JOINT INSTITUTE FOR NUCLEAR RESEARCH
1956

Bogoliubov Laboratory of Theoretical Physics

Amplitudes in the Wess-Zumino Model with Quartic Interaction

Нет войне!

D.Kazakov

In collaboration with L.Bork

BLTP JINR

JOINT INSTITUTE FOR NUCLEAR RESEARCH
1956

Bogoliubov Laboratory of Theoretical Physics

Amplitudes in the Wess-Zumino Model with Quartic Interaction

D.Kazakov

In collaboration with L.Bork

BLTP JINR

Introduction

- R-operation equally works for NR theories and leads to local counter terms
- Due to locality all higher order divergences are related to the lower ones

- These properties allow one to write down the RG equations for the scattering amplitudes which sum up the leading divergences (logarithms) and to find out the high energy behaviour

<u>Examples:</u>

- Maximally supersymmetric gauge theory in D=6,8,10 dimensions SYM_D
- Scalar field theory in D=4,6,8,10 dimensions ϕ^4_D

Published

- Gauge theory in D=4,6,8 dimensions YM_D

To appear

This paper

- Supersymmetric Wess-Zumino model with quartic superpotential in D=4 Φ^4_4

These are the toy models for (super) gravity - our aim

The Model

Lagrangian of the Wess-Zumino model

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta}\ \bar{\Phi}\Phi + \int d^2\bar{\theta}\ \frac{g}{4!}\Phi^4 + \int d^2\theta\ \frac{g}{4!}\bar{\Phi}^4$$

Chiral superfields:

$$\Phi(x,\theta,\bar{\theta}) \qquad \bar{\Phi}(x,\theta,\bar{\theta}) \qquad \bar{D}^2\bar{\Phi} = 0, \quad D^2\Phi = 0$$

Covariant derivatives:

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\frac{1}{2}\bar{\theta}^{\dot{\alpha}}\frac{\partial}{\partial x^{\alpha\dot{\alpha}}}, \quad \bar{D}_{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\frac{1}{2}\theta^\alpha\frac{\partial}{\partial x^{\alpha\dot{\alpha}}}$$

$$D^2 = 1/4\ D^\alpha D_\alpha$$
$$\bar{D}^2 = 1/4\ \bar{D}^{\dot{\alpha}}\bar{D}_{\dot{\alpha}}$$

Interaction in components:

$$g\ \psi\psi\phi\phi \quad \text{and} \quad g^2\phi^6$$

Amplitudes:

$$\langle\Phi\Phi\Phi\Phi\rangle, \quad \langle\bar{\Phi}\bar{\Phi}\bar{\Phi}\bar{\Phi}\rangle, \quad \langle\bar{\Phi}\bar{\Phi}\Phi\Phi\rangle.$$

Chiral C AntiChiral C̄ Mixed M

Four-point Amplitude:

$$A_4 = (\text{Polarisation factor}) \times (\text{Universal scalar function } C \text{ or } M)$$

UV divergences of the four point scattering amplitude

Amplitudes:

Chiral C
$$\langle\Phi_1\Phi_2\Phi_3\Phi_4\rangle \sim \int d^2\theta \prod_{i=1}^{4} d^4p_i\ \Phi(p_i,\theta)\ C(s,t,u,g),$$

AntiChiral $\bar{\text{C}}$
$$\langle\bar\Phi_1\bar\Phi_2\bar\Phi_3\bar\Phi_4\rangle \sim \int d^2\bar\theta \prod_{i=1}^{4} d^4p_i\ \bar\Phi(p_i,\theta)\ \bar C(s,t,u,g)$$

Mixed M
$$\langle\Phi_1\Phi_2\bar\Phi_3\bar\Phi_4\rangle \sim \int\ d^4\theta \prod_{i=1}^{2} d^4p_i\Phi(p_i,\theta)\prod_{i=3}^{4} d^4p_i\bar\Phi(p_i,\bar\theta)\ MS(s,t,u,g)$$

$$M(s,t,u,g) = MS(s,t,u,g) + MT(s,t,u,g) + MU(s,t,u,g).$$

$$MT(s,t,u,g) = MS(t,u,s,g),\ MU(s,t,u,g) = MS(u,s,t,g).$$

Perturbation expansion:

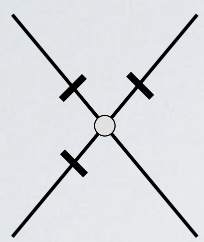
$$C(s,t,u,g) = \frac{g}{4!}\sum_{l=0} g^{2l}\ C^{(l)}(s,t,u), \quad M(s,t,u,g) = \frac{1}{4}\sum_{l=1} g^{2l}\ M^{(l)}(s,t,u).$$

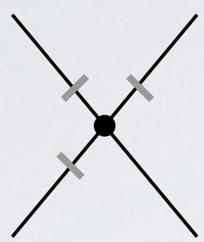
$$\begin{aligned} C^{(l)}(s,t,u) &= \qquad CS^{(2l)}(s,t,u) + CT^{(2l)}(s,t,u) + CU^{(2l)}(s,t,u), \\ M^{(l)}(s,t,u) &= MS^{(2l+1)}(s,t,u) + MT^{(2l+1)}(s,t,u) + MU^{(2l+1)}(s,t,u), \end{aligned}$$

UV divergences of the four point scattering amplitude

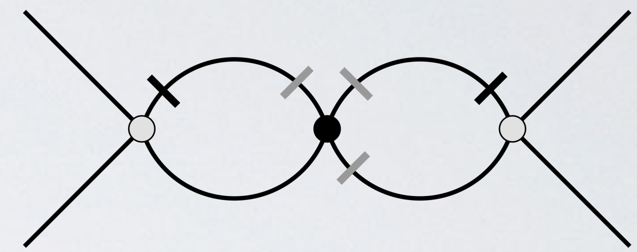
Feynman rules in superspace:

