

Divergences in Maximal SYM Theories in Diverse Dimensions

Dmitri Kazakov

**in collaboration with L. Bork, M. Kompaniets,
D.Tolkachev and D.Vlasenko**

Alikhanov Institute for Theoretical and Experimental Physics, Moscow, Russia

Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Russia

Moscow Institute of Physics and Technology, Dolgoprudny, Russia

Center for Fundamental and Applied Research, All-Russian Institute of Automatics, Moscow, Russia

Department of Theoretical Physics, St. Petersburg State University, St. Petersburg, Russia

Department of Physics, Southern Federal University, Rostov-Don, Russia

Gomel State University, Gomel, Belarus

Based on:

Phys.Lett. B 734 (2014) 111, arXiv:1404.6998 [hep-th]

JHEP1511 (2015) 059, arXiv:1508.05570 [hep-th]

arXiv:1603.0550 [hep-th]

Motivation

Maximal SYM

D=4 N=4

D=6 N=2

D=8 N=1

D=10 N=1

- Partial or total cancellation of UV divergences (all bubble and triangle diagrams cancel)
- First UV divergent diagrams at $D=4+6/L$
- Conformal or dual conformal symmetry
- Common structure of the integrands

Bern, Dixon & Co 10
Drummond, Henn,
Korchensky, Sokatchev 10
Arkani-Hamed 12

Object: Helicity Amplitudes on mass shell
with arbitrary number of legs and loops

The case: Planar limit $N_c \rightarrow \infty$, $g_{YM}^2 \rightarrow 0$ and $g_{YM}^2 N_c$ - fixed

The aim: to get all loop (exact) result

Colour decomposition

Colour ordered amplitude

$$\mathcal{A}_n^{a_1 \dots a_n}(p_1^{\lambda_1} \dots p_n^{\lambda_n}) = \sum_{\sigma \in S_n / Z_n} \text{Tr}[\sigma(T^{a_1} \dots T^{a_n})] A_n(\sigma(p_1^{\lambda_1} \dots p_n^{\lambda_n})) + \mathcal{O}(1/N_c)$$

Planar Limit $N_c \rightarrow \infty$, $g_{YM}^2 \rightarrow 0$ and $g_{YM}^2 N_c$ - fixed

This is what we calculate

Four-point amplitude

$$A_4^{(1), \text{phys.}}(1,2,3,4) = T^1 A_4^{(0)}(1,2,3,4) M^{(1)}(s,t) + T^2 A_4^{(0)}(1,2,4,3) M^{(1)}(s,u) + T^3 A_4^{(0)}(1,4,2,3) M^{(1)}(t,u).$$

$$T^1 = \text{Tr}(T^{a_1} T^{a_2} T^{a_3} T^{a_4}) + \text{Tr}(T^{a_1} T^{a_4} T^{a_3} T^{a_2}),$$

$$T^2 = \text{Tr}(T^{a_1} T^{a_2} T^{a_4} T^{a_3}) + \text{Tr}(T^{a_1} T^{a_3} T^{a_4} T^{a_2}),$$

$$T^3 = \text{Tr}(T^{a_1} T^{a_4} T^{a_2} T^{a_3}) + \text{Tr}(T^{a_1} T^{a_3} T^{a_2} T^{a_4})$$

Tree level amplitude usually has a simple universal form proportional to the delta function (conservation of momenta), in SUSY case - conservation of supercharge in on shell momentum superspace

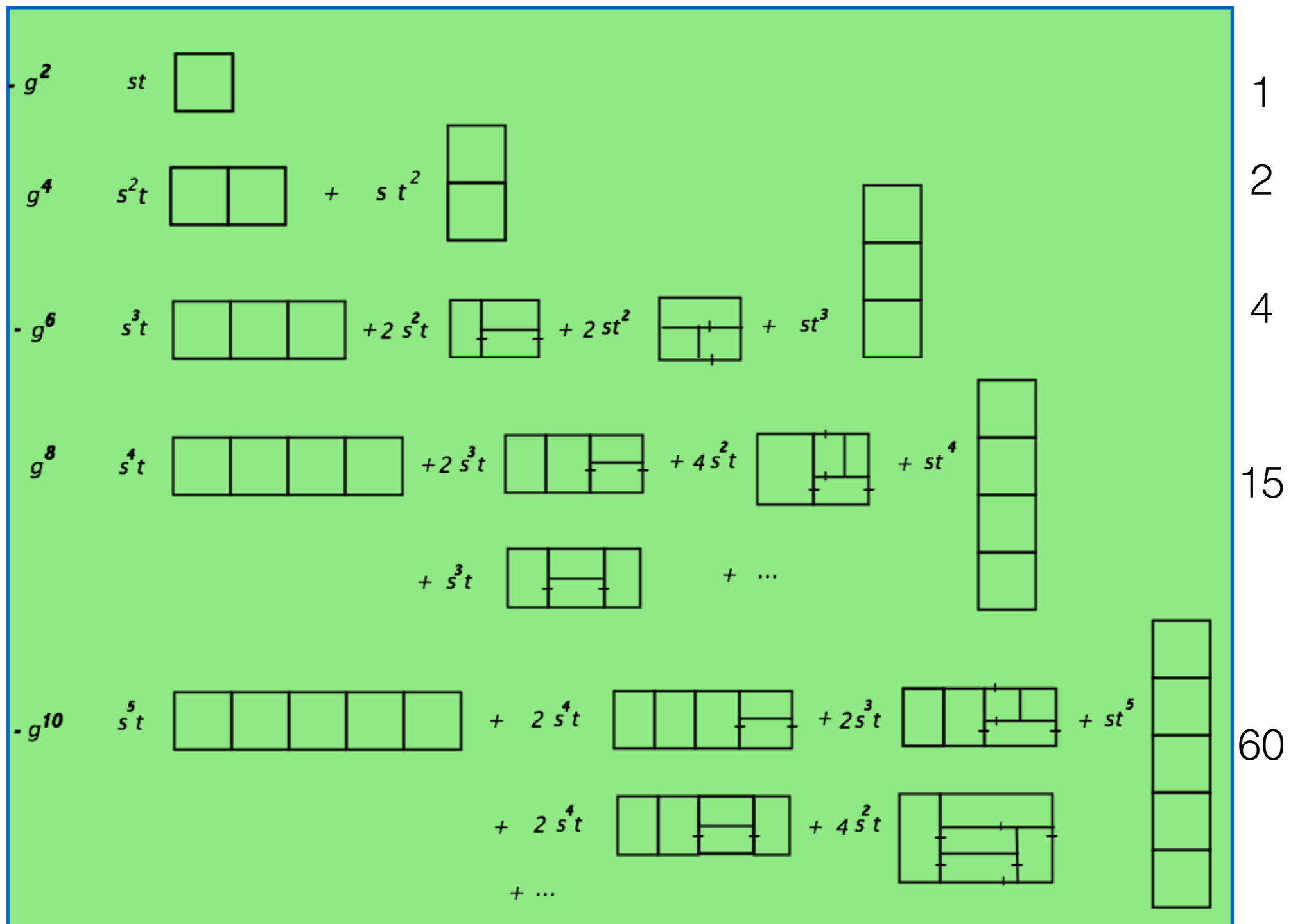
Perturbation Expansion for the Amplitudes for any D

