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QUANTUM CORRECTIONS TO COSMOLOGICAL POTENTIALS AND THE ORIGIN OF THE COSMOLOGICAL CONSTANT

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Quantum corrections to Scalar Potential

Colemann-Weinberg Mechanism

Classical potential $V_0(\phi) = g\phi^4/4!$

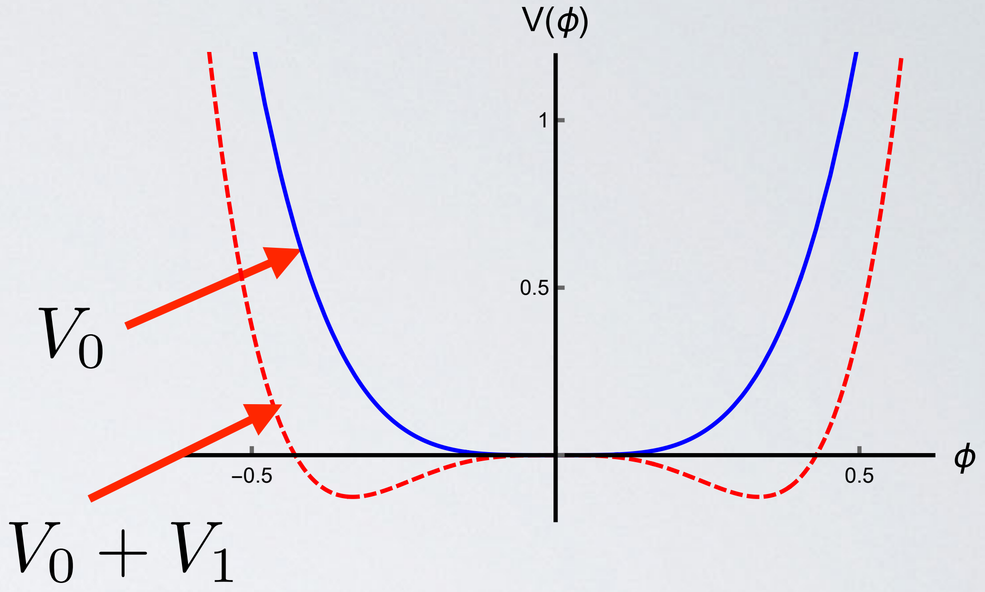
One-loop correction

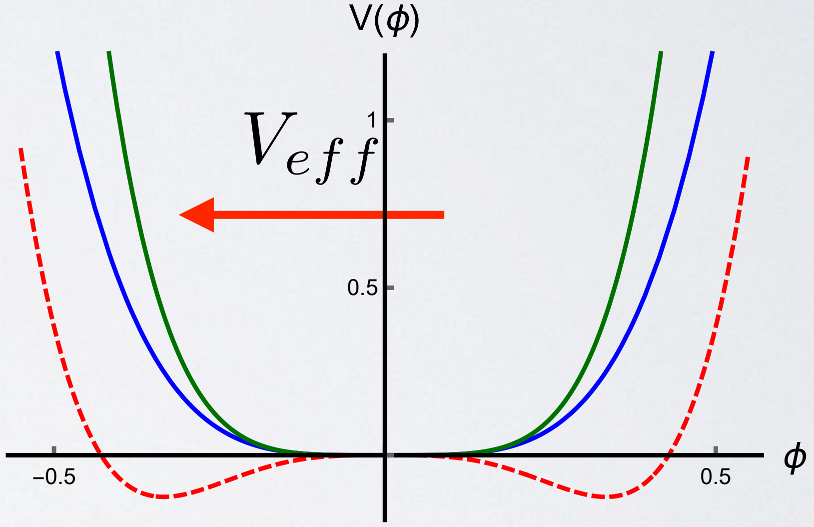
$$V_1(\phi) = \frac{g^2}{16\pi^2}\phi^4 \frac{1}{16} \log \phi^2/\mu^2$$

Negative for small field

RG summed all-loop leading terms

$$V_{eff}^{RG} = \frac{g\phi^4/4!}{1 - \frac{g}{16\pi^2}\frac{3}{2}\log \phi^2/\mu^2}$$





New minimum appears and disappears !

Effective Potential in Scalar Theory

Generating functional for Green functions

$$Z(J) = \int \mathcal{D}\phi \; \exp\left(i \int d^4x \; \mathcal{L}(\phi, d\phi) + J\phi \right)$$

$$W(J) = -i \log Z(J)$$ IPI generating functional

Effective action

$$\Gamma(\phi) = W(J) - \int d^4x J(x)\phi(x)$$ Legendre transformation

$$e^{i\Gamma(\Phi)} = \int \; \mathcal{D}\hat{\Phi} e^{i(S[\Phi+\hat{\Phi}]-\hat{\Phi}S'[\Phi])}$$