Example of chiral amplitude in two loops





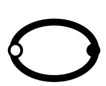
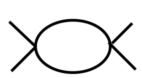
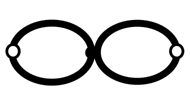
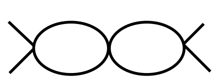
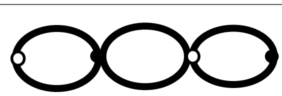
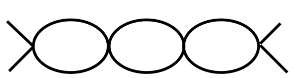
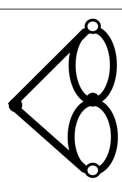
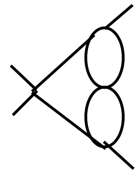
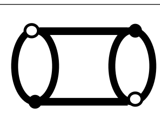
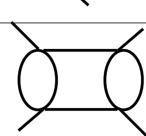
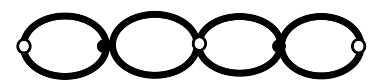
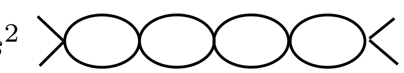
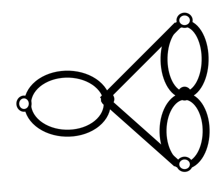
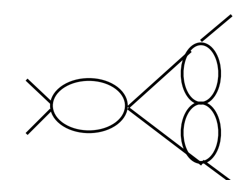
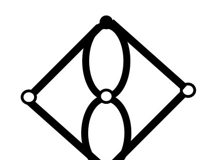
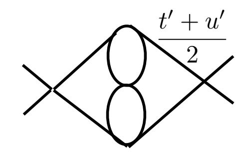
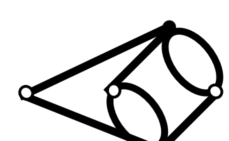
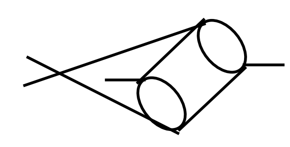
Massless propagator



Chiral vertex   AntiChiral vertex

$$\langle \Phi\bar{\Phi}\rangle = i\frac{\delta^2(\theta)\delta^2(\bar{\theta})}{p^2}, \quad \langle \Phi\Phi\rangle = 0, \quad \langle \bar{\Phi}\bar{\Phi}\rangle = 0,$$

Chiral and mixed diagrams up to four loops

Super Diagram	Scalar Diagram	Highest Pole	Comb
		$1/\epsilon$	1/2
	s 	s/ϵ^2	1/4
	s 	s/ϵ^3	1/8
	 $\frac{t'+u'}{2}$	- 1/2 $s/3/\epsilon^3$	1/2 x 2
	- $\frac{t+u}{2}$ 	1/2 $s/3/\epsilon^3$	1
	s^2 	s^2/ϵ^4	1/16
	s  $\frac{t'+u'}{2}$	-1/2 $s^2/3/\epsilon^4$	1/4
	s  $\frac{t'+u'}{2}$	-1/2 $s^2/6/\epsilon^4$	1/8
	1/4 s^2 	1/4 $s^2/12/\epsilon^4$	1

UV divergences of the four point scattering amplitude

Leading divergences (dimensional regularisation/reduction)

$$C^{(2l)}(s,t,u) = \frac{C_{2l}(s,t,u)}{\epsilon^{2l}}, \quad M^{(2l+1)}(s,t,u) = \frac{M_{2l+1}(s,t,u)}{\epsilon^{2l+1}}, \ etc.$$

$$C(s,t,u,g) = \frac{g}{4!}\left\{1 + \frac{g^2}{4}[\frac{s}{\epsilon^2} + \frac{t}{\epsilon^2} + \frac{u}{\epsilon^2}] + \frac{g^4}{32}[\frac{s^2}{\epsilon^4} + \frac{t^2}{\epsilon^4} + \frac{u^2}{\epsilon^4}] + ...\right\} = \bar{C}$$

Two loops Four loops

$$M(s,t,u,g) = \frac{1}{4}\left\{\frac{g^2}{2}[\frac{1}{\epsilon} + \frac{1}{\epsilon} + \frac{1}{\epsilon}]\right.$$ One loop

$$\left.+g^4\left[\frac{s}{8\epsilon^3} + \frac{t}{8\epsilon^3} + \frac{u}{8\epsilon^3} + \left(-\frac{s}{2}\frac{1}{3\epsilon^3} - \frac{t}{2}\frac{1}{3\epsilon^3} - \frac{u}{2}\frac{1}{3\epsilon^3}\right) + \left(\frac{s}{2}\frac{1}{3\epsilon^3} + \frac{t}{2}\frac{1}{3\epsilon^3} + \frac{u}{2}\frac{1}{3\epsilon^3}\right)\right] + ...\right\}$$

Three loops

Note peculiar cancellations on mass shell (s+t+u=0) in two and three loops

These cancellations do not lead to finite results in higher loops, however

Non-renormalisation theorems for arbitrary superpotential

Effective action is an integral over the full superspace

$$\Gamma_n[\Phi,\bar\Phi] = \int d^4\theta \prod_{i=1}^{n} d^4p_i F(\Phi(p_i,\theta),\bar\Phi(p_i,\bar\theta), D^\alpha\Phi(p_i,\theta),\bar D_{\dot\alpha}\bar\Phi(p_i),...)\ \mathcal{F}_n(p_1,\dots,p_n)$$