$$A_4/A_4^{tree}$$

No bubbles
No Triangles

First UV div at
 $L = \lceil 6/(D-4) \rceil$ loops

IR finite



T. Dennen Yu-yin Huang 10,
S. Caron-Huot D.O'Connell 10

Universal expansion for any D in maximal SYM due to Dual conformal invariance

Leading Divergences from Generalized «Renormalization Group»

- In renormalizable theories the leading divergences can be found from the 1-loop term due to the renormalization group, in particular, for a single coupling theory the coefficient of $1/\epsilon^n$ in n loops is given by

$$a_n^{(n)} = (a_1^{(1)})^n$$

- In non-renormalizable theories the leading divergences can be also found from 1-loop due to locality and R-operation

$$\mathcal{R}'G = 1 - \sum_{\gamma} K\mathcal{R}'_{\gamma} + \sum_{\gamma, \gamma'} K\mathcal{R}'_{\gamma}K\mathcal{R}'_{\gamma'} - \dots,$$

$$\mathcal{R}'G_n = \frac{A_n^{(n)} (\mu^2)^{n\epsilon}}{\epsilon^n} + \frac{A_{n-1}^{(n)} (\mu^2)^{(n-1)\epsilon}}{\epsilon^n} + \dots + \frac{A_1^{(n)} (\mu^2)^{\epsilon}}{\epsilon^n}$$

$$+ \frac{B_n^{(n)} (\mu^2)^{n\epsilon}}{\epsilon^{n-1}} + \frac{B_{n-1}^{(n)} (\mu^2)^{(n-1)\epsilon}}{\epsilon^{n-1}} + \dots + \frac{B_1^{(n)} (\mu^2)^{\epsilon}}{\epsilon^{n-1}}$$

+lower order terms

Leading pole

SubLeading pole

$A_1^{(n)}, B_1^{(n)}$

$B_2^{(n)}$

1-loop graph

2-loop graph

SubLeading Divergences from Generalized «Renormalization Group»

- In non-renormalizable theories the leading divergences can be also found from 2-loop due to locality and R-operation

All terms like $(\log \mu^2)^m / \epsilon^k$ should cancel

$$A_n^{(n)} = (-1)^{n+1} \frac{A_1^{(n)}}{n},$$

$$B_n^{(n)} = (-1)^n \left(\frac{2}{n} B_2^{(n)} + \frac{n-2}{n} B_1^{(n)} \right)$$

Leading pole
from 1 loop
diagrams

SubLeading pole
from 2 loop
diagrams

$$\mathcal{KR}'G_n = \sum_{k=1}^n \left(\frac{A_k^{(n)}}{\epsilon^n} + \frac{B_k^{(n)}}{\epsilon^{n-1}} \right) \equiv \frac{A_n^{(n)'}}{\epsilon^n} + \frac{B_n^{(n)'}}{\epsilon^{n-1}}.$$

$$A_n^{(n)'} = (-1)^{n+1} A_n^{(n)} = \frac{A_1^{(n)}}{n},$$

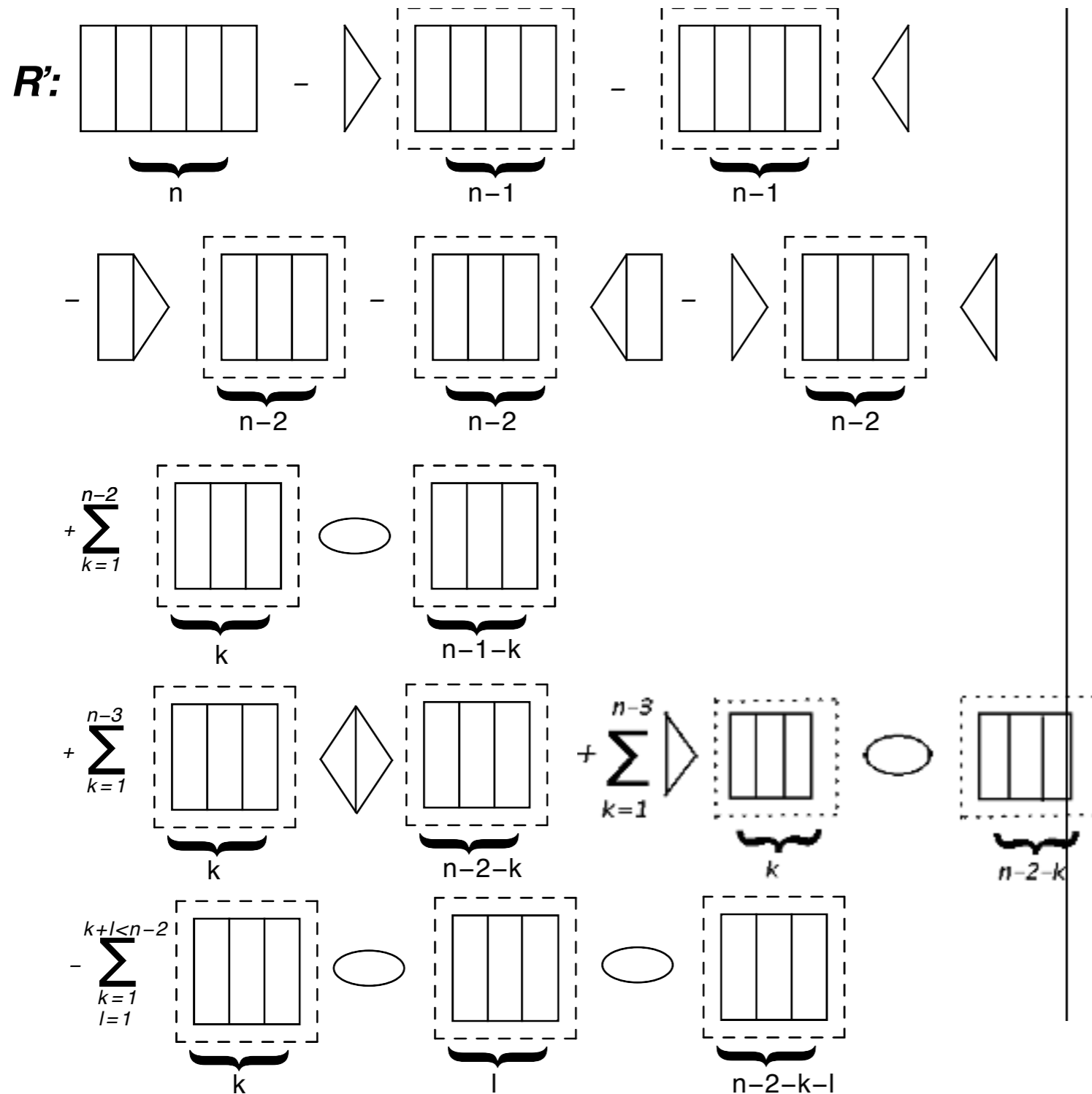
$$B_n^{(n)'} = \left(\frac{2}{n(n-1)} B_2^{(n)} + \frac{2}{n} B_1^{(n)} \right)$$