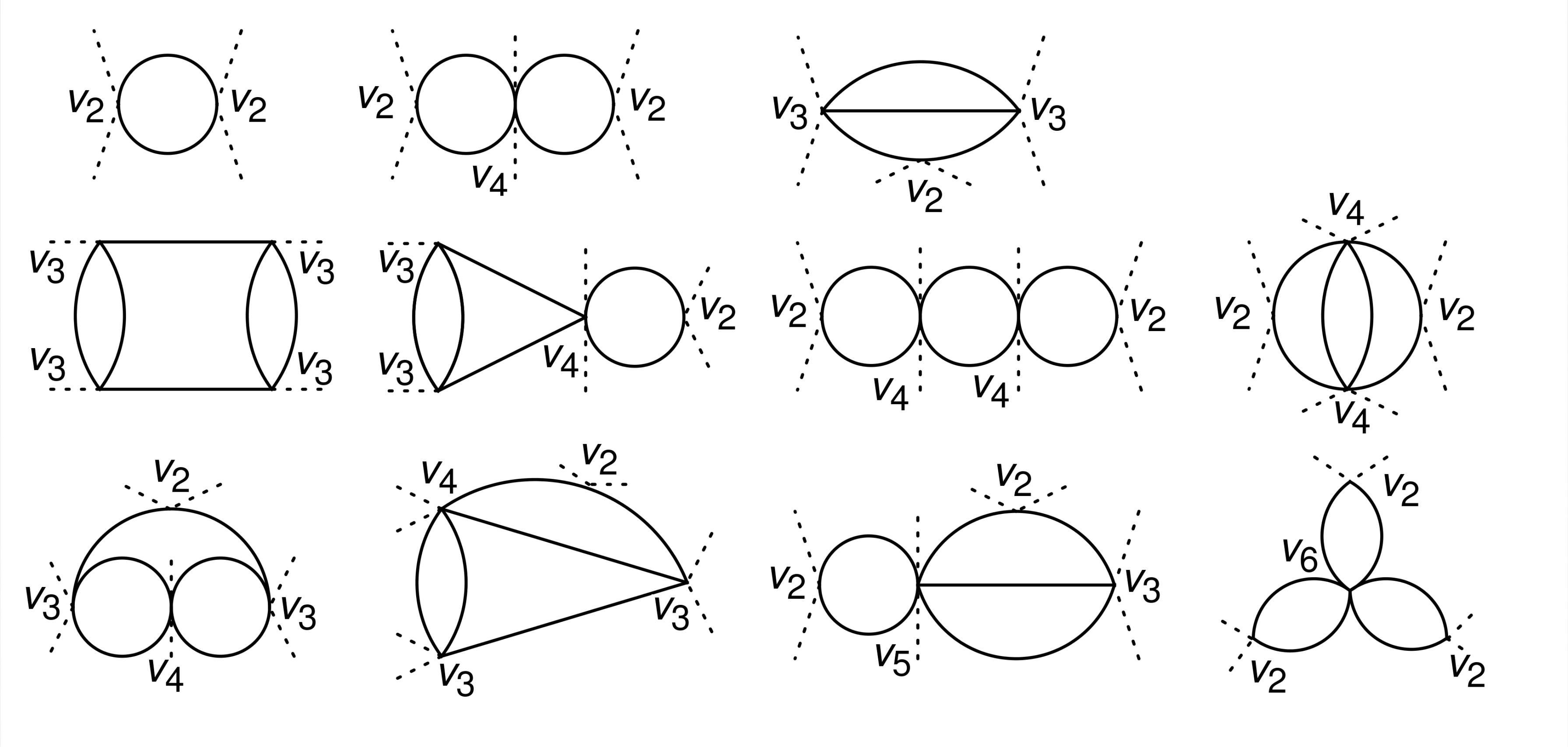
Shifted Classical action

$$S[\Phi + \hat{\Phi}] = \underset{\substack{\uparrow \\ \text{Classical external field}}}{S[\Phi]} + \hat{\Phi}S'[\Phi] + \frac{1}{2}\hat{\Phi}^2 \underset{\substack{\uparrow \\ \text{Field dependent mass}}}{S''[\Phi]} + \frac{1}{3!}\hat{\Phi}^3 \underset{\substack{\uparrow \\ \text{Interaction vertex}}}{S'''[\Phi]} + \ldots$$

Effective Potential in Scalar Theory

V_{eff} Is the sum of all vacuum 1PI diagrams

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - gV_0(\phi)$$



$$v_2(\phi) \equiv \frac{d^2 V_0(\phi)}{d\phi^2}$$

$$v_n \equiv d^n V_0 / d\phi^n$$

Shown are UV divergent vacuum diagrams in arbitrary scalar theory up to three loops

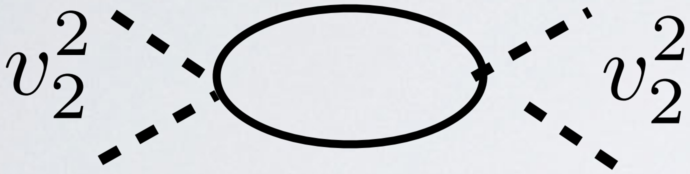
$$V_{eff} = g \sum_{n=0}^{\infty} (-g)^n V_n.$$

