ОИЯИ
ОБЪЕДИНЁННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

November '24

Non-renormalizable interactions: RG equations for the scattering amplitudes and effective potential

В ожидании_е.

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Motivation:

- The Standard Model is renormalizable
- Gravity is not renormalizable

Non-renormalizable theories are not accepted due to:

- UV divergences are not under control - infinite number of new types of divergences
- The amplitudes increase with energy (in PT) and violate unitarity

However:

- R-operation equally works for NR theories and leads to local counter terms
- Due to locality all higher order divergences are related to the lower ones

- These properties allow one to write down the RG equations for the scattering amplitudes, effective potential, etc which sum up the leading divergences (logarithms) and to find out the high energy/field behaviour

Based on: Phys. Lett. B734 (2014) 111, arXiv:1404.6998 [hep-th]
JHEP 11 (2015) 059, arXiv:1508.05570 [hep-th]
JHEP 12 (2016) 154, arXiv:1610.05549v2 [hep-th]
Phys.Rev. D95 (2017) no.4, 045006 arXiv:1603.05501 [hep-th]
Phys.Rev. D97 (2018) no.12, 125008, arXiv:1712.04348 [hep-th],
Phys.Lett. B786 (2018) 327-331, arXiv:1804.08387 [hep-th]
Symmetry 11 (2019) 1, 104, arXhiv:1812.11084 [hep-th]
Phys.Lett.B 797 (2019) 134801, arXiv:1904.08690 [hep-th]
Труды Мат. Инст. им. В.А. Стеклова, 2020, т. 308, с. 1–8
JHEP 06 (2022) 141, arXiv:2112.03091 [hep-th]
JHEP 04 (2023) 128, arXiv: 2209.08019 [hep-th]
JCAP 09 (2023) 049, arXiv: 2308.03872 [hep-th]
arXiv: 2405.18818 [hep-th]

In collaboration with L.Bork, A.Borlakov, R. Iakhibbaev, D.Tolkachev and D.Vlasenko

Renormalization

Bogolyubov-Parasiuk Theorem: In any local quantum field theory to get the UV finite S-matrix one has to introduce local counter terms to the Lagrangian in each order of perturbation theory - R-operation

$$\mathcal{L} \Rightarrow \mathcal{L} + \Delta\mathcal{L}$$

BPHZ R-operation $$RG = (1-K)R'G$$

In renormalizable case this is equivalent to the operation of multiplication by a renormalization constant Z

$$Z = 1 - \sum_i KR'G_i$$

In non-renormalizable case the BP theorem is still valid and the counter terms are also local (at maximum are polynomial over momenta)

Kazakov,18

- Multiplication operation is replaced by acting of an operator $Z \to \hat{Z}$

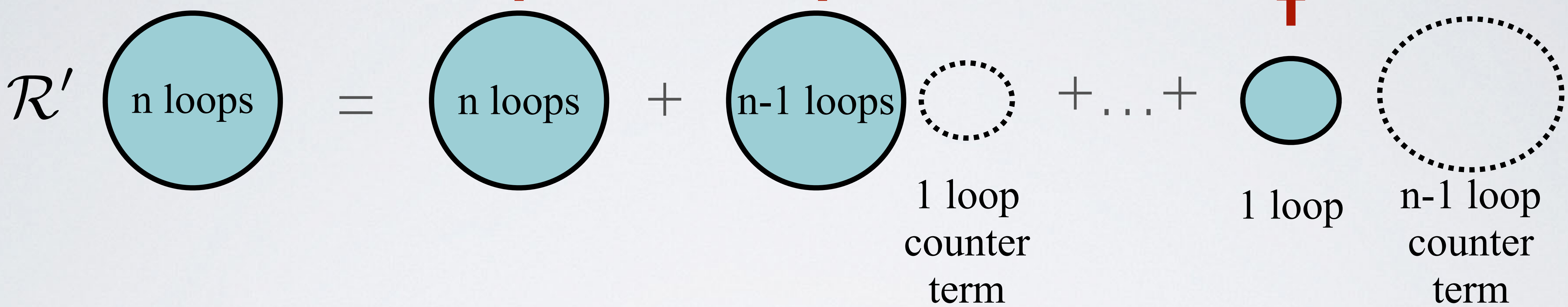
$\hat{Z}$ is a function (polynomial) of momenta (s,t,u for the 4-point case) and/or the fields

BPHZ R-operation

Locality:

$$\mathcal{R}'G_n = \frac{A_n^{(n)}(\mu^2)^{n\epsilon}}{\epsilon^n} + \frac{A_{n-1}^{(n)}(\mu^2)^{(n-1)\epsilon}}{\epsilon^n} + \ldots + \frac{A_1^{(n)}(\mu^2)^{\epsilon}}{\epsilon^n}$$

$+$ lower pole terms



$A_k^{(n)}(\mu^2)^{k\epsilon}$ terms appear after subtraction of (n-k) loop counter terms

Statement: $R'G_n$ is local, i.e. terms like $\log^k \mu^2/\epsilon^m$ should cancel for any k and m

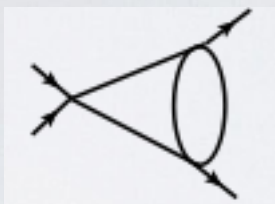
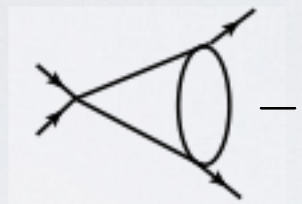
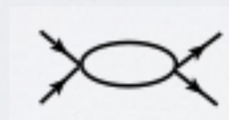
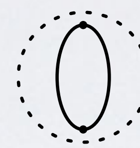
Consequence: $A_n^{(n)} = (-1)^{n+1}\frac{A_1^{(n)}}{n}$

The leading divergences are governed by 1 loop diagrams!

Two loop example

ϕ^4_4

 $= \left(\frac{A_2^{(2)}}{\epsilon^2} + \frac{A_1^{(2)}}{\epsilon} \right) (\frac{\mu^2}{s})^{2\epsilon}$

$\mathcal{R}'$  $=$  $-$   $= \left(\frac{A_2^{(2)}}{\epsilon^2} + \frac{A_1^{(2)}}{\epsilon} \right) (\frac{\mu^2}{s})^{2\epsilon} - \frac{A_1^{(1)}}{\epsilon} (\frac{\mu^2}{s})^{\epsilon} \frac{A_1^{(1)}}{\epsilon}$

$$= \frac{A_2^{(2)}}{\epsilon^2} - \frac{(A_1^{(1)})^2}{\epsilon^2} + 2\frac{A_2^{(2)}}{\epsilon} \log(\mu^2/s) - \frac{(A_1^{(1)})^2}{\epsilon} \log(\mu^2/s) \quad = -\frac{1}{2}\frac{(A_1^{(1)})^2}{\epsilon^2} + \ldots$$

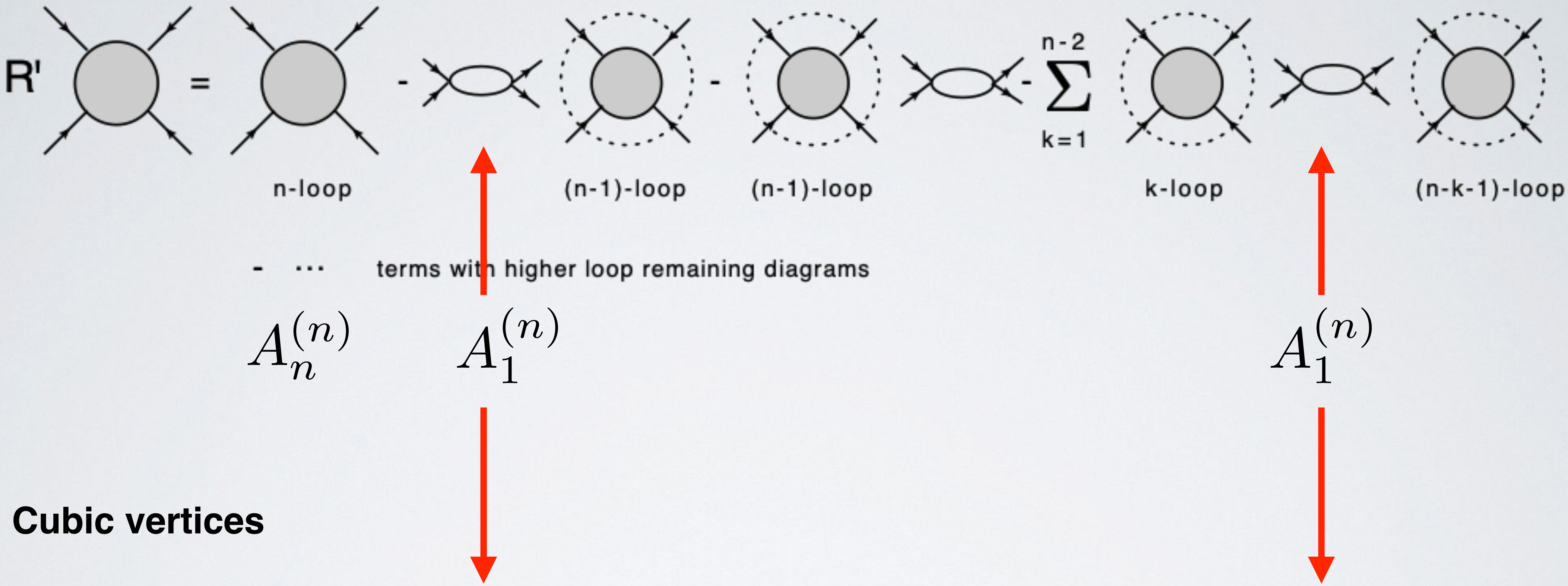
non-local terms to be cancelled

Leading divergence is given by the one-loop term $A_2^{(2)} = \frac{1}{2}(A_1^{(1)})^2$