Just like in
renormalizable
theories one can
deduce the
leading,
subleading, etc
divergences from
1, 2, etc diagrams

R-operation and Recurrence Relation

D=8 N=1

Horizontal boxes



$A_1^{(n)} B_1^{(n)}$

$B_2^{(n)}$

$A_1^{(n)} B_1^{(n)}$

$B_2^{(n)}$

$B_2^{(n)}$

Gives all leading divs

Gives all subleading divs

The leading Divergences

MI	Comb	$D = 6$	$D = 8$	$D = 10$
$I_1^{(1)}$	st	conv	$\frac{1}{3!\epsilon}$	$\frac{s+t}{5!\epsilon}$
$I_1^{(2)}$	s^2t	conv	$-\frac{s}{3!4!\epsilon^2}$	$\frac{-s^2(8s+2t)}{5!7!\epsilon^2}$
$I_1^{(3)}$	s^3t	conv	$\frac{s^2}{4!5!\epsilon^3}$	$\frac{-2s^4(135s+11t)}{5!7!7!3\epsilon^3}$
$I_2^{(3)}$	$2s^2t$	$-\frac{1}{6\epsilon}$	$\frac{s(3s^2-2st+t^2)}{3!4!5!9\epsilon^3}$	$\frac{-s^2(14s^4-10s^3t+\frac{33}{5}s^2t^2-\frac{19}{5}st^3+\frac{8}{5}t^4)}{5!7!7!9\epsilon^3}$
$I_1^{(4)}$	s^4t	conv	$-\frac{210s^3}{3!4!5!6!\epsilon^4}$	$\frac{-32s^6(99s+2t)}{5!7!7!7!3\epsilon^4}$
$I_2^{(4)}$	$2s^3t$	$\frac{1}{48\epsilon^2}$	$\frac{s^2(-\frac{430}{21}s^2+\frac{4}{9}st-\frac{1}{18}t^2)}{3!4!5!6!\epsilon^4}$	$\frac{-2s^4\left(\frac{1502144}{33}s^4-\frac{1085791}{33}s^3t+\frac{2044}{5}s^2t^2-\frac{1001}{15}st^3+\frac{112}{15}t^4\right)}{5!7!7!7!7!\epsilon^4}$
$I_3^{(4)}$	s^3t	$\frac{1}{24\epsilon^2}$	$\frac{s^2(-\frac{20}{3}s^2+\frac{8}{9}st-\frac{1}{9}t^2)}{3!4!5!6!\epsilon^4}$	$\frac{-28s^4\left(8512s^4-1043s^3t+\frac{876}{5}s^2t^2-\frac{143}{5}st^3+\frac{16}{5}t^4\right)}{5!7!7!7!7!3\epsilon^4}$
$I_4^{(4)}$	$2s^2t$	$\sim \frac{1}{\epsilon}$	$\frac{s\left(-\frac{45}{14}s^4+\frac{18}{7}s^3t-\frac{27}{14}s^2t^2+\frac{9}{7}st^3-\frac{9}{14}t^4\right)}{3!4!5!6!\epsilon^4}$	$\frac{-s^2\left(-\frac{7504}{1287}s^7+\frac{7819}{1716}s^6t-\frac{1475}{429}s^5t^2+\frac{12745}{5148}s^4t^3-\frac{716}{429}s^3t^4+\frac{1747}{1716}s^2t^5-\frac{673}{1287}st^6+\frac{105}{572}t^7\right)}{5!7!7!7!\epsilon^4}$
$I_5^{(4)}$	$4s^2t$	$\frac{t-s}{3\cdot 48\epsilon^2}$	$\frac{s\left(-\frac{15}{28}s^4+\frac{25}{63}s^3t-\frac{65}{252}s^2t^2+\frac{5}{42}st^3-\frac{1}{28}t^4\right)}{3!4!5!6!\epsilon^4}$	$\frac{-4s^2\left(-\frac{95200}{143}s^7+\frac{67634}{143}s^6t-\frac{225008}{715}s^5t^2+\frac{136514}{715}s^4t^3-\frac{6608}{65}s^3t^4+\frac{6706}{143}s^2t^5-\frac{7420}{429}st^6+\frac{1715}{429}t^7\right)}{5!7!7!7!\epsilon^4}$