Divergent terms and Logs

General scalar field theory in D=4

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - gV_0(\phi)$$

UV divergences in dimensional regularisation $D = 4 - 2\epsilon$



$$Diag \sim \frac{1}{\epsilon} v_2^2 (\frac{\mu^2}{m^2})^\epsilon \to v_2^2(\frac{1}{\epsilon} - \log\frac{m^2}{\mu^2}), \quad m^2 = gv_2^2$$

UV divergence

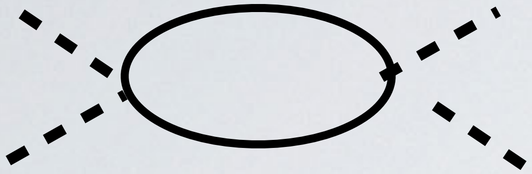
One loop

$$\Delta V_1 = \frac{g^2}{16\pi^2} v_2^2 \log\frac{gv_2}{\mu^2}$$

ϕ^4
$$\Delta V_1 = \frac{g^2}{16\pi^2}\phi^4 \log\frac{g\phi^2}{\mu^2}$$

ϕ^6
$$\Delta V_1 = \frac{g^2}{16\pi^2}\phi^8 \log\frac{g\phi^4}{\mu^2}$$

Divergences and Log ϕ behaviour



$$Diag \sim \frac{1}{\epsilon} v_2^2 (\frac{\mu^2}{m^2})^\epsilon \to v_2^2 (\frac{1}{\epsilon} - \log \frac{m^2}{\mu^2}), \quad m^2 = g v_2^2$$

The leading divergences ⟷ The leading logs

- In non-renormalizable theories divergences cannot be absorbed into the renormalization of the couplings and fields.

- If they are subtracted some way one is left with infinite arbitrariness.

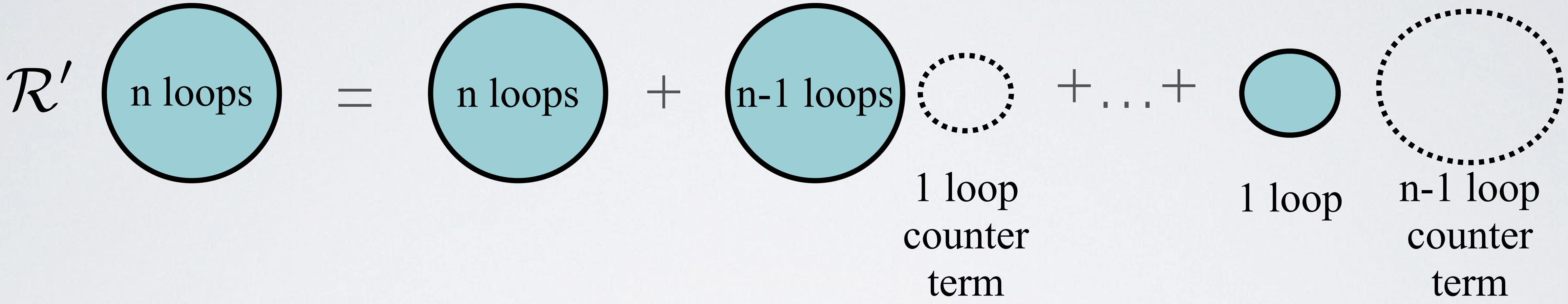
- Coefficients of the leading divergences (logs) do not depend on this arbitrariness !

The aim is to calculate the leading divergences ~ $\frac{1}{\epsilon^n}$ in n-th order of PT

BPHZ R-operation

$$\mathcal{R}'G_n = \frac{A_n^{(n)}(\mu^2)^{n\epsilon}}{\epsilon^n} + \frac{A_{n-1}^{(n)}(\mu^2)^{(n-1)\epsilon}}{\epsilon^n} + \ldots + \boxed{\frac{A_1^{(n)}(\mu^2)^{\epsilon}}{\epsilon^n}}$$

$$+ \qquad \text{lower pole terms}$$



$A_k^{(n)}(\mu^2)^{k\epsilon}$ terms appear after subtraction of (n-k) loop counter terms

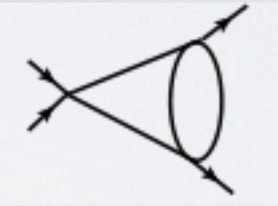
Statement: $R'G_n$ is local, i.e. terms like $\log^k \mu^2/\epsilon^m$ should cancel for any k and m

Consequence: $A_n^{(n)} = (-1)^{n+1}\frac{A_1^{(n)}}{n} \quad \Longleftarrow$

The leading divergences are governed by 1 loop diagrams!

