

Infrared-finite Observables in N=4 Super Yang-Mills Theory

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 - N=4 Super Yang Mills Theory
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- 2 Infrared Divergences
 - Weak Coupling Case
 - Strong Coupling Case
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 - Toy model: electron-quark scattering
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- 4 Summary and Outlook

$\mathcal{N}=4$ Super Yang-Mills Theory

- $\mathcal{N} = 4$ Super Yang-Mills theory is the most supersymmetric theory possible without gravity
- Field content: 1 massless gauge boson, 4 massless (Majorana) spin 1/2 fermions, 6 real (or 3 complex) massless spin 0 bosons
All fields are in adjoint representation of the gauge group (Take $SU(N_c)$)
- The theory is exactly scale invariant, conformal field theory at quantum level, i.e. the β function identically vanishes at all orders of PT

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AdS/CFT Correspondence

- $N_c \rightarrow \infty$ (planar limit) is expected to be integrable and solvable
- Maldacena's conjecture: Planar Limit of N=4 SYM at strong coupling is dual to weakly coupled type II b supergravity in 10 dimensional $AdS_5 * S_5$ space.
- How might PT series be organized to produce simple strong coupling result?
- The amplitudes on shell possess IR singularities which should cancel in observables. What are the observables in the strong coupling limit?

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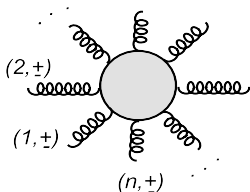
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Gluon scattering amplitudes



All outgoing gluons with helicity + or -
on mass shell

In the leading N_c order (planar limit)

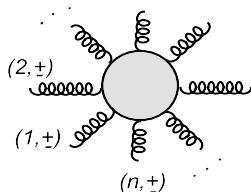
- Colour decomposition of amplitudes in N=4 SYM theory for $N_c \rightarrow \infty$

$$\mathcal{A}_n^{(l)} = g^{n-2} \left(\frac{g^2 N_c}{16\pi^2} \right)^l \sum_{perm} \text{Tr}(T^{a_{\sigma(1)}}, \dots, T^{a_{\sigma(n)}}) A_n^{(l)}(a_{\sigma(1)}, \dots, a_{\sigma(n)}),$$

where \mathcal{A}_n - physical amplitude, A_n - partial amplitude, a_i - is color index of i -th external "gluon"

- Maximal helicity violating (MHV) amplitudes (two negative helicities and the rest positive) have observed a simple structure on tree level (and even in loops) and one can speculate that this is the consequence of SUSY

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Perturbation theory

- Bern, Dixon & Smirnov's conjecture: $M_n^{(L)}(\varepsilon) \equiv A_n^{(L)}/A_n^{(0)}$

$$\mathcal{M}_n \equiv 1 + \sum_{L=1}^{\infty} \left(\frac{g^2 N_c}{16\pi^2} \right)^L M_n^{(L)}(\varepsilon) = \exp \left[\sum_{l=1}^{\infty} \left(\frac{g^2 N_c}{16\pi^2} \right)^l \left(f^{(l)}(\varepsilon) M_n^{(1)}(l\varepsilon) + C^{(l)} + E_n^{(l)}(\varepsilon) \right) \right]$$

$$f^{(l)}(\varepsilon) = f_0^{(l)}(\varepsilon) + \varepsilon f_1^{(l)}(\varepsilon) + \varepsilon^2 f_2^{(l)}(\varepsilon)$$

$$\mathcal{M}_n(\varepsilon) = \exp \left[-\frac{1}{8} \sum_{l=1}^{\infty} \left(\frac{g^2 N_c}{16\pi^2} \right)^l \left(\frac{\gamma_K^{(l)}}{(l\varepsilon)^2} + \frac{2G_0^{(l)}}{l\varepsilon} \right) \sum_{i=1}^n \left(\frac{\mu^2}{-s_{i,i+1}} \right)^{l\varepsilon} + \frac{1}{4} \sum_{l=1}^{\infty} \left(\frac{g^2 N_c}{16\pi^2} \right)^l \gamma_K^{(l)} F_n^{(1)}(0) \right]$$

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- **Cusp anomalous dimension** appears in RG eq. for the expectation value of a Wilson line with a cusp

Loop expansion $\gamma_K = \sum_{l=1}^{\infty} \left(\frac{g^2 N_c}{16\pi^2} \right)^l \gamma_K^{(l)}$

$$\gamma_K^{(1)} = 8, \quad \gamma_K^{(2)} = -16\zeta_2, \quad \gamma_K^{(3)} = 176\zeta_4, \dots$$

- It also controls the large spin limit of anomalous dimension of leading-twist operators

$$O_j \equiv \bar{q}(\gamma_+ \mathcal{D}^+)^j q$$

$$\gamma_j = \frac{1}{2} \gamma_K(\alpha) \log j + \mathcal{O}(j^0), \quad j \rightarrow \infty$$

- and large x limit of the DGLAP kernel for p.d.f.

$$P_{gg} = \frac{1}{2} \frac{\gamma_K(\alpha)}{(1-x)_+} + \dots, \quad x \rightarrow 1, \quad \gamma(j) = - \int_0^1 dx x^{j-1} P(x)$$

