Random Walk in Superspace: from Grassmannian Algebra to Gravity and Dark Matter

Dmitri Kazakov Joint Institute for Nuclear Research Dubna, Russia



Symposium on «Exciting Physics», Makutsi, 13-20 November 2011

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Electromagnetic Strong

Weak



HEP Paradox







 θ θ 4D space-tme θ

Grassmannian extra dim





Grassmannian extra dim





Grassmannian extra dim









(Super) Algebra

Lorentz Algebra

 $[P_{\mu}, P_{\nu}] = 0, \ [P_{\mu}, M_{\rho\sigma}] = i(g_{\mu\rho}P_{\sigma} - g_{\mu\sigma}P_{\rho}),$ $[M_{\mu\nu}, M_{\rho\sigma}] = i(g_{\nu\rho}M_{\mu\sigma} - g_{\nu\sigma}M_{\mu\rho} - g_{\mu\rho}M_{\nu\sigma} + g_{\mu\sigma}M_{\nu\rho}),$

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SUSY Algebra $\begin{bmatrix} Q_{\alpha}^{i}, P_{\mu} \end{bmatrix} = \begin{bmatrix} \overline{Q}_{\dot{\alpha}}^{i}, P_{\mu} \end{bmatrix} = 0,$ $\begin{bmatrix} Q_{\alpha}^{i}, M_{\mu\nu} \end{bmatrix} = \frac{1}{2} (\sigma_{\mu\nu})_{\alpha}^{\beta} Q_{\beta}^{i}, \ \begin{bmatrix} \overline{Q}_{\dot{\alpha}}^{i}, M_{\mu\nu} \end{bmatrix} = -\frac{1}{2} \overline{Q}_{\dot{\beta}}^{i} (\overline{\sigma}_{\mu\nu})_{\dot{\alpha}}^{\dot{\beta}},$ $\{ Q_{\alpha}^{i}, \overline{Q}_{\beta}^{j} \} = 2\delta^{ij} (\sigma^{\mu})_{\alpha\beta} P_{\mu},$ $\alpha, \dot{\alpha}, \beta, \dot{\beta} = 1, 2; \ i, j = 1, 2, ..., N.$

(Super) Algebra

Lorentz Algebra

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SUSY Algebra

$$\begin{split} &[\mathcal{Q}_{\alpha}^{i}, P_{\mu}] = [\overline{\mathcal{Q}}_{\alpha}^{i}, P_{\mu}] = 0, \\ &[\mathcal{Q}_{\alpha}^{i}, M_{\mu\nu}] = \frac{1}{2} (\sigma_{\mu\nu})_{\alpha}^{\beta} \mathcal{Q}_{\beta}^{i}, \ [\overline{\mathcal{Q}}_{\alpha}^{i}, M_{\mu\nu}] = -\frac{1}{2} \overline{\mathcal{Q}}_{\beta}^{i} (\overline{\sigma}_{\mu\nu})_{\alpha}^{\beta}, \\ &\{\mathcal{Q}_{\alpha}^{i}, \overline{\mathcal{Q}}_{\beta}^{j}\} = 2\delta^{ij} (\sigma^{\mu})_{\alpha\beta} P_{\mu} \\ &\alpha, \dot{\alpha}, \beta, \dot{\beta} = 1, 2; \ i, j = 1, 2, ..., N. \end{split}$$