$d^4\theta = d^2\theta d^2\bar\theta$ local function Bosonic function

Follows from Feynman rules in N=1 superspace: Each N=1 superspace Feynman diagram is constructed from the propagators which are proportional to the full fermionic delta function $\delta^4(\theta_i-\theta_{i+1})$ and the vertices which contain supercovariant derivatives D^2 or D^2 acting on adjacent propagators, and integration over the full N=1 superspace. Using integration by parts, the covariant derivatives from the vertices can be rearranged into the combinations such as . $\delta^4(\dots)[D^2\bar D^2\delta^4(\dots)]$ which can be simplified according to the following identity

$$\delta^4(\theta_i-\theta_{i+1})[D^2\bar D^2\delta^4(\theta_i-\theta_{i+1})] = \delta^4(\theta_i-\theta_{i+1})$$

Consequence: Non-renormalization of superpotential

$$\mathcal{L} = \int d^4\theta\ \bar\Phi\Phi + \int d^2\bar\theta\ \mathcal{W}(\Phi) + \int d^2\theta\ \mathcal{W}(\bar\Phi) \qquad \mathcal{W}(\Phi) = \frac{1}{2!}m\Phi^2 + \frac{1}{3!}g\Phi^3$$

Superpotential is not renormalised since it is an integral one the chiral superspace!

This is not true for finite parts since they are non-local

Non-renormalisation theorems for arbitrary superpotential

Possible loop hole in this reasoning: $d^4\theta = d^2\theta\bar{D}^2$

$$\Gamma_n[\Phi] = \int d^2\theta\bar{D}^2\prod_{i=1}^{n} d^4p_i F(\Phi(p_i,\theta), D^\alpha\Phi(p_i,\theta), ...)\ \mathcal{F}_n(p_1,\ldots,p_n)$$

If one has covariant derivatives then one may use the relation

$$\bar{D}^2D^2(\Phi(p_1)\Phi(p_2)...) = -(p_1+p_2+...)^2(\Phi(p_1)\Phi(p_2)...)$$

and transform the integration over the full superspace into the chiral one

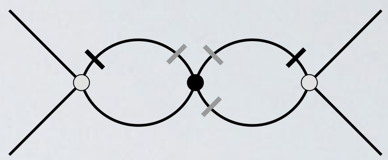
This is only possible if one has additional covariant derivative which is forbidden for the cubic superpotential on dimensional grounds. However, it becomes possible for a superpotential with dimensional couplings.

This may also happen for the finite parts which contain non-local terms, so that

$$\int d^4\theta\ f(\Phi)\frac{D^2}{-Q^2}g(\Phi) = \int d^2\theta\ f(\Phi)g(\Phi)$$

Non-renormalisation theorems for arbitrary superpotential

WZ model with quartic superpotential: two loop chiral diagram



$$\Gamma_4^{(1)} = g^3 \int \prod_{i=1}^{4} d^4\theta_i d^4 p_i \Phi_i(\theta_i)\ \delta^4(\sum_{i=1}^{4} p_i) \int \frac{d^D l_1\ \delta_{12}[\bar{D}^2 D^2 \delta_{12}]}{l_1^2 (p_{12} - l_1)^2} \int \frac{d^D l_2\ [D^2 \delta_{23}][D^2 \bar{D}^2 \delta_{23}]}{l_2^2 (p_{12} - l_2)^2}$$

$$\delta^4(\theta_i - \theta_j) \equiv \delta_{ij} \qquad p_i + p_j \equiv p_{ij}$$

Add D^2 due to negative dim of the coupling

Integration by parts

$$\Gamma_4^{(1)} = g^3 \int \prod_{i=1}^{4} d^4 p_i\ d^4\theta\ \Phi_1(\theta)\Phi_2(\theta) D^2[\Phi_3(\theta)\Phi_4(\theta)] \delta^4(\sum_{i=1}^{4} p_i) \left(\int \frac{d^D l}{l^2 (p_{12} - l)^2} \right)^2 .$$

$$\Gamma_4^{(1)} = g^3 \int \prod_{i=1}^{4} d^4 p_i\ d^2\theta\ \Phi_1(\theta)\Phi_2(\theta) \bar{D}^2 D^2[\Phi_3(\theta)\Phi_4(\theta)] \delta^4(\sum_{i=1}^{4} p_i) \left(\int \frac{d^D l}{l^2 (p_{12} - l)^2} \right)^2$$

$$\bar{D}^2 D^2 [\Phi_i(\theta)\Phi_j(\theta)] = -p_{ij}^2 \Phi_i(\theta)\Phi_j(\theta)$$

$$\Gamma_4^{(1)} = g^3 \int d^2\theta\ \prod_{i=1}^{4} d^4 p_i \Phi_i(\theta)\ \delta^4(\sum_{i=1}^{4} p_i)\ p_{34}^2 \left(\int \frac{d^D l}{l^2 (p_{12} - l)^2} \right)^2 .$$

As a result one has a divergent contribution to the chiral part of the effective action, however, not to the superpotential, but to the next term containing derivatives

BPHZ R-operation

$$\mathcal{R}'G_n = \frac{A_n^{(n)}(\mu^2)^{n\epsilon}}{\epsilon^n} + \frac{A_{n-1}^{(n)}(\mu^2)^{(n-1)\epsilon}}{\epsilon^n} + \ldots + \frac{A_1^{(n)}(\mu^2)^{\epsilon}}{\epsilon^n}$$

$$+ \quad \text{lower pole terms}$$

$A_k^{(n)}(\mu^2)^{k\epsilon}$ terms appear after subtraction of (n-k) loop counter terms

<u>Statement:</u> $R'G_n$ is local, i.e. terms like $\log^k \mu^2/\epsilon^m$ should cancel for any k and m