Two loop example

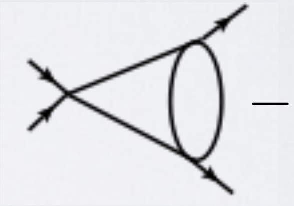
ϕ_4^4



$$= v_2 v_3^2 \left(\frac{A_2^{(2)}}{\epsilon^2} + \frac{A_1^{(2)}}{\epsilon} \right) \left(\frac{\mu^2}{m^2}\right)^{2\epsilon} \qquad m^2 = v_2$$

$\mathcal{R}'$

$=$



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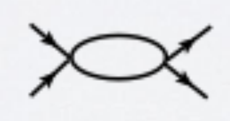

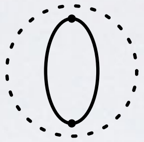

$$= \left(\frac{A_2^{(2)}}{\epsilon^2} + \frac{A_1^{(2)}}{\epsilon} \right) \left(\frac{\mu^2}{m^2}\right)^{2\epsilon} - \frac{A_1^{(1)}}{\epsilon} \left(\frac{\mu^2}{m^2}\right)^{\epsilon} \frac{A_1^{(1)}}{\epsilon}$$

$$= \frac{A_2^{(2)}}{\epsilon^2} - \frac{(A_1^{(1)})^2}{\epsilon^2} + 2\frac{A_2^{(2)}}{\epsilon} \underbrace{\log(\mu^2/m^2) - \frac{(A_1^{(1)})^2}{\epsilon} \log(\mu^2/m^2)}_{\text{non-local terms to be cancelled}} = -\frac{1}{2}\frac{(A_1^{(1)})^2}{\epsilon^2} + \ldots$$

Leading divergence is given by the one-loop term $\boxed{A_2^{(2)} = \frac{1}{2}(A_1^{(1)})^2}$

- These statements are universal and are valid in non-renormalizable theories as well.
- The only difference is that the counter term $A_1^{(1)}$ depends on the field
- As a result $A_2^{(2)}$ is not the square of $A_1^{(1)}$ anymore but is the derivative.
- This last statement is the general feature of any QFT irrespective of renormalizability

Recurrence relations for the leading poles

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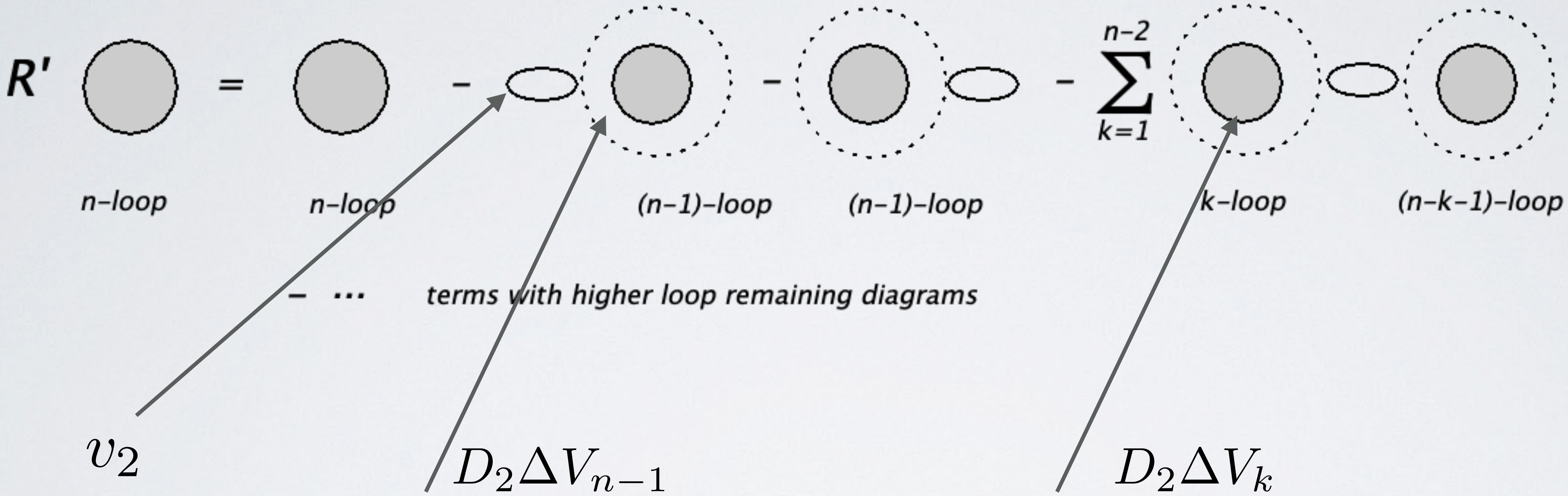
Action of R’-operation on divergent diagram



$$n\Delta V_n = \frac{1}{2} v_2 D_2 \Delta V_{n-1} + \frac{1}{4} \sum_{k=1}^{n-2} D_2 \Delta V_k D_2 \Delta V_{n-1-k}, \quad n \geq 2 \quad \Delta V_1 = \frac{1}{4} v_2^2$$

$$n\Delta V_n = \frac{1}{4} \sum_{k=0}^{n-1} D_2 \Delta V_k D_2 \Delta V_{n-1-k}, \quad n \geq 1, \quad \Delta V_0 = V_0$$

RG pole equation for arbitrary potential

$$\Sigma(z,\phi) = \sum_{n=0}^{\infty} (-z)^n \Delta V_n(\phi) \qquad z = \frac{g}{\epsilon}$$

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RG pole equation

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$$\boxed{\frac{d\Sigma}{dz} = -\frac{1}{4}(D_2\Sigma)^2 \qquad \Sigma(0,\phi) = V_0(\phi)}$$

This a non-linear partial differential equation!