Perturbation Expansion for the Amplitudes

D=8 N=1

Leading Divergences

Result up to 4 loops

$$\begin{aligned}
 L.P. = & -st \left[g^2 \frac{1}{3!\epsilon} + g^4 \frac{s^2 + t^2}{3!4!\epsilon^2} + g^6 \frac{4}{3} \frac{15s^4 - s^3t + s^2t^2 - st^3 + 15t^4}{3!4!5!\epsilon^3} \right. \\
 & \left. + g^8 \frac{1}{63} \frac{16770s^6 - 536s^5t + 412s^4t^2 - 384s^3t^3 + 412s^2t^4 - 536st^5 + 16770t^6}{3!4!5!6!\epsilon^4} \right].
 \end{aligned}$$

D=10 N=1

Leading Divergences

Result up to 4 loops

$$\begin{aligned}
 L.P. = & -st \left[g^2 \frac{s+t}{5!\epsilon} + g^4 \frac{8s^4 + 2s^3t + 2st^3 + 8t^4}{5!7!\epsilon^2} \right. \\
 & + g^6 \frac{2(2095s^7 + 115s^6t + 33s^5t^2 - 11s^4t^3 - 11s^3t^4 + 33s^2t^5 + 115st^6 + 2095t^7)}{5!7!7!45\epsilon^3} \\
 & + g^8 \frac{32(211218880s^{10} + 753490s^9t - 1395096s^8t^2 + 1125763s^7t^3 - 916916s^6t^4} \\
 & \left. + 843630s^5t^5 - 916916s^4t^6 + 1125763s^3t^7 - 1395096s^2t^8 + 753490st^9 + 211218880t^{10})}{13!7!7!5!5\epsilon^4} \right].
 \end{aligned}$$

**Doesn't look like Geom progression anymore,
however, coefficients grow slowly**

Ladder diagrams (leading divs)

D=8 N=1

Horizontal boxes

$$A_n^{(n)} = s^{n-1} A_n$$

$$nA_n = -\frac{2}{4!}A_{n-1} + \frac{2}{5!} \sum_{k=1}^{n-2} A_k A_{n-1-k}, \quad n \geq 3$$

$$A_1 = 1/6$$

1 loop box

Summation

$$\Sigma_m(z) = \sum_{n=m}^{\infty} A_n (-z)^n$$

$$-\frac{d}{dz} \Sigma_3 = -\frac{2}{4!} \Sigma_2 + \frac{2}{5!} \Sigma_1 \Sigma_1. \quad \Sigma_3 = \Sigma_1 + A_1 z - A_2 z^2, \quad \Sigma_2 = \Sigma_1 + A_1 z, \quad A_1 = \frac{1}{3!}, \quad A_2 = -\frac{1}{3!4!}$$

$$\Sigma_A \equiv \Sigma_1$$

Diff eqn

$$\frac{d}{dz} \Sigma_A = -\frac{1}{3!} + \frac{2}{4!} \Sigma_A - \frac{2}{5!} \Sigma_A^2$$

$$z = g^2 s^2 / \epsilon$$

$$\Sigma_A(z) = -\sqrt{5/3} \frac{4 \tan(z/(8\sqrt{15}))}{1 - \tan(z/(8\sqrt{15}))\sqrt{5/3}} = \sqrt{10} \frac{\sin(z/(8\sqrt{15}))}{\sin(z/(8\sqrt{15}) - z_0)}$$

$$\Sigma(z) = -(z/6 + z^2/144 + z^3/2880 + 7z^4/414720 + \dots)$$

$$z_0 = \arcsin(\sqrt{3/8})$$

Ladder diagrams (subleading divs)

$$A_n^{(n)} = s^{n-1} A_n, A_n^{(n)'} = s^{n-1} A_n',$$

$$B_n^{(n)} = s^{n-1} B_{sn} + s^{n-2} t B_{tn}, B_n^{(n)'} = s^{n-1} B'_{sn} + s^{n-2} t B'_{tn}$$

$$\begin{aligned}
 B_1^{(n)} = & -A'_{n-1} s^{n-2} \left(-\frac{s}{4!}\right) \frac{19}{6} 2 - B'_{sn-1} s^{n-2} \left(-\frac{s}{4!}\right) 2 - B'_{tn-1} s^{n-3} \left(-\frac{s}{5!}\right) (t - 2s) 2 \\
 & + \sum_{k=1}^{n-2} A'_k s^{k-1} A'_{n-1-k} s^{n-2-k} \left(\frac{2s^2}{5!}\right) \frac{46}{15} \\
 & + \sum_{k=1}^{n-2} A'_k s^{k-1} B'_{sn-1-k} s^{n-2-k} \left(\frac{2s^2}{5!}\right) 2 + \sum_{k=1}^{n-2} A'_k s^{k-1} B'_{tn-1-k} s^{n-3-k} \left(\frac{-s^3}{5!}\right) 2
 \end{aligned}$$

Ladder diagrams (subleading divs)

$$\begin{aligned}
 B_2^{(n)} = & -A'_{n-2} s^{n-3} \left(\frac{s^2}{3!4!}\right) \frac{5063}{2400} 2 - B'_{sn-2} s^{n-3} \left(\frac{s^2}{3!4!}\right) \frac{13}{40} 2 - B'_{tn-1} s^{n-4} \left(\frac{s^2}{5!5!}\right) \frac{t-32s}{2} 2 \\
 & -A'_{n-2} s^{n-3} \left(-\frac{s}{4!}\right) \left(-\frac{s}{4!} \frac{19}{6}\right) 2 - B'_{sn-2} s^{n-3} \left(-\frac{s}{4!}\right) \left(-\frac{s}{4!}\right) - B'_{tn-2} s^{n-4} \left(-\frac{s^2}{5!5!}\right) (12s-t) \\
 & + \sum_{k=1}^{n-3} A'_k s^{k-1} A'_{n-2-k} s^{n-3-k} \left(-\frac{s^3}{5!12}\right) \frac{94}{15} \\
 & + \sum_{k=1}^{n-3} A'_k s^{k-1} B'_{sn-2-k} s^{n-3-k} \left(-\frac{s^3}{5!12}\right) 2 + \sum_{k=1}^{n-3} A'_k s^{k-1} B'_{tn-2-k} s^{n-4-k} \left(\frac{s^4}{4!5!12}\right) 2 \\
 & - \sum_{k,l=1}^{n-k+l < n-2} A'_k s^{k-1} A'_l s^{l-1} A'_{n-2-k-l} s^{n-3-k-l} \left(\frac{2s^2}{5!}\right) \left(\frac{2s^2}{5!}\right) \frac{46}{15} 2 \\
 & - \sum_{k,l=1}^{n-k+l < n-2} A'_k s^{k-1} A'_l s^{l-1} B'_{sn-2-k-l} s^{n-3-k-l} \left(\frac{2s^2}{5!}\right) \left(\frac{2s^2}{5!}\right) 3 \\
 & - \sum_{k,l=1}^{n-k+l < n-2} A'_k s^{k-1} A'_l s^{l-1} B'_{tn-2-k-l} s^{n-4-k-l} \left(\frac{2s^2}{5!}\right) \left(-\frac{s^3}{5!}\right) 2 \\
 & - \sum_{k,l=1}^{n-k+l < n-2} A'_k s^{k-1} B'_{tl} s^{l-2} A'_{n-2-k-l} s^{n-3-k-l} \left(\frac{-2s^5}{5!5!}\right)
 \end{aligned}$$

