

From UV to IR Finiteness of Conformal Field Theories

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Conformal Quantum Theories in D=4

- N=4 Super Yang-Mills Theory - UV Finite to all loops
- N=2 Super Yang-Mills Theory - Only one-loop UV divergences
UV Finite if

$$\sum_R T(R) = C_2(G)$$

- N=1 Super Yang-Mills Theory
One-loop UV finiteness

$$\sum_R T(R) = 3C_2(G), \quad Y_{ijk} Y^{kjl} = \delta_i^l C_2(R), \quad \text{for all } R$$

All loop UV finiteness via PT expansion

$$Y_{ijk} = \sum_{n=0}^{\infty} C_{ijk}^n g^{2n+1}$$

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Conformal Quantum Theory in D=4

- No-scale classical theory, no mass parameters
- Conformal anomaly $\sim \beta$ -function =0
- Absence of UV divergences in gauge invariant quantities
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N=4 Super Yang-Mills Theory

- $\mathcal{N} = 4$ Super Yang-Mills theory is the most supersymmetric theory possible without gravity
- Field content: 1 massless gauge boson, 4 massless (Majorana) spin 1/2 fermions, 6 real (or 3 complex) massless spin 0 bosons
All fields are in adjoint representation of the gauge group (Take $SU(N_c)$)
- The theory is exactly scale invariant, conformal field theory at quantum level, i.e. the β function identically vanishes at all orders of PT

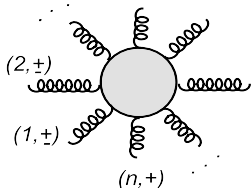
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Gluon scattering amplitudes



All outgoing gluons with helicity + or -
on mass shell

In the leading N_c order (planar limit)

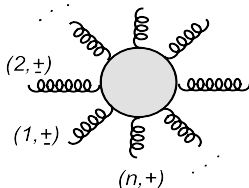
- Colour decomposition of amplitudes in N=4 SYM theory for $N_c \rightarrow \infty$

$$\mathcal{A}_n^{(l)} = g^{n-2} \left(\frac{g^2 N_c}{16\pi^2} \right)^l \sum_{perm} \text{Tr}(T^{a_{\sigma(1)}}, \dots, T^{a_{\sigma(n)}}) A_n^{(l)}(a_{\sigma(1)}, \dots, a_{\sigma(n)}),$$

where \mathcal{A}_n - physical amplitude, A_n - partial amplitude, a_i - is color index of i -th external "gluon"

- Maximal helicity violating (MHV) amplitudes (two negative helicities and the rest positive) have observed a simple structure on tree level (and even in loops) and one can speculate that this is the consequence of SUSY

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Perturbation theory

- Bern, Dixon & Smirnov's conjecture: $M_n^{(L)}(\varepsilon) \equiv A_n^{(L)}/A_n^{(0)}$

$$\mathcal{M}_n \equiv 1 + \sum_{L=1}^{\infty} \left(\frac{g^2 N_c}{16\pi^2} \right)^L M_n^{(L)}(\varepsilon) = \exp \left[\sum_{l=1}^{\infty} \left(\frac{g^2 N_c}{16\pi^2} \right)^l \left(f^{(l)}(\varepsilon) M_n^{(1)}(l\varepsilon) + C^{(l)} + E_n^{(l)}(\varepsilon) \right) \right]$$

$$f^{(l)}(\varepsilon) = f_0^{(l)}(\varepsilon) + \varepsilon f_1^{(l)}(\varepsilon) + \varepsilon^2 f_2^{(l)}(\varepsilon)$$

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$$F_4^{(1)}(0) = \frac{1}{2} \log^2 \left(\frac{-t}{-s} \right) + 4\zeta_2$$

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Loop expansion
$$\gamma_K = \sum_{l=1}^{\infty} \left(\frac{g^2 N_c}{16\pi^2} \right)^l \gamma_K^{(l)}$$

$$\gamma_K^{(1)} = 4, \quad \gamma_K^{(2)} = -8\zeta_2, \quad \gamma_K^{(3)} = 88\zeta_4, \dots$$