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- Classical solution (Alday & Maldacena) for the scattering amplitude

$$\mathcal{M}_4(\varepsilon) = \exp[-S_{cl}^E]$$

$$\bullet S_{cl}^E = \frac{1}{\varepsilon^2} \frac{\sqrt{g^2 N_c}}{\pi} \left[\left(\frac{\mu_{IR}^2}{-s} \right)^{\varepsilon/2} + \left(\frac{\mu_{IR}^2}{-t} \right)^{\varepsilon/2} \right]$$

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Violation of BDS ansatz

- For $n = 4, 5$ the BDS ansatz goes through all checks, namely the amplitudes were calculated up to three loops for four gluons and up to two loops for five gluons.
- However, starting from $n = 6$ it fails.
 - ▶ In the strong coupling calculation in the limit $n \rightarrow \infty$ discrepancy with the BDS formula was found.
 - ▶ Starting from $n = 6$ the Regge limit factorization of the amplitude in some physical regions fails. This was also shown by explicit two-loop calculation.
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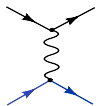
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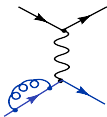
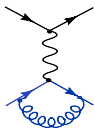
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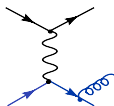
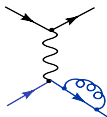
Electron-quark scattering



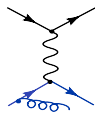
Born



Virtual



Real Emission



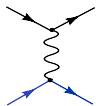
- Virtual Correction

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{\alpha^2}{2E^2} \left(\frac{s^2 + u^2 - \epsilon t^2}{t^2}\right) \left(\frac{\mu^2}{s}\right)^\epsilon$$

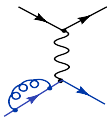
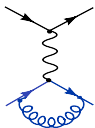
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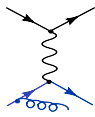
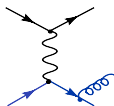
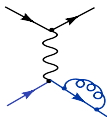
Electron-quark scattering



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Virtual



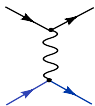
Real Emission

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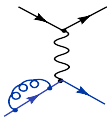
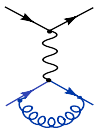
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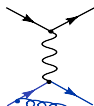
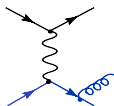
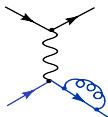
Electron-quark scattering



Born



Virtual



Real Emission

- Virtual Correction

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{\alpha^2}{2E^2} \left(\frac{s^2 + u^2 - \varepsilon t^2}{t^2}\right) \left(\frac{\mu^2}{s}\right)^\varepsilon$$

-

$$\left(\frac{d\sigma}{d\Omega}\right)_{virt} = \left(\frac{d\sigma}{d\Omega}\right)_0 \left[1 - 2C_F \frac{\alpha_s}{4\pi} \left(\frac{\mu^2}{-t}\right)^\varepsilon \left(\frac{2}{\varepsilon^2} + \frac{3}{\varepsilon} + 8\right) \right]$$

Real Emission



$$\begin{aligned} \left(\frac{d\sigma}{d\Omega}\right)_{real} &= \left(\frac{d\sigma}{d\Omega}\right)_0 \left[2C_F \frac{\alpha_s}{4\pi} \left(\frac{\mu^2}{-t}\right)^\epsilon \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + 8\right) \right] \\ &+ C_F \frac{\alpha_s^2}{E^2} \frac{\alpha_s}{4\pi} \left(\frac{\mu^2}{s}\right)^\epsilon \left(\frac{\mu^2}{-t}\right)^\epsilon \left(\frac{f_1}{\epsilon} + f_2\right), \end{aligned}$$

- where the functions f_1 and f_2 in the c.m. frame are ($c = \cos \theta$)

$$f_1 = -2 \frac{(c^3 + 5c^2 - 3c + 5) \log\left(\frac{1-c}{2}\right) + (1-c^2)(c-11)/4}{(1-c)(1+c)^2}$$

$$\begin{aligned} f_2 &= -\frac{1}{(1-c^2)^2} \left[(1-c)(c^3 + 5c^2 - 3c + 5) \log^2\left(\frac{1-c}{2}\right) \right. \\ &+ \frac{1}{2}(1-c)(3c^3 + 15c^2 + 77c - 31) \log\left(\frac{1-c}{2}\right) + \frac{1}{2}(1-c^2)(5c^2 - 42c - 23) \\ &\left. + (1+c)^2(c^2 + 5c + 3)\pi^2 - 12(9c^2 + 2c + 5) Li_2\left(\frac{1+c}{2}\right) \right]. \end{aligned}$$

Real Emission



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Splitting of Massless states

- There are NO isolated massless asymptotic states!
- Massless particle can split into two (or more) indistinguishable collinear particles.
- One has to consider coherent states of parallel massless particles.
- Distribution function

$$q_i(z) = \delta(1 - z) + \frac{\alpha}{2\pi\epsilon} \left(\frac{\mu^2}{Q_f^2} \right)^\epsilon \sum_j P_{ij}(z)$$