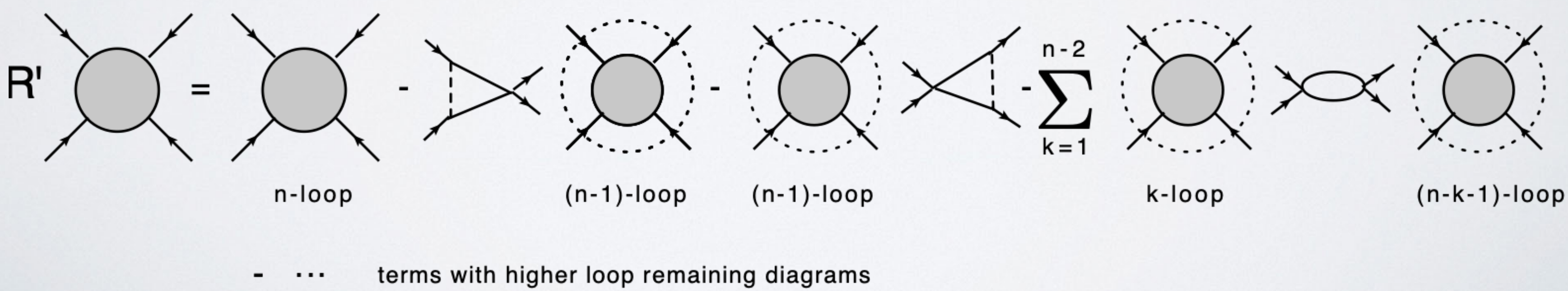
- These statements are universal and are valid in non-renormalizable theories as well.
- The only difference is that the counter term $A_1^{(1)}$ depends on kinematics and has to be integrated through the remaining one-loop graph.
- As a result $A_2^{(2)}$ is not the square of $A_1^{(1)}$ anymore but is the integrated square .
- This last statement is the general feature of any QFT irrespective of renormalizability

Leading divergences in Scattering Amplitudes

Quartic vertices

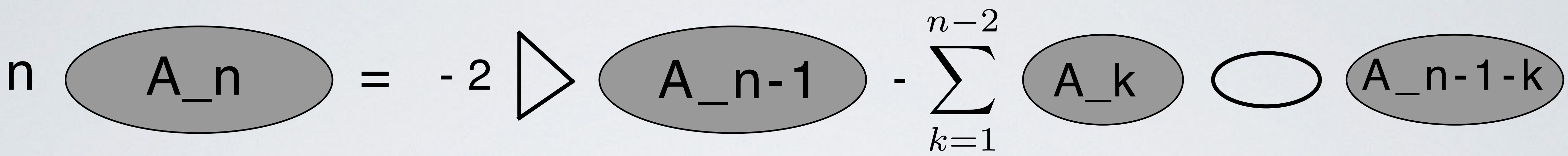


Cubic vertices



The Recurrence Relation for the Scattering Amplitude

Kazakov,20



- This is the general recurrence relation that reflects the locality of the counter terms in any theory
- In renormalizable theories A_n is a constant and this relation is reduced to the algebraic one
- In non-renormalizable theories A_n depends on kinematics and one has to integrate through the one loop diagrams

Taking the sum $\sum_n A_n(-z)^n = A(z)$ one can transform the recurrence relation into integro-diff equation

$$\frac{d}{dz}A(z) = b_0\{-1 - 2\int_\triangle A(z) - \int_\bigcirc A^2(z)\}$$

$$\frac{d}{dz} = \frac{d}{d\log\mu^2}$$

This is the generalized RG equation valid in any (even non-renormalizable) theory!

Examples:

- Maximally supersymmetric gauge theory in D=6,8,10 dimensions SYM_D
- Scalar field theory in D=4,6,8,10 dimensions ϕ^4_D
- Gauge theory in D=4,6,8 dimensions YM
- Supersymmetric Wess-Zumino model with quartic superpotential in D=4 Φ^4_4

These are the toy models for (super) gravity - our aim

SYM_D

Perturbation Expansion for the 4-point Amplitudes for any D

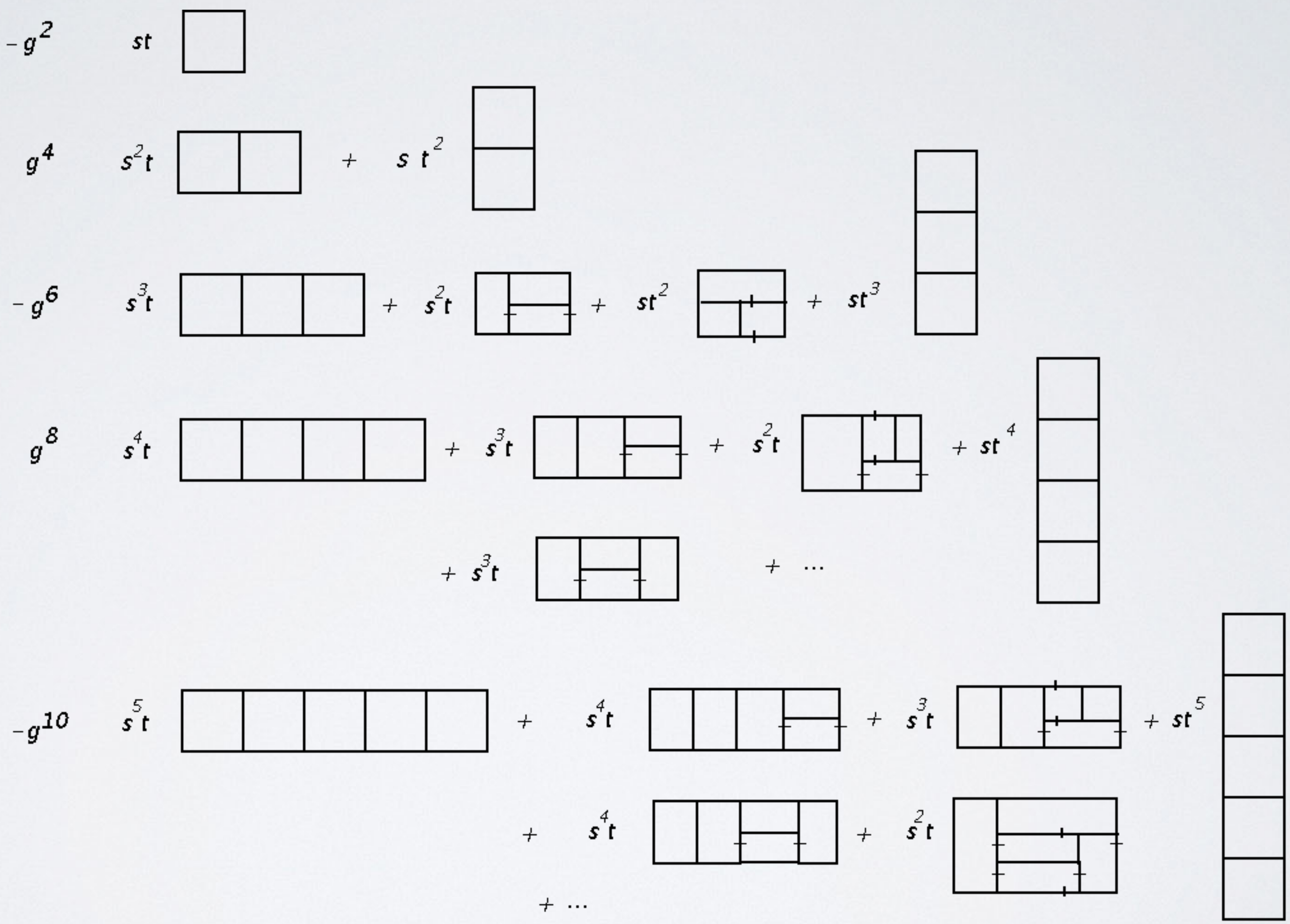
T. Dennen Yu-yin Huang 10 ,
S.Caron-Huot D.O'Connell 10

$$A_4/A_4^{tree}$$

No bubbles
No Triangles

First UV div at
L=[6/(D-4)] loops

IR finite



1

2

4

15

60

Universal expansion for any D in maximal SYM due to Dual conformal invariance

Bork,Kazakov,Kompaneets,Vlasenko, 13

Borlakov,,Kazakov,Tolkachev,Vlasenko, 15

SYM_D

D=6 N=2

S-channel $S_n(s,t)$ **T-channel** $T_n(s,t)$ $T_n(s,t) = S_n(t,s)$

Exact all-loop recurrence relation $S_3 = -s/3,\ T_3 = -t/3$

$$nS_n(s,t) = -2s \int_0^1 dx \int_0^x dy\ (S_{n-1}(s,t') + T_{n-1}(s,t'))$$

$n \geq 4$

$t' = t(x-y) - sy$

D=8 N=1

S-channel $S_n(s,t)$ **T-channel** $T_n(s,t)$ $T_n(s,t) = S_n(t,s)$

Exact all-loop recurrence relation $S_1 = \frac{1}{12},\ T_1 = \frac{1}{12}$

$$nS_n(s,t) = -2s^2 \int_0^1 dx \int_0^x dy\ y(1-x)\ (S_{n-1}(s,t') + T_{n-1}(s,t'))|_{t'=tx+yu}$$

$$+ \quad s^4 \int_0^1 dx\ x^2(1-x)^2 \sum_{k=1}^{n-2} \sum_{p=0}^{2k-2} \frac{1}{p!(p+2)!} \frac{d^p}{dt'^p}(S_k(s,t') + T_k(s,t')) \times$$

$$\times \frac{d^p}{dt'^p}(S_{n-1-k}(s,t') + T_{n-1-k}(s,t'))|_{t'=-sx}\ (tsx(1-x))^p$$