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Lorentz Algebra

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SUSY Algebra $[Q_{\alpha}^{i}, P_{\mu}] = [\overline{Q}_{\dot{\alpha}}^{i}, P_{\mu}] = 0,$ $[Q_{\alpha}^{i}, M_{\mu\nu}] = \frac{1}{2} (\sigma_{\mu\nu})_{\alpha}^{\beta} Q_{\beta}^{i}, [\overline{Q}_{\dot{\alpha}}^{i}, M_{\mu\nu}] = -\frac{1}{2} \overline{Q}_{\beta}^{i} (\overline{\sigma}_{\mu\nu})_{\dot{\alpha}}^{\dot{\beta}},$ $\{Q_{\alpha}^{i}, \overline{Q}_{\beta}^{j}\} = 2\delta^{ij} (\sigma^{\mu})_{\alpha\beta} P_{\mu}$ $\alpha, \dot{\alpha}, \beta, \dot{\beta} = 1, 2; i, j = 1, 2, ..., N.$ Superspace
 $x_{\mu} \rightarrow x_{\mu}, \theta_{\alpha}, \bar{\theta}_{\dot{\alpha}}$ Grassmannian $\alpha, \dot{\alpha} = 1, 2$
parameters $\vartheta_{\alpha}^2 = 0, \ \bar{\vartheta}_{\dot{\alpha}}^2 = 0$

(Super) Algebra

Lorentz Algebra

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Superspace $x_{\mu} \to x_{\mu}, \theta_{\alpha}, \overline{\theta}_{\dot{\alpha}}$ $\alpha, \dot{\alpha} = 1, 2$ Grassmannian parameters $\vartheta_{\alpha}^2 = 0, \ \overline{\vartheta}_{\dot{\alpha}}^2 = 0$ **SUSY** Generators $Q_{\alpha} = \frac{\partial}{\partial \vartheta_{\alpha}} - i\sigma_{\alpha\dot{\alpha}}^{\mu}\overline{\theta}^{\alpha}\partial_{\mu}$ $\overline{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial \overline{\mathfrak{H}}_{\dot{\alpha}}} + i \theta_{\alpha} \sigma^{\mu}_{\alpha \dot{\alpha}} \partial_{\mu}$

$$Q_{\alpha}^2 = 0, \ \overline{Q}_{\dot{\alpha}}^2 = 0$$

(Super) Algebra

Lorentz Algebra

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Superspace $x_{\mu} \to x_{\mu}, \theta_{\alpha}, \bar{\theta}_{\dot{\alpha}}$ $\alpha, \dot{\alpha} = 1, 2$ Grassmannian parameters $\vartheta_{\alpha}^2 = 0, \ \overline{\vartheta}_{\dot{\alpha}}^2 = 0$ **SUSY** Generators $Q_{\alpha} = \frac{\partial}{\partial \vartheta_{\alpha}} - i\sigma_{\alpha\dot{\alpha}}^{\mu}\overline{\theta}^{\alpha}\partial_{\mu}$ $\overline{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial \overline{\vartheta}_{\dot{\alpha}}} + i \theta_{\alpha} \sigma^{\mu}_{\alpha \dot{\alpha}} \partial_{\mu}$ $Q_{\alpha}^{2} = 0, \ \overline{Q}_{\dot{\alpha}}^{2} = 0$ Supertranslation $x_{\mu} \to x_{\mu} + i\theta\sigma_{\mu}\bar{\xi} - i\xi\sigma_{\mu}\bar{\theta}.$ $\theta \to \theta + \xi$,

 $\bar{\theta} \to \bar{\theta} + \bar{\xi}$

Unification with Gravity

Unification with Gravity

SUSY transformation

 $Q \mid boson >= \mid fermion > \quad Q \mid fermion >= \mid boson >$ spin 2 \Rightarrow spin 3/2 \Rightarrow spin 1 \Rightarrow spin 1/2 \Rightarrow spin 0

Unification entry

SUSY transformation

 $Q \mid boson >= \mid fermion > \quad Q \mid fermion >= \mid boson >$ spin 2 \implies spin 3/2 \implies spin 1 \implies spin 1/2 \implies spin 0

Unification of matter (fermions) with forces (bosons) naturally arises from an attempt to unify gravity with the other interactions

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$$\{Q_{\alpha}^{i}, \overline{Q}_{\beta}^{j}\} = 2\delta^{ij} (\sigma^{\mu})_{\alpha\beta} P_{\mu} \implies \{\delta_{\varepsilon}, \overline{\delta_{\varepsilon}}\} = 2(\varepsilon \sigma^{\mu} \overline{\varepsilon}) P_{\mu}$$

$$\varepsilon = \varepsilon(x) \text{ local coordinate transf.} \implies (\text{super)gravity}$$

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Local supersymmetry = general relativity !