$P_{ij}(z)$ - Splitting function, Q_f^2 - transverse momentum cutoff

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Initial state splitting



$$\left(\frac{d\sigma}{d\Omega}\right)_{split} = \frac{1}{\epsilon} \frac{\alpha_s}{2\pi} \int_0^1 dz \left(\frac{\mu^2}{Q_f^2}\right)^\epsilon P_{qq}(z) \frac{d\sigma_0}{d\Omega}(pz)$$



$$\left(\frac{d\sigma}{d\Omega}\right)_{split} = C_F \frac{\alpha^2}{2E^2} \frac{\alpha_s}{2\pi} \left(\frac{\mu^2}{s}\right)^\epsilon \left(\frac{\mu^2}{-t}\right)^\epsilon \left(-\frac{f_1}{\epsilon} + f_3\right),$$

- where for $Q_f^2 = \hat{t}$

$$f_3 = -\frac{1}{(1-c)^2(1+c)^2} \left[2(1-c)(c^3 + c^2 - 33c + 7) \log\left(\frac{1-c}{2}\right) + 12(9c^2 + 2c + 5) \text{Li}_2\left(\frac{1+c}{2}\right) - (1+c)^2(c^2 + 5c + 3)\pi^2 - \frac{1}{2}(1-c)(1+c)(11c^2 - 19) \right].$$

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Infrared-free observable = inclusive cross-section

$$\begin{aligned}
 \left(\frac{d\sigma}{d\Omega}\right)_{\text{observ}} &= \left(\frac{d\sigma}{d\Omega}\right)_{\text{virt}}^{2\rightarrow 2} + \left(\frac{d\sigma}{d\Omega}\right)_{\text{real}}^{2\rightarrow 3} + \left(\frac{d\sigma}{d\Omega}\right)_{\text{split}}^{2\rightarrow 2} \\
 &= \frac{\alpha^2}{2E^2} \left\{ \frac{c^2 + 2c + 5}{(1-c)^2} \right. \\
 &\quad - \frac{\alpha_s}{2\pi} \frac{C_F}{(1-c)(1+c)^2} \left[(c^3 + 5c^2 - 3c + 5) \log^2 \frac{1-c}{2} \right. \\
 &\quad \left. \left. + \frac{1}{2}(7c^3 + 19c^2 - 55c - 3) \log \frac{1-c}{2} - (1+c)(3c^2 + 21c + 2) \right] \right\}
 \end{aligned}$$

From partial amplitudes to cross-sections

To obtain the cross sections from partial amplitudes one have to compute the square of them. In the the planar limit it is just:

$$\Phi_n(p_1^\pm, \dots, p_n^\pm) = g^{2n-4} \left(\frac{g^2 N_c}{16\pi^2} \right)^{2l} \sum_{\text{colors}} \mathcal{A}_n^{(l)} \mathcal{A}_n^{(l)*} =$$

$$2g^{2n-4} N_c^{n-2} (N_c^2 - 1) \left(\frac{g^2 N_c}{16\pi^2} \right)^{2l} \sum_{\text{perm}} |\mathcal{A}_n^{(l)}(\mathbf{a}_{\sigma(1)}, \dots, \mathbf{a}_{\sigma(n-1)}, \mathbf{a}_n)|^2$$

Then the cross-section is

$$d\sigma_n(p_{in}) = \Phi_n(p_1^\pm, \dots, p_n^\pm) d\phi_k,$$

where $d\phi_k$ is the phase space of the outgoing particles:

$$d\phi_k \sim \delta^D(p_{in} - p_{fin}) \mathcal{S}_n \prod_k \delta^+(p_k^2) d^D p_k,$$

where \mathcal{S}_n - is so called measurement function and integration goes over $D = 4 - 2\epsilon$ dimensions.

Virtual Correction (MHV)

- Born Term

$$c \equiv \cos \theta$$

$$\left(\frac{d\sigma}{d\Omega}\right)_0^{-++} = \frac{\alpha^2 N_c^2}{8E^2} \frac{s^4 (s^2 + t^2 + u^2)}{s^2 t^2 u^2} \left(\frac{\mu^2}{s}\right)^\epsilon = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^\epsilon \frac{3 + c^2}{(1 - c^2)^2}$$