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$$\gamma_K(g^2) \sim \frac{\sqrt{g^2 N_c}}{\pi}, \quad G_0(g^2) \sim \sqrt{g^2 N_c} \frac{1 - \log 2}{2\pi}, \quad \text{for } g^2 N_c \rightarrow \infty$$

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Violation of BDS ansatz

- For $n = 4, 5$ the BDS ansatz goes through all checks, namely the amplitudes were calculated up to three loops for four gluons and up to two loops for five gluons.
- However, starting from $n = 6$ it fails.
 - ▶ In the strong coupling calculation in the limit $n \rightarrow \infty$ discrepancy with the BDS formula was found.
 - ▶ Starting from $n = 6$ the Regge limit factorization of the amplitude in some physical regions fails. This was also shown by explicit two-loop calculation.

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Formfactors of gauge invariant operators

Local operator

$$F(p_1^{\lambda_1}, p_2^{\lambda_2}, \dots, p_n^{\lambda_n}) = \langle 0 | \mathcal{O} | p_1^{\lambda_1} p_2^{\lambda_2} \dots p_n^{\lambda_n} \rangle$$

Using N=1 superfield notation for chiral and vector fields

$$C_{IJ} = \text{Tr}(\Phi_I \Phi_J), I \neq J$$

$$\mathcal{V}_I^J = \text{Tr}(e^{-gV} \bar{\Phi}^J e^{gV} \Phi_I), I \neq J$$

$$\mathcal{O}_I^{(n)} = \text{Tr}((\Phi_I)^n),$$

$$\mathcal{K} = \sum_I \text{Tr}(e^{-gV} \bar{\Phi}^I e^{gV} \Phi_I).$$

Exponentiation of IR divergencies in 1 and 2 loops

$$\mathcal{F}(p_1 \dots p_n) = \mathcal{F}_{tree}(p_1 \dots p_n)(1 + \text{loops})$$

$$\mathcal{M} = \frac{\mathcal{F}}{\mathcal{F}_{tree}} = (1 + \text{loops}) = \sum_{l=0} (g^2 N_c)^l \mathcal{M}^{(l)}.$$

$$\mathcal{M} = \text{Exp} \left[\sum_{i=1}^2 \frac{1}{2} \left(\hat{M}(s_{i,i+1}/\mu^2) \right) + O(\epsilon) \right] (1 + \text{finite}).$$

$$\hat{M}(s_{i,i+1}/\mu^2) = -\frac{1}{2} \sum_l \left(\frac{g^2 N_c}{16\pi^2} \right)^l \left(\frac{\gamma_{cusp}^{(l)}}{(l\epsilon)^2} + \frac{G^{(l)}}{l\epsilon} + C^{(l)} \right) \left(\frac{s_{i,i+1}}{\mu^2} \right)^{l\epsilon}$$

$$s_{i,i+1} = (p_i + p_{i+1})^2$$

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- $O^{(n)}, n > 2$

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$$\begin{aligned} \log(\mathcal{M}) = & -\frac{1}{4} \left\{ \sum_{i=1}^2 \left(\frac{g^3 N_c}{16\pi^2} \right) \frac{\mu^2}{s_{i i+1}} \left(\frac{4}{\epsilon^2} + 2\zeta_2 \right) \right. \\ & + \sum_{i=1}^n \left(\frac{g^3 N_c}{16\pi^2} \right)^2 \left(\frac{\mu^2}{s_{i i+1}} \right)^2 \left(\frac{42\zeta_2 + 3 \log^2 \frac{s_{12}}{s_{13}}}{96\epsilon^2} + \frac{23\zeta_3}{8\epsilon} \right. \\ & \left. \left. - \frac{1}{2880} \left(75 \log^4 \frac{s_{12}}{s_{13}} + 120 \log^2 \frac{s_{12}}{s_{13}} - 317\pi^4 \right) \right) \right\}. \end{aligned}$$