Length = N



Length = N Length ~ N^{ν}, $\nu < 1$ N $\rightarrow \infty$



Length = N
Length ~ N^{$$\nu$$}, $\nu < 1$
N $\rightarrow \infty$



Walk in superspace



Length = N
Length ~ N^{$$\nu$$}, $\nu < 1$
N $\rightarrow \infty$
Field theory \Rightarrow
Anamalous dimension
of the field

of the field at the critical point



Walk in superspace



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Length ~ N^{$$\nu$$}, $\nu < 1$
N $\rightarrow \infty$
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$$\left\langle \begin{array}{c} \\ \\ \\ \\ \end{array} \right\rangle \sim \theta^2 = 0$$

Walk in superspace

Exact Solution:
$$\nu = \frac{3}{d+2}$$

 $d = 1$ $d = 2$ $d = 3$ $d = 4$
 $\nu = 1$ $\nu = \frac{3}{4}$ $\nu = \frac{3}{5}$ $\nu = \frac{1}{2}$



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Experiment : 0.76 ± 0.03 0.589 ± 0.003



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N=1 SUSY Chiral supermultiplet:

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Expansion over grassmannian parameter

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 $\Phi(y,\theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)$
N=1 SUSY Chiral supermultiplet:

$$\Phi(y,\theta) = A(y) + \sqrt{2\theta}\psi(y) + \theta\theta F(y)$$
$$\theta^2 = \theta_1\theta_2, \ \theta_1^2 = \theta_2^2 = 0!$$

N=1 SUSY Chiral supermultiplet:

superfield

$$\mathbf{\hat{\Phi}}(y,\theta) = A(y) + \sqrt{2\theta\psi(y)} + \theta\theta F(y)$$
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Superfield in Superspace
N=1 SUSY Chiral supermultiplet:
Superfield

$$\Phi(y,\theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)$$

 $\Phi(y,\theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)$
 $(y = x + i\theta\sigma\overline{\theta})$
 $= A(x) + i\theta\sigma^{\mu}\overline{\theta}\partial_{\mu}A(x) + \frac{1}{4}\theta\theta\overline{\theta}\overline{\theta}\Box A(x)$
 $+\sqrt{2}\theta\psi(x) - i/\sqrt{2}\theta\theta\partial_{\mu}\psi(x)\sigma^{\mu}\overline{\theta} + \theta\theta F(x)$
Gauge supermultiplet:
 $V(x,\theta,\overline{\theta}) = C(x) + i\theta\chi(x) - i\overline{\theta}\chi(x) + i\theta\theta M(x) - i\overline{\theta}\overline{\theta}M^{+}(x)$
 $-\theta\sigma^{\mu}\overline{\theta}v_{\mu}(x) + i\theta\theta\overline{\theta}[\overline{\lambda}(x) + i\sigma^{\mu}\partial_{\mu}\chi(x)] - i\overline{\theta}\overline{\theta}\theta[\lambda(x) + i\sigma^{\mu}\partial_{\mu}\overline{\chi}(x)]$
 $+\frac{1}{2}\theta\theta\overline{\theta}\overline{\theta}[D(x) + \frac{1}{2}\Box C(x)]$