- Virtual Correction

$$\begin{aligned} \left(\frac{d\sigma}{d\Omega}\right)_{\text{virt}}^{-++} &= \frac{\alpha^2 N_c^2}{8E^2} \left(\frac{\mu^2}{s}\right)^\epsilon \left\{ \frac{\alpha N_c}{2\pi} \frac{s^4}{s^2 t^2 u^2} \left[-\frac{8}{\epsilon^2} \left(\left(\frac{\mu^2}{-t}\right)^\epsilon + \left(\frac{\mu^2}{-u}\right)^\epsilon \right) s^2 \right. \right. \\ &\quad \left. \left. + \left(\left(\frac{\mu^2}{s}\right)^\epsilon + \left(\frac{\mu^2}{-t}\right)^\epsilon \right) u^2 + \left(\left(\frac{\mu^2}{s}\right)^\epsilon + \left(\frac{\mu^2}{-u}\right)^\epsilon \right) t^2 \right] \right. \\ &\quad \left. + \frac{16}{3} \pi^2 (s^2 + t^2 + u^2) + 4(u^2 \log^2(\frac{-s}{t}) + t^2 \log^2(\frac{-s}{u}) + s^2 \log^2(\frac{t}{u})) \right\} \\ &= \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^{2\epsilon} \left\{ \frac{\alpha N_c}{2\pi} \left[-\frac{16}{\epsilon^2} \frac{3 + c^2}{(1 - c^2)^2} + \frac{4}{\epsilon} \left(\frac{5 + 2c + c^2}{(1 - c^2)^2} \log\left(\frac{1 - c}{2}\right) \right. \right. \right. \\ &\quad \left. \left. + (c \leftrightarrow -c) \right) + \frac{16(3 + c^2)\pi^2}{3(1 - c^2)^2} - \frac{16}{(1 - c^2)^2} \log\left(\frac{1 - c}{2}\right) \log\left(\frac{1 + c}{2}\right) \right\} \end{aligned}$$

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● Virtual Correction

$$\begin{aligned} \left(\frac{d\sigma}{d\Omega}\right)_{\text{virt}}^{-+} &= \frac{\alpha^2 N_c^2}{8E^2} \left(\frac{\mu^2}{s}\right)^\epsilon \left\{ \frac{\alpha N_c}{2\pi} \frac{s^4}{s^2 t^2 u^2} \left[-\frac{8}{\epsilon^2} \left(\left(\frac{\mu^2}{-t}\right)^\epsilon + \left(\frac{\mu^2}{-u}\right)^\epsilon \right) s^2 \right. \right. \\ &\quad \left. \left. + \left(\left(\frac{\mu^2}{s}\right)^\epsilon + \left(\frac{\mu^2}{-t}\right)^\epsilon \right) u^2 + \left(\left(\frac{\mu^2}{s}\right)^\epsilon + \left(\frac{\mu^2}{-u}\right)^\epsilon \right) t^2 \right] \right. \\ &\quad \left. + \frac{16}{3} \pi^2 (s^2 + t^2 + u^2) + 4(u^2 \log^2(\frac{-s}{t}) + t^2 \log^2(\frac{-s}{u}) + s^2 \log^2(\frac{t}{u})) \right\} \\ &= \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^{2\epsilon} \left\{ \frac{\alpha N_c}{2\pi} \left[-\frac{16}{\epsilon^2} \frac{3 + c^2}{(1 - c^2)^2} + \frac{4}{\epsilon} \left(\frac{5 + 2c + c^2}{(1 - c^2)^2} \log\left(\frac{1 - c}{2}\right) \right. \right. \right. \\ &\quad \left. \left. + (c \leftrightarrow -c) \right) + \frac{16(3 + c^2)\pi^2}{3(1 - c^2)^2} - \frac{16}{(1 - c^2)^2} \log\left(\frac{1 - c}{2}\right) \log\left(\frac{1 + c}{2}\right) \right] \right\} \end{aligned}$$

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- Virtual Correction

$$\begin{aligned} \left(\frac{d\sigma}{d\Omega}\right)_{virt}^{-++} &= \frac{\alpha^2 N_c^2}{8E^2} \left(\frac{\mu^2}{s}\right)^\epsilon \left\{ \frac{\alpha N_c}{2\pi} \frac{s^4}{s^2 t^2 u^2} \left[-\frac{8}{\epsilon^2} \left(\left(\frac{\mu^2}{-t}\right)^\epsilon + \left(\frac{\mu^2}{-u}\right)^\epsilon \right) s^2 \right. \right. \\ &\quad \left. \left. + \left(\left(\frac{\mu^2}{s}\right)^\epsilon + \left(\frac{\mu^2}{-t}\right)^\epsilon \right) u^2 + \left(\left(\frac{\mu^2}{s}\right)^\epsilon + \left(\frac{\mu^2}{-u}\right)^\epsilon \right) t^2 \right] \right. \\ &\quad \left. + \frac{16}{3} \pi^2 (s^2 + t^2 + u^2) + 4(u^2 \log^2(\frac{-s}{t}) + t^2 \log^2(\frac{-s}{u}) + s^2 \log^2(\frac{t}{u})) \right\} \\ &= \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^{2\epsilon} \left\{ \frac{\alpha N_c}{2\pi} \left[-\frac{16}{\epsilon^2} \frac{3 + c^2}{(1 - c^2)^2} + \frac{4}{\epsilon} \left(\frac{5 + 2c + c^2}{(1 - c^2)^2} \log\left(\frac{1 - c}{2}\right) \right. \right. \right. \\ &\quad \left. \left. + (c \leftrightarrow -c) \right) + \frac{16(3 + c^2)\pi^2}{3(1 - c^2)^2} - \frac{16}{(1 - c^2)^2} \log\left(\frac{1 - c}{2}\right) \log\left(\frac{1 + c}{2}\right) \right] \right\} \end{aligned}$$