Ladder diagrams (subleading divs)

$$\begin{aligned}
 B'_{tn} &= -\frac{2}{n(n-1)} B'_{tn-2} \frac{2}{5!5!} + \frac{2}{n} B'_{tn-1} \frac{2}{5!}, \\
 B'_{sn} &= \frac{2}{n(n-1)} \left[-A'_{n-2} \frac{2321}{5!5!2} - B'_{sn-2} \frac{18}{4!5!} + B'_{tn-2} \frac{44}{5!5!} \right. \\
 &\quad - \sum_{k=1}^{n-3} A'_k A'_{n-2-k} \frac{2}{5!3} \frac{47}{15} - \sum_{k=1}^{n-3} A'_k B'_{sn-2-k} \frac{2}{5!12} + \sum_{k=1}^{n-3} A'_k B'_{tn-2-k} \frac{2}{4!5!12} \\
 &\quad - \sum_{k,l=1}^{n-k+l < n-2} A'_k A'_l A'_{n-2-k-l} \frac{8}{5!5!} \frac{46}{15} - \sum_{k,l=1}^{n-k+l < n-2} A'_k A'_l B'_{sn-2-k-l} \frac{12}{5!5!} \\
 &\quad \left. + \sum_{k,l=1}^{n-k+l < n-2} A'_k A'_l B'_{tn-2-k-l} \frac{4}{5!5!} + \sum_{k,l=1}^{n-k+l < n-2} B'_k A'_l A'_{sn-2-k-l} \frac{2}{5!5!} \right] \\
 &\quad + \frac{2}{n} \left[A'_{n-1} \frac{19}{34!} + B'_{sn-1} \frac{2}{4!} - B'_{tn-1} \frac{4}{5!} \right. \\
 &\quad \left. + \sum_{k=1}^{n-2} A'_k A'_{n-2-k} \frac{2}{5!} \frac{46}{15} + \sum_{k=1}^{n-2} A'_k B'_{sn-1-k} \frac{4}{5!} - \sum_{k=1}^{n-2} A'_k B'_{tn-1-k} \frac{2}{5!} \right]. \\
 B_{tn} &= (-1)^n \left[-\frac{2}{n} B'_{tn-2} \frac{2}{5!5!} + \frac{n-2}{n} B'_{tn-1} \frac{2}{5!} \right]
 \end{aligned}$$

Sum of Ladder diagrams (subleading divs)

$$\Sigma'_{tB} = \sum_{n=2}^{\infty} z^n B'_{tn}$$

$$\frac{d^2 \Sigma'_{tB}(z)}{dz^2} - \frac{1}{30} \frac{d \Sigma'_{tB}(z)}{dz} + \frac{\Sigma'_{tB}(z)}{3600} = -\frac{1}{432}$$

$$z = \frac{g^2 s^2}{\epsilon}$$

$$\Sigma'_{tB}(z) = \frac{25}{3} \left[e^{z/60} \left(1 - \frac{z}{60} \right) - 1 \right]$$

$$\Sigma_{tB} = \sum_{n=2}^{\infty} z^n B_{tn}$$

$$\frac{d \Sigma_{tB}(z)}{dz} = \frac{1}{60} z \frac{d \Sigma'_{tB}(z)}{dz} - \frac{\Sigma'_{tB}(z)}{60} - z \frac{\Sigma'_{tB}(z)}{3600}$$

$$\Sigma_{tB}(z) = -\frac{25}{3} \left[e^{z/60} - 1 - \frac{z}{60} \right]$$

Sum of Ladder diagrams (subleading divs)

$$\Sigma'_{sB} = \sum_{n=2}^{\infty} z^n B'_{sn}$$

$$\frac{d^2 \Sigma'_{sB}(z)}{dz^2} + f_1(z) \frac{d \Sigma'_{sB}(z)}{dz} + f_2(z) \Sigma'_{sB}(z) = f_3(z)$$

$$f_1(z) = -\frac{1}{6} + \frac{\Sigma_A}{15},$$

$$f_2(z) = \frac{1}{80} - \frac{\Sigma_A}{360} + \frac{\Sigma_A^2}{600} + \frac{1}{15} \frac{d \Sigma_A}{dz},$$

$$f_3(z) = \frac{2321}{5!5!2} \Sigma_A + \frac{11}{1800} \Sigma'_{tB} - \frac{47}{5!45} \Sigma_A^2 - \frac{1}{5!72} \Sigma_A \Sigma'_{tB} + \frac{23}{6750} \Sigma_A^3 + \frac{1}{1200} \Sigma_A^2 \Sigma'_{tB}$$