Effective potential

$$V_{eff}(g,\phi) = g\Sigma(z,\phi)|_{z\to -\frac{g}{16\pi^2}\log g v_2/\mu^2} \cdot \qquad v_2(\phi) \equiv \frac{d^2V_0(\phi)}{d\phi^2}$$

α -attractor Inflaton Potential

Kallosh, Linde 13

Inflaton action with hyperbolic geometry

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{2}R(g) + \frac{1}{2}\frac{\partial_\mu\phi\partial^\mu\phi}{1-\frac{\phi^2}{6\alpha}} - V(\phi)\right]$$

Transition to the standard kinetic term

$$\partial\phi/\sqrt{1-\frac{\phi^2}{1-6\alpha}} = \partial\varphi \qquad \phi = \sqrt{6\alpha}\tanh\left(\frac{\varphi}{\sqrt{6\alpha}}\right)$$

Inflaton action of α -attractor model

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{2}R(g) + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V\left(\sqrt{6\alpha}\tanh\left(\frac{\varphi}{\sqrt{6\alpha}}\right)\right)\right].$$

T- model

$$gV_T(\varphi) = g\tanh^n\left(\frac{\varphi}{\sqrt{6\alpha}M_{Pl}}\right)$$

n=2 $\quad T_2$ - model

RG Equation for the T-model Effective potential

$$\frac{d\Sigma}{dz} = -\frac{1}{4}(D_2\Sigma)^2$$

Dimensionless variables

$$x = z/M_{Pl}^4 \quad y = \tanh^n(\varphi/\sqrt{6\alpha}M_{Pl})$$

$$\Sigma(z/M_{Pl}^4, \tanh^n(\varphi/\sqrt{6\alpha}M_{Pl})) \equiv S(x,y)$$

$$S_x = -\frac{n^2 y^{2-\frac{4}{n}}\left(y^{2/n}-1\right)^2}{144\alpha^2}\left(\left(y^{2/n}+n\left(y^{2/n}-1\right)+1\right)S_y + ny\left(y^{2/n}-1\right)S_{yy}\right)^2$$

This is a nonlinear partial differential equation!