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- Virtual Correction

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Real Emission

- MHV

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{Born}^{(--++++)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^{2\epsilon} \frac{\alpha N_c}{2\pi} \left\{ \frac{1}{\epsilon^2} \frac{8(3+c^2)}{(1-c^2)^2} + \frac{1}{\epsilon} \left[\frac{2}{(1+c)^2} \log\left(\frac{1-c}{2}\right) + \frac{2}{(1-c)^2} \log\left(\frac{1+c}{2}\right) + \frac{16\delta(2\delta-3)}{(1-c^2)^2(1-\delta)^2} + \frac{12(3+c^2)}{(1-c^2)^2} \log\left(\frac{1-\delta}{\delta}\right) \right] + \text{Finite part} \right\}$$

- Anti MHV

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{Born}^{(--++--)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^{2\epsilon} \frac{\alpha N_c}{\pi} \left\{ \frac{1}{\epsilon^2} \frac{8(3+c^2)}{(1-c^2)^2} + \frac{1}{\epsilon} \left[-\frac{12(c^2+3) \log \delta}{(1-c^2)^2} + \frac{64(12c^2+17)}{3(1-c^2)^3} + \frac{2\delta}{(1-c^2)^2} \left(\frac{2}{3}(5+3c^2)\delta^2 - (c^2+19)\delta + 2(5c^2+43) \right) + \frac{2(3c^2-24c+85)}{(1-c)(1+c)^3} \log\left(\frac{1-c}{2}\right) - \frac{8(c^2-6c+21)}{(1-c)(1+c)^3} \log\left(\frac{1+\delta-(1-\delta)c}{2}\right) - \frac{32(c^2-4c+7)}{(1+c)^3(1-c)(1+\delta-c(1-\delta))} + \frac{32(2-c)}{(1+c)^3(1+\delta-c(1-\delta))^2} - \frac{64(1-c)}{3(1+c)^3(1+\delta-c(1-\delta))^3} + (c \leftrightarrow -c) \right] + \text{Finite part} \right\}$$

Real Emission

- MHV

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{Born}^{(--++++)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^{2\epsilon} \frac{\alpha N_c}{2\pi} \left\{ \frac{1}{\epsilon^2} \frac{8(3+c^2)}{(1-c^2)^2} + \frac{1}{\epsilon} \left[\frac{2}{(1+c)^2} \log\left(\frac{1-c}{2}\right) + \frac{2}{(1-c)^2} \log\left(\frac{1+c}{2}\right) + \frac{16\delta(2\delta-3)}{(1-c^2)^2(1-\delta)^2} + \frac{12(3+c^2)}{(1-c^2)^2} \log\left(\frac{1-\delta}{\delta}\right) \right] + \text{Finite part} \right\}$$

- Anti MHV

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{Born}^{(--++--)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^{2\epsilon} \frac{\alpha N_c}{\pi} \left\{ \frac{1}{\epsilon^2} \frac{8(3+c^2)}{(1-c^2)^2} + \frac{1}{\epsilon} \left[-\frac{12(c^2+3)\log\delta}{(1-c^2)^2} + \frac{64(12c^2+17)}{3(1-c^2)^3} + \frac{2\delta}{(1-c^2)^2} \left(\frac{2}{3}(5+3c^2)\delta^2 - (c^2+19)\delta + 2(5c^2+43) \right) + \frac{2(3c^2-24c+85)}{(1-c)(1+c)^3} \log\left(\frac{1-c}{2}\right) - \frac{8(c^2-6c+21)}{(1-c)(1+c)^3} \log\left(\frac{1+\delta-(1-\delta)c}{2}\right) - \frac{32(c^2-4c+7)}{(1+c)^3(1-c)(1+\delta-c(1-\delta))} + \frac{32(2-c)}{(1+c)^3(1+\delta-c(1-\delta))^2} - \frac{64(1-c)}{3(1+c)^3(1+\delta-c(1-\delta))^3} + (c \leftrightarrow -c) \right] + \text{Finite part} \right\}$$

Real Emission

- MHV

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{Born}^{(--++++)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^{2\epsilon} \frac{\alpha N_c}{2\pi} \left\{ \frac{1}{\epsilon^2} \frac{8(3+c^2)}{(1-c^2)^2} + \frac{1}{\epsilon} \left[\frac{2}{(1+c)^2} \log\left(\frac{1-c}{2}\right) + \frac{2}{(1-c)^2} \log\left(\frac{1+c}{2}\right) + \frac{16\delta(2\delta-3)}{(1-c^2)^2(1-\delta)^2} + \frac{12(3+c^2)}{(1-c^2)^2} \log\left(\frac{1-\delta}{\delta}\right) \right] + \text{Finite part} \right\}$$

