Microscopic Model for Chemical Freeze-Out in Heavy-Ion Collisions

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\[^{1}\text{Collaboration: J. Berdermann, J. Cleymans, D. Prorok, K. Redlich, L. Turko}
\]
Beam energy scan (BES) programs in the QCD phase diagram

Highest baryon densities at freeze-out shall be reached for $\sqrt{s_{NN}} \sim 8 \text{GeV} \rightarrow \text{QGP phase transition}$?
Chemical Freeze-out in the QCD Phase Diagram

"Old" freeze-out data from RHIC (red), SPS (blue), AG (black), SIS (green).

"New" freeze-out data from STAR BES @ RHIC.
Centrality dependence!

Lokesh Kumar (STAR Collab.), arxiv:1201.4203 [nucl-ex]
Chemical freeze-out condition

\[ \tau_{\text{exp}}(T, \mu) = \tau_{\text{coll}}(T, \mu) \]

\[ \tau_{\text{coll}}^{-1}(T, \mu) = \sum_{i,j} \sigma_{ij} n_j \]

\[ \sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle \]


D. Blaschke, Chiral Condensate and Chemical Freezeout

B. Povh, J. Hüfner, PRD 46 (1992) 990
Hadronic radii and chiral condensate

\[ r_\pi^2(T, \mu) = \frac{3}{4\pi^2} F_\pi^{-2}(T, \mu) . \]

\[ F_\pi^2(T, \mu) = -m_0 \langle \bar{q}q \rangle_{T,\mu}/m_\pi^2. \]

\[ r_\pi^2(T, \mu) = \frac{3m_\pi^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_{T,\mu}|^{-1}. \]

\[ r_N^2(T, \mu) = r_0^2 + r_\pi^2(T, \mu), \]

Expansion time from entropy conservation

\[ S = s(T, \mu) \quad V(\tau_{\text{exp}}) = \text{const} \]

\[ \tau_{\text{exp}}(T, \mu) = a \cdot s^{-1/3}(T, \mu), \]


D.B., J. Berdermann, J. Cleymans, K. Redlich, Few

Body Systems (2011) [arxiv:1109.5391]
Chiral Condensate in a Hadron Resonance Gas

\[
\frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_{\text{vac}}} = 1 - \frac{m_0}{F_\pi^2 m_\pi^2} \left\{ 4N_c \int \frac{dp \, p^2}{2\pi^2} \frac{m}{\varepsilon_p} \left[ f_\Phi^+ + f_\Phi^- \right] 
+ \sum_{M=f_0,\omega,\ldots} d_M (2 - N_s) \int \frac{dp \, p^2}{2\pi^2} \frac{m_M}{E_M(p)} f_M(E_M(p)) 
+ \sum_{B=N,\Lambda,\ldots} d_B (3 - N_s) \int \frac{dp \, p^2}{2\pi^2} \frac{m_B}{E_B(p)} \left[ f_B^+(E_B(p)) + f_B^-(E_B(p)) \right] \right\} 
- \sum_{G=\pi,K,\eta,\eta'} \frac{d_G r_G}{4\pi^2 F_G^2} \int dp \, \frac{p^2}{E_G(p)} f_G(E_G(p))
\]

Chemical freeze-out vs. Condensate

Chemical freeze-out from kinetic condition, schematic model

Chemical freeze-out vs. Condensate

Chemical freeze-out from kinetic condition, $a \sim$ inverse system size

Strong T-Dependence of (inelastic) Collision Time

The model works astonishingly well!

Improvements are plenty:
- Hadron mass formulae, e.g. from holographic QCD ...
- Spectral functions - generalized Beth-Uhlenbeck
- Thermodynamics ... hydrodynamics.

Beyond freeze-out towards the deconfined phase: Mott-Hagedorn model
Theoretical laboratory of QCD

The energy density normalized by $T^4$
as a function of the temperature
on $N_t = 6, 8$ and $10$ lattices.

S. Borsanyi et al. “The QCD equation of state with dynamical quarks,”
JHEP 1011, 077 (2010)

The pressure normalized by $T^4$
as a function of the temperature
on $N_t = 6, 8$ and $10$ lattices.
Hagedorn resonance gas: hadrons with finite widths

The energy density per degree of freedom with the mass \( M \)

\[
\varepsilon(T, \mu_B, \mu_S) = \sum_{i: \ m_i < m_0} g_i \varepsilon_i(T, \mu_i; m_i) \\
+ \sum_{i: \ m_i \geq m_0} g_i \int_{m_0^2}^{\infty} d(M^2) \ A(M, m_i) \ \varepsilon_i(T, \mu_i; M),
\]

Spectral function

\[
A(M, m) = N_M \frac{\Gamma \cdot m}{(M^2 - m^2)^2 + \Gamma^2 \cdot m^2},
\]

\[
\Gamma(T) = C_{\Gamma} \left( \frac{m}{T_H} \right)^{N_m} \left( \frac{T}{T_H} \right)^{N_T} \exp \left( \frac{m}{T_H} \right)
\]
Hagedorn resonance gas: hadrons with finite widths

\[ P(T) = T \int_{0}^{T} dT' \frac{\varepsilon(T')}{T'^2} \]

\( N_m \) in the range from \( N_m = 2.5 \) (dashed line) to \( N_m = 3.0 \) (solid line).