Superfield in Superspace
N=1 SUSY Chiral supermultiplet:
Superfield
$$gin=0$$
 $gin=1/2$
 $\Phi(y,\theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)$
 $(y = x + i\theta\sigma\bar{\theta})$ $\theta^2 = \theta_1\theta_2, \ \theta_1^2 = \theta_2^2 = 0!$ Auxiliary field
 $(y = x + i\theta\sigma\bar{\theta})$ $\theta^2 = \theta_1\theta_2, \ \theta_1^2 = \theta_2^2 = 0!$ Auxiliary field
 $(unphysical d.o.f. needed
to close SYSY algebra)$
 $= A(x) + i\theta\sigma^{\mu}\bar{\theta}\partial_{\mu}A(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box A(x)$
 $+\sqrt{2}\theta\psi(x) - i/\sqrt{2}\theta\bar{\theta}\partial_{\mu}\psi(x)\sigma^{\mu}\bar{\theta} + \theta\theta F(x)$ component fields
Gauge supermultiplet:
 $V(x,\theta,\bar{\theta}) = C(x) + i\theta\chi(x) - i\bar{\theta}\chi(x) + i\theta\theta M(x) - i\bar{\theta}\bar{\theta}M^+(x)$
 $-\theta\sigma^{\mu}\bar{\theta}v_{\mu}(x) + i\theta\theta\bar{\theta}[\bar{\lambda}(x) + i\sigma^{\mu}\partial_{\mu}\chi(x)] - i\bar{\theta}\bar{\theta}\bar{\theta}[\lambda(x) + i\sigma^{\mu}\partial_{\mu}\chi(x)]$
 $gin=1$ $+\frac{1}{2}\theta\bar{\theta}\bar{\theta}\bar{\theta}[D(x) + \frac{1}{2}\Box C(x)]$

Superfield in Superspace
N=1 SUSY Chiral supermultiplet:

$$\begin{array}{c} \text{spin=0} \\ \Phi(y,\theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \\ \Phi(y,\theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \\ \Phi(y,\theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_2^2 = 0! \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_2^2 = 0! \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_2^2 = 0! \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_2^2 = 0! \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_2^2 = 0! \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_2^2 = 0! \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_2^2 = 0! \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_2^2 = 0! \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_2^2 = 0! \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_2^2 = 0! \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_2^2 = 0! \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_2^2 = 0! \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_2^2 = 0! \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_2^2 = 0! \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_2^2 = 0! \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_2^2 = 0! \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_2^2 = 0! \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_2^2 = 0! \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_2^2 = 0! \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_2^2 = 0! \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_2^2 = 0! \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_2^2 = 0! \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_2^2 = 0! \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_2^2 = 0! \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_1^2 \theta_2 \theta_2 \theta_2 \theta_2 \\ \Phi(y) = \theta_1 \theta_2, \quad \theta_1^2 = \theta_1^2 \theta_2 \theta_2 \theta_2 \theta_2 \\ \Phi(y) = \theta_1^2 \theta_2, \quad \theta_1^2 = \theta_1^2 \theta_2 \theta_2 \theta_2 \\ \Phi(y) = \theta_1^2 \theta_2, \quad \theta_1^2 = \theta_1^2 \theta_2 \theta_2 \\ \Phi(y) = \theta_1^2 \theta_2, \quad \theta_1^2 = \theta_1^2 \theta_2 \theta_2 \\ \Phi(y) = \theta_1^2 \theta_2, \quad \theta_1^2 = \theta_1^2 \theta_2 \theta_2 \\ \Phi(y) = \theta_1^2 \theta_2 \\ \Phi(y) = \theta_1$$



SUSY Multiplets

SUSY Multiplets Chiral multiplet N = 1 h = 0# of states 1 2 1

SUSY Multiplets Chiral multiplet N = 1 h = 0 $\begin{array}{c} helicity -1/2 \ 0 \ 1/2 \\ \# \text{ of states } 1 \ 2 \ 1 \end{array}$ $\begin{array}{c} scalar \ spinor \\ (\varphi, \psi) \end{array}$

SUSY Multiplets
Chiral multiplet
$$N = 1$$
 $h = 0$
Nector multiplet $N = 1$ $h = 1/2$
Melicity $-1/2$ 0 1/2
of states 1 2 1
Melicity $-1/2$ 1/2 (ϕ, ψ)
Nector multiplet $N = 1$ $h = 1/2$
Melicity -1 -1/2 1/2 1
of states 1 1 1 1