- Anti MHV

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{Born}^{(--++--)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^{2\epsilon} \frac{\alpha N_c}{\pi} \left\{ \frac{1}{\epsilon^2} \frac{8(3+c^2)}{(1-c^2)^2} + \frac{1}{\epsilon} \left[-\frac{12(c^2+3)\log\delta}{(1-c^2)^2} + \frac{64(12c^2+17)}{3(1-c^2)^3} + \frac{2\delta}{(1-c^2)^2} \left(\frac{2}{3}(5+3c^2)\delta^2 - (c^2+19)\delta + 2(5c^2+43) \right) + \frac{2(3c^2-24c+85)}{(1-c)(1+c)^3} \log\left(\frac{1-c}{2}\right) - \frac{8(c^2-6c+21)}{(1-c)(1+c)^3} \log\left(\frac{1+\delta-(1-\delta)c}{2}\right) - \frac{32(c^2-4c+7)}{(1+c)^3(1-c)(1+\delta-c(1-\delta))} + \frac{32(2-c)}{(1+c)^3(1+\delta-c(1-\delta))^2} - \frac{64(1-c)}{3(1+c)^3(1+\delta-c(1-\delta))^3} + (c \leftrightarrow -c) \right] + \text{Finite part} \right\}$$

Real Emission (Matter)

- Fermions

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{Real}^{(--+\bar{q}q)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^{2\epsilon} \frac{\alpha N_c}{2\pi} \left\{ \frac{16}{\epsilon} \left[\frac{(79 - 25c^2)}{3(1 - c^2)^2} \right. \right. \\ \left. \left. + \frac{2(3 - c)^2}{(1 - c)(1 + c)^3} \right) \log\left(\frac{1 - c}{2}\right) + \frac{2(3 + c)^2}{(1 - c)^3(1 + c)} \log\left(\frac{1 + c}{2}\right) \right] + \text{Finite part} \right\}$$

- Sfermions

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{Real}^{(--+\bar{q}\bar{q})} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^{2\epsilon} \frac{\alpha N_c}{2\pi} \left\{ \frac{8}{\epsilon} \left[-\frac{2(10 + 7c^2)}{(1 - c^2)^2} \right. \right. \\ \left. \left. - \frac{3(5 - c)}{(1 + c)^3} \log\left(\frac{1 - c}{2}\right) - \frac{3(5 + c)}{(1 - c)^3} \log\left(\frac{1 + c}{2}\right) \right] + \text{Finite part} \right\}$$

Real Emission (Matter)

- Fermions

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{Real}^{(--+\bar{q}q)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^{2\epsilon} \frac{\alpha N_c}{2\pi} \left\{ \frac{16}{\epsilon} \left[\frac{(79 - 25c^2)}{3(1 - c^2)^2} \right. \right. \\ \left. \left. + \frac{2(3 - c)^2}{(1 - c)(1 + c)^3} \right) \log\left(\frac{1 - c}{2}\right) + \frac{2(3 + c)^2}{(1 - c)^3(1 + c)} \log\left(\frac{1 + c}{2}\right) \right] + \text{Finite part} \right\}$$

- Sfermions

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{Real}^{(--+\bar{q}\bar{q})} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^{2\epsilon} \frac{\alpha N_c}{2\pi} \left\{ \frac{8}{\epsilon} \left[-\frac{2(10 + 7c^2)}{(1 - c^2)^2} \right. \right. \\ \left. \left. - \frac{3(5 - c)}{(1 + c)^3} \log\left(\frac{1 - c}{2}\right) - \frac{3(5 + c)}{(1 - c)^3} \log\left(\frac{1 + c}{2}\right) \right] + \text{Finite part} \right\}$$

Real Emission (Matter)

- Fermions

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{Real}^{(--+\bar{q}q)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^{2\epsilon} \frac{\alpha N_c}{2\pi} \left\{ \frac{16}{\epsilon} \left[\frac{(79 - 25c^2)}{3(1 - c^2)^2} \right. \right. \\ \left. \left. + \frac{2(3 - c)^2}{(1 - c)(1 + c)^3} \right) \log\left(\frac{1 - c}{2}\right) + \frac{2(3 + c)^2}{(1 - c)^3(1 + c)} \log\left(\frac{1 + c}{2}\right) \right] + \text{Finite part} \right\}$$

- Sfermions

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{Real}^{(--+\bar{q}\bar{q})} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^{2\epsilon} \frac{\alpha N_c}{2\pi} \left\{ \frac{8}{\epsilon} \left[-\frac{2(10 + 7c^2)}{(1 - c^2)^2} \right. \right. \\ \left. \left. - \frac{3(5 - c)}{(1 + c)^3} \log\left(\frac{1 - c}{2}\right) - \frac{3(5 + c)}{(1 - c)^3} \log\left(\frac{1 + c}{2}\right) \right] + \text{Finite part} \right\}$$

Splitting Contribution

$$d\sigma_{2\rightarrow 2}^{spl,init} = \frac{\alpha}{2\pi} \frac{1}{\epsilon} \left(\frac{\mu^2}{Q_f^2} \right)^\epsilon \int_0^1 dz \sum_{l=g,q,\Lambda} P_{gl}(z) \sum_{i,j=1,2; i\neq j} d\sigma_{2\rightarrow 2}(z p_i, p_j, p_3, p_4) S_2^{spl,init}(z)$$

$$d\sigma_{2\rightarrow 2}^{spl,fin} = \frac{\alpha}{2\pi} \frac{1}{\epsilon} \left(\frac{\mu^2}{Q_f^2} \right)^\epsilon d\sigma_{2\rightarrow 2}(p_1, p_2, p_3, p_4) \int_0^1 dz \sum_{l=g,q,\Lambda} P_{gl}(z) S_2^{spl,fin}(z)$$