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- What is left after cancellation of IR divergences?
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From partial amplitudes to cross-sections

To obtain the cross sections from partial amplitudes one has to compute the square of them. In the the planar limit it is just:

$$\Phi_n(p_1^\pm, \dots, p_n^\pm) = g^{2n-4} \left(\frac{g^2 N_c}{16\pi^2}\right)^{2l} \sum_{\text{colors}} \mathcal{A}_n^{(l)} \mathcal{A}_n^{(l)*} =$$

$$2g^{2n-4} N_c^{n-2} (N_c^2 - 1) \left(\frac{g^2 N_c}{16\pi^2}\right)^{2l} \sum_{\text{perm}} |\mathcal{A}_n^{(l)}(\mathbf{a}_{\sigma(1)}, \dots, \mathbf{a}_{\sigma(n-1)}, \mathbf{a}_n)|^2$$

Then the cross-section is

$$d\sigma_n(p_{in}) = \Phi_n(p_1^\pm, \dots, p_n^\pm) d\phi_k,$$

where $d\phi_k$ is the phase space of the outgoing particles:

$$d\phi_k \sim \delta^D(p_{in} - p_{fin}) S_n \prod_k \delta^+(p_k^2) d^D p_k,$$

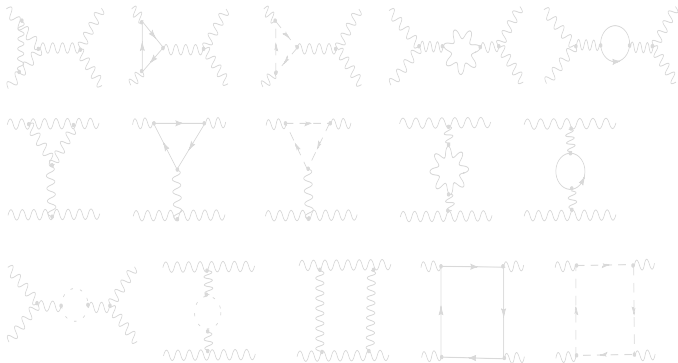
S_n - is the measurement function and integration goes over $D = 4 - 2\varepsilon$ dimensions.

2×2 gluon scattering. Feynman Diagrams

- Tree level



- 1loop



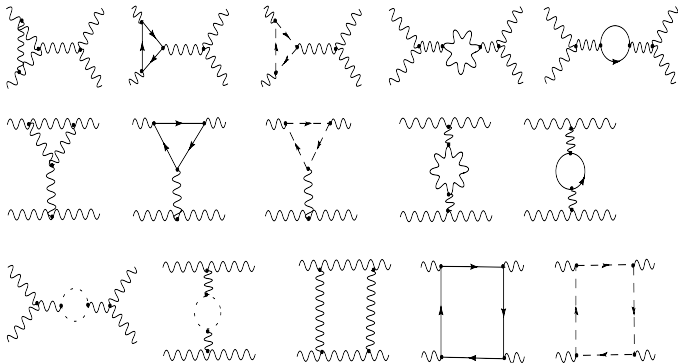
+ permutations

2×2 gluon scattering. Feynman Diagrams

- Tree level



- 1loop



+ permutations

Virtual Correction (MHV)

- Born Term

$$\left(\frac{d\sigma}{d\Omega}\right)_0^{-++} = \frac{\alpha^2 N_c^2}{8E^2} \frac{s^4 (s^2 + t^2 + u^2)}{s^2 t^2 u^2} \left(\frac{\mu^2}{s}\right)^\epsilon = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^\epsilon \frac{3 + c^2}{(1 - c^2)^2} \quad c \equiv \cos \theta$$

