Quantum flavor kinetics and chemical freeze-out
(Hadronization as Mott-Anderson localization)

David Blaschke
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M. Srednicki,
“Chaos and Quantum Thermalization”,
PRE 50 (1994) 888

F. Becattini @ ECT* 2014

11th Polish Workshop on Heavy Ion Collisions, Warsaw, January 17, 2015
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Dictionary:
Quantum: hadrons = bound states of quarks
Flavor kinetics: quark exchange between hadrons
Freeze-out: localization of bound states in expanding, cooling system
(inverse of delocalization by compression: Mott effect)

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1. Mott-Anderson localization model for chemical freeze-out
   - idea
   - inputs
   - results
2. Chiral QM for hadron Mott transition
3. Thermodynamics of Mott-HRG and lattice QCD data

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Mott-Anderson localization model for chemical freeze-out


The basic idea: Localization of (certain) multiquark states ("cluster") = hadronization;
Reverse process = delocalization by quark exchange between hadrons

Freeze-out criterion:

\[ \tau_{\text{exp}}(T, \mu) = \tau_{\text{coll}}(T, \mu) \]

\[ \tau_{\text{coll}}^{-1}(T, \mu) = \sum_{i,j} \sigma_{ij} n_j \]

\[ \sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle \]

\[ r_\pi^2(T, \mu) = \frac{3}{4\pi^2} f_\pi^2(T, \mu) \]

\[ f_\pi^2(T, \mu) = -m_0 \langle \bar{q}q \rangle_{T, \mu}/M_\pi^2 \]

\[ r_\pi^2(T, \mu) = \frac{3M_\pi^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_{T, \mu}|^{-1} \]

\[ \langle \bar{q}q \rangle = \langle \bar{q}q \rangle_{\text{MF}} \left[ 1 - \frac{T^2}{8f_\pi^2(T, \mu)} - \frac{\sigma_N n_{s,N}(T, \mu)}{M_\pi^2 f_\pi^2(T, \mu)} \right] \]

Povh-Huefner law,
PRC 46 (1992) 990

Hippe & Klevansky, PRC 52 (1995) 2172
Mott-Anderson localization model for chemical freeze-out


Povh-Huefner law behaviour for quark exchange between hadrons

Quark exchange model for charmonium dissociation in hot hadronic matter

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(Received 15 November 1994)

\[ \langle \sigma_{\text{abs}} v_{\text{rel}} \rangle \propto \langle r^2 \rangle_{Q \bar{Q}} \langle r^2 \rangle_{q \bar{q}} \]

Flavor exchange processes

\[
\phi + \pi \rightarrow K + \bar{K}, \quad K^- + p \rightarrow \Lambda + X, \quad K^+ + p \rightarrow \Lambda + X,
\]

Nonrelativistic \rightarrow rel. quark loop integrals

\[ M_{f1} = \]

\[ \text{Diagram with quark loop integrals} \]
Quark exchange in meson-meson scattering


Povh-Huefner law behaviour for quark exchange between hadrons?

\[ \sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle \]
\[ r_{\pi}^2(T, \mu) = \frac{3 M_{\pi}^2}{4 \pi^2 m_q} |\langle \bar{q} q \rangle_{T, \mu}|^{-1} \]

\[ \psi_{ss}(12, 1'2') = \frac{16}{3 \sqrt{3}} C_{SFC}(12, 1'2') \frac{(2\pi)^3}{\Omega_0} \frac{\alpha_s}{3 \pi^2 m_q^2} \exp \left( -\frac{1}{4b^2} (k'^2 + \frac{1}{2} k^2) \right) \delta_{K,K'} \]

Quark exchange process in M-M scattering

Nonrelativistic \rightarrow rel. quark loop integrals

\[ M_{fi} = \]

Quark exchange process in M-M scattering

Nonrelativistic \rightarrow rel. quark loop integrals

\[ M_{fi} = \]
Mott-Anderson localization model for chemical freeze-out


Model results:

\[ \tau_{\text{exp}}(T, \mu) = \tau_{\text{coll}}(T, \mu) \]

Collision time strongly T, \( \mu \) dependent!

