INFRARED-FINITE OBSERVABLES IN N=4 SUPER YANG-MILLS THEORY

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Outline

N=4 Super Yang-Mills Theory

- $\mathcal{N} = 4$ Super Yang-Mills theory is the most supersymmetric theory possible without gravity
- Field content: 1 massless gauge boson, 4 massless (Majorana) spin 1/2 fermions, 6 real (or 3 complex) massless spin 0 bosons All fields are in adjoint representation of the gauge group (Take $SU(N_c)$)
- The theory is exactly scale invariant, conformal field theory at quantum level, i.e. the β function identically vanishes at all orders of PT

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AdS/CFT Correspondence

- $N_c \to \infty$ (planar limit) is expected to be integrable and solvable
- Maldacena's conjecture: Planar Limit of N=4 SYM at strong coupling is dual to weakly coupled type II b supergravity in 10 dimensional $AdS_5 * S_5$ space.
- What are the quantities that reveal the integrability properties and might be calculated both ways?
- How might PT series be organized to produce simple strong coupling result?

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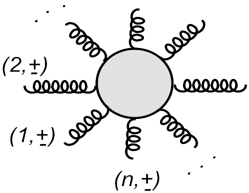
Gluon scattering amplitudes



All outgoing gluons with helicity + or -
on mass shell
In the leading N_c order (planar limit)

- Colour decomposition of amplitudes in N=4 SYM theory for $N_c \to \infty$

$$\mathcal{A}_n^{(l)} = g^{n-2}\left(\frac{g^2 N_c}{16\pi^2}\right)^l \sum_{perm} Tr(T^{a_{\sigma(1)}}, ..., T^{a_{\sigma(n)}}) A_n^{(l)}(a_{\sigma(1)}, ..., a_{\sigma(n)}),$$

where $\mathcal{A}_n$ - physical amplitude, A_n - partial amplitude, a_i - is color index of $i-th$ external "gluon"

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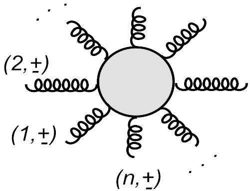

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Perturbation theory

- Bern, Dixon & Smirnov's conjecture: $M_n^{(L)}(\varepsilon) \equiv A_n^{(L)}/A_n^{(0)}$

$$\mathcal{M}_n \equiv 1 + \sum_{L=1}^{\infty}\left(\frac{g^2N_c}{16\pi^2}\right)^L M_n^{(L)}(\varepsilon) = exp\left[\sum_{l=1}^{\infty}\left(\frac{g^2N_c}{16\pi^2}\right)^l\left(f^{(l)}(\varepsilon)M_n^{(1)}(l\varepsilon)+C^{(l)}+E_n^{(l)}(\varepsilon)\right)\right]$$

$$f^{(l)}(\varepsilon) = f_0^{(l)}(\varepsilon) + \varepsilon f_1^{(l)}(\varepsilon) + \varepsilon^2 f_2^{(l)}(\varepsilon)$$

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- Cusp anomalous dimension appears in RG eq. for the expectation value of a Wilson line with a cusp

  Loop expansion $$\gamma_K = \sum_{l=1}^{\infty} \left(\frac{g^2 N_c}{16\pi^2}\right)^l \gamma_K^{(l)}$$

  $$\gamma_K^{(1)} = 8, \quad \gamma_K^{(2)} = -16\zeta_2, \quad \gamma_K^{(3)} = 176\zeta_4, \ldots$$

- Classical solution (Alday & Maldacena) for the scattering amplitude

  $$\gamma_K(g^2) \sim \frac{\sqrt{g^2 N_c}}{\pi}, \quad G_0(g^2) \sim \sqrt{g^2 N_c}\,\frac{1-\log 2}{2\pi}, \quad for\ g^2 N_c \to \infty$$

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Violation of BDS ansatz

- For $n = 4, 5$ the BDS ansatz goes through all checks, namely the amplitudes were calculated up to three loops for four gluons and up to two loops for five gluons.
- However, starting from $n = 6$ it fails.
  - In the strong coupling calculation in the limit $n \to \infty$ discrepancy with the BDS formula was found.
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From partial amplitudes to cross-sections