Members of a supermultiplet are called superpartners

SUSY Multiplets

Chiral multiplet N = 1 h = 0

Vector multiplet N = 1 h = 1/2 helicity -1 -# of states 1

helicity
$$-1/2 \ 0 \ 1/2$$

of states 1 2 1
helicity $-1 - 1/2 \ 1/2 \ 1$
of states 1 1 1 1

spinor vector

scalar spinor

Members of a supermultiplet are called **superpartners**

| N=4 | SUSY YM | helicity | -1 -1/2 0 1/2 1 |
|-----|---------|-------------|-------------------------------|
| | h = -1 | # of states | 1 4 6 4 1 |
| N=8 | SUGRA | helicity | -2 -3/2 -1 -1/2 0 1/2 1 3/2 2 |
| | h = -2 | # of states | 1 8 28 56 70 56 28 8 1 |

SUSY Multiplets

Chiral multiplet N = 1 h = 0 helicity -1/2 0 1/2# of states 1 2 1

Vector multiplet N = 1 h = 1/2 helicity

spinor vector

 (λ, A_{μ})

scalar spinor

 $(\dot{\varphi}, \dot{\psi})$

Members of a supermultiplet are called **superpartners**

| N=4 | SUSY YM | helicity | -1 -1/2 0 1/2 1 |
|--|---------|-------------|-------------------------------|
| | h = -1 | # of states | 1 4 6 4 1 |
| N=8 | SUGRA | helicity | -2 -3/2 -1 -1/2 0 1/2 1 3/2 2 |
| | h = -2 | # of states | 1 8 28 56 70 56 28 8 1 |
| $N \le 4S$ spin $N \le 4$ For renormalizable theories (YM) $N \le 8$ For (super)gravity | | | |

N=1 SUSY: Cancellation of quadratic divergences Λ^2

N=1 SUSY: Cancellation of quadratic divergences Λ^2

N=4 SUSY: Cancellation of all divergences LogA²



- N=1 SUSY: Cancellation of quadratic divergences Λ^2
- N=4 SUSY: Cancellation of all divergences LogA²



N=4 SUSY: Integrability (Exact solution)?

- N=1 SUSY: Cancellation of quadratic divergences Λ^2
- N=4 SUSY: Cancellation of all divergences $LogA^2$



- N=4 SUSY: Integrability (Exact solution)?
- N=8 SUGRA: Finiteness (Construction of quantum) gravity)??

Bosons and Fermions come in pairs

Bosons and Fermions come in pairs

(φ,ψ) Spin 0 Spin 1/2

Bosons and Fermions come in pairs

(φ,ψ)

Spin 0 Spin 1/2

schur chinnen

Bosons and Fermions come in pairs

$$(\varphi, \psi)$$
 (λ, A_{μ})

Spin 0 Spin I/2 Spin I/2 Spin I Scalar (highlight)

Bosons and Fermions come in pairs

$$(\varphi, \psi)$$
 (λ, A_{μ})

Spin 0 Spin 1/2 Spin 1/2 Spin 1

schult with a sono sector

Bosons and Fermions come in pairs

$$(\varphi, \psi)$$
 (λ, A_{μ})