Schematic resonance gas: \( d_p \) pions, \( d_N \) nucleons

Expansion time scale from entropy conservation:

\[ s(T, \mu) V(\tau_{\text{exp}}) = \text{const} \]

\[ \tau_{\text{exp}}(T, \mu) = a s^{-1/3}(T, \mu), \]

Thermodynamics consistent with phenomenological Freeze-out rules:
Mott-Anderson localization model for chemical freeze-out


Model results:
Full hadron resonance gas model

Collision time follows a power law
\[ t_{\text{coll}} \sim (T/T_c)^a \]

with a large exponent \( a \sim 20 \)

Mott-Anderson localization model for chemical freeze-out


Model results:
Full hadron resonance gas model

See also: S. Leupold, J. Phys. G (2006)

\[
\frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_{\text{vac}}} = 1 - \frac{m_0}{F^2 \pi m} \left[ 4N_c \int \frac{dp}{2\pi^2} \frac{p^2}{E} \left[ f_{\Phi}^+ + f_{\Phi}^- \right] \right] 
+ \sum_{M=J_0, J_1, \ldots} d_M (2 - N_S) \int \frac{dp}{2\pi^2} \frac{m_M}{E_M(p)} f_M(E_M(p)) 
+ \sum_{B=J, \Lambda, \ldots} d_B (3 - N_S) \int \frac{dp}{2\pi^2} \frac{m_B}{E_B(p)} \left[ f_B^+ (E_B(p)) + f_B^- (E_B(p)) \right] 
- \sum_{G=\pi, K, \eta, \eta'} \frac{d_{G^*}}{4\pi^2 F^2_G} \int \frac{dp}{E_G(p)} f_G(E_G(p)).
\]

\[
\sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle ; \quad r_N^2(T, \mu) = r_0^2 + r_{\pi}^2(T, \mu)
\]

The factor \(a\) stands for the inverse system size in the formula for the 3D expansion time scale assuming entropy conservation.
Mott-Anderson localization model for $K^+ / \pi^+$ “horn”


\[
\langle \bar{q}q \rangle_{T,\mu} = \langle \bar{q}q \rangle_{T,\mu}^{MF} + \sum_{h=M,B} \frac{\sigma_h}{m_q} n_h(T, \mu),
\]

\[
n_h(T, \mu) = \frac{d_h}{2\pi^2} \int_0^\infty dk k^2 \frac{m_h}{E_h} \frac{1}{e^{(E_h - \mu_h)/T} + 1},
\]

\[
\tau_{\text{coll}}^{-1}(T, \mu) = \sum_{i,j} \sigma_{ij} n_j ; \quad \sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle
\]

\[
\langle r_\pi^2 \rangle_{T,\mu} \simeq \frac{3}{4\pi^2} f_\pi^{-2}(T, \mu) = \frac{3M_\pi^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_{T,\mu}|^{-1}
\]

\[
\langle r_K^2 \rangle_{T,\mu} \simeq \frac{3M_K^2}{\pi^2 (m_q + m_s)} |\langle \bar{q}q \rangle_{T,\mu} + \langle \bar{s}s \rangle_{T,\mu}|^{-1}
\]

The factor $a$ stands for the inverse system size in the formula

\[
\tau_{\text{exp}}(T, \mu) = \tau_{\text{coll}}(T, \mu)
\]

for the 3D expansion time scale assuming entropy conservation
Mott Dissociation of Hadrons in Hadron Matter

- Partition function as a Path Integral (imaginary time \( \tau = i t, 0 \leq \tau \leq \beta = 1/T \))

\[
Z[T, V, \mu] = \int D\bar{\psi} D\psi DA \exp \left\{ - \int_0^\beta d\tau \int V d^3x \mathcal{L}_{QCD}(\psi, \bar{\psi}, A) \right\}
\]

- QCD Lagrangian, non-Abelian gluon field strength:

\[
F^{a\mu}_{\nu}(A) = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc}[A^b_\mu, A^c_\nu]
\]

\[
\mathcal{L}_{QCD}(\psi, \bar{\psi}, A) = \bar{\psi} [i \gamma^\mu (\partial_\mu - ig A^a_\mu) - m - \gamma^0 \mu] \psi - \frac{1}{4} F^{a\mu}_{\nu}(A) F^{a,\mu\nu}(A)
\]

