Quark-Gluon Plasma Formation in Heavy Ion Collisions in Holographic Description

Irina Aref'eva
Steklov Mathematical Institute, RAN, Moscow

JINR, Dubna

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Outlook

• Quark-Gluon Plasma (QGP) in heavy-ions collisions (HIC)
• Holography description of QGP in equilibrium
• Holography description of formation of QGP in HIC $\iff$ Black Holes formation in AdS
• Thermalization time/Dethermalization time
• Non-central collisions in holography description
Quark-Gluon Plasma (QGP): a new state of matter

QGP is a state of matter formed from deconfined quarks, antiquarks, and gluons at high temperature.

QCD: asymptotic freedom, quark confinement

T increases, or density increases

nuclear matter → Deconfined phase

LHC
RHIC
170 MeV

Quark gluon plasma

Hadrinic matter

Color superconductor

300 MeV Nuclei
Neutron stars
Experiments: Heavy Ions collisions produced a medium

HIC are studied in several experiments:
• started in the 1990's at the Brookhaven Alternating Gradient Synchrotron (AGS),
• the CERN Super Proton Synchrotron (SPS)
• the Brookhaven Relativistic Heavy-Ion Collider (RHIC)
• the LHC collider at CERN.

\[ \sqrt{s_{NN}} = 4.75 GeV \]
\[ \sqrt{s_{NN}} = 17.2 GeV \]
\[ \sqrt{s_{NN}} = 200 GeV \]
\[ \sqrt{s_{NN}} = 2.76 TeV \]

Fireball at the LHC is denser, larger and longer lived than at RHIC.

\[ \epsilon \sim 10 GeV/fm^3, \quad V \sim 4800 fm^3, \quad \tau_{life} \sim 10 fm/c \]

There are strong experimental evidences that RHIC or LHC have created some medium which behaves collectively:
• modification of particle spectra (compared to p+p)
• jet quenching
• high p_T-suppression of hadrons
• elliptic flow
• suppression of quarkonium production

Study of this medium is also related with study of Early Universe
One of the fundamental questions in physics is: what happens to matter at extreme densities and temperatures as may have existed in the first microseconds after the Big Bang.

The aim of heavy-ion physics is to create such a state of matter in the laboratory.
pp collisions vs heavy ions collisions
Jet quenching

Central collision

P. Sorensen, Highlights from Heavy Ion Collisions at RHIC….., 1201.0784[nucl-ex]

I.A., Holographic Description of Heavy Ion Collisions, PoS ICMP2012 (2012) 025
Elliptic flow

\[ \frac{dN}{d\varphi} = \frac{N}{2\pi} \left( 1 + v_2(p_\perp, b) \cos(2\varphi) + \ldots \right) \]

Imprints of anisotropies are more essential for small shear viscosity, since usually large viscosity erases stronger irregularity

\[ \eta/s \approx 0.03 - 0.15 \]
The nuclear modification factor

\[ R^h_{AB}(p_T, \eta, \text{centrality}) = \frac{\frac{dN_{AB\rightarrow h}}{dp_T\,d\eta}}{\langle N_{AB} \rangle \frac{dN_{pp\rightarrow h}^{\text{vacuum}}}{dp_T\,d\eta}} \]
**Multiplicity:** Landau’s/Holographic formula vs experimental data

Landau formula

\[ M \sim S_{NN}^{1/4} \]

Plot from: ATLAS Collaboration 1108.6027

\[ dN_{ch}/d\eta|_{\eta=0} \]
The QCD equation of state with dynamical quarks, 

S. Borsanyi et al., "The QCD equation of state with dynamical quarks," arXiv:1007.2580
QGP as a strongly coupled fluid

- Conclusion from the RHIC and LHC experiments: appearance of QGP (not a weakly coupled gas of quarks and gluons, but a strongly coupled fluid).

- This makes **perturbative methods** inapplicable.

- The **lattice formulation** of QCD does not work, since we have to study real-time phenomena.

- This has provided a motivation to try to understand the dynamics of QGP through the **gauge/string duality**.
Dual description of QGP as a part of Gauge/string duality

- There is not yet exist a gravity dual construction for QCD.
- Differences between $N = 4$ SYM and QCD are less significant, when quarks and gluons are in the deconfined phase (because of the conformal symmetry at the quantum level $N = 4$ SYM theory does not exhibit confinement.)

