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Bogoliubov Laboratory of Theoretical Physics

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adiative corrections
divergences
egularization
enormalization
enormalization group

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BLTP JINR

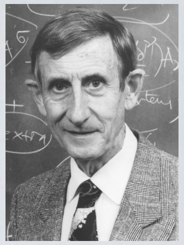
21 September 2023

Quantum Electrodynamics - The first Quantum Field Theory



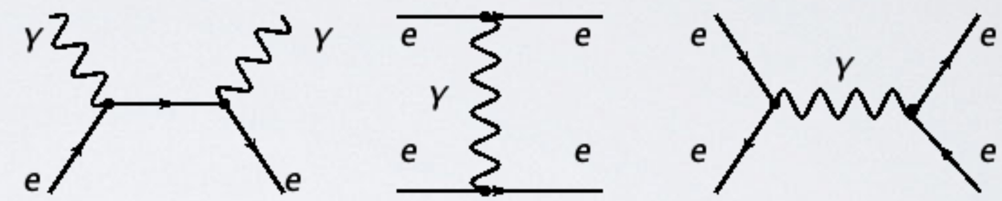

R. Feynman S.Tomonaga J. Shwinger

Nobel Prize motivation: “for their fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles”

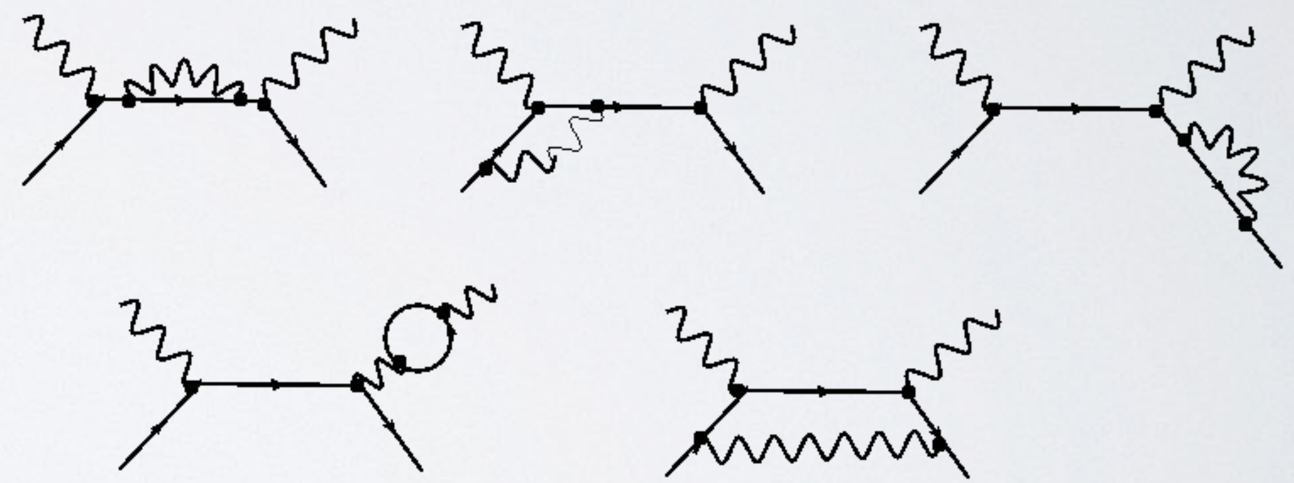


F. Dyson

- Covariant Feynman Rules

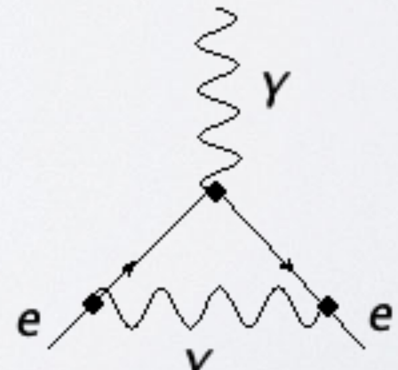


- The first UV divergences: electron self energy, vacuum polarization
- The idea of renormalization
- Multiplicative renormalization - Dyson transformations