Spin 0 Spin 1/2 Spin 1/2 Spin 1

scalar with a solution sector

Spin 3/2 Spin 2

Bosons and Fermions come in pairs

$$\begin{pmatrix} \varphi, \psi \end{pmatrix} & (\lambda, A_{\mu}) & (g, g) \\ \text{Spin 0 Spin 1/2 Spin 1/2 Spin 1 Spin 3/2 Spin 2} \\ \text{Spin 0 Spin 1/2 Spin 1/2 Spin 1 Spin 3/2 Spin 2} \\ \text{Spin 0 Spin 1/2 Spin 1/2 Spin 1 Spin 3/2 Spin 2} \\ \text{Spin 0 Spin 1/2 Spin 1/2 Spin 1 Spin 3/2 Spin 2} \\ \text{Spin 0 Spin 1/2 Spin 1/2 Spin 1 Spin 3/2 Spin 2} \\ \text{Spin 0 Spin 1/2 Spin 1/2 Spin 1 Spin 3/2 Spin 2} \\ \text{Spin 0 Spin 1/2 Spin 1/2 Spin 1 Spin 3/2 Spin 2} \\ \text{Spin 0 Spin 1/2 Spin 1/2 Spin 1 Spin 3/2 Spin 2} \\ \text{Spin 0 Spin 1/2 Spin 1/2 Spin 1 Spin 3/2 Spin 2} \\ \text{Spin 0 Spin 1/2 Spin 1/2 Spin 1 Spin 3/2 Spin 2} \\ \text{Spin 0 Spin 1/2 Spin 1/2 Spin 1 Spin 3/2 Spin 2} \\ \text{Spin 0 Spin 1/2 Spin 1/2 Spin 1 Spin 3/2 Spin 2} \\ \text{Spin 0 Spin 1/2 Spin 1/2 Spin 1 Spin 3/2 Spin 2} \\ \text{Spin 0 Spin 1/2 Spin 1/2 Spin 1 Spin 3/2 Spin 2} \\ \text{Spin 0 Spin 1/2 Spin 1/2 Spin 1 Spin 3/2 Spin 2} \\ \text{Spin 0 Spin 3/2 Spin 3/2 Spin 2} \\ \text{Spin 0 Spin 3/2 Spin$$

Particle Content of the MSSM

| Superfield | Bosons | Fermions | $SU_{c}(3)$ | $SU_L(2)$ | $U_{\gamma}(1)$ |
|----------------------|--|--|-------------|-----------|-----------------|
| Gauge | | | | | |
| G^{a} | <i>gluon</i> g ^a | gluino ĝ ^a | 8 | 1 | 0 |
| V^k | Weak $W^{k}(W^{\pm}, Z)$ | wino, zino $	ilde{w}^k(ilde{w}^{\pm},	ilde{z})$ | 1 | 3 | 0 |
| V' | Hypercharge $B(\gamma)$ | bino $\tilde{b}(\tilde{\gamma})$ | 1 | 1 | 0 |
| Matter | | | | | |
| $L_{i \text{ slep}}$ | $\tilde{L}_i = (\tilde{v}, \tilde{e})_L$ | $L_i = (\mathbf{v}, e)_L$ | 1 | 2 | -1 |
| E_i | $\tilde{E}_i = \tilde{e}_R$ | $\sum_{i=1}^{n} E_{i} = e_{R}$ | 1 | 1 | 2 |
| Q_i | $\tilde{Q}_i = (\tilde{u}, \tilde{d})_L$ | $Q_i = (u, d)_L$ | 3 | 2 | 1/3 |
| $U_i^{}$ squa | $\mathbf{rks} \ \ \vec{U}_i = \vec{u}_R \qquad \mathbf{q}$ | uarks $U_i = u_R^c$ | 3* | 1 | -4/3 |
| D_i | $\tilde{D}_i = \tilde{d}_R$ | $D_i = d_R^c$ | 3* | 1 | 2/3 |
| Higgs | | | | | |
| H_1 Hi | H_1 higg | $\frac{1}{1}$ sinos $\left\{ \tilde{H}_{1} \right\}$ | 1 | 2 | -1 |
| H_2 | H_2 | \tilde{H}_2 | 1 | 2 | 1 |