To obtain the cross sections from partial amplitudes one has to compute the square of them. In the the planar limit it is just:

$$\Phi_n(p_1^\pm, ..., p_n^\pm) = g^{2n-4}(\frac{g^2 N_c}{16\pi^2})^{2l} \sum_{colors} \mathcal{A}_n^{(l)} \mathcal{A}_n^{(l)*} =$$

$$2g^{2n-4} N_c^{n-2}(N_c^2 - 1)(\frac{g^2 N_c}{16\pi^2})^{2l} \sum_{perm} |A_n^{(l)}(a_{\sigma(1)}, ..., a_{\sigma(n-1)}, a_n)|^2$$

Then the cross-section is

$$d\sigma_n(p_{in}) = \Phi_n(p_1^\pm, ..., p_n^\pm) d\phi_k,$$

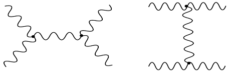
where $d\phi_k$ is the phase space of the outgoing particles:

$$d\phi_k \sim \delta^D(p_{in} - p_{fin}) \mathcal{S}_n \prod_k \delta^+(p_k^2) d^D p_k,$$

$\mathcal{S}_n$ - is the measurement function and integration goes over $D = 4 - 2\varepsilon$ dimensions.

2 × 2 gluon scattering. Feynman Diagrams

- Tree level



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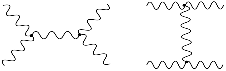

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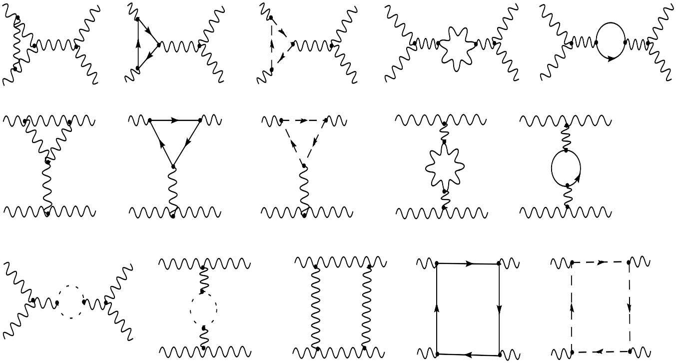

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Virtual Correction (MHV)

- Born Term $\qquad c \equiv \cos\theta$

$$\left(\frac{d\sigma}{d\Omega}\right)_0^{--++} = \frac{\alpha^2 N_c^2}{8E^2}\frac{s^4(s^2+t^2+u^2)}{s^2t^2u^2}\left(\frac{\mu^2}{s}\right)^\varepsilon = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^\varepsilon \frac{3+c^2}{(1-c^2)^2}$$

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$$\left. +((\frac{\mu^2}{s})^\varepsilon + (\frac{\mu^2}{-t})^\varepsilon)u^2 + ((\frac{\mu^2}{s})^\varepsilon + (\frac{\mu^2}{-u})^\varepsilon)t^2\right)$$
$$\left.\left. +\frac{16}{3}\pi^2(s^2+t^2+u^2) + 4(u^2\log^2(\frac{-s}{t}) + t^2\log^2(\frac{-s}{u}) + s^2\log^2(\frac{t}{u}))\right]\right\}$$

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$$\left.\left.\left. +(c\leftrightarrow -c)\right) + \frac{16(3+c^2)\pi^2}{3(1-c^2)^2} - \frac{16}{(1-c^2)^2}\log(\frac{1-c}{2})\log(\frac{1+c}{2})\right]\right\}$$