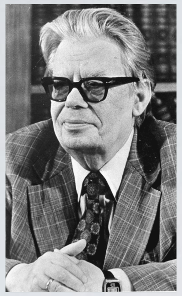


$$\psi \to \sqrt{Z_2}\psi, \quad A \to \sqrt{Z_3}A, \quad e \to \sqrt{Z_3}e$$

- QED - renormalizable quantum field theory
- First radiative corrections: anomalous magnetic moment of electron $a_e = \frac{\alpha}{2\pi}$



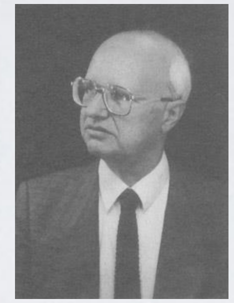
$\mathcal{R}$enormalization operation



N. Bogoliubov

O. Parasyuk

K. Hepp



W. Zimmerman

- BPHZ $\mathcal{R}$-operation — the basis for obtaining finite expressions for the Green functions and S-matrix elements in generic quantum field theory

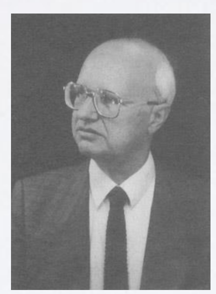
$$\mathcal{R}\mathcal{G} = (1 - K\mathcal{R}')\mathcal{G}$$

$$\mathcal{R}'\mathcal{G} = \mathcal{G} - \sum_{\gamma} K\mathcal{R}'_{\gamma}\mathcal{G}/_{\gamma} + \sum_{\gamma,\gamma'} K\mathcal{R}'_{\gamma}K\mathcal{R}'_{\gamma'}\mathcal{G}/_{\gamma\gamma'} - ...,$$

- Bogoliubov-Parasiuk theorem: In any local quantum field theory after subtraction of subdivergences the overall UV divergence is always local in coordinate space

H. Lehman

K. Symanzik



W. Zimmerman

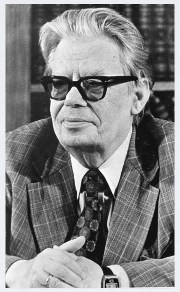
$$S^{conn}_{q_1,\ldots,q_n;p_1,\ldots,p_{n'}} = \frac{1}{c^n (c^*)^{n'}} \prod_{k=1}^{n} \frac{q_k^2 - m^2_{phys}}{i} \prod_{j=1}^{n} \frac{p_j^2 - m^2_{phys}}{i} G^{conn}_{n+n'}(p_1, \ldots, p_{n'}; -q_1, \ldots, -q_{n-1})$$

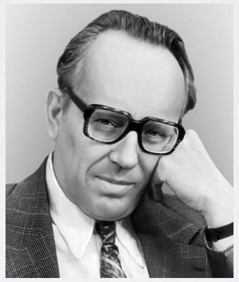
- LSZ reduction formula shows how to obtain the S-matrix from time-ordered Green functions. It is fundamental to practical calculations of scattering processes.

Renormalization group

Normalization group generated by infinitesimal operators P_i connected with renormalization of the coupling constant e

E. Stuekelberg A. Peterman





M. Gell-Mann F. Low N. Bogoliubov D. Shirkov C. Callan K. Symanzik

$$d(\frac{k^2}{\lambda^2}, e_2^2) = \frac{d_c(k^2/m^2, e_1^2)}{d_c(\lambda^2/m^2, e_1^2)}, \quad e_2^2 = e_1^2 d_c(\lambda^2/m^2, e_1^2)$$

$$d(x, y; e^2) = d(t, y; e^2) d(\frac{x}{t}, \frac{y}{t}; e^2 d(t, y; e^2))$$

$$\log x = \int_{e^2}^{e^2 d} \frac{dy}{\psi(y)} \qquad \psi(e^2) = \frac{d(e^2 d)}{d \log x}|_{x=1}$$

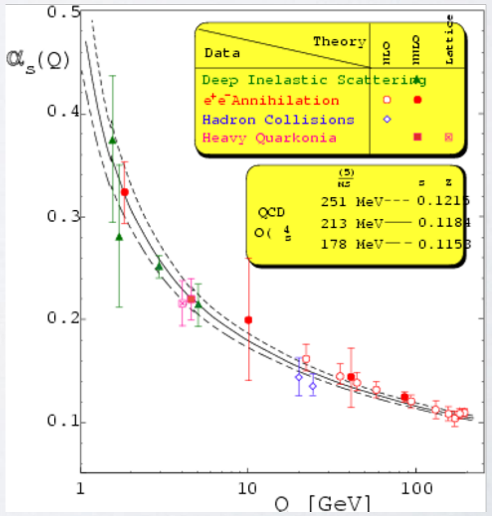
$$\left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} - \beta(y, \alpha)\frac{\partial}{\partial \alpha}\right) \bar{\alpha}(x, y, \alpha) = 0$$