<u>Consequence:</u> $A_n^{(n)} = (-1)^{n+1}\frac{A_1^{(n)}}{n}$

$$KR'G_n = \sum_{k=1}^{n}\left(\frac{A_k^{(n)}}{\epsilon^n}\right) \equiv \frac{A_n^{(n)'}}{\epsilon^n} \qquad A_n^{(n)'} = (-1)^{n+1}A_n^{(n)} = \frac{A_1^{(n)}}{n}.$$

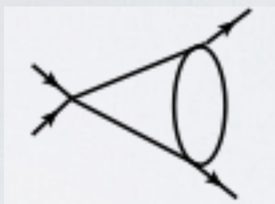
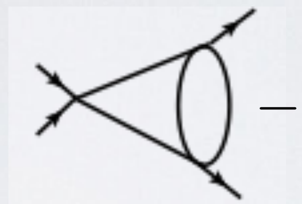
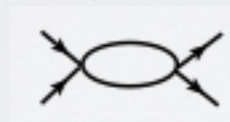
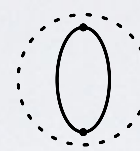
$A_1^{(n)}$ is the contribution to the leading pole in n-loops from the diagrams appearing in due corse of R-operation after subtraction of (n-1) loop counter terms

The leading divergences are governed by 1 loop diagrams!

Two loop example

ϕ^4_4

 $= \left(\frac{A_2^{(2)}}{\epsilon^2} + \frac{A_1^{(2)}}{\epsilon} \right) (\frac{\mu^2}{s})^{2\epsilon}$

$\mathcal{R}'$  $=$  $-$   $= \left(\frac{A_2^{(2)}}{\epsilon^2} + \frac{A_1^{(2)}}{\epsilon} \right) (\frac{\mu^2}{s})^{2\epsilon} - \frac{A_1^{(1)}}{\epsilon} (\frac{\mu^2}{s})^{\epsilon} \frac{A_1^{(1)}}{\epsilon}$

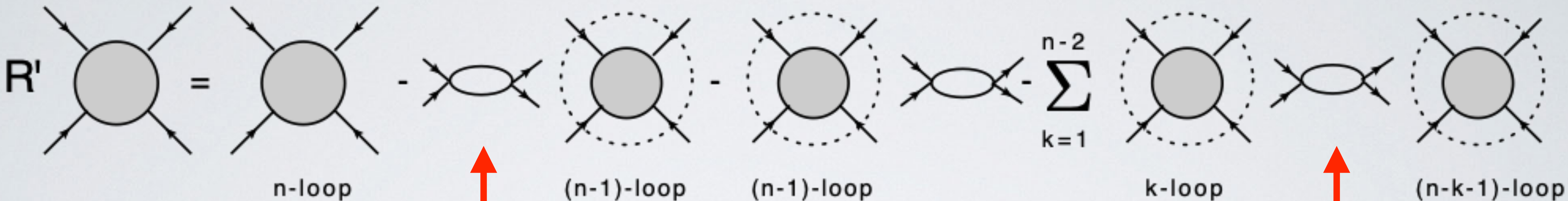
$$= \frac{A_2^{(2)}}{\epsilon^2} - \frac{(A_1^{(1)})^2}{\epsilon^2} + \underbrace{2\frac{A_2^{(2)}}{\epsilon} \log(\mu^2/s) - \frac{(A_1^{(1)})^2}{\epsilon} \log(\mu^2/s)}_{\text{non-local terms to be cancelled}} = -\frac{1}{2}\frac{(A_1^{(1)})^2}{\epsilon^2} + \dots$$

Leading divergence is given by the one-loop term $\boxed{A_2^{(2)} = \frac{1}{2}(A_1^{(1)})^2}$

- These statements are universal and are valid in non-renormalizable theories as well.
- The only difference is that the counter term $A_1^{(1)}$ depends on kinematics and has to be integrated through the remaining one-loop graph.
- As a result $A_2^{(2)}$ is not the square of $A_1^{(1)}$ anymore but is the integrated square.
- This last statement is the general feature of any QFT irrespective of renormalizability

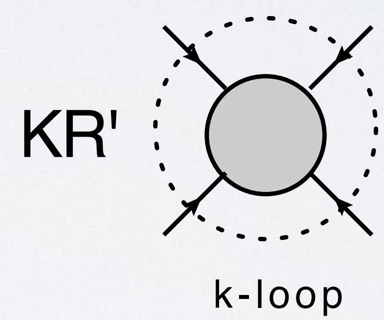
Leading divergences

Quartic vertices



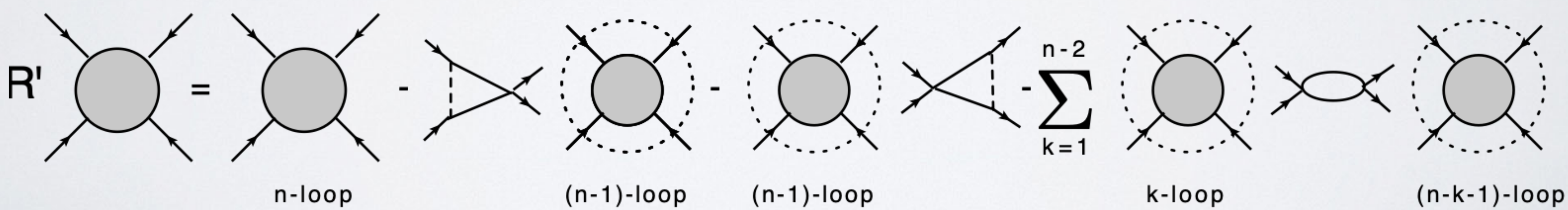
- ⋯ terms with higher loop remaining diagrams