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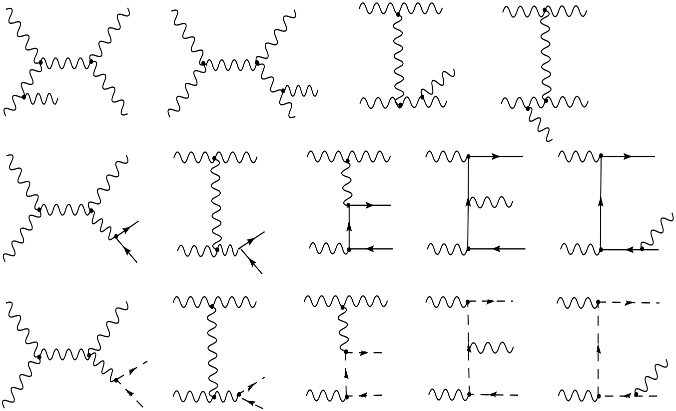
$$\left.\left.+\frac{16}{3}\pi^2(s^2+t^2+u^2)+4(u^2\log^2(\frac{-s}{t})+t^2\log^2(\frac{-s}{u})+s^2\log^2(\frac{t}{u}))\right]\right\}$$

$$= \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\left\{\frac{\alpha N_c}{2\pi}\left[-\frac{16}{\varepsilon^2}\frac{3+c^2}{(1-c^2)^2}+\frac{4}{\varepsilon}\left(\frac{5+2c+c^2}{(1-c^2)^2}\log(\frac{1-c}{2})\right.\right.\right.$$
$$\left.\left.\left.+(c\leftrightarrow -c)\right)+\frac{16(3+c^2)\pi^2}{3(1-c^2)^2}-\frac{16}{(1-c^2)^2}\log(\frac{1-c}{2})\log(\frac{1+c}{2})\right]\right\}$$

2×3 gluon scattering. Feynman Diagrams

- Tree level



Real Emission

- MHV

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{Born} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon^2}\frac{8(3+c^2)}{(1-c^2)^2}\right.$$
$$+\frac{1}{\varepsilon}\left[\frac{2}{(1+c)^2}\log(\frac{1-c}{2})+\frac{2}{(1-c)^2}\log(\frac{1+c}{2})+\frac{16\delta(2\delta-3)}{(1-c^2)^2(1-\delta)^2}\right.$$
$$\left.\left.+\frac{12(3+c^2)}{(1-c^2)^2}\log\frac{(1-\delta)}{\delta}\right]+\text{Finite part}\right\}$$

- Anti MHV

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{Born} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\epsilon}\frac{\alpha N_c}{\pi}\left\{\frac{1}{\epsilon^2}\frac{8(3+c^2)}{(1-c^2)^2}+\frac{1}{\epsilon}\left[-\frac{12(c^2+3)\log\delta}{(1-c^2)^2}+\frac{64(12c^2+17)}{3(1-c^2)^3}\right.\right.$$
$$+\frac{2\delta}{(1-c^2)^2}\left(\frac{2}{3}(5+3c^2)\delta^2-(c^2+19)\delta+2(5c^2+43)\right)+\left(\frac{2(3c^2-24c+85)}{(1-c)(1+c)^3}\log(\frac{1-c}{2})\right.$$
$$-\frac{8(c^2-6c+21)}{(1-c)(1+c)^3}\log(\frac{1+\delta-(1-\delta)c}{2})-\frac{32(c^2-4c+7)}{(1+c)^3(1-c)(1+\delta-c(1-\delta))}$$
$$\left.\left.\left.+\frac{32(2-c)}{(1+c)^3(1+\delta-c(1-\delta))^2}-\frac{64(1-c)}{3(1+c)^3(1+\delta-c(1-\delta))^3}+(c\leftrightarrow -c)\right)\right]+\text{Finite part}\right\}$$

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Real Emission (Matter)

- Fermions

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q)}_{Real} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{16}{\varepsilon}\left[\frac{(79-25c^2)}{3(1-c^2)^2)}\right.\right.$$
$$\left.\left.+\frac{2(3-c)^2}{(1-c)(1+c)^3})\log(\frac{1-c}{2}) + \frac{2(3+c)^2}{(1-c)^3(1+c)}\log(\frac{1+c}{2})\right] + \text{Finite part}\right\}$$