Summation of the leading asymptotic (leading logs) with the help of the RG equations

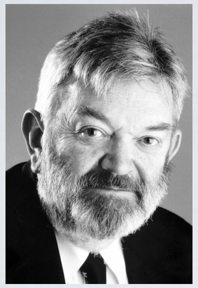
Running coupling

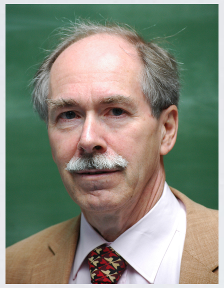
$$\frac{\partial \bar{\alpha}(x, y; \alpha)}{\partial \log x} = \beta(\frac{y}{x}, \bar{\alpha}(x, y; \alpha))$$

Asymptotic freedom of the strong coupling



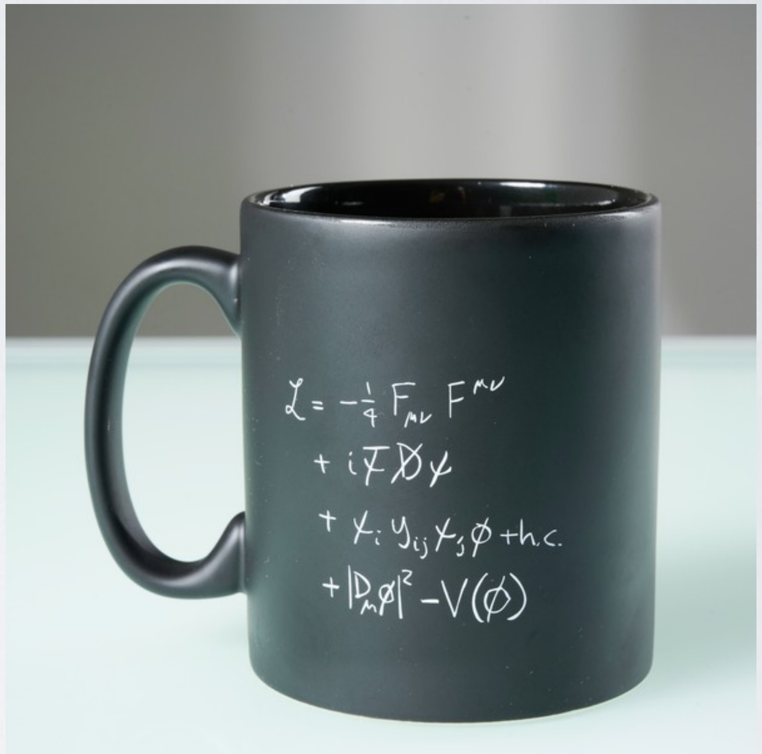
Renormalizability in Quantum Field Theory