Particle Content of the MSSM

| Superfield | Bosons | Fermions | $SU_c(3)$ | $SU_L(2)$ | $U_{\gamma}(1)$ |
|----------------------|--|--|-----------|-----------|-----------------|
| Gauge | | | | | |
| G^{a} | gluon g ^a | gluino ĝ ^a | 8 | 1 | 0 |
| V^k | Weak $W^{k}(W^{\pm}, Z)$ | wino, zino $	ilde{w}^k(ilde{w}^{\pm},	ilde{z})$ |) 1 | 3 | 0 |
| V' | Hypercharge $B(\gamma)$ | bino $\tilde{b}(\tilde{\gamma})$ | 1 | 1 | 0 |
| Matter | | | | | |
| $L_{i \text{ slep}}$ | $\tilde{L}_i = (\tilde{v}, \tilde{e})_L$ | $\int L_i = (\mathbf{v}, e)_L$ | 1 | 2 | -1 |
| E_i | $\tilde{E}_i = \tilde{e}_R$ | $E_i = e_R$ | 1 | 1 | 2 |
| Q_i | $\tilde{Q}_i = (\tilde{u}, \tilde{d})_L$ | $Q_i = (u, d)_L$ | 3 | 2 | 1/3 |
| $U_i^{}$ squa | $\mathbf{urks} \in \tilde{U}_i = \tilde{u}_R \qquad \mathbf{q}$ | uarks $\downarrow U_i = u_R^c$ | 3* | 1 | -4/3 |
| D_i | $\tilde{D}_i = \tilde{d}_R$ | $D_i = d_R^c$ | 3* | 1 | 2/3 |
| Higgs | | | | | |
| H_1 Hi | $\frac{\int H_1}{higg}$ | $\sin \delta = \tilde{H}_1$ | 1 | 2 | -1 |
| H_2 | H_2 | \tilde{H}_2 | 1 | 2 | 1 |

Particle Content of the MSSM

| Superfield | Bosons | Fermions | $SU_c(3)$ | $SU_L(2)$ | $U_{Y}(1)$ |
|----------------------|---|--|-----------|-----------|------------|
| Gauge | | | | | |
| G^{a} | gluon g ^a | gluino $	ilde{g}^{a}$ | 8 | 1 | 0 |
| V^k | Weak $W^{k}(W^{\pm}, Z)$ | wino, zino $\tilde{w}^k(\tilde{w}^{\pm}, \tilde{z})$ |) 1 | 3 | 0 |
| V' | Hypercharge $B(\gamma)$ | bino $\tilde{b}(\tilde{\gamma})$ | 1 | 1 | 0 |
| Matter | | | | | |
| $L_{i \text{ slep}}$ | $\int \tilde{L}_i = (\tilde{v}, \tilde{e})_L$ | $\int L_i = (\mathbf{v}, e)_L$ | 1 | 2 | -1 |
| E_i | $\tilde{E}_i = \tilde{e}_R$ | $\sum_{i=1}^{n} E_i = e_R$ | 1 | 1 | 2 |
| Q_i | $\tilde{Q}_i = (\tilde{u}, \tilde{d})_L$ | $Q_i = (u,d)_L$ | 3 | 2 | 1/3 |
| $U_i^{}$ squa | $\mathbf{rks} \neq \tilde{U}_i = \tilde{u}_R \qquad \mathbf{q}$ | uarks $U_i = u_R^c$ | 3* | 1 | -4/3 |
| D_i | $\tilde{D}_i = \tilde{d}_R$ | $D_i = d_R^c$ | 3* | 1 | 2/3 |
| Higgs | | | | | |
| H_1 Hi | $\frac{\int H_1}{higg}$ | $\sin sin S \left\{ \tilde{H}_{1} \right\}$ | 1 | 2 | -1 |
| H_2 | H_2 | \tilde{H}_2 | 1 | 2 | 1 |
Particle Content of the MSSM