- Sfermions

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$$\left.\left.-\frac{3(5-c)}{(1+c)^3}\log(\frac{1-c}{2}) - \frac{3(5+c)}{(1-c)^3}\log(\frac{1+c}{2})\right] + \text{Finite part}\right\}$$

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Splitting of Massless states

- There are NO isolated massless asymptotic states!
- Massless particle can split into two (or more) indistinguishable collinear particles.
- One has to consider coherent states of parallel massless particles.
- Distribution function

$$g_i(z) = \delta(1-z) + \frac{\alpha}{2\pi\epsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}\sum_j P_{ij}(z)$$

$P_{ij}(z)$ - Splitting function, Q_f^2 - transverse momentum cutoff

- Initial splitting

$$d\sigma_{2\to 2}^{spl,init} = \frac{\alpha}{2\pi}\frac{1}{\epsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\epsilon}\int_0^1 dz \sum_{l=g,q,\Lambda} P_{gl}(z) \sum_{i,j=1,2;\ i\neq j} d\sigma_{2\to 2}(zp_i, p_j, p_3, p_4)\mathcal{S}_2^{spl,init}(z)$$

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IR versus UV divergences

- Anomalous dimensions (UV) of leading-twist operators

$$\begin{aligned}
\mathcal{O}^{g}_{\mu_1\ldots\mu_n} &= \hat{S} G^{a}_{\rho\mu_1} D_{\mu_2} \ldots D_{\mu_{n-1}} G^{a}_{\rho\mu_n} \\
\mathcal{O}^{\lambda}_{\mu_1\ldots\mu_n} &= \hat{S} \bar{\lambda}^{a}_{i} \gamma^{\mu_1} D_{\mu_2} \ldots D_{\mu_n} \lambda^{a}_{i} \\
\mathcal{O}^{\phi}_{\mu_1\ldots\mu_n} &= \hat{S} \bar{\phi}^{a}_{r} D_{\mu_1} \ldots D_{\mu_n} \phi^{a}_{r}
\end{aligned}$$

$$\gamma^{(n)}_{ij} = \int_0^1 dx\, x^{n-1} P_{ij}(x), \qquad i,j = g,\lambda,\phi$$

P_{ij} - Splitting functions - kernels of the DGLAP equations

- Collinear divergences (IR) are governed by collinear counterterms

$$Collinear\ Counterterm = -\frac{1}{\epsilon}\frac{\alpha_s}{2\pi} P_{ij}(x) \left(\frac{\mu^2}{Q_f^2}\right)^{\epsilon}$$

- Cusp anom dim controls the large spin limit of the leading-twist operators

$$O_j \equiv \bar{q}(\gamma_+ \mathcal{D}^+)^j q$$

$$\gamma_j = \frac{1}{2}\gamma_K(\alpha) \log j + \mathcal{O}(j^0), \quad j \to \infty$$

$$P_{gg} = \frac{1}{2}\frac{\gamma_K(\alpha)}{(1-x)_+} + \ldots, \quad x \to 1, \quad \gamma(j) = -\int_0^1 dx\, x^{j-1} P(x)$$

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Initial and final state splitting (MHV)

- Initial

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{InSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}$$

$$\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon}\left[-\frac{4(c^2+3)}{(1-c^2)^2}\left(\log\frac{1-c}{2}+\log\frac{1+c}{2}\right)-\frac{8(c^2+3)}{(1-c^2)^2}\log\frac{1-\delta}{\delta}\right.\right.$$

$$\left.\left.-\frac{16\delta(2\delta-3)}{(1-c^2)^2(1-\delta)^2}\right]+\text{Finite part}\right\}$$

- Final

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Initial and final state splitting (Anti MHV)