RG Equation

SYM_D

D=6 N=2

$$\Sigma(s,t,z) = z^{-2}\sum_{n=3}^{\infty}(-z)^n S_n(s,t)$$

$$\frac{d}{dz}\Sigma(s,t,z) = s - \frac{2}{z}\Sigma(s,t,z) + 2s\int_0^1 dx \int_0^x dy\ (\Sigma(s,t',z)+\Sigma(t',s,z))|_{t'=xt+yu}$$

Linear equation

D=8 N=1

$$\Sigma(s,t,z) = \sum_{n=1}^{\infty}(-z)^n S_n(s,t)$$

$$\frac{d}{dz}\Sigma(s,t,z) = -\frac{1}{12} + 2s^2\int_0^1 dx \int_0^x dy\ y(1-x)\ (\Sigma(s,t',z)+\Sigma(t',s,z))|_{t'=tx+yu}$$
$$-s^4\int_0^1 dx\ x^2(1-x)^2\sum_{p=0}^{\infty}\frac{1}{p!(p+2)!}\left(\frac{d^p}{dt'^p}(\Sigma(s,t',z)+\Sigma(t',s,z))|_{t'=-sx}\right)^2\ (tsx(1-x))^p.$$

Non-linear equation

The Scalar theory example

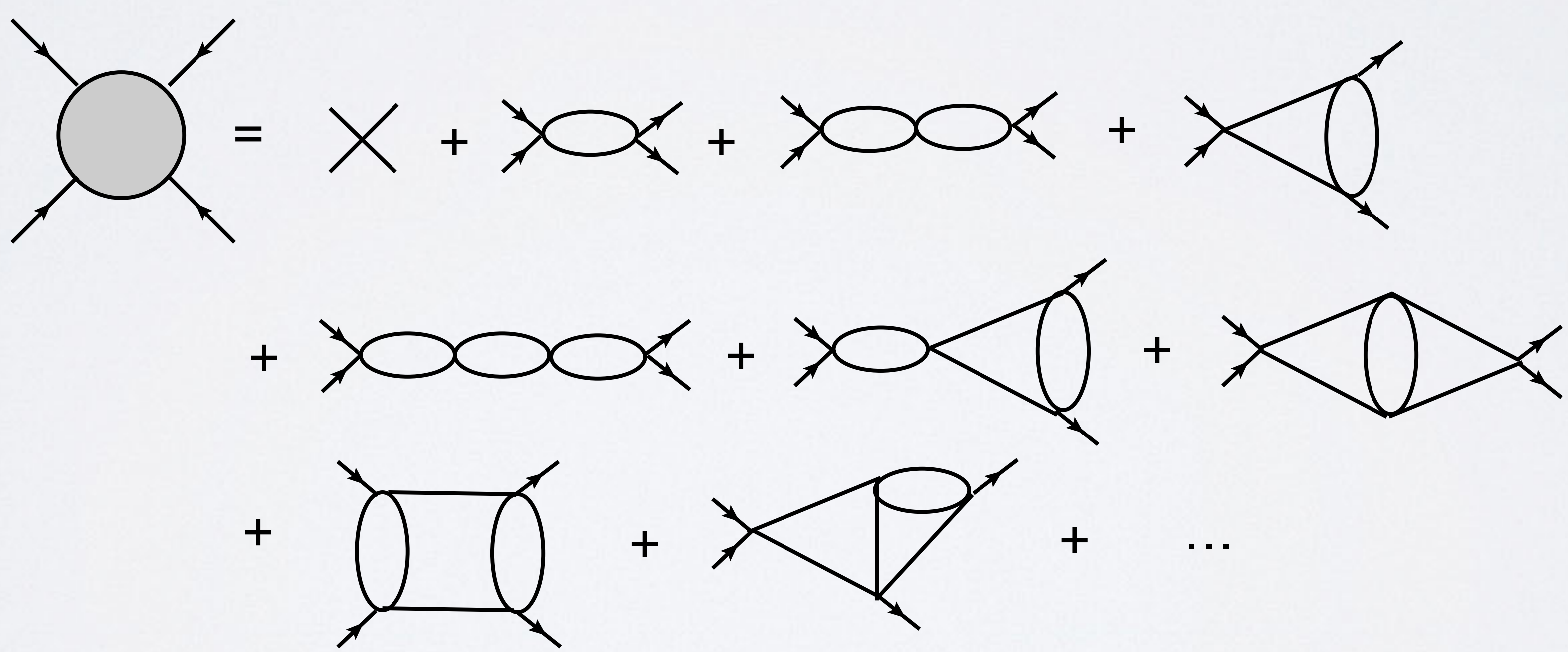
ϕ^4_D $\quad D = 4, 6, 8, 10 \quad [\lambda] = 2 - D/2$

Kazakov,19

2->2 scattering amplitude on shell $\quad m = 0 \quad s + t + u = 0$

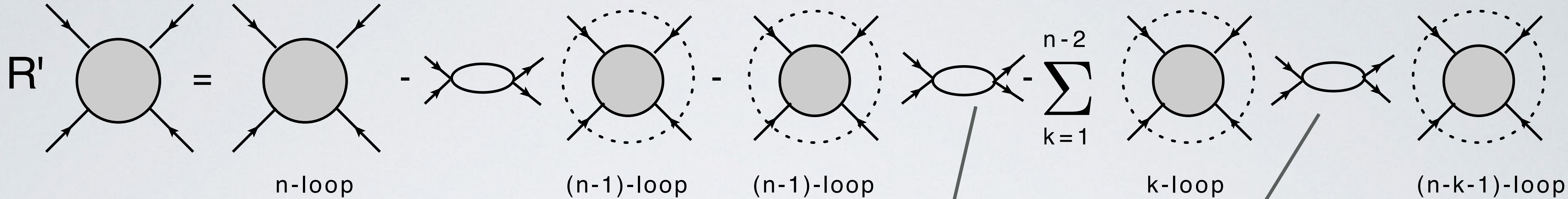
$$\Gamma_4(s,t,u) = \lambda(1 + \Gamma_s(s,t,u) + \Gamma_t(s,t,u) + \Gamma_u(s,t,u))$$

PT: $$\Gamma_s = \sum_{n=1}^{\infty}(-z)^n S_n, \quad \Gamma_t = \sum_{n=1}^{\infty}(-z)^n T_n, \quad \Gamma_u = \sum_{n=1}^{\infty}(-z)^n U_n, \quad z \equiv \frac{\lambda}{\epsilon}$$

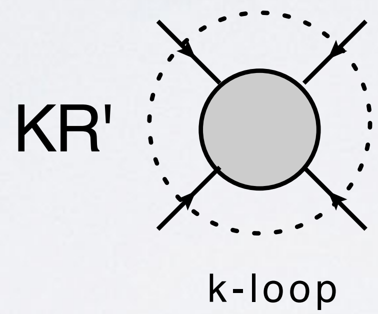


PT expansion (only s-channel is shown)

Recurrence Relations for the Leading Poles



- ⋯ terms with higher loop remaining diagrams



$$= \frac{A_k^{(k)'}}{\epsilon^k} = (-1)^{k+1}\frac{A_k^{(k)}}{\epsilon^k}$$

$$\begin{aligned} nS_n(s,t,u) \;&=\; \frac{s^{D/2-2}}{\Gamma(D/2-1)}\int_0^1 dx[x(1-x)]^{D/2-2}\left(S_{n-1}(s,t',u')+T_{n-1}(s,t',u')+U_{n-1}(s,t',u')\right) \\ &+\; \frac{1}{2}\frac{s^{D/2-2}}{\Gamma(D/2-1)}\int_0^1 dx[x(1-x)]^{D/2-2}\sum_{k=1}^{n-2}\sum_{p=0}^{(D/2-2)k}\sum_{l=0}^{p}\frac{1}{p!(p+D/2-2)!}\times \\ &\times\; \frac{d^p}{dt'^l du'^{p-l}}(S_k+T_k+U_k)\frac{d^p}{dt'^l du'^{p-l}}(S_{n-k-1}+T_{n-k-1}+U_{n-k-1})s^p[x(1-x)]^p t^l u^{p-l} \end{aligned}$$

$$t' = -xs,\; u' = -(1-x)s$$

Differential Equation

Summing up the recurrence relation $\sum_{n=2}^{\infty}(-z)^n$ one gets the diff equation

$$-\frac{d\Gamma_s(s,t,u)}{dz} = \frac{1}{2}\frac{\Gamma(D/2-1)}{\Gamma(D-2)}s^{D/2-2} \qquad \Gamma_s(z=0)=0$$

$$+\frac{s^{D/2-2}}{\Gamma(D/2-1)}\int_0^1 dx[x(1-x)]^{D/2-2}\,[\Gamma_s(s,t',u')+\Gamma_t(s,t',u')+\Gamma_u(s,t',u')]\Big|_{\substack{t'=-xs,\\ u'=-(1-x)s}}$$

$$+\quad \frac{1}{2}\frac{s^{D/2-2}}{\Gamma(D/2-1)}\int_0^1 dx[x(1-x)]^{D/2-2}\sum_{p=0}^{\infty}\sum_{l=0}^{p}\frac{1}{p!(p+D/2-2)!}\times$$