- Lattice calculations show that QCD exhibits a quasi-conformal behavior at temperatures $T > 300$ MeV and the equation of state can be approximated by $E = 3P$ (a traceless conformal energy-momentum tensor).

- The above observations, have motivated to use the AdS/CFT correspondence as a tool to get non-perturbative dynamics of QGP.

- There is the considerable success in description of the static QGP.

Review: Solana, Liu, Mateos, Rajagopal, Wiedemann, 1101.0618
“Holographic description of quark-gluon plasma”

- Holographic description of quark-gluon plasma in equilibrium

- Holography description of quark-gluon plasma formation in heavy-ions collisions
Holographic description of QGP

(QGP in equilibrium)

Holography for thermal states

TQFT in $M_D$-spacetime

= Black hole in AdS$D+1$-space-time

TQFT = QFT with temperature
AdS/CFT correspondence in Euclidean space. $T \neq 0$

To compute the Matsubara correlator at finite temperature

$$G^E(k_E) = \int d^4 x_E \ e^{-i k_E \cdot x_E} \langle T_E \hat{O}(x_E) \hat{O}(0) \rangle$$

Here $T_E$ denotes Euclidean time ordering.

Euclidean time coordinate $\tau$ is periodic, $\tau \sim \tau + T^{-1}$

$$\phi(\tau, \vec{x}, z), \quad S_g[\phi], \quad \delta S_g[\phi_c] = 0 \quad \phi_c \big|_{\partial M} = \phi_0 \quad \phi_c = \phi_c(\phi_0)$$

Boundary condition $\phi|_{z=0} = \phi_0$ + requirement of regularity at horizon

$$g: \quad ds^2 = \frac{R^2}{z^2} \left( f(z) d\tau^2 + dx^2 + \frac{dz^2}{f(z)} \right) + R^2 d\Omega_5^2$$

$$f(z) = 1 - z^4/z_H^4 \quad \quad z_H = (\pi T)^{-1}$$

$T$ is the Hawking temperature

$$0 < z < z_H$$
Correlators with $T \neq 0$ AdS/CFT

Example. $D=2$

\[
\langle O(t, x)O(t, x') \rangle_{ren} \sim e^{-\Delta \delta \mathcal{L}} \quad x-x' = \ell
\]

\[
\delta \mathcal{L} \equiv \mathcal{L} + 2 \ln(z_0/2)
\]

Vacuum correlators $\text{M}=\text{AdS}$

\[
\delta \mathcal{L}_{\text{vacuum}}(\ell) = 2 \ln \frac{\ell}{2}
\]

Temperature $\text{M}=\text{BHAdS}$ with $r_H$

\[
\delta \mathcal{L}_{\text{thermal}}(\ell) = 2 \ln \frac{\sinh \frac{r_H \ell}{2}}{r_H}
\]

Bose gas
Holographic thermalization

Thermalization of QFT in Minkowski D-dim space-time

Black Hole formation in Anti de Sitter (D+1)-dim space-time

Profit:

Time of thermalization in HIC

Multiplicity in HIC

Studies of BH formation in $\text{AdS}_{D+1}$
Formation of BH in AdS. Deformations of AdS metric leading to BH formation

- colliding gravitational shock waves
  
  Gubser, Pufu, Yarom, Phys.Rev., 2008 (I)
  Gubser, Pufu, Yarom, JHEP, 2009 (II)
  Alvarez-Gaume, C. Gomez, Vera, Tavanfar, Vazquez-Mozo, PRL, 2009
  Kiritsis, Taliotis, 2011 JHEP

- drop of a shell of matter with vanishing rest mass
  
  "null dust",
  infalling shell geometry = Vaidya metric

  Danielsson, Keski-Vakkuri, Kruczenski, 1999
  ...