Boundary conditions

$$S(0,y) = y,\ \ S(x,1) = 1,\ \ S_y(x,1) = 0.$$

n=2 case

$$S_x = -\frac{(y-1)^2\left((3y-1)S_y + 2(y-1)yS_{yy}\right)^2}{36\alpha^2}$$

Numerical Solution for T_2 - model

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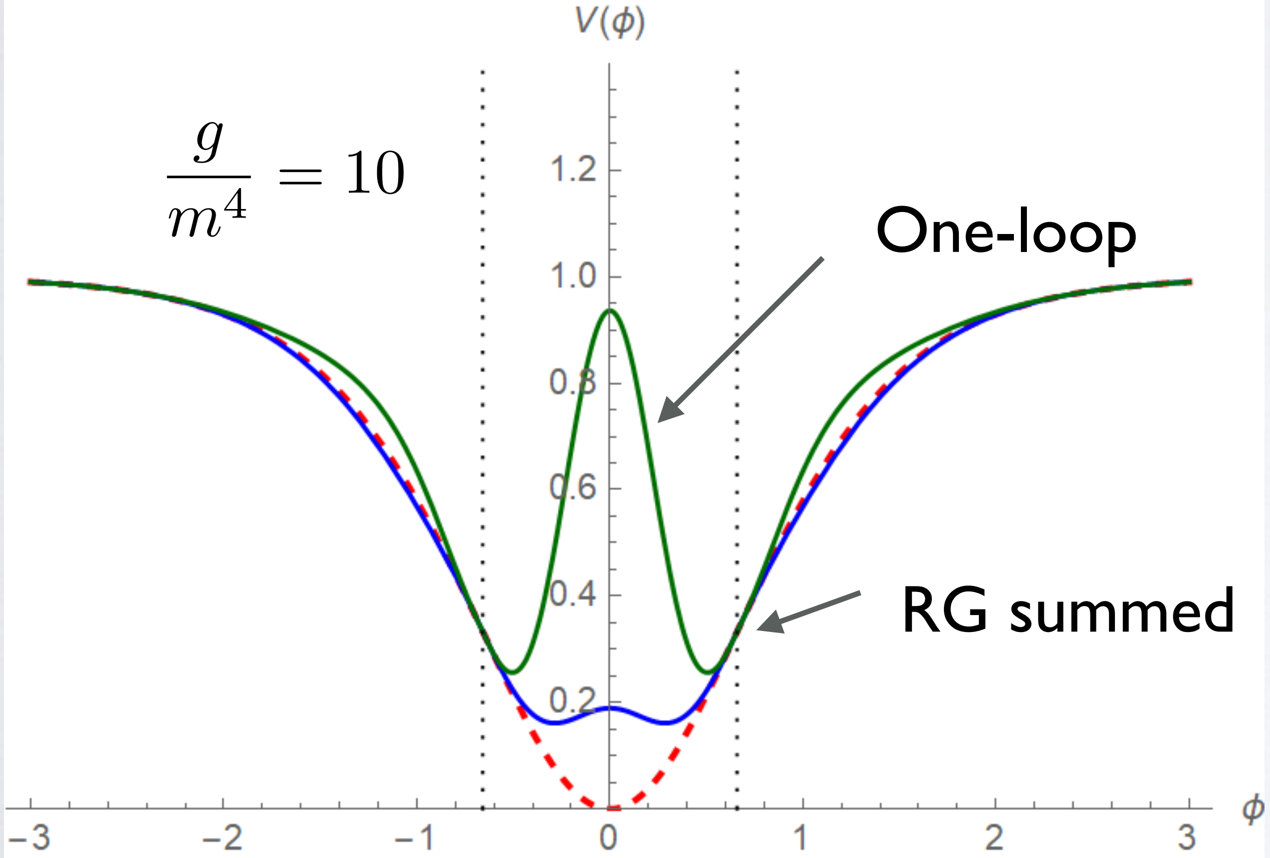
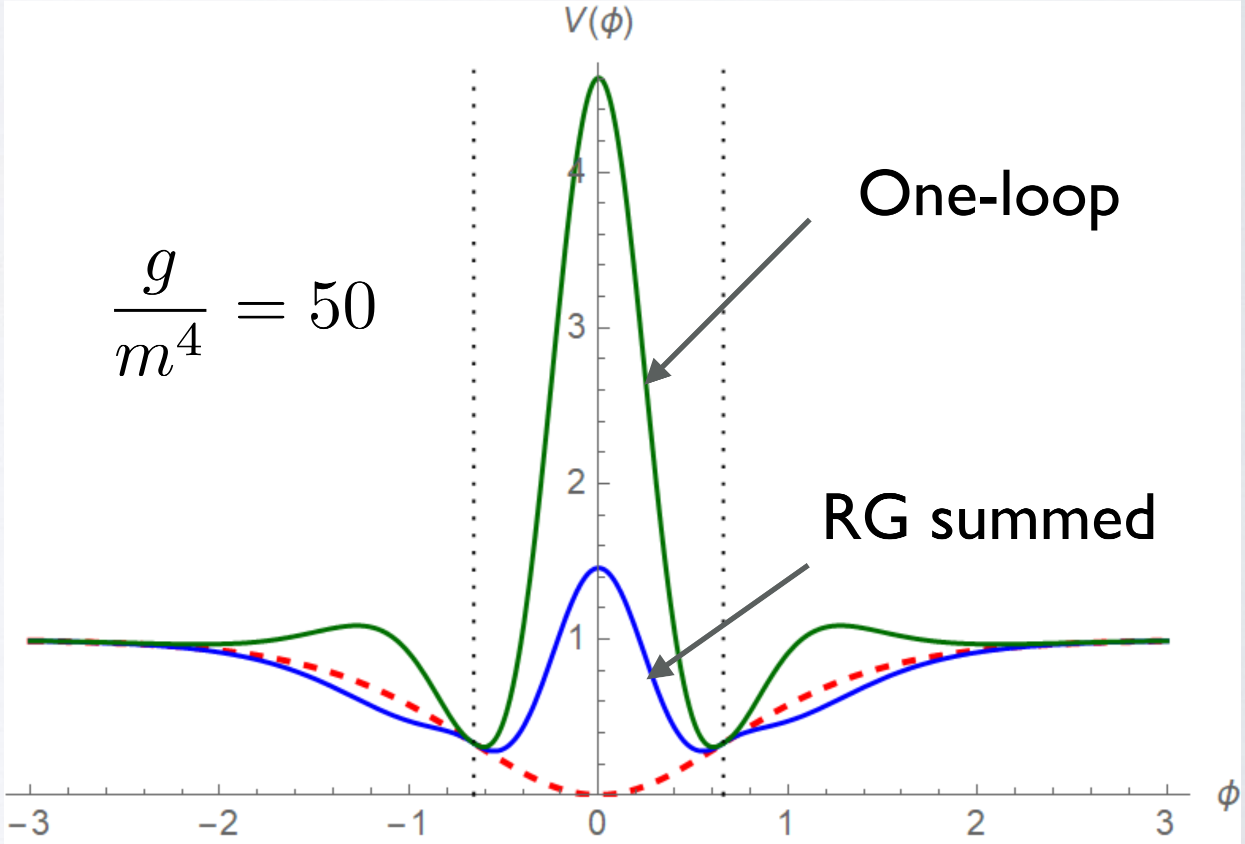
ArXiv: 2308.03872