$$\times\quad \left(\frac{d^p}{dt'^l du'^{p-l}}(\Gamma_s+\Gamma_t+\Gamma_u)\Big|_{\substack{t'=-xs,\\ u'=-(1-x)s}}\right)^2 s^p[x(1-x)]^p t^l u^{p-l}$$

$$\frac{d\Gamma_s(s,t,u)}{d\log\mu^2} = -\frac{\lambda}{2}\frac{s^{D/2-2}}{\Gamma(D/2-1)}\int_0^1 dx[x(1-x)]^{D/2-2}\sum_{p=0}^{\infty}\sum_{l=0}^{p}\frac{1}{p!(p+D/2-2)!}\times$$

$$\times\left(\frac{d^p\bar{\Gamma}_4(s,t',u')}{dt'^l du'^{p-l}}\Big|_{\substack{t'=-xs,\\ u'=-(1-x)s}}\right)^2 s^p[x(1-x)]^p t^l u^{p-l}$$

$$\Gamma_s(\log\mu^2=0)=0$$

- YM_D Both cubic and quartic vertices

Equation is more complicated but has the same main features

- Wess-Zumino modern in D=4

$$\mathcal{L} = \int d^2\theta d^2\bar\theta\ \bar\Phi\Phi + \int d^2\bar\theta\ \frac{g}{4!}\bar\Phi^4 + \int d^2\theta\ \frac{g}{4!}\Phi^4,$$

$$C =< \Phi\Phi\Phi\Phi >, \quad \bar C =< \bar\Phi\bar\Phi\bar\Phi\bar\Phi >, \quad M =< \bar\Phi\bar\Phi\Phi\Phi > . \qquad C = CS + CT + CU, \quad M + MS + MT + MU$$

RG Equations

$$\frac{dCS}{dz} = sg^2 MS \otimes (CS + CT + CU)\,,$$

$$\frac{dMS}{dz} = \frac{1}{2}[sg^2(MS \otimes MS + MT \otimes MT + MU \otimes MU)$$
$$+\bar CS \otimes CS + \bar CT \otimes CT + \bar CU \otimes CU],$$

$$A(s,t,u) \otimes B(s,t,u) = \int_0^1 dx \sum_{p=0}^{\infty} \sum_{l=0}^{p} \frac{1}{p!p!} \frac{d^p}{dt'^l du'^{p-l}} A(s,t',u') \frac{d^p}{dt'^l du'^{p-l}} B(s,t',u')|_{\substack{t' = -xs, \\ u' = -(1-x)s}} \quad s^p[x(1-x)]^p t$$

Solution of RG Equations - Genaral Case

$$\frac{d}{dz}A(z) = b_0\{-1 - 2\int_\Delta A(z) - \int_\bigcirc A^2(z)\}$$

In the r.h.s. one has a second degree polynomial:

- Two real roots - solution is an exponent (decreasing or increasing depending on a theory and kinematics) SYM_6

- Degenerate real root - solution with a pole at low (Asymptotic Freedom) or high (Zero Charge) energies depending on a kinematics ϕ^4_D

- Two complex roots - solution with infinite number of periodic poles in both directions SYM_8

Solution of RG equation

D=8 N=1

Borlakov,Kazakov,Tolkachev,Vlasenko, 16

Horizontal ladder

Diff equation

$$\frac{d}{dz}\Sigma_A = -\frac{1}{3!} + \frac{2}{4!}\Sigma_A - \frac{2}{5!}\Sigma_A^2$$

$$z = g^2 s^2/\epsilon$$

$$\Sigma_A(z) = -\sqrt{5/3}\frac{4\tan(z/(8\sqrt{15}))}{1-\tan(z/(8\sqrt{15}))\sqrt{5/3}} = \sqrt{10}\frac{\sin(z/(8\sqrt{15}))}{\sin(z/(8\sqrt{15})-z_0)}$$

$$\Sigma(z) = -(z/6 + z^2/144 + z^3/2880 + 7z^4/414720 + \ldots)$$

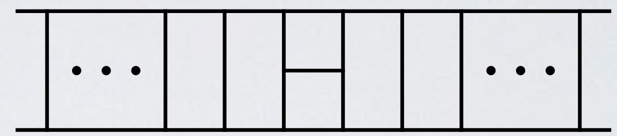
$$z_0 = \arcsin(\sqrt{3/8})$$

infinite number of poles

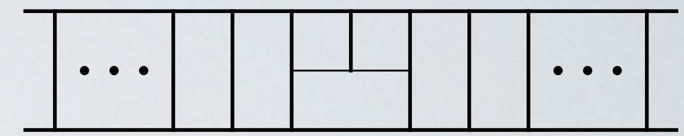
In general case - numerical solution similar to the ladder approximation possessing infinite number of poles in both directions

Solution of RG equation D=6 N=2

Horizontal ladder + tennis court



Ladder



Lddder 2

$$\Sigma_L(s,z) = \frac{2}{s^2z^2}(e^{sz} - 1 - sz - \frac{s^2z^2}{2})$$

$$\Sigma_{L2} = \frac{1}{2s^2z^2}\left[27(e^{z/3} - 1 - \frac{z}{3} - \frac{1}{2}\frac{z^2}{9} - \frac{1}{6}\frac{z^3}{27})(1+2\frac{t}{s}) - (e^z - 1 - sz - \frac{1}{2}z^2 - \frac{1}{6}z^3)\right]$$

In general case - numerical solution similar to the ladder approximation

$$\Sigma_s + \Sigma_t \sim e^{(s+t)z}$$

$$z \to \infty \qquad \begin{aligned} s+t=-u>0, &\quad \Sigma \to \infty \\ s+u=-t>0, &\quad \Sigma \to \infty \\ t+u=-s<0, &\quad \Sigma \to const \end{aligned}$$

Effective Potential in Scalar Theory

Generating functional for Green functions

$$Z(J) = \int \mathcal{D}\phi \ \exp\left(i \int d^4x \ \mathcal{L}(\phi, d\phi) + J\phi\right)$$

$$W(J) = -i \log Z(J)$$ IPI generating functional

Effective action

$$\Gamma(\phi) = W(J) - \int d^4x J(x)\phi(x)$$ Legendre transformation

$$e^{i\Gamma(\Phi)} = \int \mathcal{D}\hat{\Phi} e^{i(S[\Phi+\hat{\Phi}]-\hat{\Phi}S'[\Phi])}$$

Shifted Classical action

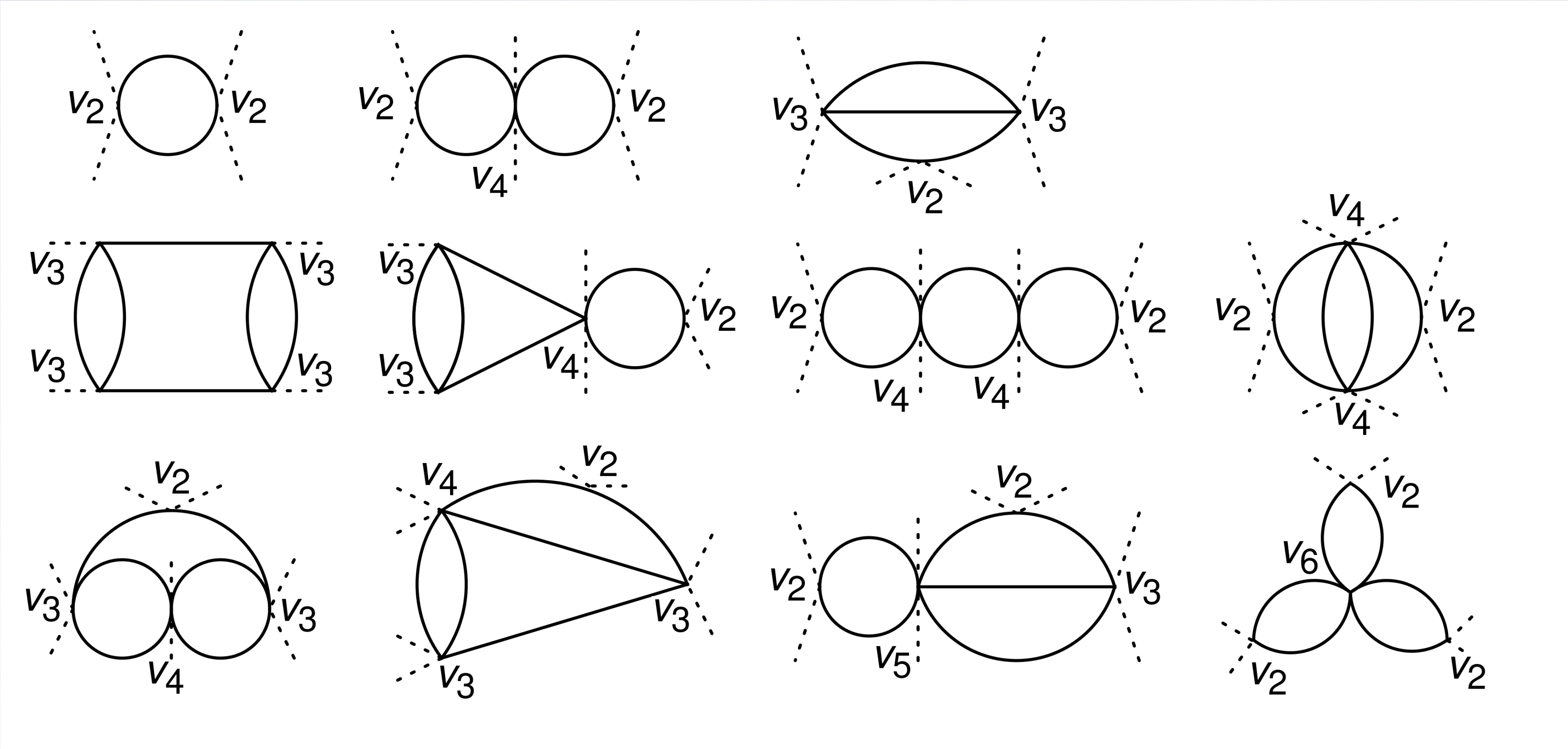
$$S[\Phi + \hat{\Phi}] = S[\Phi] + \hat{\Phi}S'[\Phi] + \frac{1}{2}\hat{\Phi}^2 S''[\Phi] + \frac{1}{3!}\hat{\Phi}^3 S'''[\Phi] + \ldots$$