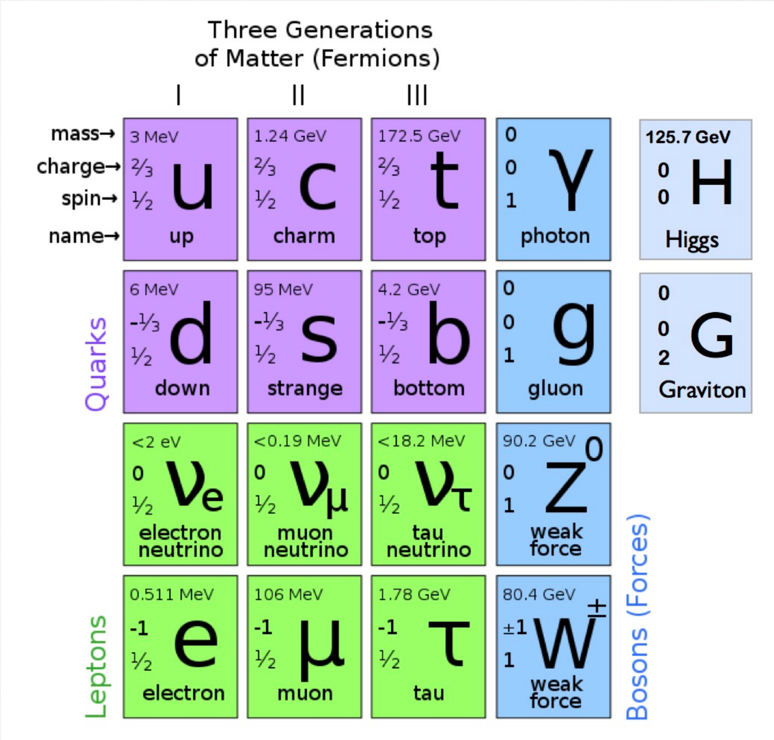


M.Veltman G. ’t Hooft

Proof of renormalizability of spontaneously broken Yang-Mills theory



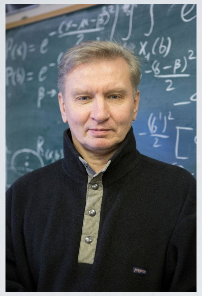
Gauge choice	Unitarity v	Renormalizability
$$G_{\mu\nu}(k) = -i\frac{g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}}{k^2 - M^2}$$	−	+
$$G_{\mu\nu}(k) = -i\frac{g^{\mu\nu} - \frac{k^\mu k^\nu}{M^2}}{k^2 - M^2}$$	+	−
$$G_{\mu\nu}(k) = -i\frac{g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2 - \xi M^2}(1-\xi)}{k^2 - M^2}$$	+	+



The Standard Model is a renormalizable gauge quantum field theory!

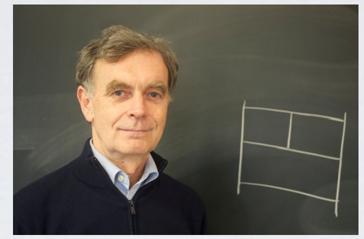
$$SU(3) \times SU(2) \times U(1)$$

$\mathcal{R}^*$-operation



F. Tkachov

K. Chetyrkin



V. Smirnov

$$R^* = \tilde{R}R$$

UV	IR
$\Delta(\gamma)$ UV counterterm P_γ Contraction of subgraph Locality in coordinate space R-operation	$\tilde{\Delta}(\gamma)$ IR counterterm $\tilde{P}_{\tilde{\gamma}}$ Removal of subgraph Locality in momentum space $\tilde{\text{R}}$-operation

R* is a generalisation of the R-operation which includes the IR divergent subgraphs
- new powerful technique to calculate multiloop Feynman diagrams

Non-renormalizable interactions

- The Standard Model is renormalizable
- Gravity is not renormalizable
- All cosmological models are non-renormalizable

Non-renormalizable theories are not accepted due to:

- UV divergences are not under control - infinite number of new types of divergences
- The amplitudes increase with energy (in PT) and violate unitarity

<u>However:</u>

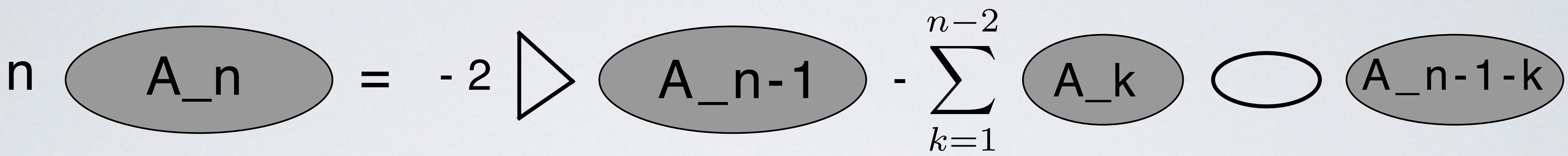
- R-operation equally works for NR theories and leads to local counter terms
- Due to locality all higher order divergences are related to the lower ones

- These properties allow one to write down the RG equations for the scattering amplitudes, effective potential, etc which sum up the leading divergences (logarithms) and to find out the high energy/field behaviour

The Recurrence Relation

Leading terms

Kazakov,20



- This is the general recurrence relation that reflects the locality of the counter terms in any theory
- In <u>renormalizable</u> theories A_n is a constant and this relation is reduced to the algebraic one
- In <u>non-renormalizable</u> theories A_n depends on kinematics and one has to integrate through the one loop diagrams

Taking the sum $\sum_n A_n(-z)^n = A(z)$ one can transform the recurrence relation into <u>integro-diff</u> equation

$$\frac{d}{dz}A(z) = b_0\{-1 - 2\int_{\triangle} A(z) - \int_{\bigcirc} A^2(z)\} \qquad \frac{d}{dz} = \frac{d}{d\log\mu^2}$$

This is the generalized RG equation valid in any (even non-renormalizable) theory!