Initial and final state splitting (MHV)

- Initial

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{InSplit}^{(--++++)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^\epsilon \left(\frac{\mu^2}{Q_f^2}\right)^\epsilon$$

$$\frac{\alpha N_c}{2\pi} \left\{ \frac{1}{\epsilon} \left[-\frac{4(c^2+3)}{(1-c^2)^2} \left(\log \frac{1-c}{2} + \log \frac{1+c}{2} \right) - \frac{8(c^2+3)}{(1-c^2)^2} \log \frac{1-\delta}{\delta} \right. \right.$$

$$\left. \left. - \frac{16\delta(2\delta-3)}{(1-c^2)^2(1-\delta)^2} \right] + \text{Finite part} \right\}$$

- Final

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{FnSplit}^{(--++++)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^\epsilon \left(\frac{\mu^2}{Q_f^2}\right)^\epsilon$$

$$\frac{\alpha N_c}{2\pi} \left\{ -\frac{1}{\epsilon} \frac{4(c^2+3)}{(1-c^2)^2} \log \frac{1-\delta}{\delta} + \text{Finite part} \right\}$$

Initial and final state splitting (MHV)

- Initial

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{InSplit}^{(--++++)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^\epsilon \left(\frac{\mu^2}{Q_f^2}\right)^\epsilon$$

$$\frac{\alpha N_c}{2\pi} \left\{ \frac{1}{\epsilon} \left[-\frac{4(c^2+3)}{(1-c^2)^2} \left(\log \frac{1-c}{2} + \log \frac{1+c}{2} \right) - \frac{8(c^2+3)}{(1-c^2)^2} \log \frac{1-\delta}{\delta} \right. \right.$$

$$\left. \left. - \frac{16\delta(2\delta-3)}{(1-c^2)^2(1-\delta)^2} \right] + \text{Finite part} \right\}$$

- Final

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{FnSplit}^{(--++++)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^\epsilon \left(\frac{\mu^2}{Q_f^2}\right)^\epsilon$$

$$\frac{\alpha N_c}{2\pi} \left\{ -\frac{1}{\epsilon} \frac{4(c^2+3)}{(1-c^2)^2} \log \frac{1-\delta}{\delta} + \text{Finite part} \right\}$$

Initial and final state splitting (MHV)

- Initial

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{InSplit}^{(--++++)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^\epsilon \left(\frac{\mu^2}{Q_f^2}\right)^\epsilon$$

$$\frac{\alpha N_c}{2\pi} \left\{ \frac{1}{\epsilon} \left[-\frac{4(c^2+3)}{(1-c^2)^2} \left(\log \frac{1-c}{2} + \log \frac{1+c}{2} \right) - \frac{8(c^2+3)}{(1-c^2)^2} \log \frac{1-\delta}{\delta} \right. \right.$$

$$\left. \left. - \frac{16\delta(2\delta-3)}{(1-c^2)^2(1-\delta)^2} \right] + \text{Finite part} \right\}$$

- Final

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{FnSplit}^{(--++++)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^\epsilon \left(\frac{\mu^2}{Q_f^2}\right)^\epsilon$$

$$\frac{\alpha N_c}{2\pi} \left\{ -\frac{1}{\epsilon} \frac{4(c^2+3)}{(1-c^2)^2} \log \frac{1-\delta}{\delta} + \text{Finite part} \right\}$$

Initial and final state splitting (Anti MHV)

● Initial

$$\begin{aligned} \left(\frac{d\sigma}{d\Omega_{14}} \right)_{InSplit}^{(--+--+)} &= \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s} \right)^\epsilon \left(\frac{\mu^2}{Q_f^2} \right)^\epsilon \frac{\alpha N_c}{\pi} \left\{ \frac{1}{\epsilon} \left[\frac{8(c^2+3)}{(1-c^2)^2} \log \delta - \frac{64(12c^2+17)}{3(1-c^2)^3} \right. \right. \\ &- \frac{4\delta}{(1-c^2)^2} \left(\frac{2}{3}(1+c^2)\delta^2 + (c^2-5)\delta + 2(c^2+17) \right) + \left(\frac{4(c^3-15c^2+51c-45)}{(1-c)^2(1+c)^3} \log \frac{1-c}{2} \right. \\ &+ \frac{8(c^2-6c+21)}{(1-c)(1+c)^3} \log \frac{1+\delta-c(1-\delta)}{2} + \frac{32(c^2-4c+7)}{(1+c)^3(1-c)(1+\delta-c(1-\delta))} \\ &\left. \left. - \frac{32(2-c)}{(1+c)^3(1+\delta-c(1-\delta))^2} + \frac{64(1-c)}{3(1+c)^3(1+\delta-c(1-\delta))^3} + (c \leftrightarrow -c) \right) \right] + \text{Finite part} \left. \right\} \end{aligned}$$