Classical external field — Field dependent mass — Interaction vertex

Effective Potential in Scalar Theory

V_{eff} Is the sum of all vacuum 1PI diagrams

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - gV_0(\phi)$$



$$v_2(\phi) \equiv \frac{d^2 V_0(\phi)}{d\phi^2}$$

$$v_n \equiv d^n V_0 / d\phi^n$$

Shown are UV divergent vacuum diagrams in arbitrary scalar theory up to three loops

$$V_{eff} = g \sum_{n=0}^{\infty} (-g)^n V_n .$$

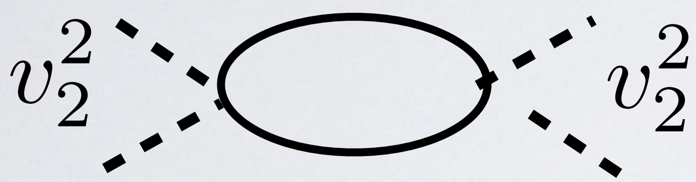
Divergent terms and Logs

General scalar field theory in D=4

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - gV_0(\phi)$$

UV divergences in dimensional regularisation

$$D = 4 - 2\epsilon$$



$$Diag \sim \frac{1}{\epsilon} v_2^2 (\frac{\mu^2}{m^2})^\epsilon \to v_2^2 (\frac{1}{\epsilon} - \log \frac{m^2}{\mu^2}), \quad m^2 = g v_2^2$$

UV divergence

One loop

$$\Delta V_1 = \frac{g^2}{16\pi^2} v_2^2 \log \frac{g v_2}{\mu^2}$$

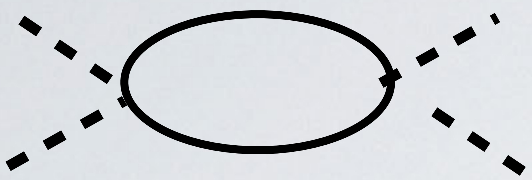
ϕ^4

$$\Delta V_1 = \frac{g^2}{16\pi^2} \phi^4 \log \frac{g\phi^2}{\mu^2}$$

ϕ^6

$$\Delta V_1 = \frac{g^2}{16\pi^2} \phi^8 \log \frac{g\phi^4}{\mu^2}$$

Divergences and Log ϕ behaviour



$$Diag \sim \frac{1}{\epsilon} v_2^2 (\frac{\mu^2}{m^2})^\epsilon \to v_2^2 (\frac{1}{\epsilon} - \log \frac{m^2}{\mu^2}), \quad m^2 = g v_2^2$$

The leading divergences ⟷ The leading logs

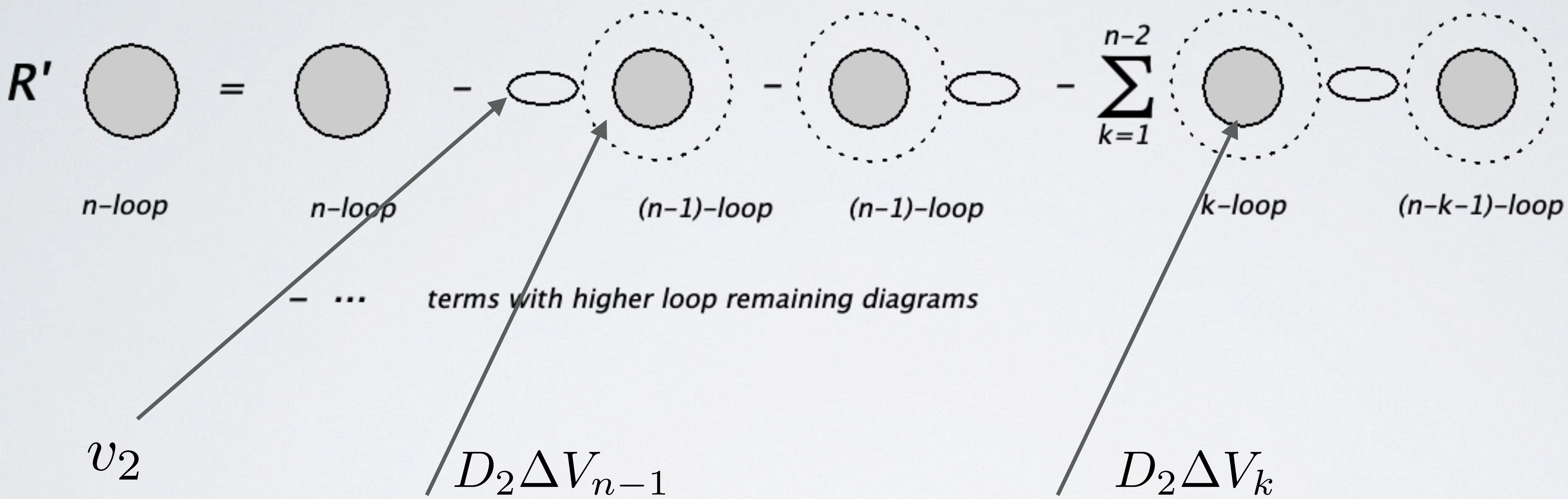
- In non-renormalizable theories divergences cannot be absorbed into the renormalization of the couplings and fields.
- If they are subtracted some way one is left with <u>infinite</u> arbitrariness.
- Coefficients of the leading divergences (logs) <u>do not depend</u> on this arbitrariness !

The aim is to calculate the leading divergences ~ $\frac{1}{\epsilon^n}$ in n-th order of PT

Recurrence relations for the leading poles

Kazakov, Iakhibbaev, Tolkachev 22

Action of R'-operation on divergent diagram



$$n\Delta V_n = \frac{1}{2}v_2 D_2\Delta V_{n-1} + \frac{1}{4}\sum_{k=1}^{n-2} D_2\Delta V_k D_2\Delta V_{n-1-k}, \quad n \geq 2 \qquad \Delta V_1 = \frac{1}{4}v_2^2$$

$$n\Delta V_n = \frac{1}{4}\sum_{k=0}^{n-1} D_2\Delta V_k D_2\Delta V_{n-1-k}, \quad n \geq 1, \quad \Delta V_0 = V_0$$

RG pole equation for arbitrary potential

$$\Sigma(z,\phi) = \sum_{n=0}^{\infty} (-z)^n \Delta V_n(\phi) \qquad\qquad z = \frac{g}{\epsilon}$$

Kazakov, Iakhibbaev, Tolkachev 22

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RG pole equation

$$\frac{d\Sigma}{dz} = -\frac{1}{4}(D_2\Sigma)^2 \qquad\qquad \Sigma(0,\phi) = V_0(\phi)$$

This a non-linear partial differential equation!

Effective potential

$$V_{eff}(g,\phi) = g\Sigma(z,\phi)|_{z\to -\frac{g}{16\pi^2}\log g v_2/\mu^2}. \qquad\qquad v_2(\phi) \equiv \frac{d^2V_0(\phi)}{d\phi^2}$$

Example I: Power like Potential

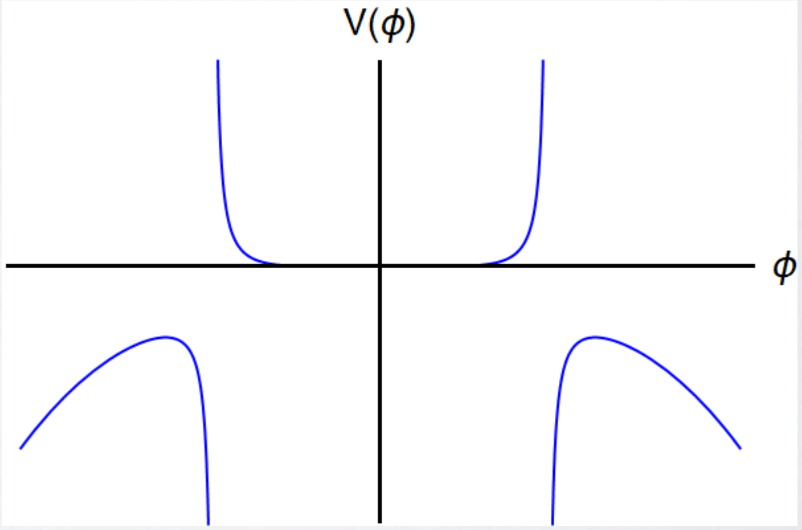
$$gV_0(\phi) = g\frac{\phi^p}{p!} \qquad y = g\phi^{p-4} \qquad \Sigma(z,\phi) = \frac{\phi^p}{p!} f(z\phi^{p-4})$$

$$f'(y) = -\frac{1}{4p!}\left[p(p-1)f(y) + (p-4)(3p-5)yf'(y) + (p-4)^2 y^2 f''(y)\right]^2$$

$$f(0) = 1,\, f'(0) = -\frac{1}{4}\frac{p(p-1)}{(p-2)!}$$

p=4

$$f'(y) = -\frac{3}{2}f(y)^2 \qquad f(y) = \frac{1}{1+\frac{3}{2}y}$$



$$V_{eff}(\phi) = \frac{g\phi^4/4!}{1 - \frac{3}{2}\frac{g}{16\pi^2}\log\left(\frac{g\phi^2}{2\mu^2}\right)}.$$