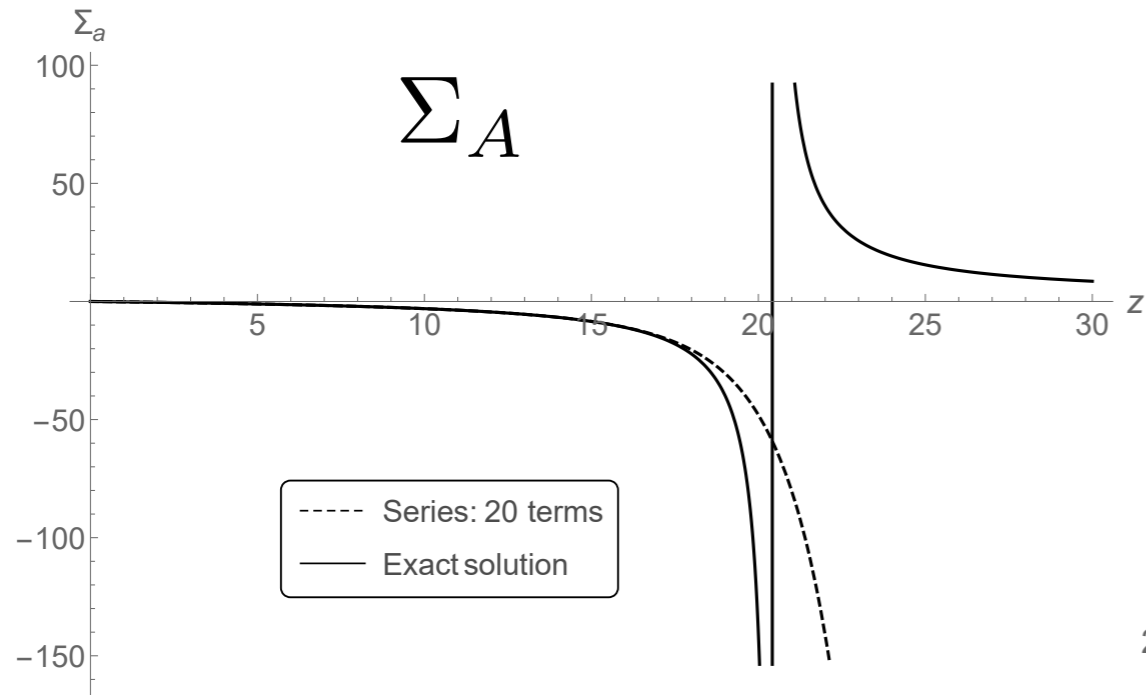
$$- \frac{19}{36} \frac{d \Sigma_A}{dz} - \frac{1}{15} \frac{d \Sigma'_{tB}}{dz} + \frac{23}{225} \frac{d \Sigma_A^2}{dz} + \frac{1}{30} \frac{d(\Sigma_A \Sigma'_{tB})}{dz} - \frac{3}{32}$$

$$\Sigma'_{sB} = \sum_{n=2}^{\infty} z^n B'_{sn}$$

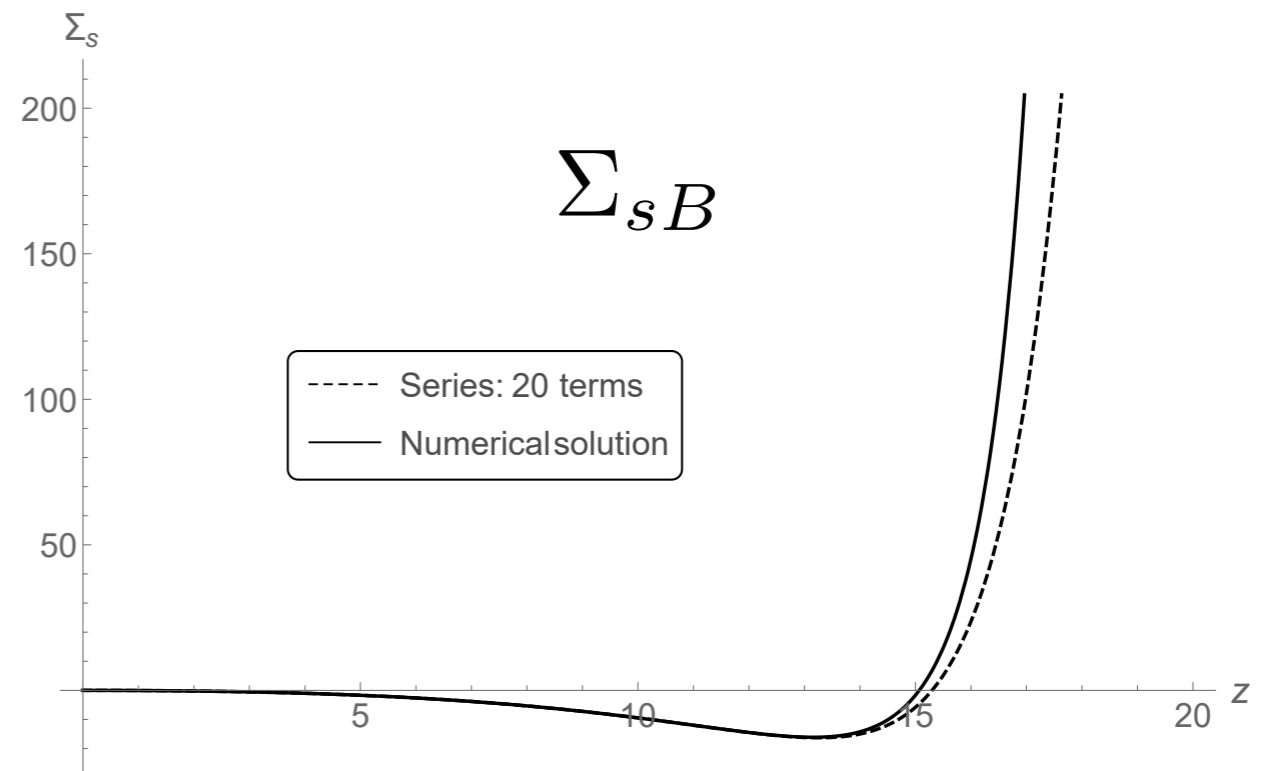
$$\Sigma_{sB} = \left(z \frac{d}{dz} - 1 \right) \Sigma'_{sB} - z \left(-\frac{19}{72} \Sigma_A + \frac{1}{12} \Sigma'_{sB} - \frac{1}{30} \Sigma'_{tB} + \frac{23}{450} \Sigma_A^2 - \frac{1}{30} \Sigma_A \Sigma'_{sB} + \frac{1}{60} \Sigma_A \Sigma'_{tB} \right)$$

Sum of Ladder diagrams

Leading divs



Subleading divs



Infinite number of poles

Infinite number of poles at the same position

All loop Exact Recurrence Relation

D=8 N=1

s-channel term $S_n(s, t)$ **t-channel term** $T_n(s, t)$ $T_n(s, t) = S_n(t, s)$