- Numerical evaluation: Lattice gauge theory simulations (hotQCD, Wuppertal-Budapest)
Mott Dissociation of Hadrons in Hadron Matter

\[ P_{\text{tot}}(T, \{\mu_j\}) = P_{\text{PNJL}}(T, \{\mu_i\}) + \sum_{r=M,B} \delta_r g_r \int ds A_r(s, m_r; T) \int \frac{d^3p}{(2\pi)^3} T \ln \left\{ 1 + \delta_r \exp \left( \frac{\sqrt{p^2 + s - \mu_r}}{T} \right) \right\} \]

Spectral function for hadronic resonances:

\[ A_r(s, m; T) = N_s \frac{m \Gamma_r(T)}{(s - m^2)^2 + m^2 \Gamma_r^2(T)} \]

Ansatz motivated by chemical freeze-out model:

\[ \Gamma_r(T) = \tau_r^{-1}(T) = \sum_{h} \lambda < r_r^2 >_T < r_h^2 >_T n_h(T) \]

Apparent phase transition at \( T_c \approx 165 \text{ MeV} \)

Hadron resonances present up to \( T_{\text{max}} \approx 250 \text{ MeV} \)

Blaschke & Bugaev, Fizika B13, 491 (2004)

Hadronic states above \( T_c \) ! See also: Ratti, Bellwied et al., arXiv:1109.6243 [hep-ph]
Mott Dissociation of Hadrons in Hadron Matter

Possible application: parton fraction in the EoS at the hadronization transition

Compare:
Mott Dissociation of Mesons in Quark Matter


- Partition function as a Path Integral (imaginary time $\tau = i t$)

$$Z[T, V, \mu] = \int \mathcal{D}q \mathcal{D}q \exp \left\{ - \int_{V} dt \int d^3x [\bar{q}(i\gamma^\mu \partial_\mu - m_0 - \gamma^0 \mu)q + \sum_{M=\pi,\sigma} G_M(\bar{q}\Gamma_M q)^2] \right\}$$

- Couplings: $G_\pi = G_\sigma = G_S$ (chiral symmetry)

- Vertices: $\Gamma_\sigma = 1_D \otimes 1_f \otimes 1_c$; $\Gamma_\pi = i\gamma_5 \otimes \vec{r} \otimes 1_c$

- Bosonization (Hubbard-Stratonovitch Transformation)

$$\exp \left[ G_S(\bar{q}\Gamma_\sigma q)^2 \right] = \text{const.} \int \mathcal{D}\sigma \exp \left[ \frac{\sigma^2}{4G_S} + \bar{q}\Gamma_\sigma q\sigma \right]$$

- Integrate out quark fields $\rightarrow$ bosonized partition function

$$Z[T, V, \mu] = \int \mathcal{D}\sigma \mathcal{D}\pi \exp \left\{ -\frac{\sigma^2 + \pi^2}{4G_S} + \frac{1}{2} \text{Tr} \ln S^{-1}[\sigma, \pi] \right\}$$

- Systematic evaluation: Mean fields + Fluctuations

  - Mean-field approximation: order parameters for phase transitions (gap equations)
  - Lowest order fluctuations: hadronic correlations (bound & scattering states)
Mott Dissociation of Mesons in Quark Matter

- Separate the mean-field part of the quark determinant
  \[
  \text{Tr} \ln S^{-1}[\sigma, \pi] = \text{Tr} \ln S^{-1}_{\text{MF}}[m] + \text{Tr} \ln [1 + (\sigma + i\gamma_5 \vec{\tau} \vec{\pi})S_{\text{MF}}[m]]
  \]

- Mean-field quark propagator
  \[
  S_{\text{MF}}(\vec{p}, i\omega_n; m) = \frac{\gamma_0 (i\omega_n + \mu) - \vec{\gamma} \cdot \vec{p} + m}{(i\omega_n + \mu)^2 - E_p^2}
  \]

- Expand the logarithm: \( \ln(1 + x) = -\sum_{n=1}^{\infty} (-1)^n x^n / n = x - x^2 / 2 + \ldots \)