Example I: Power like Potential

p>4

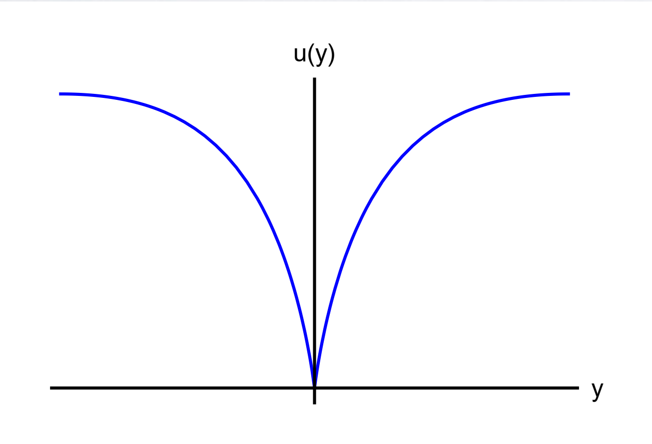
$$gV_0(\phi) = g\frac{\phi^p}{p!}$$

$$f(y) = u(y)/y$$

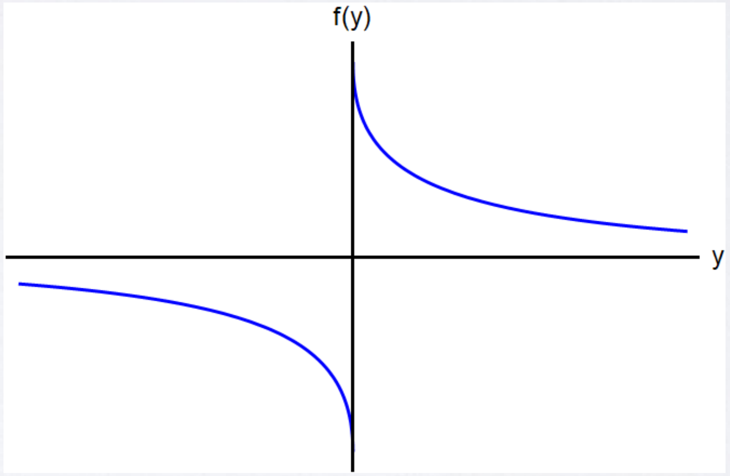
$$yu'(y) - u(y) = -\frac{1}{4p!}[12u(y) + (p-4)(p+3)yu'(y) + (p-4)^2y^2u''(y)]^2$$

$$u(\pm 0) = 0, u'(\pm 0) = \pm 1$$

Discontinuity at y=0



$y = 0$

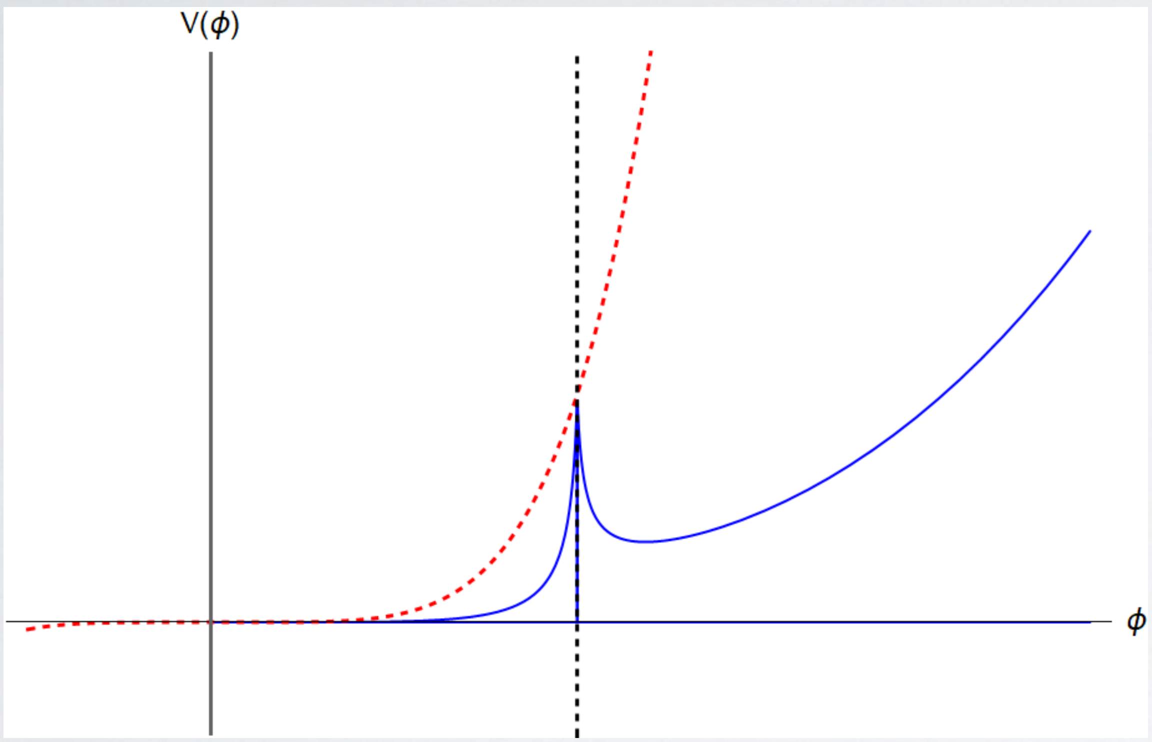


$y = 0$

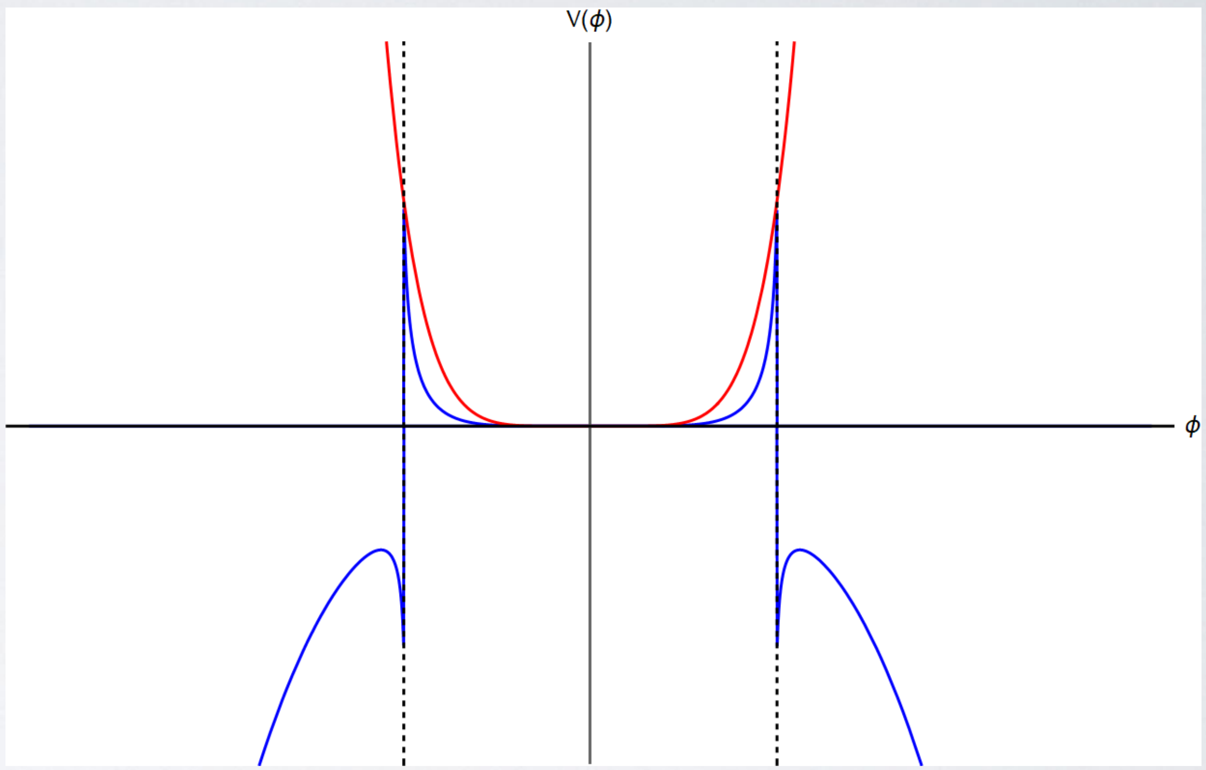
$$y \to -\frac{g}{16\pi^2}\phi^{p-4}\log\frac{g\phi^{p-2}}{\mu^2/(p-2)!}$$

Example I: Power like Potential

p=5



p=6



- Finite gap instead of an infinite barrier as for p=4
- Metastability of the quantum state
- No new minima

α -attractor Inflaton Potential

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Inflaton action with hyperbolic geometry

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{2}R(g) + \frac{1}{2}\frac{\partial_\mu\phi\partial^\mu\phi}{1-\frac{\phi^2}{6\alpha}} - V(\phi)\right]$$