- sudden perturbations of the metric near the boundary that propagate into the bulk

  Chesler, Yaffe, PRL, 2011
d+1-dimensional infalling shell geometry is described in Poincar'e coordinates by the Vaidya metric

\[ ds^2 = \frac{1}{\zeta^2} \left[ - \left(1 - m(v) \zeta^d \right) dv^2 - 2d\zeta dv + d\bm{x}^2 \right] \]

\( v \) labels ingoing null trajectories

1) For constant \( m(v) = M \), the coordinate transformation \( dv = dt - \frac{dz}{1 - M \zeta^d} \) brings the form

\[ ds^2 = \frac{1}{\zeta^2} \left[ - \left(1 - M \zeta^d \right) dt^2 + \frac{dz^2}{1 - M \zeta^d} + d\bm{x}^2 \right] \]

2) \( m(v) = \frac{M}{2} \left(1 + \tanh \frac{v}{v_0} \right) \)

interpolates between vacuum AdS inside the shell and an AdS black brane
Correlators via Geodesics in AdS/CFT

\[
\langle \mathcal{O}_\Delta(\tau_1, x_1) \mathcal{O}_\Delta(\tau_2, x_2) \rangle = \int DP \ e^{i\Delta L(P)}
\]

\[P \in M\]

\[(\tau_1, x_1) \in \partial M\]

\[L(P) = \int (-g_{\mu\nu} \dot{X}^\mu \dot{X}^\nu)^{1/2}\]

\[(\tau_2, x_2) \in \partial M\]

Vacuum correlators: M=AdS

Temperature: M=BHAdS
Thermalization with Vadya AdS

Equal-time correlators
Evaporation vs thermalization

\[ m(v) = M\theta(v) - M_1\theta(v - v_1) \]

\[ M = M_1 \]

No thermalization for large \( l \)
\[
\tau_{\text{therm}} = \int_J^\infty \frac{dr}{r^2 (1 - \frac{M}{r^d})}
\]

\[
\ell = 2J \int_J^\infty \frac{dr}{r^2 \sqrt{(r^2 - J^2) (1 - \frac{M}{r^d})}}.
\]

\[
\frac{\tau_{\text{ther}}}{\tau_{\text{dether}}} = F(m^2, d) \quad F(m^2, d) = \int_1^\infty \frac{d\rho}{\rho^2 (1 - \frac{m^2}{\rho^d})} = \frac{1}{2} \int_1^\infty \frac{d\rho}{\rho^2 \sqrt{(\rho^2 - 1) (1 - \frac{m^2}{\rho^d})}}.
\]
\[ \frac{\tau_{ther}}{\tau_{dether}} = F(m^2, d) \]

\[ 0.78 < \frac{\tau_{ther}}{\tau_{det}} < 1 \]

Data: \[ \frac{\tau_{ther}}{\tau_{det}} \sim 0.1 - 0.05 \]
Data: \( \tau_{\text{ther}} \sim 1 \text{ fm/c} \)

thermal scale \( l \sim \hbar/T \) \( T \sim 300 - 400 \text{ MeV} \)

\( \tau_{\text{therm}} \sim 0.3 \text{ fm/c} \),

\( l \sim 2 \text{ fm} \)

\( r_{Pb} \approx 7 \text{ fm} \) can pack 208 (A=208 for Pb) balls with radius

\[
r_n = 3 \sqrt[3]{\frac{\eta_K}{208}} \quad r_{Pb} \approx 1.07 \text{ fm}
\]

\( \eta_K \) is the Kepler number \( \eta_K = \frac{\pi}{\sqrt{18}} \approx 0.74 \)
Data: \( \frac{\tau_{\text{ther}}}{\tau_{\text{det}}} \sim 0.1 - 0.05 \)

\[ l_{\text{det}} \sim 2r_{\text{Pb}} \sim 14 \text{ fm.} \]

\[ l_{\text{therm}} \sim 2 \text{ fm} \]

\[ \tau_{\text{det}} \sim 7 \text{ fm/c} \]

\[
\frac{\tau_{\text{ther}}}{\tau_{\text{det}}} = \frac{\tau_{\text{ther}}}{0.5 \cdot l_{\text{ther}}} \cdot \frac{l_{\text{ther}}}{l_{\text{det}}} = 0.39 \cdot \frac{2}{14} \approx 0.056
\]
Thermalization Time and Centricity