- Thermodynamic potential in Gaussian approximation
  \[
  \Omega(T, \mu) = -T \ln Z(T, \mu) = \Omega_{\text{MF}}(T, \mu) + \sum_M \Omega^{(2)}_M(T, \mu) + \mathcal{O}[\phi_M^3]
  \]
  \[
  \Omega^{(2)}_M(T, \mu) = \frac{N_M}{2} \int \frac{d^2 \vec{p}}{(2\pi)^3} \frac{1}{\beta} \sum_n e^{i\nu_n^M n} \ln S^{-1}_M(\vec{p}, i\nu_n), \quad N_\sigma = 1, \quad N_\pi = 3
  \]

- Meson propagator \( S_M(\vec{p}, i\nu_n) = 1/[1/(2G_S) - \Pi_M(\vec{p}, i\nu_n)] \)

- Mesonic polarization loop
  \[
  \Pi_M(\vec{p}, i\nu_n) = -\frac{1}{\beta} \sum_{n'} e^{i\nu_n \eta} \int \frac{d^2 \vec{k}}{(2\pi)^3} \text{Tr} \left[ \Gamma_M S_{\text{MF}}(-\vec{k}, -i\omega_n') \Gamma_M S_{\text{MF}}(\vec{k} + \vec{p}, i\omega_{n'} + i\nu_n) \right]
  \]
Mott Dissociation of Mesons in Quark Matter

- Polar representation of the analytically continued quark propagator
  \[ S_M = |S_M| e^{i\delta_M} = S_R + iS_I \]
- Phase shift
  \[ \delta_M(\omega, q) = -\text{Im} \ln S_M^{-1}(\omega - \mu_M + i\eta, q) \]
- Thermodynamic potential for mesonic modes
  \[
  \Omega_M(T, \mu) = \text{Tr} \ln S_M^{-1}(i\xi_n, q) = d_M T \sum_n \int \frac{d^3q}{(2\pi)^3} \ln S_M^{-1}(i\xi_n, q),
  \]
  \[
  = -d_M T \sum_n \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{i\xi_n - \omega} \text{Im} \ln S_M^{-1}(\omega + i\eta, q)
  \]
- Perform Matsubara summation
  \[ \Omega_M(T, \mu) = d_M \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} n_M^-(\omega) \delta_M(\omega, q) \]
- Using symmetries of Bose function
  \[ n_M^-(\omega) = -[1 + n_M^+(\omega)] \]
  and polarization loop
  \[ \Omega_M(T, \mu) = d_M \int \frac{d^3q}{(2\pi)^3} \int_0^{\infty} \frac{d\omega}{2\pi} [1 + n_M^-(\omega) + n_M^+(\omega)] \delta_M(\omega, q) \]
- Partial integration gives field theoretic Beth-Uhlenbeck formula
  \[ \Omega_M = -d_M \int \frac{d^3q}{(2\pi)^3} \int_0^{\infty} \frac{d\omega}{2\pi} \left[ \omega + T \ln \left( 1 - e^{-(\omega - \mu_M)/T} \right) + T \ln \left( 1 - e^{-(\omega + \mu_M)/T} \right) \right] \frac{d\delta_M(\omega, q)}{d\omega} \]
Mott Dissociation of Mesons in Quark Matter

- When polarization loop integral can be expressed in the form
  \[ \Pi_M(z, q) = \Pi_{M,0} + \Pi_{M,2}(z, q) \]

- Factorization of two-particle propagator possible with
  \[ R_M(z, q) = \frac{1 - G_M \Pi_{M,0}}{G_M \Pi_{M,2}(z, q)} \]

\[ S_M(z, q) = \frac{1}{G^{-1}_M - \Pi_{M,0} - \Pi_{M,2}(z, q)} = \frac{1}{\Pi_{M,2}(z, q)} \frac{1}{R_M(z, q) - 1} \]

- This entails \( \ln S_M(z, q)^{-1} = \ln \Pi_{M,2}(z, q) + \ln [R_M(z, q) - 1] \)
  and thus a separation of the phase shift in two contributions
  \[ \delta_M(\omega, q) = \delta_{X,c}(\omega, q) + \delta_{X,R}(\omega, q) \]