- Virtual Correction

$$\begin{aligned} \left(\frac{d\sigma}{d\Omega}\right)_{virt}^{-++} &= \frac{\alpha^2 N_c^2}{8E^2} \left(\frac{\mu^2}{s}\right)^\epsilon \left\{ \frac{\alpha N_c}{2\pi} \frac{s^4}{s^2 t^2 u^2} \left[-\frac{8}{\epsilon^2} \left(\left(\frac{\mu^2}{-t}\right)^\epsilon + \left(\frac{\mu^2}{-u}\right)^\epsilon \right) s^2 \right. \right. \\ &\quad \left. \left. + \left(\left(\frac{\mu^2}{s}\right)^\epsilon + \left(\frac{\mu^2}{-t}\right)^\epsilon \right) u^2 + \left(\left(\frac{\mu^2}{s}\right)^\epsilon + \left(\frac{\mu^2}{-u}\right)^\epsilon \right) t^2 \right] \right. \\ &\quad \left. + \frac{16}{3} \pi^2 (s^2 + t^2 + u^2) + 4(u^2 \log^2(\frac{-s}{t}) + t^2 \log^2(\frac{-s}{u}) + s^2 \log^2(\frac{t}{u})) \right\} \\ &= \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^{2\epsilon} \left\{ \frac{\alpha N_c}{2\pi} \left[-\frac{16}{\epsilon^2} \frac{3 + c^2}{(1 - c^2)^2} + \frac{4}{\epsilon} \left(\frac{5 + 2c + c^2}{(1 - c^2)^2} \log\left(\frac{1 - c}{2}\right) \right. \right. \right. \\ &\quad \left. \left. + (c \leftrightarrow -c) \right) + \frac{16(3 + c^2)\pi^2}{3(1 - c^2)^2} - \frac{16}{(1 - c^2)^2} \log\left(\frac{1 - c}{2}\right) \log\left(\frac{1 + c}{2}\right) \right\} \end{aligned}$$

Virtual Correction (MHV)

- Born Term

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$$\left(\frac{d\sigma}{d\Omega}\right)_0^{-+} = \frac{\alpha^2 N_c^2}{8E^2} \frac{s^4 (s^2 + t^2 + u^2)}{s^2 t^2 u^2} \left(\frac{\mu^2}{s}\right)^\epsilon = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^\epsilon \frac{3 + c^2}{(1 - c^2)^2}$$

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$$\begin{aligned} &= \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^{2\epsilon} \left\{ \frac{\alpha N_c}{2\pi} \left[-\frac{16}{\epsilon^2} \frac{3 + c^2}{(1 - c^2)^2} + \frac{4}{\epsilon} \left(\frac{5 + 2c + c^2}{(1 - c^2)^2} \log\left(\frac{1 - c}{2}\right) \right. \right. \right. \\ &\quad \left. \left. + (c \leftrightarrow -c) \right) + \frac{16(3 + c^2)\pi^2}{3(1 - c^2)^2} - \frac{16}{(1 - c^2)^2} \log\left(\frac{1 - c}{2}\right) \log\left(\frac{1 + c}{2}\right) \right\} \end{aligned}$$

Virtual Correction (MHV)

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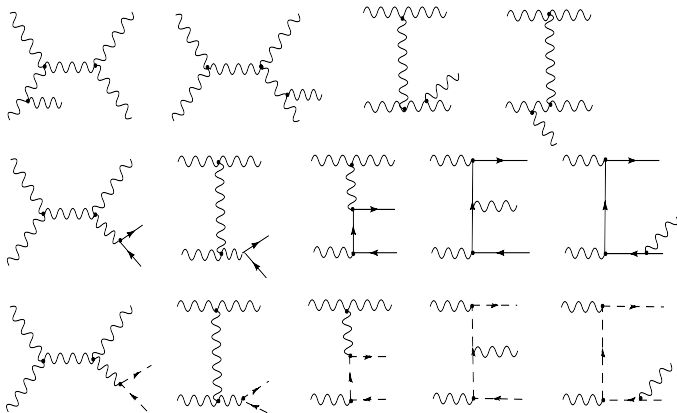
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$$\begin{aligned} &= \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^{2\epsilon} \left\{ \frac{\alpha N_c}{2\pi} \left[-\frac{16}{\epsilon^2} \frac{3 + c^2}{(1 - c^2)^2} + \frac{4}{\epsilon} \left(\frac{5 + 2c + c^2}{(1 - c^2)^2} \log\left(\frac{1 - c}{2}\right) \right. \right. \right. \\ &\quad \left. \left. + (c \leftrightarrow -c) \right) + \frac{16(3 + c^2)\pi^2}{3(1 - c^2)^2} - \frac{16}{(1 - c^2)^2} \log\left(\frac{1 - c}{2}\right) \log\left(\frac{1 + c}{2}\right) \right\} \end{aligned}$$