RG Equation

SYM_D

D=6 N=2

$$\Sigma(s,t,z) = z^{-2}\sum_{n=3}^{\infty}(-z)^n S_n(s,t)$$

$$\frac{d}{dz}\Sigma(s,t,z) = s - \frac{2}{z}\Sigma(s,t,z) + 2s\int_0^1 dx \int_0^x dy\ (\Sigma(s,t',z)+\Sigma(t',s,z))|_{t'=xt+yu}$$

Linear equation

Bork,Kazakov,Kompaneets,Vlasenko, 13

D=8 N=1

$$\Sigma(s,t,z) = \sum_{n=1}^{\infty}(-z)^n S_n(s,t)$$

$$\frac{d}{dz}\Sigma(s,t,z) = -\frac{1}{12} + 2s^2\int_0^1 dx \int_0^x dy\ y(1-x)\ (\Sigma(s,t',z)+\Sigma(t',s,z))|_{t'=tx+yu}$$
$$-s^4\int_0^1 dx\ x^2(1-x)^2\sum_{p=0}^{\infty}\frac{1}{p!(p+2)!}\left(\frac{d^p}{dt'^p}(\Sigma(s,t',z)+\Sigma(t',s,z))|_{t'=-sx}\right)^2\ (tsx(1-x))^p.$$

Borlakov,,Kazakov,Tolkachev,Vlasenko, 15

Non-linear equation

Examples:

- Maximally supersymmetric gauge theory in D=6,8,10 dimensions SYM_D
- Scalar field theory in D=4,6,8,10 dimensions ϕ^4_D
- Gauge theory in D=4,6,8 dimensions YM
- Supersymmetric Wess-Zumino model with quartic superpotential in D=4 Φ^4_4

Based on:
Phys. Lett. B734 (2014) 111, arXiv:1404.6998 [hep-th]
JHEP 11 (2015) 059, arXiv:1508.05570 [hep-th]
JHEP 12 (2016) 154, arXiv:1610.05549v2 [hep-th]
Phys.Rev. D95 (2017) no.4, 045006 arXiv:1603.05501 [hep-th]
Phys.Rev. D97 (2018) no.12, 125008, arXiv:1712.04348 [hep-th],
Phys.Lett. B786 (2018) 327-331, arXiv:1804.08387 [hep-th]
Symmetry 11 (2019) 1, 104, arXhiv:1812.11084 [hep-th]
Phys.Lett.B 797 (2019) 134801, arXiv:1904.08690 [hep-th]
Труды Мат. Инст. им. В.А. Стеклова, 2020, т. 308, с. 1–8
JHEP 06 (2022) 141, arXiv:2112.03091 [hep-th]

These are the toy models for (super) gravity - our aim

Effective Potential in Arbitrary Scalar Theory in D=4

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$$V_{eff}(g,\phi)=\sum_{k=0}^{\infty}(-g)^k V_k(\phi)$$

$V_0(\phi)$ - Classical potential

RG pole equation (in the leading log approximation)

Kazakov, Tolkachev, Iakhibbaev 22

$$\frac{d\Sigma}{dz}=-\frac{1}{4}(D_2\Sigma)^2 \qquad \Sigma(0,\phi)=V_0(\phi)$$

This a non-linear partial differential equation!

This equations sums the leading logs in all orders of PT!