\( C_\Gamma = 10^{-4} \)

\( N_T = 6.5 \)

\( T_H = 165 \text{ MeV} \)

\[ \Gamma(T) = C_\Gamma \left( \frac{m}{T_H} \right)^{N_m} \left( \frac{T}{T_H} \right)^{N_T} \exp \left( \frac{m}{T_H} \right) \]

Mott-Hagedorn resonance gas

State-dependent hadron resonance width

\[ A_i(M, m_i) = N_M \frac{\Gamma_i \cdot m_i}{(M^2 - m_i^2)^2 + \Gamma_i^2 \cdot m_i^2}, \]

\[ \Gamma_i(T) = \tau^{-1}_{\text{coll},i}(T) = \sum_j \lambda \langle r_i^2 \rangle_T \langle r_j^2 \rangle_T n_j(T) \]


For pions (mesons)

\[ r_\pi^2(T, \mu) = \frac{3M_\pi^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_T|^{-1}; \quad \langle \bar{q}q \rangle_T = 304.8 [1 - \tanh (0.002 T - 1)] \]

For nucleons (baryons)

\[ r_N^2(T, \mu) = r_0^2 + r_\pi^2(T, \mu); \quad r_0 = 0.45 \text{fm pion cloud.} \]
Mott-Hagedorn resonance gas: Pressure and energy density for three values of the mass threshold

$m_0 = 1.0$ GeV (solid lines)
$m_0 = 0.98$ GeV (dashed lines)
and
$m_0 = 0$ (dash-dotted lines)

Quarks and gluons are missing!
Systematic expansion of the pressure as the thermodynamical potential in the grand canonical ensemble for a chiral quark model of the PNJL type beyond its mean field description $P_{\text{PNJL,MF}}(T)$ by including perturbative corrections

$$P(T) = P^*_{\text{HRG}}(T) + P_{\text{PNJL,MF}}(T) + P_2(T) ,$$

$$P^*_{\text{HRG}}(T) = \frac{P_{\text{HRG}}(T)}{1 + (P_{\text{HRG}}(T)/(aT^4))^{\alpha}} ,$$

with $a = 2.7$ and $\alpha = 1.8$.

**Quark and gluon contributions**

$$P_2(T) = P_{2}^{\text{quark}}(T) + P_{2}^{\text{gluon}}(T)$$
Quark and gluon contributions

\[ P_2^{\text{quark}}(T) \]

\[ P_2^{\text{gluon}}(T) \]

Total perturbative QCD correction

\[
P_2 = -\frac{8}{\pi} \alpha_s T^4 (I_\Lambda^+ + \frac{3}{\pi^2} ((I_\Lambda^+)^2 + (I_\Lambda^-)^2))
\]

\[
\xrightarrow{\Lambda/T \to 0} -\frac{3\pi}{2} \alpha_s T^4
\]

where

\[
I_\Lambda^\pm = \int_{\Lambda/T}^{\infty} \frac{dx}{e^x \pm 1}
\]

\cdot Energy corrections

\[
\varepsilon_2(T) = T \frac{dP_2(T)}{dT} - P_2(T)
\]
$P_{\text{MHRG}}(T) = \sum_i \delta_i d_i \int \frac{d^3p}{(2\pi)^3} \int dM A_i(M,m_i) T \ln \left\{ 1 + \delta_i e^{-\frac{\sqrt{p^2 + M^2 - \mu_i}}{T}} \right\}$,

- Quark-gluon plasma contributions are described within the improved PNJL model with $\alpha_s$ corrections.
- Heavy hadrons are described within the resonance gas with finite width exhibiting a Mott effect at the coincident chiral and deconfinement transitions.
- Contribution restricted to the region around the chiral/deconfinement transition 170-250 MeV
- Fit formula for the pressure

\[
P = aT^4 + bT^{4.4} \tanh(cT - d),
\]

\[
a = 1.0724, \quad b = 0.2254, \quad c = 0.00943, \quad d = 1.6287
\]
Conclusions - part II

- An effective model description of QCD thermodynamics at finite temperatures which properly accounts for the fact that in the QCD transition region it is dominated by a tower of hadronic resonances.

- A generalization of the Hagedorn resonance gas thermodynamics which includes the finite lifetime of hadronic resonances in a hot and dense medium

To do

- Join hadron resonance gas with quark-gluon model.
- Calculate kurtosis and compare with lattice QCD.
- Spectral function for low-lying hadrons from microphysics (PNJL model ...).
Invitation to upcoming events

DIAS-TH: Dubna International Advanced School for Theoretical Physics

Helmholtz International Summer School
Dense Matter in Heavy Ion Collisions and Astrophysics: Theory and Experiment

Dubna, Russia, August 28 - September 8, 2012

Topics
• Equation of state & QCD phase transitions
• Transport properties in dense QCD matter
• Hadronization & freeze-out in heavy ion collisions (HIC)
• Astrophysics of compact stars (CS)
• Simulations of dense QCD, HIC and CS
• Experiments and observational programs

Organisers
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A. Sonin (JINR)
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Local Organisers
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The New Physics of Compact Stars: CompStar School 2012

Equation of State for Compact Star Interiors and Supernovae

Zadar, Croatia
September 24 - 28, 2012

Topics
Theory for hot and dense QCD and nuclear matter
Heavy - ion collision physics
Astrophysics of compact stars and supernovae

Lecturers
David Blaschke Review of EoS and Compact Stars
[University of Wroclaw, Poland]
Fiorella Burgio Theoretical approaches towards a modern nuclear EoS
[MPG, Germany]
Paweł Danielewicz Heavy ion experiments and EoS constraints
[University of Gdansk]
Tobias Fischer EoS and neutrino transport for supernova explosions
[University of Tübingen]
Mariano Mendez Observations of masses and radii of compact stars
[University of Granada]
Bernd-Jochen Schaefer Aspects of the QCD phase diagram and the EoS
[University of Göttingen]

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Registration deadline 06. 07. 2012.

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Chiral Condensate and Chemical Freezeout