2×3 gluon scattering. Feynman Diagrams

- Tree level



Real Emission

- MHV

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{Born}^{(--\dots)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^{2\epsilon} \frac{\alpha N_c}{2\pi} \left\{ \frac{1}{\epsilon^2} \frac{8(3+c^2)}{(1-c^2)^2} \right. \\ \left. + \frac{1}{\epsilon} \left[\frac{2}{(1+c)^2} \log\left(\frac{1-c}{2}\right) + \frac{2}{(1-c)^2} \log\left(\frac{1+c}{2}\right) + \frac{16\delta(2\delta-3)}{(1-c^2)^2(1-\delta)^2} \right. \right. \\ \left. \left. + \frac{12(3+c^2)}{(1-c^2)^2} \log\left(\frac{1-\delta}{\delta}\right) \right] + \text{Finite part} \right\}$$

- Fermions

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{Real}^{(--+\bar{q}q)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^{2\epsilon} \frac{\alpha N_c}{2\pi} \left\{ \frac{16}{\epsilon} \left[\frac{(79-25c^2)}{3(1-c^2)^2} \right. \right. \\ \left. \left. + \frac{2(3-c)^2}{(1-c)(1+c)^3} \log\left(\frac{1-c}{2}\right) + \frac{2(3+c)^2}{(1-c)^3(1+c)} \log\left(\frac{1+c}{2}\right) \right] + \text{Finite part} \right\}$$

- Sfermions

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{Real}^{(--+\bar{q}\bar{q})} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^{2\epsilon} \frac{\alpha N_c}{2\pi} \left\{ \frac{8}{\epsilon} \left[-\frac{2(10+7c^2)}{(1-c^2)^2} \right. \right. \\ \left. \left. - \frac{3(5-c)}{(1+c)^3} \log\left(\frac{1-c}{2}\right) - \frac{3(5+c)}{(1-c)^3} \log\left(\frac{1+c}{2}\right) \right] + \text{Finite part} \right\}$$

Real Emission

- MHV

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{Born}^{(--++++)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^{2\epsilon} \frac{\alpha N_c}{2\pi} \left\{ \frac{1}{\epsilon^2} \frac{8(3+c^2)}{(1-c^2)^2} \right. \\ \left. + \frac{1}{\epsilon} \left[\frac{2}{(1+c)^2} \log\left(\frac{1-c}{2}\right) + \frac{2}{(1-c)^2} \log\left(\frac{1+c}{2}\right) + \frac{16\delta(2\delta-3)}{(1-c^2)^2(1-\delta)^2} \right. \right. \\ \left. \left. + \frac{12(3+c^2)}{(1-c^2)^2} \log\left(\frac{1-\delta}{\delta}\right) \right] + \text{Finite part} \right\}$$

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Real Emission

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$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{Born}^{(--\text{++++})} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^{2\epsilon} \frac{\alpha N_c}{2\pi} \left\{ \frac{1}{\epsilon^2} \frac{8(3+c^2)}{(1-c^2)^2} \right. \\ \left. + \frac{1}{\epsilon} \left[\frac{2}{(1+c)^2} \log\left(\frac{1-c}{2}\right) + \frac{2}{(1-c)^2} \log\left(\frac{1+c}{2}\right) + \frac{16\delta(2\delta-3)}{(1-c^2)^2(1-\delta)^2} \right. \right. \\ \left. \left. + \frac{12(3+c^2)}{(1-c^2)^2} \log\left(\frac{1-\delta}{\delta}\right) \right] + \text{Finite part} \right\}$$