$A_n^{(n)}$ $A_1^{(n)}$ $A_1^{(n)}$



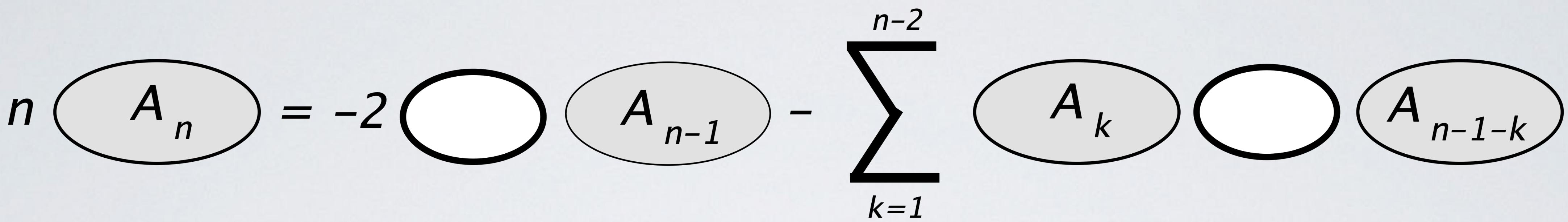
$$= \frac{A_k^{(k)\prime}}{\epsilon^k} = (-1)^{k+1}\frac{A_k^{(k)}}{\epsilon^k}$$

Cubic vertices



- ⋯ terms with higher loop remaining diagrams

Recurrence relations



- This is the general recurrence relation that reflects the locality of the counter terms in any theory
- In <u>renormalizable</u> theories A_n is a constant and this relation is reduced to the algebraic one
- In <u>non-renormalizable</u> theories A_n depends on kinematics and one has to integrate through the one loop diagrams

  - The leading divergences are defined by the one loop diagrams
  - Integration through the live loop can be made explicitly introducing Feynman parameters
  - One has to integrate momentum polynomials over Feynman parameters

Recurrence relations

- $$2nCS_{2n} = \frac{1}{2}\left[2s\int_0^1 dx MS_{2n-1}(s,t',u')|_{t'=-xs,u'=-(1-x)s}\right.$$

$$+\,2s\int_0^1 dx \sum_{k=1}^{n-1}\sum_{p=0}^{k-1}\sum_{l=0}^{p}\frac{1}{p!p!}s^p[x(1-x)]^p t^l u^{p-l}\frac{d^p}{dt'^l du'^{p-l}}MS_{2k-1}(s,t',u')\times$$

$$\left.\times\frac{d^p}{dt'^l du'^{p-l}}\left(CS_{2n-2k}(s,t',u')+CT_{2n-2k}(s,t',u')+CU_{2n-2k}(s,t',u')\right)|_{t'=-xs,u'=-(1-x)s}\right]$$

- $$(2n+1)MS_{2n+1} = \frac{1}{2}\left[\int_0^1 dx\sum_{k=0}^{n}\sum_{p=0}^{k-1}\sum_{l=0}^{p}\frac{1}{p!p!}s^p[x(1-x)]^p t^l u^{p-l}\times\right.$$

$$\times\left(\frac{d^p}{dt'^l du'^{p-l}}\bar{C}S_{2k}\frac{d^p}{dt'^l du'^{p-l}}CS_{2n-2k}+\frac{d^p}{dt'^l du'^{p-l}}\bar{C}T_{2k}\frac{d^p}{dt'^l du'^{p-l}}CT_{2n-2k}\right.$$

$$\left.+\frac{d^p}{dt'^l du'^{p-l}}\bar{C}U_{2k}\frac{d^p}{dt'^l du'^{p-l}}CU_{2n-2k}\right)|_{t'=-xs,u'=-(1-x)s}$$

$$+\,s\int_0^1 dx\sum_{k=1}^{n}\sum_{p=0}^{k}\sum_{l=0}^{p}\frac{1}{p!p!}s^p[x(1-x)]^p t^l u^{p-l}\times$$

$$\times\left(\frac{d^p}{dt'^l du'^{p-l}}MS_{2k-1}\frac{d^p}{dt'^l du'^{p-l}}MS_{2n-2k+1}+\frac{d^p}{dt'^l du'^{p-l}}MT_{2k-1}\frac{d^p}{dt'^l du'^{p-l}}MT_{2n-2k+1}\right.$$

$$\left.\left.+\frac{d^p}{dt'^l du'^{p-l}}MU_{2k-1}\frac{d^p}{dt'^l du'^{p-l}}MU_{2n-2k+1}\right)|_{t'=-xs,u'=-(1-x)s}\right]$$

Recurrence relations in lower orders

- $$\begin{aligned}
&CS_0 = 1,\\
&MS_1 = \frac{1}{2},\\
&2CS_2 = \frac{1}{2}\left[2s\int_0^1 dx MS_1\right] = \frac{1}{2}\left[2s\int_0^1 dx\frac{1}{2}\right] = \frac{s}{2} \Longrightarrow CS_2 = \frac{s}{4},\\
&3MS_3 = \frac{1}{2}\Bigg[\int_0^1 dx\left(CS_2 + \bar{C}S_2 + CT_2 + \bar{C}T_2 + CU_2 + \bar{C}U_2\right)\\
&+s\int_0^1 dx\left(MS_1MS_1 + MT_1MT_1 + MU_1MU_1\right)\Bigg]\\
&= \frac{1}{2}\left[\int_0^1 dx\left(\frac{s}{4} + \frac{s}{4} + \frac{t'}{4} + \frac{t'}{4} + \frac{u'}{4} + \frac{u'}{4}\right) + s\int_0^1 dx\left(\frac{1}{2}\frac{1}{2} + \frac{1}{2}\frac{1}{2} + \frac{1}{2}\frac{1}{2})\right)\right]\\
&= \frac{1}{2}\left[\frac{s - s/2 - s/2}{2} + 3\frac{s}{4}\right] = 3\frac{s}{8} \Longrightarrow MS_3 = \frac{s}{8},\\
&4CS_4 = \frac{1}{2}\left[2s\int_0^1 dx MS_3 + 2s\int_0^1 dx MS_1(CS_2 + CT_2 + CU_2)\right]\\
&= \frac{1}{2}\left[2s\int_0^1 dx\frac{s}{8} + 2s\int_0^1 dx\frac{1}{2}(\frac{s}{4} + \frac{t'}{4} + \frac{u'}{4})\right] = \frac{s^2}{8} \Longrightarrow CS_4 = \frac{s^2}{32}
\end{aligned}$$