Transition to the standard kinetic term

$$\partial\phi/\sqrt{1-\frac{\phi^2}{1-6\alpha}} = \partial\varphi \qquad \phi = \sqrt{6\alpha}\tanh\left(\frac{\varphi}{\sqrt{6\alpha}}\right)$$

Inflaton action of α -attractor model

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{2}R(g) + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V\left(\sqrt{6\alpha}\tanh\left(\frac{\varphi}{\sqrt{6\alpha}}\right)\right)\right].$$

T- model

$$gV_T(\varphi) = g\tanh^n\left(\frac{\varphi}{\sqrt{6\alpha}M_{Pl}}\right)$$

n=2 T_2 - model

RG Equation for the T-model Effective potential

$$\frac{d\Sigma}{dz} = -\frac{1}{4}(D_2\Sigma)^2$$

Dimensionless variables

$$x = z/M_{Pl}^4 \quad y = \tanh^n(\varphi/\sqrt{6\alpha}M_{Pl})$$

$$\Sigma(z/M_{Pl}^4, \tanh^n(\varphi/\sqrt{6\alpha}M_{Pl})) \equiv S(x,y)$$

$$S_x = -\frac{n^2 y^{2-\frac{4}{n}}\left(y^{2/n}-1\right)^2}{144\alpha^2}\left(\left(y^{2/n}+n\left(y^{2/n}-1\right)+1\right)S_y + ny\left(y^{2/n}-1\right)S_{yy}\right)^2$$

This is a nonlinear partial differential equation!

Boundary conditions

$$S(0,y) = y,\ S(x,1) = 1,\ S_y(x,1) = 0.$$

n=2 case

$$S_x = -\frac{(y-1)^2\left((3y-1)S_y + 2(y-1)yS_{yy}\right)^2}{36\alpha^2}$$

Numerical Solution for T_2 - model

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ArXiv: 2308.03872

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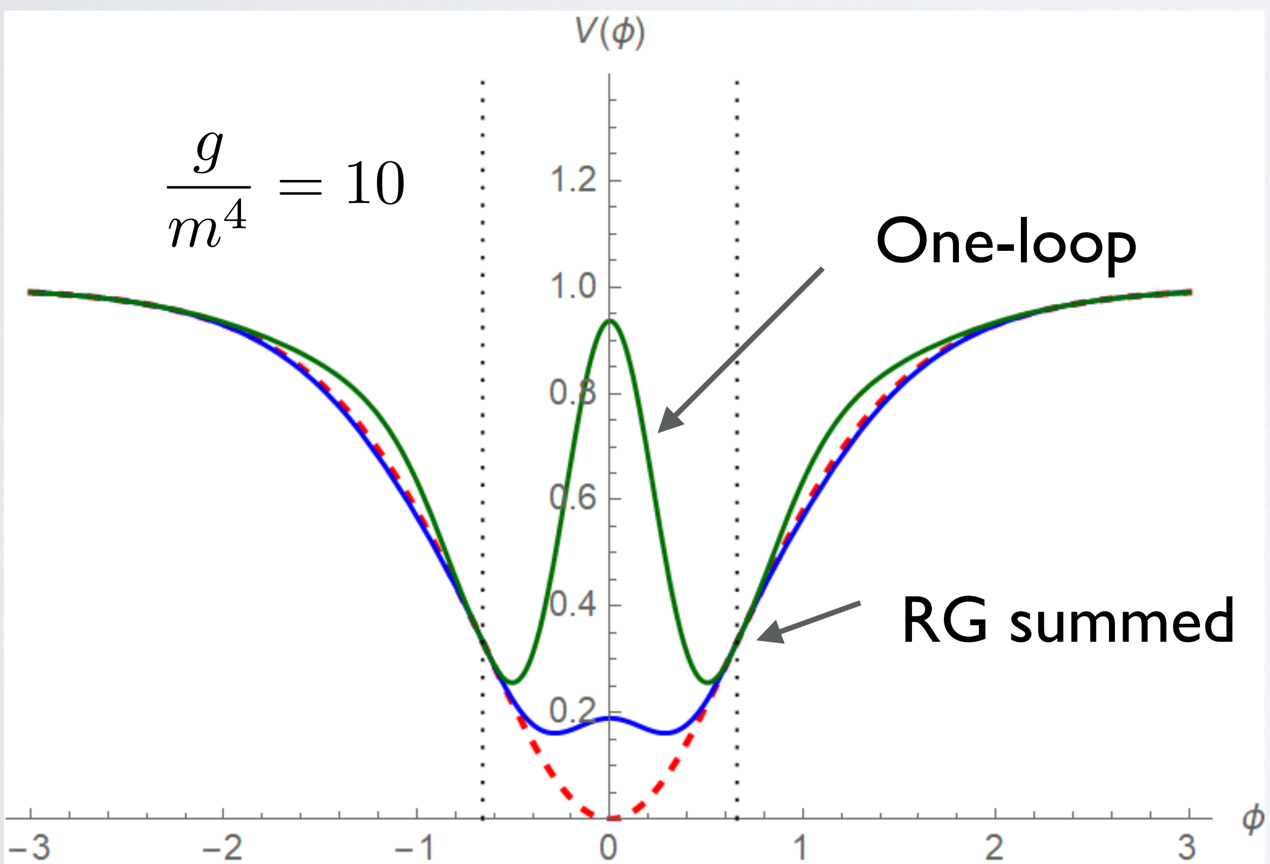
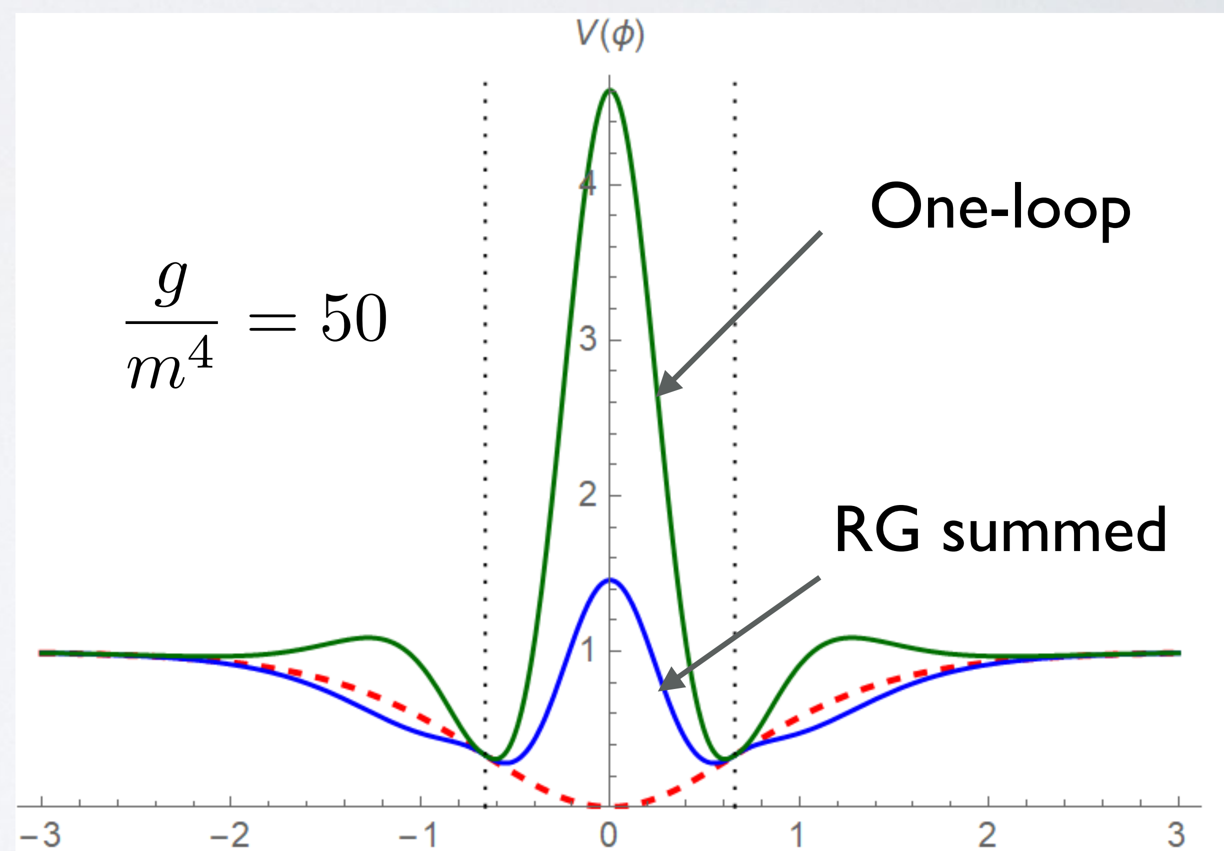
$$gV_0 = g\tanh^2(\phi/m)$$