Effective potential

$$V_{eff}(g,\phi)=g\Sigma(z,\phi)|_{z\to-\frac{g}{16\pi^2}\log gv_2/\mu^2}. \qquad v_2(\phi)\equiv\frac{d^2V_0(\phi)}{d\phi^2}$$

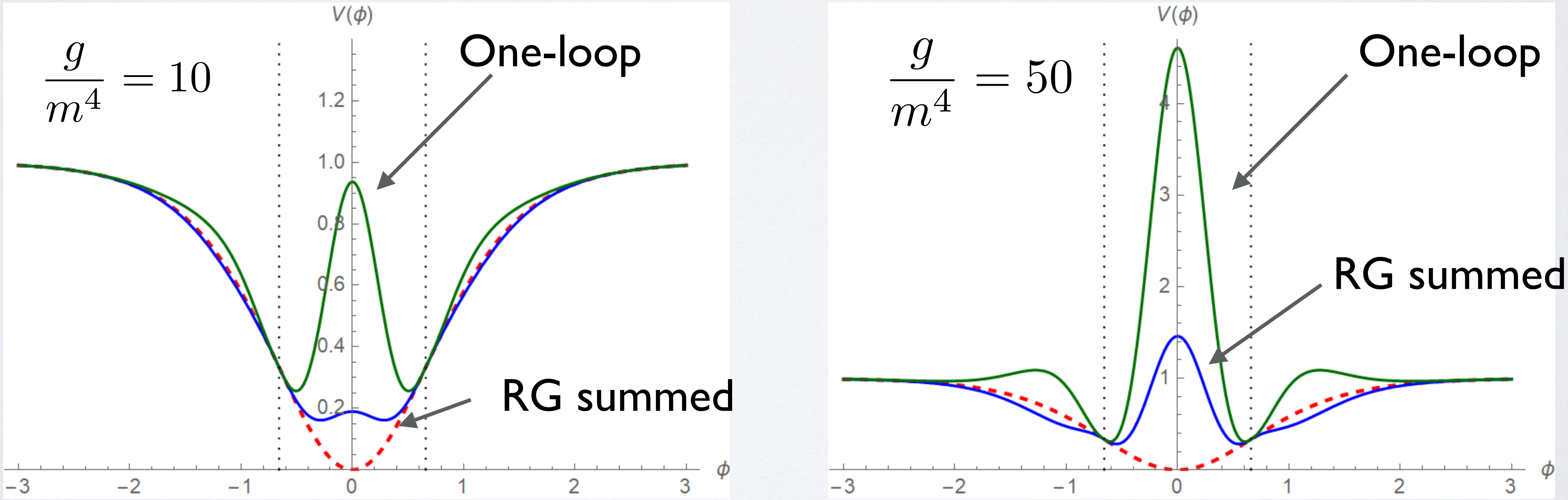
Example III: Inflation Potential

Kazakov, Tolkachev, Iakhibbaev 23

$$gV_0 = g\tanh^2(\phi/m)$$

$$\frac{d\Sigma}{dz} = -\frac{1}{4}\left(\frac{d^2\Sigma}{d\phi^2}\right)^2 \qquad \Sigma\left(\frac{z}{m^4},\frac{\phi}{m}\right) \qquad \Sigma|_{\phi\to\infty} \to 1 \quad \Sigma'|_{\phi\to\infty} \to 0 \qquad z = \frac{g}{\epsilon}$$

$$V_{eff}(g,\phi) = g\Sigma(z,\phi)|_{z\to -\frac{g}{16\pi^2}\log g v_2/\mu^2}. \qquad v_2(\phi) \equiv \frac{d^2V_0(\phi)}{d\phi^2}$$



- Peak at the origin
- Additional minima

General Resume

- The UV divergences in non-renormalizable theories are local and can be removed by local counter terms like in renormalizable ones

- The main difference is that the renormalization constant Z depends on kinematics and acts like an operator rather than simple multiplication

- Based on locality of the counter terms due to the Bogoliubov-Parasiuk theorem one can construct the recurrence relations that define all loop divergences starting from one loop

- The recurrence relations can be converted into the generalized RG equations just like in renormalizable theories

- The RG equations allow one to sum up the leading (subleading, etc) divergences in all loops and define the high-energy/field behaviour

General Resume

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- The UV divergences in non-renormalizable theories are local and can be removed by local counter terms like in renormalizable ones
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- The recurrence relations can be converted into the generalized RG equations just like in renormalizable theories
- The RG equations allow one to sum up the leading (subleading, etc) divergences in all loops and define the high-energy/field behaviour
- I have a suggestion how to handle the problem of infinite arbitrariness in NR theories but … this is the subject of another prize