Starting values

- $$CS_0 = 1, CS_2 = \frac{s}{4}, CS_4 = \frac{1}{2}\left(\frac{s}{4}\right)^2, CS_6 = \frac{5}{9}\left(\frac{s}{4}\right)^3, CS_8 = \frac{61}{126}\left(\frac{s}{4}\right)^4, CS_{10} = \frac{718}{1575}\left(\frac{s}{4}\right)^5 \dots$$

- $$\begin{aligned}
&MS_1 = \frac{1}{2},\ MS_3 = \frac{1}{2}\frac{s}{4},\ MS_5 = \frac{5}{12}\left(\frac{s}{4}\right)^2,\ MS_7 = \frac{26}{63}\left(\frac{s}{4}\right)^3,\\
&MS_9 = (\frac{s}{4})^4\left(\frac{14281}{45360} + \frac{t}{1080s} + \frac{t^2}{1080s^2}\right), MS_{11} = (\frac{s}{4})^5\left(\frac{773741}{2494800} + \frac{t}{2376s} + \frac{t^2}{2376s^2}\right)\dots
\end{aligned}$$

RG Equations

Introduce the functions $CS(s,t,u,g) = \frac{g}{4!}\sum_{n=0}^{\infty} CS_{2n} z^{2n}, \quad MS(s,t,u,g) = \frac{g}{4}\sum_{n=0}^{\infty} MS_{2n+1} z^{2n+1}, \quad z \equiv \frac{g}{\epsilon}$

and the same in t and u channels

- Taking the sum one can transform the recurrence relations into the integro-differential equation, which is the RG equation

$$\frac{dCS}{dz} = sMS \otimes (CS + CT + CU) \qquad \frac{d}{dz} = \frac{d}{d\log\mu^2},$$

$$\frac{dMS}{dz} = \frac{1}{2}[s(MS \otimes MS + MT \otimes MT + MU \otimes MU) \quad + \bar{C}S \otimes CS + \bar{C}T \otimes CT + \bar{C}U \otimes CU]$$

where the product is defined as

$$A(s,t,u) \otimes B(s,t,u) = \int_0^1 dx \sum_{p=0}^{\infty} \sum_{l=0}^{p} \frac{1}{p!p!} \times$$

$$\times \frac{d^p}{dt'^l du'^{p-l}} A(s,t',u') \frac{d^p}{dt'^l du'^{p-l}} B(s,t',u')|_{\substack{t' = -xs, \\ u' = -(1-x)s}} \quad s^p [x(1-x)]^p t^l u^{p-l}$$

- The solution of the RG equations determine the high energy behaviour of the amplitudes when $s \sim t \sim u \sim E^2 \to \infty$

Particular Solution to RG Equations

Solution for a particular chain of bubbles

Justified by the leading order in 1/N approximation in vector and matrix (1st) cases

Planar case

$$\int d^2\theta \; \frac{g}{4N}(\Phi^a\Phi^a)^2 \qquad\qquad \int d^2\theta \; \frac{g}{4N}(Tr\Phi\Phi)^2 \;\; \text{or} \;\; \int d^2\theta \; \frac{g}{4!N}(Tr\Phi\Phi\Phi\Phi)$$

Vector case   Matrix case

Pure diff eqs

$$\frac{dCS}{dy} = sMS \cdot CS,$$
$$\frac{dMS}{dy} = \frac{1}{2}[sMS^2 + CS^2], \quad \bar{C}S = CS$$

Solution

$$\boxed{CS(z) = \frac{1}{1 - sz^2/4}, \quad MS(z) = \frac{z/2}{1 - sz^2/4}}$$

High energy behaviour $\quad z \to -g \log s$

$$CS = \frac{1}{1 - g^2 s \log^2 s/4}, \; CT = \frac{1}{1 - g^2 t \log^2 t/4}, \; CU = \frac{1}{1 - g^2 u \log^2 u/4},$$
$$MS = -\frac{g \log s/2}{1 - g^2 s \log^2 s/4}, \; MT = -\frac{g \log t/2}{1 - g^2 t \log^2 t/4}, \; MU = -\frac{g \log u/2}{1 - g^2 \log^2 u/4}$$

Pole in s-channel and no poles in t- and u-channels !   Ghost state?

