle_{b,c} \langle O(t_2) \rangle_{b,c}$$
Analytical continuation problem.

- Fit by model function
- Maximum entropy method
- Linear method
- ...

Analytical continuation problem. Fit by model function.

\[ C_{12,12}(x_0, p) = \beta^5 \int_0^\infty \rho_{12,12}(\omega, p) \frac{\cosh \omega \left( \frac{1}{2} L_0 - x_0 \right)}{\sinh \frac{\omega L_0}{2}} d\omega \]

Proposed in the first work on transport coefficients:

\[ \rho(\omega)/\omega = A \left( \frac{\gamma}{(m - \omega)^2 + \gamma^2} + \frac{\gamma}{(m + \omega)^2 + \gamma^2} \right) \]

A, m, \gamma - parameters
Clearly ignores asymptotic behaviour
Analytical continuation problem. Maximum entropy method.

\[ C_{12,12}(x_0, p) = \beta^5 \int_0^\infty \rho_{12,12}(\omega, p) \frac{\cosh \omega \left( \frac{1}{2} L_0 - x_0 \right)}{\sinh \frac{\omega L_0}{2}} d\omega \]

We discretize \( \omega \), \( N_\omega \sim O(10^3) \)

\[
\rho(\omega) \xrightarrow{K(\omega,x_i)} G(x_i) = G_i
\]

\[
\chi^2 = \sum_{i,j} (G_i - G_i^{(0)})(S^{-1})_{ij}(G_j - G_j^{(0)})
\]

\[
\min \chi^2 \rightarrow \sim O(10) \text{ equations}
\]
Instead of $\chi^2$ we minimize

$$\chi^2 + \alpha S,$$

Entropy $S$ determines how our function is close to some model function $\mu(\omega)$ (which summarizes our prior knowledge about the spectral function).

$$S = \sum_{m=1}^{N_w} \left( \rho_m - \mu_m - \rho_m \log \frac{\rho_m}{\mu_m} \right)$$

Doesn’t work for small lattice sizes.
Analytical continuation problem.

Linear method:

$$\rho(\omega) = m(\omega)(1 + a(\omega)) = m(\omega)(1 + \sum_l a_l u_l(\omega)),$$

where $m(\omega)$ is an initial approximation:

$$m(\omega) = \frac{A\omega^4}{\tanh^2 \frac{\omega}{4T} \tanh \frac{\omega}{2T}},$$

and $u_l(\omega)$ are eigenmodes of $H(\omega, \omega') = \sum_i K(t_i, \omega)K(t_i, \omega')$ with

$$K(t, \omega) = m(\omega) \frac{\cosh(\omega(\frac{1}{2T} - t))}{\sinh \frac{\omega}{2T}}$$

$a_l$ are selected to minimize $\chi^2$.

\[ \rho(\omega) = m(\omega)(1 + a(\omega)) \]

Let \( \hat{\rho}(\omega) \) be a true spectral function.

\[ \hat{\rho}(\omega) = m(\omega)(1 + \hat{a}(\omega)) \xrightarrow{K(t_i,w)} G_i \xrightarrow{linear} \rho(\omega) = m(\omega)(1 + a(\omega)) \]

Resolution function:

\[ a(\omega) = \int d\omega \hat{a}(\omega) \delta(\omega, \omega') \]
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  \[ C_{12,12}(x_0, p) = \beta^5 \int_0^\infty \rho_{12,12}(\omega, p) \frac{\cosh \omega (\frac{1}{2} L_0 - x_0)}{\sinh \frac{\omega L_0}{2}} d\omega \]

- Fit by model function
- Maximum entropy method
- Linear method
- ...
Numerical setup.

- $SU(2)$-gluodynamics with Wilson action:

$$S = \frac{\beta}{2} \sum_{pl} \text{tr}(1 - U_{pl})$$

- Lattice $8 \times 32^3$
- $\beta = 2.643$
- $T / T_c \approx 1.2$
- Clover-shaped discretization for $F_{\mu\nu}$
- Two-level algorithm for measuring stress-energy tensor correlator.
Spectral function $\rho_{12,12}$. 

\[ \frac{\rho_{12,12}}{\text{Sh}(\frac{\omega}{2T})} \]
Numerical results.

\[ \frac{\eta}{s} = 0.111 \pm 0.032 \]

KSS-bound:

\[ \frac{\eta}{s} \geq \frac{1}{4\pi} \approx 0.08 \]

Perturbative result:

\[ \frac{\eta}{s} \sim 2 \]

Experimental bound and preferred value:

\[ \frac{\eta}{s} < 5\frac{1}{4\pi} \approx 0.4 \]

\[ \frac{\eta}{s} \sim (1 \leftrightarrow 3)\frac{1}{4\pi} \]
Unsatisfactory attempts to increase lattice size

![Graph showing data points and a trend line.](image-url)