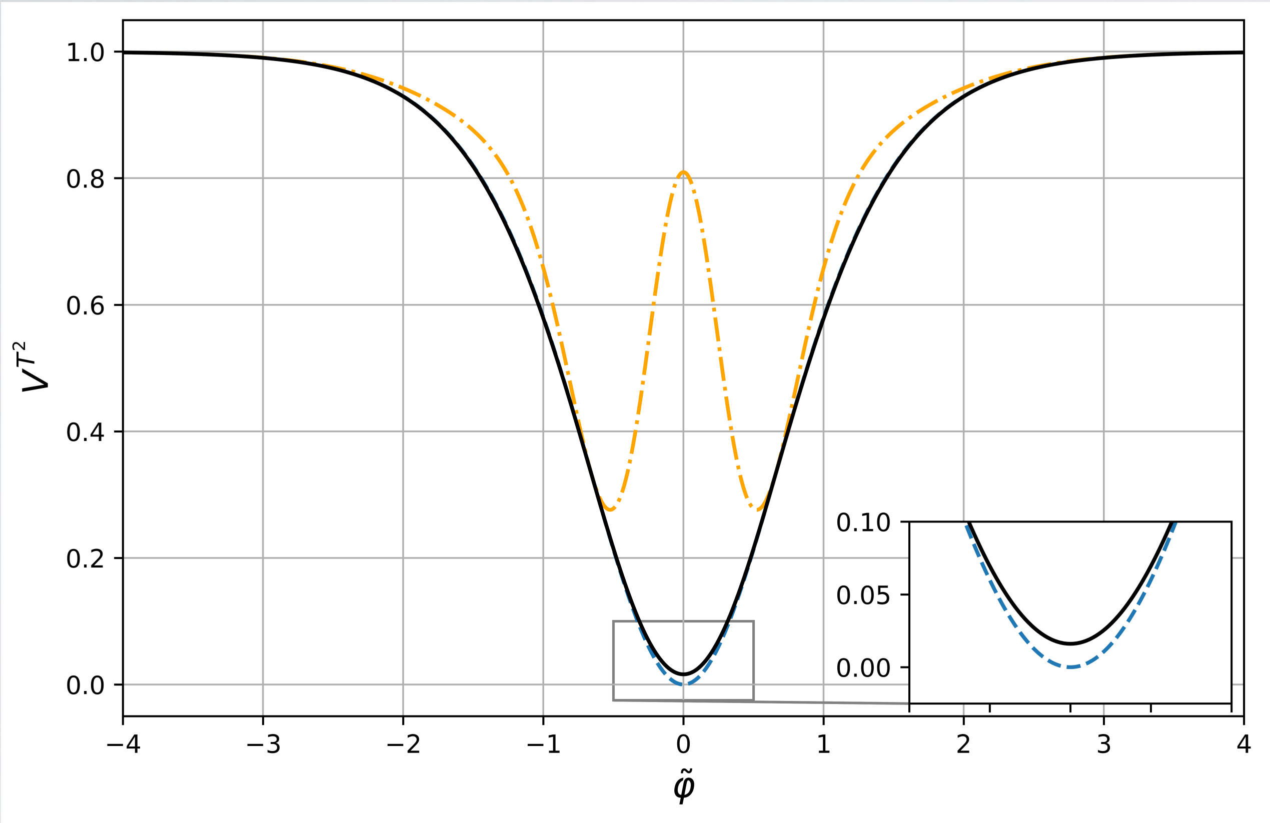
$$V_{eff}(g,\phi) = g\Sigma(z,\phi)|_{z\to -\frac{g}{16\pi^2}\log g v_2/\mu^2} \cdot \qquad v_2(\phi) \equiv \frac{d^2V_0(\phi)}{d\phi^2}$$





- Peak at the origin
- Additional minima

Lift of the Potential at the Minima - Origin of the Cosmological Constant



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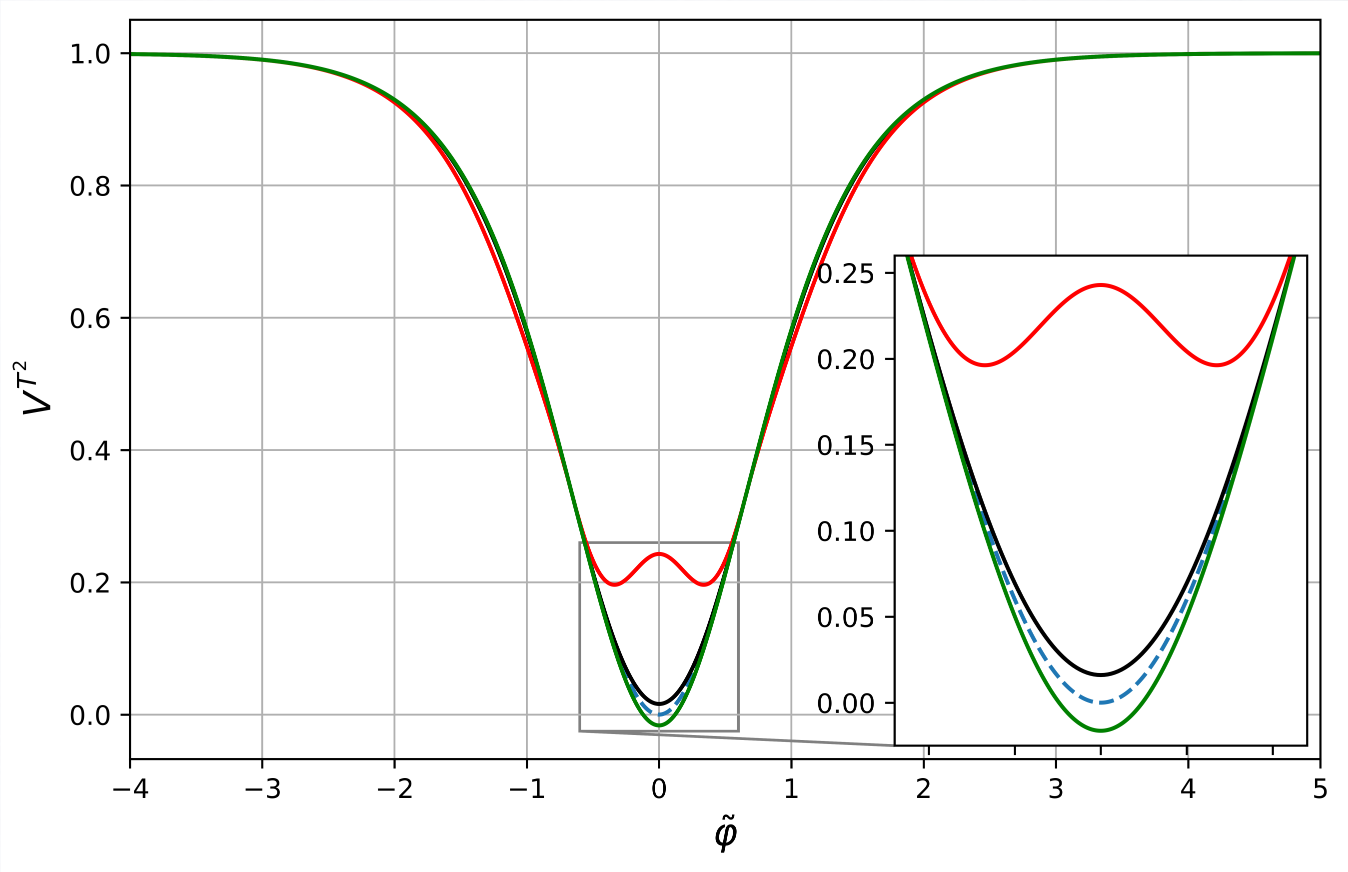
ArXiv: 2405.18818

Comparison of the classical T2-model potential (blue dashed line), the one-loop correction (orange dashed line), and the RG summed potential (black solid line) for $g \sim 1, \mu < M_{PL}$

T2-model potential: variation of μ . The classical potential (blue dashed line), the RG summed potential (solid lines) for

$\mu < M_{Pl}$ black line, $\mu \ll M_{Pl}$ red line, $\mu > M_{Pl}$ green line

$$g = 2, \ \alpha = 1$$



Estimation of the value of the Cosmological Constant

One-loop Potential

$$V_{eff} = V_0 + \frac{g^2}{16\pi^2}\frac{v_2^2}{4}\log\frac{gv_2(\varphi)}{\mu^2}$$

Cosmological constant

$$\Lambda = \left[\frac{g^2}{16\pi^2}\frac{v_2^2}{4}\log\left(\frac{gv_2}{\mu^2}\right)\right]\Bigg|_{\varphi=\varphi_{vac}}$$

μ - dependent

Inverse formula

$$\mu^2 = \frac{g}{3\alpha M_{Pl}^2} e^{-576\pi^2 \frac{\alpha^2}{g^2}\Lambda M_{Pl}^4}$$

Numerical Estimation

$$g = 10^{-10} M_{Pl}^4, \quad M_{Pl} = (8\pi G)^{-\frac{1}{2}}, \quad \alpha = 1$$

$\Lambda \sim 10^{-120} M_{Pl}^4 \quad \mu \approx 10^{-6} M_{Pl}$ - Inflaton mass

Conclusion on Effective potential

- The effective potential in the LL approximation obeys the RG master equation which is a partial non-linear differential equation
- This generalised RG equation is valid for any potential including non-renormalizable one
- In some cases this equation is simplified to the ordinary differential one and can be solved at least numerically.
- The effective potential has a minima at the origin or can have additional minima depending on the free scale parameter μ .
- This formalism is applicable to inflation cosmology potentials like the model of α -attractors T2
- At the minima the potential is lifted due to radiative correction so that the cosmological constant appears
- Properly choosing the parameters of the potential one can get the observable value of the cosmological constant

General Resume

- The UV divergences in non-renormalizable theories are local and can be removed by local counter terms like in renormalizable ones
- The main difference is that the renormalization constant Z depends on kinematics and acts like an operator rather than simple multiplication
- Based on locality of the counter terms due to the Bogoliubov-Parasiuk theorem one can construct the recurrence relations that define all loop divergences starting from one loop
- The recurrence relations can be converted into the generalized RG equations just like in renormalizable theories
- The RG equations allow one to sum up the leading (subleading, etc) divergences in all loops and define the high-energy/field behaviour