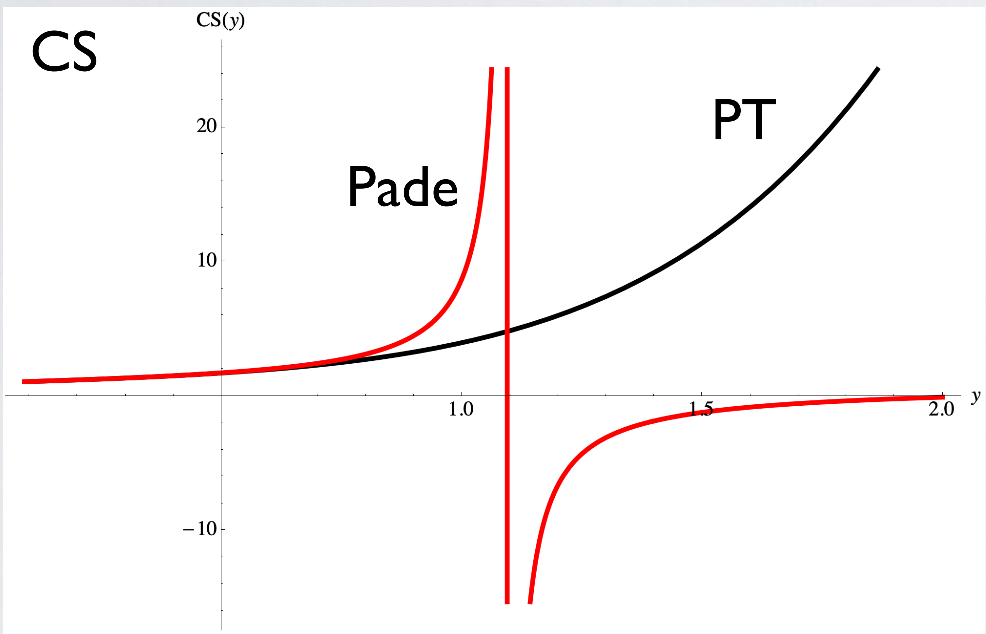
Numerical Solution to RG Equations

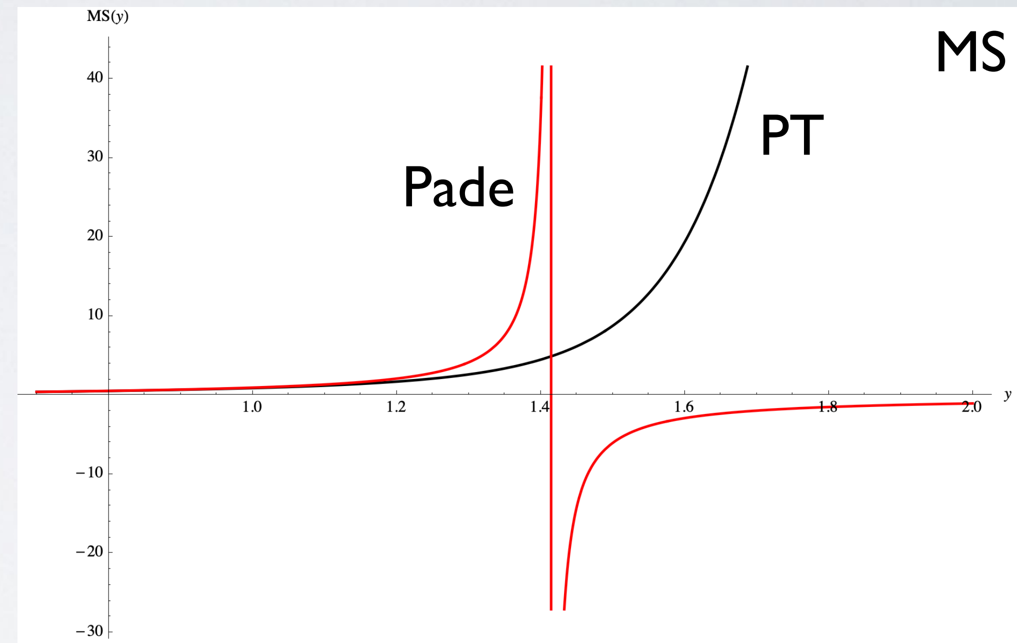
Pade versus PT

Pade [3,3] PT6

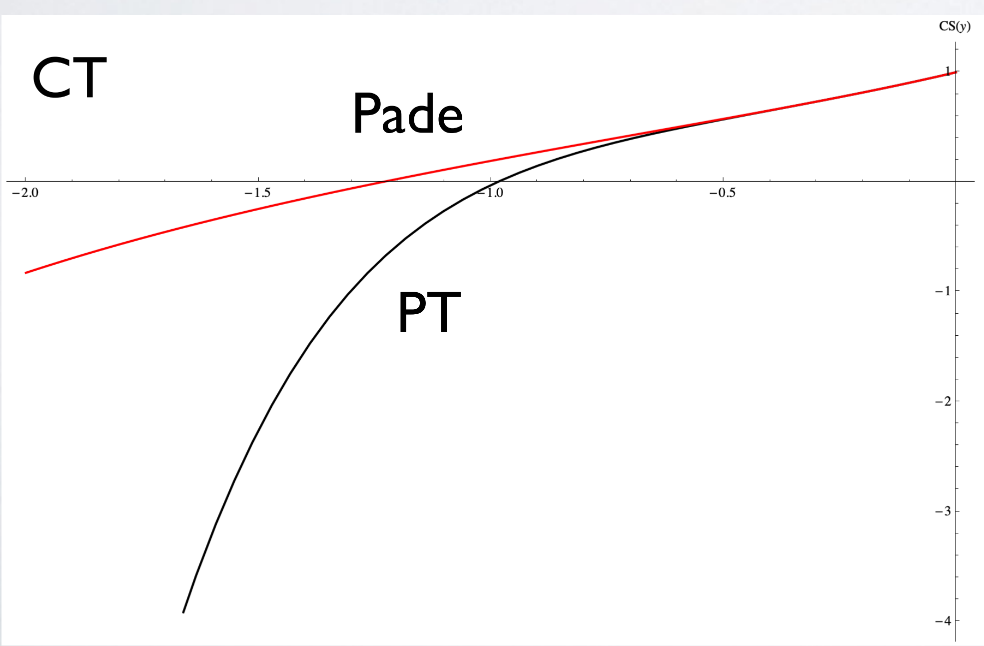
$$CS(y) = \frac{1 + \frac{919521}{17198} y + \frac{3619086}{214975} y^2 - \frac{1132734289}{54173700} y^3}{1 + \frac{902323}{17198} y - \frac{7767439}{214975} y^2 - \frac{34810827}{3009650} y^3},$$

$$MS(y) = \frac{\frac{1}{2} - \frac{60757261387}{27020023140} y - \frac{17465208191899}{4458303818100} y^2 - \frac{211448333535053}{1123492562161200} y^3}{1 - \frac{74267272957}{13510011570} y - \frac{7068734744869}{2229151909050} y^2 + \frac{105130578087131}{16049893745160} y^3},$$