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Real Emission

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Splitting of Massless states

- There are NO isolated massless asymptotic states!
- Massless particle can split into two (or more) indistinguishable collinear particles.
- One has to consider coherent states of parallel massless particles.
- Distribution function

$$g_i(z) = \delta(1-z) + \frac{\alpha}{2\pi\epsilon} \left(\frac{\mu^2}{Q_f^2} \right)^\epsilon \sum_j P_{ij}(z)$$

$P_{ij}(z)$ - Splitting function, Q_f^2 - transverse momentum cutoff

- Initial splitting

$$d\sigma_{2 \rightarrow 2}^{spl, init} = \frac{\alpha}{2\pi} \frac{1}{\epsilon} \left(\frac{\mu^2}{Q_f^2} \right)^\epsilon \int_0^1 dz \sum_{l=g,q,\Lambda} P_{gl}(z) \sum_{i,j=1,2; i \neq j} d\sigma_{2 \rightarrow 2}(z p_i, p_j, p_3, p_4) S_2^{spl, init}(z)$$

- Final splitting

$$d\sigma_{2 \rightarrow 2}^{spl, fin} = \frac{\alpha}{2\pi} \frac{1}{\epsilon} \left(\frac{\mu^2}{Q_f^2} \right)^\epsilon d\sigma_{2 \rightarrow 2}(p_1, p_2, p_3, p_4) \int_0^1 dz \sum_{l=g,q,\Lambda} P_{gl}(z) S_2^{spl, fin}(z)$$

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Initial and final state splitting (MHV)

- Initial

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{InSplit}^{(--++++)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^\epsilon \left(\frac{\mu^2}{Q_f^2}\right)^\epsilon$$

$$\frac{\alpha N_c}{2\pi} \left\{ \frac{1}{\epsilon} \left[-\frac{4(c^2+3)}{(1-c^2)^2} \left(\log \frac{1-c}{2} + \log \frac{1+c}{2} \right) - \frac{8(c^2+3)}{(1-c^2)^2} \log \frac{1-\delta}{\delta} - \frac{16\delta(2\delta-3)}{(1-c^2)^2(1-\delta)^2} \right] + \text{Finite part} \right\}$$

- Final

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{FnSplit}^{(--++++)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^\epsilon \left(\frac{\mu^2}{Q_f^2}\right)^\epsilon$$

$$\frac{\alpha N_c}{2\pi} \left\{ -\frac{1}{\epsilon} \frac{4(c^2+3)}{(1-c^2)^2} \log \frac{1-\delta}{\delta} + \text{Finite part} \right\}$$

Initial and final state splitting (MHV)

- Initial

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{InSplit}^{(--+++)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^\epsilon \left(\frac{\mu^2}{Q_f^2}\right)^\epsilon$$

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Initial state splitting (Matter) ($\delta = 1$)

- Fermions

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{\text{InSplit}}^{(--+\bar{q}\bar{q})} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^\epsilon \left(\frac{\mu^2}{Q_f^2}\right)^\epsilon \frac{\alpha N_c}{2\pi} \left\{ \frac{16}{\epsilon} \left[\frac{(79 - 25c^2)}{3(1 - c^2)^2} \right. \right. \\ \left. \left. + \frac{2(3 - c)^2}{(1 - c)(1 + c)^3} \log\left(\frac{1 - c}{2}\right) + \frac{2(3 + c)^2}{(1 - c)^3(1 + c)} \log\left(\frac{1 + c}{2}\right) \right] + \text{Finite part} \right\}$$

- Sfermions

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Infrared-free sets (for any arbitrary δ)

- $$A^{MHV} = \frac{1}{2} \left(\frac{d\sigma}{d\Omega_{14}} \right)_{Virt}^{(----)} + \left(\frac{d\sigma}{d\Omega_{14}} \right)_{Real}^{(-----)} + \left(\frac{d\sigma}{d\Omega_{14}} \right)_{InSplit}^{(-----)} + \left(\frac{d\sigma}{d\Omega_{14}} \right)_{FnSplit}^{(-----)}$$