Exact relation for ALL diagrams

$$\begin{aligned}
 nS_n(s, t) &= -2s^2 \int_0^1 dx \int_0^x dy y(1-x) (S_{n-1}(s, t') + T_{n-1}(s, t'))|_{t'=tx+yu} \\
 + s^4 \int_0^1 dx x^2(1-x)^2 \sum_{k=1}^{n-2} \sum_{p=0}^{2k-2} \frac{1}{p!(p+2)!} \frac{d^p}{dt'^p} (S_k(s, t') + T_k(s, t')) \times \\
 S_1 = \frac{1}{12}, T_1 = \frac{1}{12} &\times \frac{d^p}{dt'^p} (S_{n-1-k}(s, t') + T_{n-1-k}(s, t'))|_{t'=-sx} (tsx(1-x))^p
 \end{aligned}$$

summation $\Sigma_3(s, t, z) = \Sigma_1(s, t, z) - S_2(s, t)z^2 + S_1(s, t)z$, $\Sigma_2(s, t, z) = \Sigma_1(s, t, z) + S_1(s, t)z$

Diff eqn

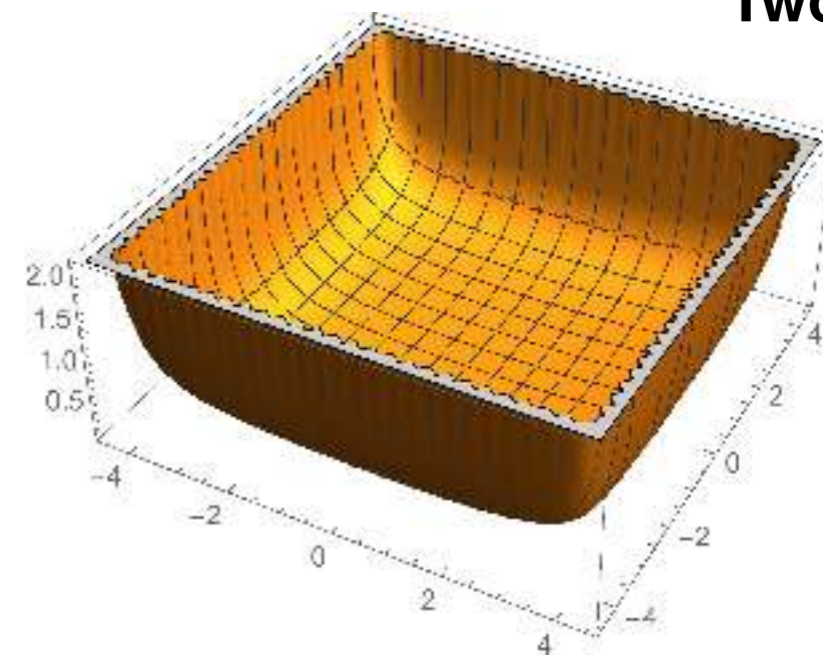
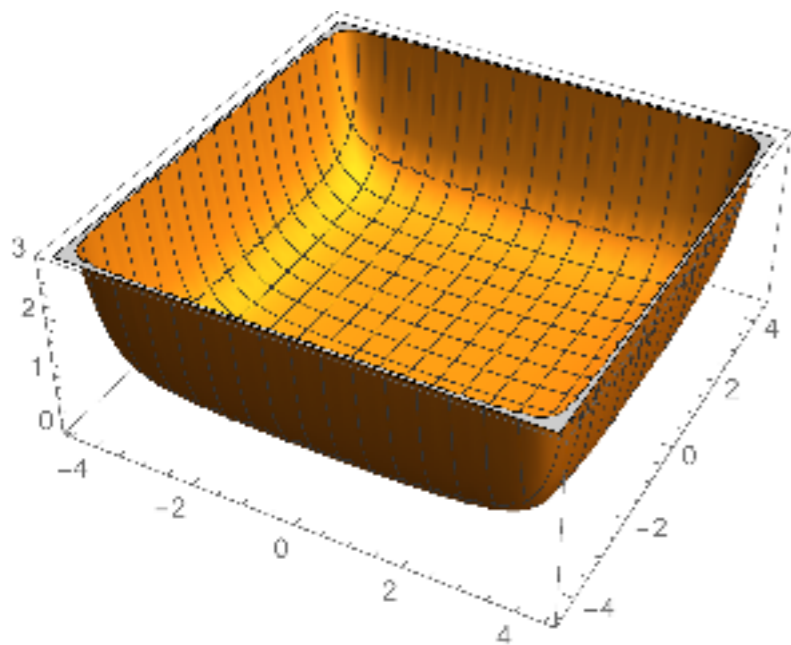
$$\begin{aligned}
 \frac{d}{dz} \Sigma(s, t, z) &= -\frac{1}{12} + 2s^2 \int_0^1 dx \int_0^x dy y(1-x) (\Sigma(s, t', z) + \Sigma(t', s, z))|_{t'=tx+yu} \\
 -s^4 \int_0^1 dx x^2(1-x)^2 \sum_{p=0}^{\infty} \frac{1}{p!(p+2)!} \left(\frac{d^p}{dt'^p} (\Sigma(s, t', z) + \Sigma(t', s, z))|_{t'=-sx} \right)^2 &(tsx(1-x))^p.
 \end{aligned}$$

All loop Solution (leading divs)

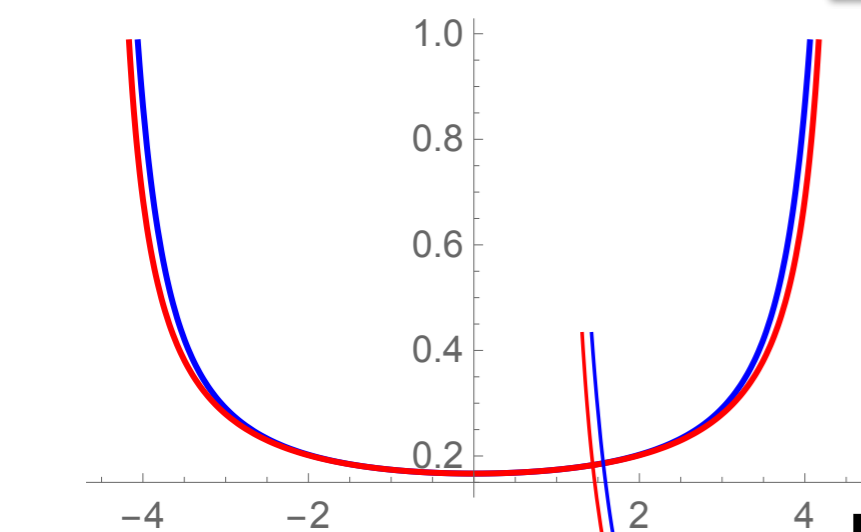
D=8 N=1

PT (15 terms)