- JCAP 09 (2023) 049

$$gV_0 = g\tanh^2(\phi/m)$$

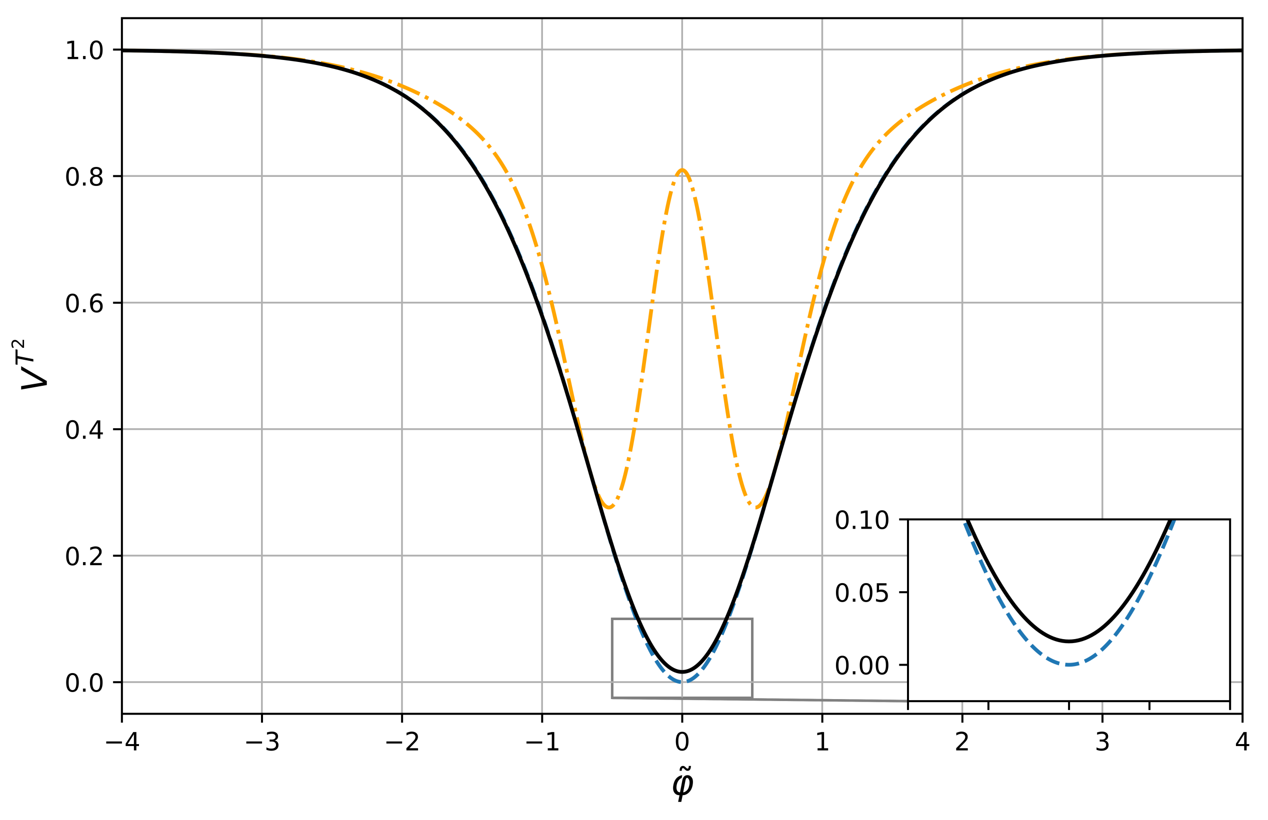
$$V_{eff}(g,\phi) = g\Sigma(z,\phi)|_{z\to -\frac{g}{16\pi^2}\log gv_2/\mu^2}\cdot \qquad v_2(\phi) \equiv \frac{d^2V_0(\phi)}{d\phi^2}$$