Non-centricity

Kerr-ADS-BH

In progress with A. Koshelev, A. Bagrov

\[ ds^2 = -\left( N^\perp(r) \right)^2 dt^2 + \frac{1}{\left( N^\perp(r) \right)^2} dr^2 + r^2 (N^\phi(r) dt + d\phi)^2 \]

\[ N^\perp = \left( -M + \left( \frac{r}{l} \right)^2 + \frac{a^2}{r^2} \right)^{1/2}, \quad N^\phi(r) = -\frac{a}{r^2} \]

\[ ds^2 = -\left( -M + \frac{r^2}{l^2} \right) dv^2 + 2dvd\hat{r} - 2advd\hat{\phi} + r^2 d\hat{\phi}^2 \]

\[ dv = dt + \frac{dr}{\left( N^\perp \right)^2} \]

\[ d\hat{\phi} = d\phi - \frac{N^\phi}{\left( N^\perp \right)^2} dr \]

\[ M(v) = M \theta(v), \quad j(v) = 2a \theta(v) \]
$\beta_{1,2} = \frac{l^2 M}{2} \left( 1 \pm \sqrt{1 - \frac{4a^2}{l^2 M^2}} \right)$

Geodesics

$t(r) = t_0 - \frac{\mathcal{E} l^3}{2} I_\pm |_{\alpha = -\frac{\mathcal{J}a}{\mathcal{E}}} , \quad \phi(r) = \phi_0 + \frac{\mathcal{J} l}{2} I_\pm |_{\alpha = \frac{\mathcal{E}a + \mathcal{J}M}{\mathcal{J}/l^2}}$

$-I_+ = \frac{1}{(\beta_1 - \beta_2)} \left[ \frac{\alpha - \beta_1}{\sqrt{B_1}} \ln(X_1 - \text{sign}(x - \beta_1) \sqrt{X_1^2 - (\gamma_1 - \gamma_2)^2}) - \frac{\alpha - \beta_2}{\sqrt{B_2}} \ln(X_2 - \text{sign}(x - \beta_2) \sqrt{X_2^2 - (\gamma_1 - \gamma_2)^2}) \right] + C = I_- + C$

$X_i = (2\beta_i - \gamma_1 - \gamma_2) + \frac{2B_2}{x - \beta_i}$

$x \equiv r^2$

$C = -\frac{2}{(\beta_1 - \beta_2)} \left[ \frac{\alpha - \beta_1}{\sqrt{B_1}} - \frac{\alpha - \beta_2}{\sqrt{B_2}} \right] \ln(\gamma_1 - \gamma_2)$

$\gamma_1 + \gamma_2 = M l^2 - l^2 \mathcal{E}^2 - + J^2$

$\gamma_1 \gamma_2 = l^2 a^2 - l^2 J(M J + 2a\mathcal{E})$
Geodesics which start and finish at $r = \infty$

\[ t_1 \equiv t_-(\infty) \quad t_2 \equiv t_+(\infty) \]

\[
t_1 = t_0 - \frac{\mathcal{E}}{2} I_-(\infty)\bigg|_{\alpha = -\frac{\mathcal{J}_a}{\mathcal{E}}} \quad \phi_1 = \phi-(\infty) = \phi_0 + \frac{\mathcal{J}}{2} I_-(\infty)\bigg|_{\alpha = \frac{\mathcal{E} a + \mathcal{J} M}{\mathcal{J}}} \]

\[
t_2 = t_0 - \frac{\mathcal{E}}{2} I_+(\infty)\bigg|_{\alpha = -\frac{\mathcal{J}_a}{\mathcal{E}}} \quad \phi_2 = \phi+(\infty) = \phi_0 + \frac{\mathcal{J}}{2} I_+(\infty)\bigg|_{\alpha = \frac{\mathcal{E} a + \mathcal{J} M}{\mathcal{J}}} \]

\[
 l = t_1 + t_2 \quad \dot{v}_* = 0 \\
 a=0 \\
 l = t_1 + t_2 - 2v_*
\]
Conclusion

Formation of QGP of 4-dim QCD $\iff$ Black Hole formation in AdS$_5$

- Multiplicity: AdS-estimations fit experimental data

$$S_{data} \propto S_{NN}^{0.15}$$

$$\frac{\tau_{ther}}{\tau_{det}} \sim 0.1 - 0.05$$

- Non-centricity decreases thermalization time.

- New phase transition $(T \text{ vs } \mu_B)$
Formation of trapped surfaces is only possible when \( Q < Q_{cr} \)

**Red** for a smeared matter

**Blue** for a point-like source