- $$B^{AntiMHV} = \frac{1}{2} \left(\frac{d\sigma}{d\Omega_{14}} \right)_{Virt}^{(----)} + \left(\frac{d\sigma}{d\Omega_{14}} \right)_{Real}^{(-----)} + \left(\frac{d\sigma}{d\Omega_{14}} \right)_{InSplit}^{(-----)} + \left(\frac{d\sigma}{d\Omega_{14}} \right)_{FnSplit}^{(-----)}$$

- $$C^{Matter} = \left(\frac{d\sigma}{d\Omega_{14}} \right)_{Real}^{(---+\bar{q}q,\bar{q}\bar{q})} + \left(\frac{d\sigma}{d\Omega_{14}} \right)_{InSplit}^{(---+\bar{q}q,\bar{q}\bar{q})} + \left(\frac{d\sigma}{d\Omega_{14}} \right)_{FnSplit}^{(---+\bar{q}q,\bar{q}\bar{q})}$$

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Infrared-free observables

- Registration of two fastest gluons of positive chirality

$$A^{MHV} \Big|_{\delta=1/3} + B^{AntiMHV} \Big|_{\delta=1}$$

- Registration of one fastest gluon of positive chirality

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- Anti MHV cross-section

$$B^{AntiMHV} \Big|_{\delta=1} + C^{Matter} \Big|_{\delta=1} \Rightarrow \text{Finite Part}$$

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The simplest IR finite answer so far ($Q_f = E$): **N=4 SYM Anti MHV**

$$\left(\frac{d\sigma}{d\Omega_{14}} \right)_{\text{AntiMHV}} = \frac{\alpha^2 N_c^2}{E^2} \left\{ \frac{3 + c^2}{(1 - c^2)^2} - \frac{\alpha N_c}{2\pi} \left[2 \frac{(c^4 + 2c^3 + 4c^2 + 6c + 19) \log^2\left(\frac{1-c}{2}\right)}{(1-c)^2(1+c)^4} + 2 \frac{(c^4 - 2c^3 + 4c^2 - 6c + 19) \log^2\left(\frac{1+c}{2}\right)}{(1-c)^4(1+c)^2} - 8 \frac{(c^2 + 1) \log\left(\frac{1+c}{2}\right) \log\left(\frac{1-c}{2}\right)}{(1-c^2)^2} + \frac{6\pi^2(3c^2 + 13) - 5(61c^2 + 99)}{9(1-c^2)^2} + 2 \frac{(11c^3 + 31c^2 - 47c + 133) \log\left(\frac{1+c}{2}\right)}{3(1-c)^3(1+c)^2} - 2 \frac{(11c^3 - 31c^2 - 47c - 133) \log\left(\frac{1-c}{2}\right)}{3(1+c)^3(1-c)^2} \right] \right\}$$

Summary

- Factorization (exponentiation) of IR divergences takes place with universal second order pole (cusp anomalous dimension) and non-universal first order pole (collinear dim).
- The finite part factorizes only in simple cases both for the gluon amplitudes and for the formfactors
- In observable cross-sections the IR divergences do cancel in accordance with Kinoshita-Lee-Nauenberg theorem

$$\begin{aligned}
 d\sigma_{obs}^{incl} &= \sum_{n=2}^{\infty} \int_0^1 dz_1 q_1(z_1, \frac{Q_f^2}{\mu^2}) \int_0^1 dz_2 q_2(z_2, \frac{Q_f^2}{\mu^2}) \prod_{i=1}^n \int_0^1 dx_i q_i(x_i, \frac{Q_f^2}{\mu^2}) \times \\
 &\times d\sigma^{2 \rightarrow n}(z_1 p_1, z_2 p_2, \dots) S_n(\{z\}, \{x\}) = g^4 \sum_{L=0}^{\infty} \left(\frac{g^2}{16\pi^2} \right)^L d\sigma_L^{Finite}(s, t, u, Q_f^2)
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