- Peak at the origin
- Additional minima

Lift of the Potential at the Minima - Origin of the Cosmological Constant



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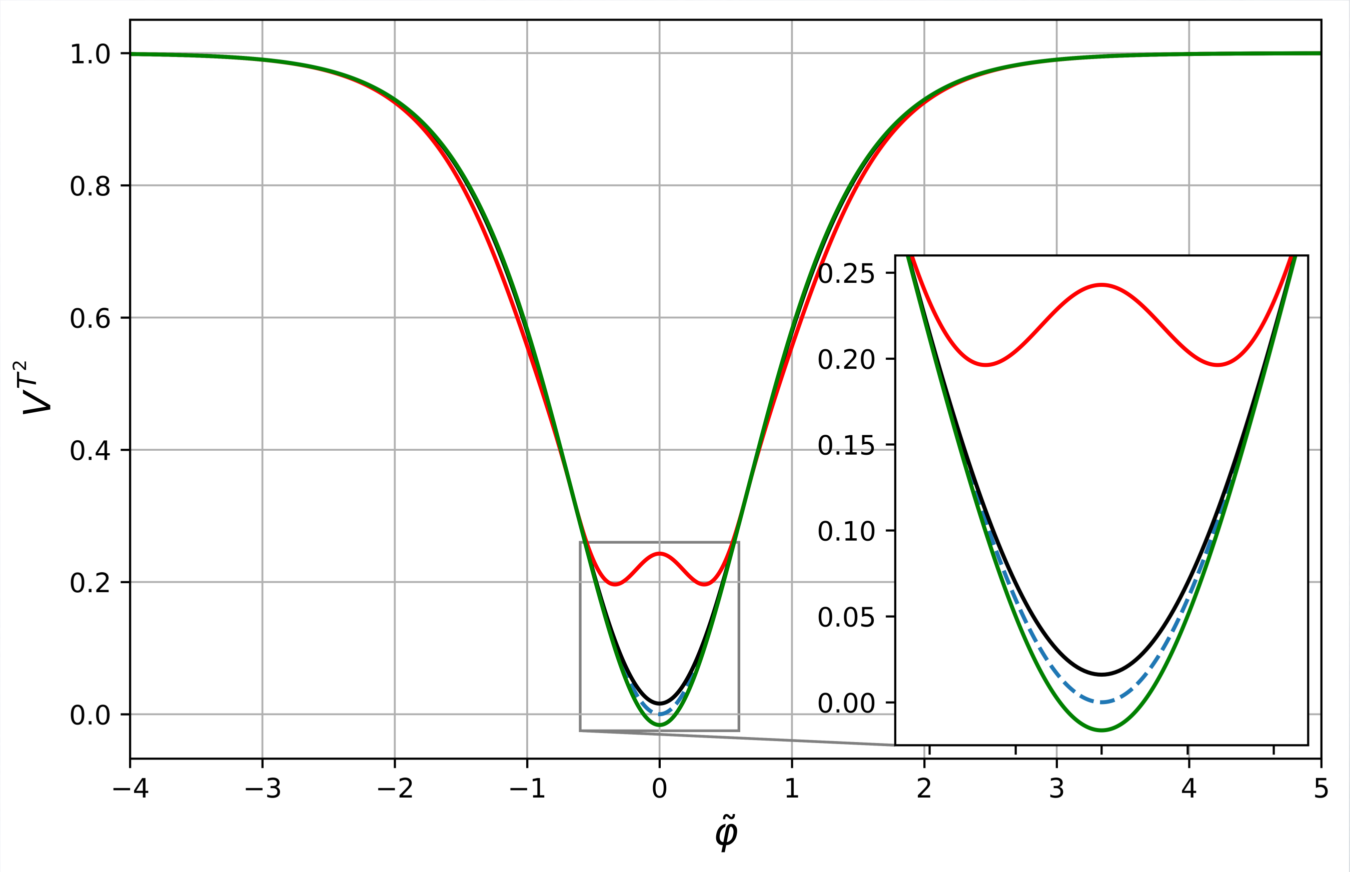
ArXiv: 2405.18818

Comparison of the classical T2-model potential (blue dashed line), the one-loop correction (orange dashed line), and the RG summed potential (black solid line) for $g \sim 1, \mu < M_{PL}$

T2-model potential: variation of μ . The classical potential (blue dashed line), the RG summed potential (solid lines) for

$\mu < M_{Pl}$	$\mu \ll M_{Pl}$	$\mu > M_{Pl}$
black line,	red line,	green line

$$g = 2, \;\; \alpha = 1$$



Estimation of the value of the Cosmological Constant

One-loop Potential

$$V_{eff} = V_0 + \frac{g^2}{16\pi^2}\frac{v_2^2}{4}\log\frac{gv_2(\varphi)}{\mu^2}$$

Cosmological constant

$$\Lambda = \left[\frac{g^2}{16\pi^2}\frac{v_2^2}{4}\log\left(\frac{gv_2}{\mu^2}\right)\right]\Big|_{\varphi=\varphi_{vac}}$$

μ - dependent

Inverse formula

$$\mu^2 = \frac{g}{3\alpha M_{Pl}^2} e^{-576\pi^2\frac{\alpha^2}{g^2}\Lambda M_{Pl}^4}$$

Numerical Estimation

$$g = 10^{-10}M_{Pl}^4, \quad M_{Pl} = (8\pi G)^{-\frac{1}{2}}, \quad \alpha = 1$$

$\Lambda \sim 10^{-120}M_{Pl}^4 \quad \mu \approx 10^{-6}M_{Pl}$ - Inflaton mass

Conclusion on Effective potential

- The effective potential in the LL approximation obeys the RG master equation which is a partial non-linear differential equation

- This generalised RG equation is valid for any potential including non-renormalizable one

- In some cases this equation is simplified to the ordinary differential one and can be solved at least numerically.

- The effective potential has a minima at the origin or can have additional minima depending on the free scale parameter μ .

- This formalism is applicable to inflation cosmology potentials like the model of α -attractors T2

- At the minima the potential is lifted due to radiative correction so that the cosmological constant appears

- Properly choosing the parameters of the potential one can get the observable value of the cosmological constant