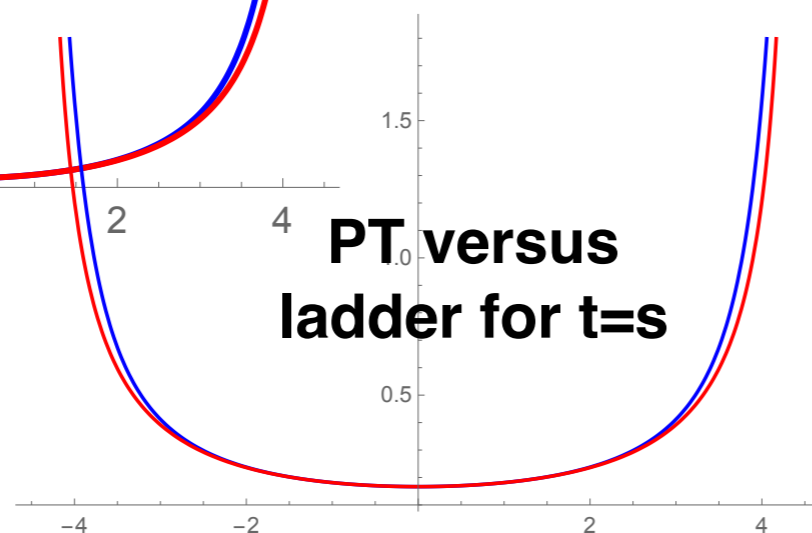
Two ladders



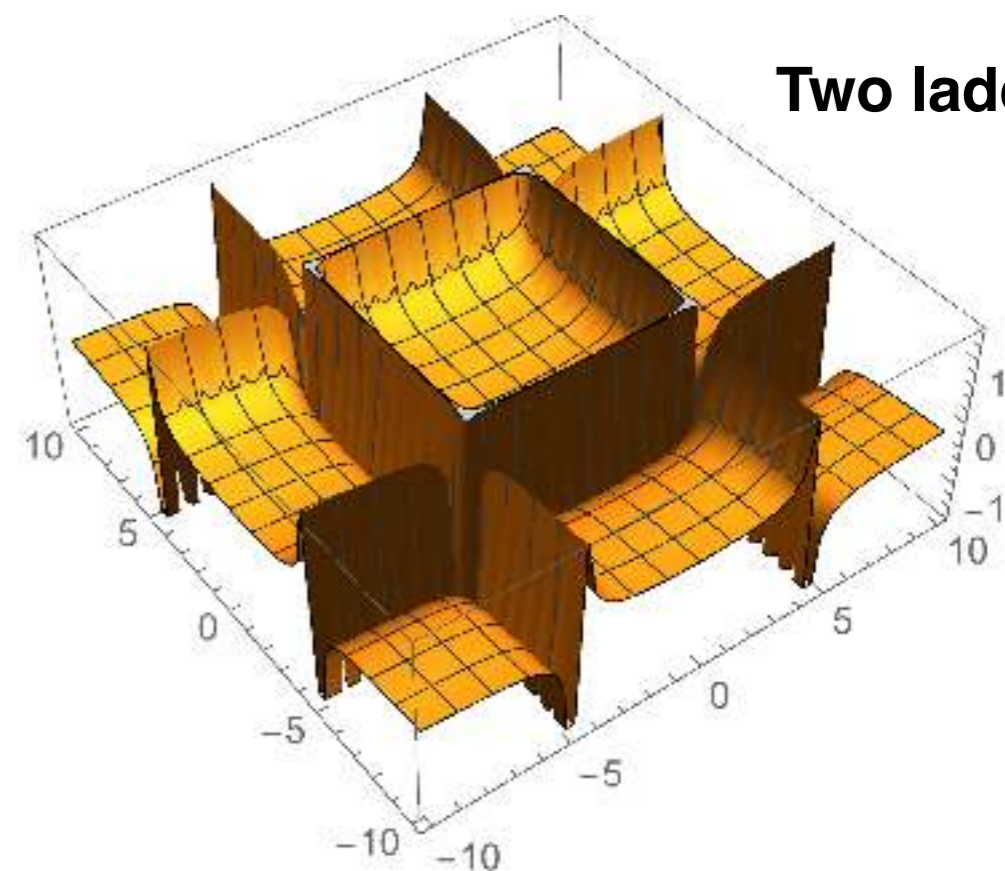
Total leading divergence as function of zs^2 and zt^2



PT versus ladder for t=0



PT versus ladder for t=s



Two ladders

Summary

$D=8$ $N=1$

- The UV divergences for the on-shell scattering amplitudes **DO NOT CANCEL** in any given order of PT
- The recurrence relations allow one to calculate the leading UV divergences in **ALL** orders of PT algebraically starting from 1 loop
- The recurrence relations allow one to calculate the sub leading UV divergences in **ALL** orders of PT algebraically starting from 1 and 2 loops
- This procedure apparently continues the same way for all divergences just like in renormalizable theories

Summary

D=8 N=1

• The sum of the leading UV divergences to ALL orders obeys the nonlinear integro-differential equation

• The solution to this equation possesses the infinite sequence of poles in s and t channels

• There is no simple limit when $\epsilon \rightarrow +0$

• The subleading divergences seem to repeat the general pattern of the leading ones.

Summary

D=8 N=1

👤 What is a possible interpretation of the UV divergences in this case?

👤 The theory is non-renormalizable in perturbative sense.

👤 If one assumes some kind of effective cut-off, then taking $1/\epsilon = \log s/M$ one gets an infinite number of bound states in s and t channels with equidistant masses² - possible link to a string theory interpretation