- They correspond to continuum (state independent) and resonant phases
  \[ \delta_{M,c}(\omega, q) = \arctan \left( \frac{\text{Im}\Pi_{M,2}(\omega - \mu_M + i\eta, q)}{\text{Re}\Pi_{M,2}(\omega - \mu_M + i\eta, q)} \right) \]
  \[ \delta_{M,R}(\omega, q) = \arctan \left( \frac{\text{Im}R_M(\omega - \mu_M + i\eta, q)}{1 - \text{Re}R_M(\omega - \mu_M + i\eta, q)} \right) \]
Mott Dissociation of Mesons in Quark Matter

Suppose $\delta_{X,R}(\omega, q)$ corresponds to a resonance at $\omega = \omega_M = \sqrt{q^2 + M^2_M}$, then the propagator shall have the representation with a complex pole at $z = z_M = \omega_M + i\Gamma_M/2$, where $\Gamma_M$ is the width of the resonance.

The position of the pole is found from the condition $\text{Re}R_M(z_M, q) = 1$, where $\delta_{M,R}(\omega \to \omega_M) \to \pi/2$ since $\tan \delta_{M,R}(\omega \to \omega_M) \to \infty$.

Expanding $R_M(z, q)$ at the complex pole $z_M$ for small width, one obtains

$$1 - \text{Re}R_M(z_M, q) = -(\omega^2 - \omega_M^2) \frac{dR_M(z, q)}{d\omega^2} \bigg|_{z=z_M}, \quad \text{Im}R_M(z_M, q) = \omega_M \Gamma_M \frac{dR_M(z, q)}{d\omega^2} \bigg|_{z=z_M}$$ (1)

The resonant shift becomes $\delta_{M,R}(\omega, q) = -\arctan \left( \frac{\omega_M \Gamma_M}{\omega^2 - \omega_M^2} \right)$ corresponding to a Breit-Wigner form of the spectral density in the Beth-Uhlenbeck EoS

$$\frac{d\delta_{M,R}}{d\omega} = \frac{2\omega \omega_M \Gamma_M}{(\omega^2 - \omega_M^2)^2 + \omega_M^2 \Gamma_M^2}.$$ 

This takes the form of a bound state spectral density for $\Gamma_M \to 0$

$$\lim_{\Gamma_M \to 0} \delta'_{M,R}(\omega) = \pi \left[ \delta(\omega - \omega_M) + \delta(\omega + \omega_M) \right]$$
Mott Dissociation of Mesons in Quark Matter

Mott Dissociation of Mesons and Diquarks in Quark Matter

DB, A. Dubinin, M. Buballa, arxiv:1411.1040 [hep-ph]
Mott Dissociation of Hadrons in Hadron Matter in a cQM


very preliminary
Summary

- Generalized Beth-Uhlenbeck approach as microphysical basis to account for hadron dissociation (Mott effect) at extreme temperatures and densities
- Benchmark: pion and sigma Mott effect within NJL model, revised within nonlocal PNJL model
- Nonlocal PNJL model calibrated with lattice quark propagator data, EoS at finite $T$ and $\mu$, Phase diagram with critical point
- Application of GBU to interprete chemical freeze-out as Mott-Anderson localization
- Effective GBU model description: Mott-Hagedorn resonance gas + PNJL model describes Lattice QCD thermodynamics

Outlook

- RMF (Walecka) model as limit of the PNJL model: chiral transition effects in nuclear EoS
- Prospects for HIC (CBM & NICA) and Supernovae: color superconducting (quarkyonic) phases accessible!
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Joint Institute for Nuclear Research

XV International conference

Strangeness in Quark Matter

6 July - 11 July 2015 Dubna, Russia

Topics:

- STRANGENESS and heavy QUARK production in nuclear collisions
- Hadronic INTERACTIONS
- Bulk MATTER PHENOMENA associated with strange and HEAVY quarks
- Strangeness in astrophysics
- OPEN questions and NEW developments

Satellite Meetings:

Summer School “Dense Matter” 29 June-11July 2015
Roundtable “Physics at NICA” 5 July 2015

http://SQM.JINR.RU
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   - Hyperon puzzle
   - Reconfiment
   - Masquerade

2. The Solution:
   Baryon finite size (compositenes):
   \( \rightarrow \) Excluded volume Appr. (EVA)

3. The Mechanism:
   Quark Pauli Blocking

4. Outlook:
   - High-Mass Twins (next talk)
   - Supernova explosion mechanism
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28 member countries!! (MP1304)