- Initial

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{InSplit}=\frac{\alpha^2N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\epsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\epsilon}\frac{\alpha N_c}{\pi}\left\{\frac{1}{\epsilon}\left[\frac{8(c^2+3)}{(1-c^2)^2}\log\delta-\frac{64(12c^2+17)}{3(1-c^2)^3}\right.\right.$$
$$-\frac{4\delta}{(1-c^2)^2}\left(\frac{2}{3}(1+c^2)\delta^2+(c^2-5)\delta+2(c^2+17)\right)+\left(\frac{4(c^3-15c^2+51c-45)}{(1-c)^2(1+c)^3}\log\frac{1-c}{2}\right.$$
$$+\frac{8(c^2-6c+21)}{(1-c)(1+c)^3}\log\frac{1+\delta-c(1-\delta)}{2}+\frac{32(c^2-4c+7)}{(1+c)^3(1-c)(1+\delta-c(1-\delta))}$$
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- Final

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{FnSplit}=\frac{\alpha^2N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon}\frac{4(c^2+3)}{(1-c^2)^2}\left[\log\delta-\frac{\delta}{3}(2\delta^2-9\delta+18)\right]+\text{F.p.}\right\}$$

Initial and final state splitting (Anti MHV)

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Initial state splitting (Matter) ($\delta = 1$)

- Fermions

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q)}_{InSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{16}{\varepsilon}\left[\frac{(79-25c^2)}{3(1-c^2)^2)}\right.\right.$$
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- Sfermions

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\tilde{\bar{q}}\tilde{q})}_{InSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{8}{\varepsilon}\left[-\frac{2(10+7c^2)}{(1-c^2)^2}\right.\right.$$
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Infrared-free sets (for any arbitrary δ)

- $$A^{MHV} = \frac{1}{2}\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++)}_{Virt} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{Real} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{InSplit} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{FnSplit}$$

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- Registration of two fastest gluons of positive chirality

$$A^{MHV}\Big|_{\delta=1/3} + B^{AntiMHV}\Big|_{\delta=1}$$

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The <u>simplest</u> IR finite answer so far ($Q_f = E$): N=4 SYM Anti MHV

$$
\begin{aligned}
\left(\frac{d\sigma}{d\Omega_{14}}\right)_{AntiMHV} &= \frac{\alpha^2 N_c^2}{E^2}\Bigg\{\frac{3+c^2}{(1-c^2)^2} \\
&-\frac{\alpha N_c}{2\pi}\Bigg[2\frac{(c^4+2c^3+4c^2+6c+19)\log^2(\frac{1-c}{2})}{(1-c)^2(1+c)^4}+2\frac{(c^4-2c^3+4c^2-6c+19)\log^2(\frac{1+c}{2})}{(1-c)^4(1+c)^2} \\
&-8\frac{(c^2+1)\log(\frac{1+c}{2})\log(\frac{1-c}{2})}{(1-c^2)^2}+\frac{6\pi^2(3c^2+13)-5(61c^2+99)}{9(1-c^2)^2} \\
&+2\frac{(11c^3+31c^2-47c+133)\log(\frac{1+c}{2})}{3(1-c)^3(1+c)^2}-2\frac{(11c^3-31c^2-47c-133)\log(\frac{1-c}{2})}{3(1+c)^3(1-c)^2}\Bigg]\Bigg\}
\end{aligned}
$$

Summary

- In observable cross-sections the IR divergences do cancel in accordance with Kinoshita-Lee-Nauenberg theorem

$$d\sigma^{incl}_{obs} = \sum_{n=2}^{\infty}\int_0^1 dz_1 q_1(z_1,\frac{Q_f^2}{\mu^2})\int_0^1 dz_2 q_2(z_2,\frac{Q_f^2}{\mu^2})\prod_{i=1}^{n}\int_0^1 dx_i q_i(x_i,\frac{Q_f^2}{\mu^2})\times$$
$$\times\, d\sigma^{2\to n}(z_1p_1,z_2p_2,...)S_n(\{z\},\{x\}) = g^4\sum_{L=0}^{\infty}\left(\frac{g^2}{16\pi^2}\right)^L d\sigma_L^{Finite}(s,t,u,Q_f^2)$$

- The simple structure of the MHV amplitude DOES NOT reveal at the level of IR finite cross-sections;
- In some cases the cancellation of complicated functions occurs, though not always;
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