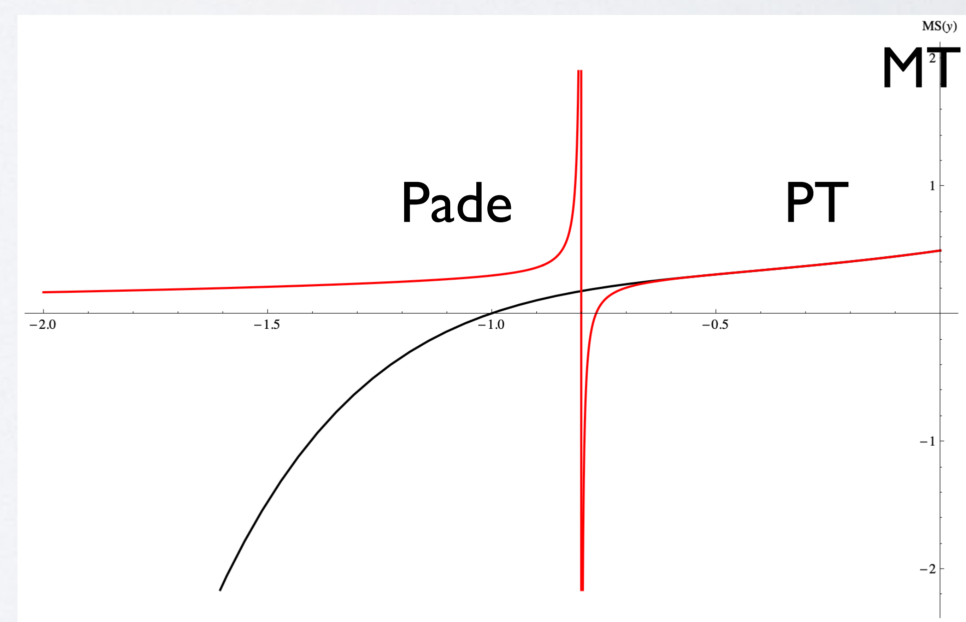




Pole



No Pole?



Resume

Resume

- The UV divergences in non-renormalizable theories are local and can be removed by local counter terms like in renormalizable ones

Resume

- The UV divergences in non-renormalizable theories are local and can be removed by local counter terms like in renormalizable ones
- In SUSY theories in non-renormalizable case the chiral part of the effective action receives divergent radiative corrections

Resume

- The UV divergences in non-renormalizable theories are local and can be removed by local counter terms like in renormalizable ones
- In SUSY theories in non-renormalizable case the chiral part of the effective action receives divergent radiative corrections
- Based on locality of the counter terms one can construct the recurrence relations that define all loop divergences starting from one loop

Resume

- The UV divergences in non-renormalizable theories are local and can be removed by local counter terms like in renormalizable ones
- In SUSY theories in non-renormalizable case the chiral part of the effective action receives divergent radiative corrections
- Based on locality of the counter terms one can construct the recurrence relations that define all loop divergences starting from one loop
- The recurrence relations can be converted into the generalized RG equations which allow one to sum up the leading divergences in all loops and define the high-energy behaviour

Resume

- The UV divergences in non-renormalizable theories are local and can be removed by local counter terms like in renormalizable ones
- In SUSY theories in non-renormalizable case the chiral part of the effective action receives divergent radiative corrections
- Based on locality of the counter terms one can construct the recurrence relations that define all loop divergences starting from one loop
- The recurrence relations can be converted into the generalized RG equations which allow one to sum up the leading divergences in all loops and define the high-energy behaviour
- In the Wess-Zumino model with quartic superpotential the bubble diagrams are summed into a geometrical progression which has a pole in the s-channel and no poles in the t- and u-channels.

Resume

- The UV divergences in non-renormalizable theories are local and can be removed by local counter terms like in renormalizable ones
- In SUSY theories in non-renormalizable case the chiral part of the effective action receives divergent radiative corrections
- Based on locality of the counter terms one can construct the recurrence relations that define all loop divergences starting from one loop
- The recurrence relations can be converted into the generalized RG equations which allow one to sum up the leading divergences in all loops and define the high-energy behaviour
- In the Wess-Zumino model with quartic superpotential the bubble diagrams are summed into a geometrical progression which has a pole in the s-channel and no poles in the t- and u-channels.
- Numerical solution of the full equation seems to have a pole in the s-channel while the the t-channel behaviour is not reliable

Resume

- The UV divergences in non-renormalizable theories are local and can be removed by local counter terms like in renormalizable ones
- In SUSY theories in non-renormalizable case the chiral part of the effective action receives divergent radiative corrections
- Based on locality of the counter terms one can construct the recurrence relations that define all loop divergences starting from one loop
- The recurrence relations can be converted into the generalized RG equations which allow one to sum up the leading divergences in all loops and define the high-energy behaviour
- In the Wess-Zumino model with quartic superpotential the bubble diagrams are summed into a geometrical progression which has a pole in the s-channel and no poles in the t- and u-channels.
- Numerical solution of the full equation seems to have a pole in the s-channel while the the t-channel behaviour is not reliable
- This pole if exists corresponds to the ghost bound state similar to QED or ϕ^4_4 theory

JOINT INSTITUTE
FOR NUCLEAR RESEARCH

JINR Laboratories

MESHCHERYAKOV
LABORATORY of
INFORMATION
TECHNOLOGIES