| Superfield | Bosons | Fermions | $SU_c(3)$ | $SU_L(2)$ | $U_{\gamma}(1)$ |
|---------------|--|---|-----------|-----------|-----------------|
| Gauge | | | | | |
| G^{a} | gluon g ^a | gluino $	ilde{\mathbf{g}}^{\mathrm{a}}$ | 8 | 1 | 0 |
| V^k | Weak $W^{k}(W^{\pm}, Z)$ | wino, zino $\tilde{w}^k(\tilde{w}^{\pm},\tilde{z})$ |) 1 | 3 | 0 |
| V' | Hypercharge $B(\gamma)$ | bino $\tilde{b}(\tilde{\gamma})$ | 1 | 1 | 0 |
| Matter | | | | | |
| L_{i} slep | $\int L_i = (\tilde{v}, \tilde{e})_L$ | $\int L_i = (\mathbf{v}, e)_L$ | 1 | 2 | -1 |
| E_i | $\tilde{E}_i = \tilde{e}_R$ | $\sum_{i=1}^{n} E_i = e_R$ | 1 | 1 | 2 |
| Q_i | $\tilde{Q}_i = (\tilde{u}, \tilde{d})_L$ | $Q_i = (u,d)_L$ | 3 | 2 | 1/3 |
| $U_i^{}$ squa | $\mathbf{rks} \mathbf{V}_i = \tilde{u}_R \qquad \mathbf{Q}$ | uarks $U_i = u_R^c$ | 3* | 1 | -4/3 |
| D_i | $\tilde{D}_i = \tilde{d}_R$ | $D_i = d_R^c$ | 3* | 1 | 2/3 |
| Higgs | | | | | |
| H_1 Hi | $\frac{1}{2} H_1$ higg | I I I I I I I I I I I I I I I I I I I | 1 | 2 | -1 |
| H_2 | H_2 | \tilde{H}_2 | 1 | 2 | 1 |

















































Strong int's

Typical SUSY signature: Missing Energy and Transverse Momentum

New precise cosmological data

$$\Omega h^{2} = 1 \quad \Leftrightarrow \quad \rho = \rho_{cri}$$



Supernova Ia explosion
CMBR thermal fluctuations (measured by WMAP)

New precise cosmological data

$$\Omega h^2 = 1 \quad \iff \rho = \rho_{crit}$$



Dark Matter in the Universe:





Supernova Ia explosion
CMBR thermal fluctuations (measured by WMAP)

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Hot DM

(not favoured by galaxy formation)



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Dark Matter in the Universe:





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Cold DM (rotation curves of Galaxies)

New precise cosmological data

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Dark Matter in the Universe:





Supernova Ia explosion
CMBR thermal fluctuations (measured by WMAP)

Hot DM

(not favoured by galaxy formation)

Cold DM (rotation curves of Galaxies)

SUSY

Dark Matter in the Universe

15



Spiral galaxies consist of a central bulge and a very thin disc, and surrounded by an approximately spherical halo of dark matter The flat rotation curves of spiral galaxies provide the most direct evidence for the existence of large amount of the dark matter.



SUSY provides a candidate for the Dark matter – a stable neutral particle

Origin of Dark Matter











$$\widetilde{\chi}^{0} = N_{1}\widetilde{\gamma} + N_{2}\widetilde{z} + N_{3}\widetilde{H}_{1}^{0} + N_{4}\widetilde{H}_{2}^{0}$$
photino zino higgsino higgsino

Origin of Dark Matter Photon Correspondence Photino THE $\tilde{\gamma}$ LIGHTESTNEUTRALINO Cosmic Microwave Relic Dark Matter Background



Origin of Dark Matter Photon Correction THE $\tilde{\gamma}$ LIGHTEST NEUTRALINO Cosmic Microwave Relic Dark Matter Background

$$\widetilde{\chi}^{0} = N_{1}\widetilde{\gamma} + N_{2}\widetilde{z} + N_{3}\widetilde{H}_{1}^{0} + N_{4}\widetilde{H}_{2}^{0}$$
photino zino higgsino higgsino
$$\chi_{1}^{0}$$
bino higgsino (u)
wino higgsino (d)









N=4 SYM and N=8 SUGRA: Exceptional models of QFT



- N=4 SYM and N=8 SUGRA: Exceptional models of QFT
- ✓ SUSY in Particle Physics:Yes or No for LHC to judge



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✓ SUSY World is beautiful


We like elegant solutions