● Final

$$\left(\frac{d\sigma}{d\Omega_{14}} \right)_{FnSplit}^{(--+--+)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s} \right)^\epsilon \left(\frac{\mu^2}{Q_f^2} \right)^\epsilon \frac{\alpha N_c}{2\pi} \left\{ \frac{1}{\epsilon} \frac{4(c^2+3)}{(1-c^2)^2} \left[\log \delta - \frac{\delta}{3}(2\delta^2-9\delta+18) \right] + \text{F.p.} \right\}$$

Initial and final state splitting (Anti MHV)

● Initial

$$\begin{aligned}
 \left(\frac{d\sigma}{d\Omega_{14}} \right)_{InSplit}^{(--+--+)} &= \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s} \right)^\epsilon \left(\frac{\mu^2}{Q_f^2} \right)^\epsilon \frac{\alpha N_c}{\pi} \left\{ \frac{1}{\epsilon} \left[\frac{8(c^2 + 3)}{(1 - c^2)^2} \log \delta - \frac{64(12c^2 + 17)}{3(1 - c^2)^3} \right. \right. \\
 &- \frac{4\delta}{(1 - c^2)^2} \left(\frac{2}{3}(1 + c^2)\delta^2 + (c^2 - 5)\delta + 2(c^2 + 17) \right) + \left(\frac{4(c^3 - 15c^2 + 51c - 45)}{(1 - c)^2(1 + c)^3} \log \frac{1 - c}{2} \right. \\
 &+ \frac{8(c^2 - 6c + 21)}{(1 - c)(1 + c)^3} \log \frac{1 + \delta - c(1 - \delta)}{2} + \frac{32(c^2 - 4c + 7)}{(1 + c)^3(1 - c)(1 + \delta - c(1 - \delta))} \\
 &\left. \left. - \frac{32(2 - c)}{(1 + c)^3(1 + \delta - c(1 - \delta))^2} + \frac{64(1 - c)}{3(1 + c)^3(1 + \delta - c(1 - \delta))^3} + (c \leftrightarrow -c) \right] + \text{Finite part} \right\}
 \end{aligned}$$

● Final

$$\left(\frac{d\sigma}{d\Omega_{14}} \right)_{FnSplit}^{(--+--+)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s} \right)^\epsilon \left(\frac{\mu^2}{Q_f^2} \right)^\epsilon \frac{\alpha N_c}{2\pi} \left\{ \frac{1}{\epsilon} \frac{4(c^2 + 3)}{(1 - c^2)^2} \left[\log \delta - \frac{\delta}{3}(2\delta^2 - 9\delta + 18) \right] + \text{F.p.} \right\}$$

Initial and final state splitting (Anti MHV)

● Initial

$$\begin{aligned} \left(\frac{d\sigma}{d\Omega_{14}} \right)_{InSplit}^{(--++-)} &= \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s} \right)^\epsilon \left(\frac{\mu^2}{Q_f^2} \right)^\epsilon \frac{\alpha N_c}{\pi} \left\{ \frac{1}{\epsilon} \left[\frac{8(c^2 + 3)}{(1 - c^2)^2} \log \delta - \frac{64(12c^2 + 17)}{3(1 - c^2)^3} \right. \right. \\ &- \frac{4\delta}{(1 - c^2)^2} \left(\frac{2}{3}(1 + c^2)\delta^2 + (c^2 - 5)\delta + 2(c^2 + 17) \right) + \left(\frac{4(c^3 - 15c^2 + 51c - 45)}{(1 - c)^2(1 + c)^3} \log \frac{1 - c}{2} \right. \\ &+ \frac{8(c^2 - 6c + 21)}{(1 - c)(1 + c)^3} \log \frac{1 + \delta - c(1 - \delta)}{2} + \frac{32(c^2 - 4c + 7)}{(1 + c)^3(1 - c)(1 + \delta - c(1 - \delta))} \\ &\left. \left. - \frac{32(2 - c)}{(1 + c)^3(1 + \delta - c(1 - \delta))^2} + \frac{64(1 - c)}{3(1 + c)^3(1 + \delta - c(1 - \delta))^3} + (c \leftrightarrow -c) \right] + \text{Finite part} \right\} \end{aligned}$$

● Final

$$\left(\frac{d\sigma}{d\Omega_{14}} \right)_{FnSplit}^{(--++-)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s} \right)^\epsilon \left(\frac{\mu^2}{Q_f^2} \right)^\epsilon \frac{\alpha N_c}{2\pi} \left\{ \frac{1}{\epsilon} \frac{4(c^2 + 3)}{(1 - c^2)^2} \left[\log \delta - \frac{\delta}{3}(2\delta^2 - 9\delta + 18) \right] + \text{F.p.} \right\}$$

Initial state splitting (Matter) ($\delta = 1$)

- Fermions

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{InSplit}^{(--+\bar{q}q)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^\epsilon \left(\frac{\mu^2}{Q_f^2}\right)^\epsilon \frac{\alpha N_c}{2\pi} \left\{ \frac{16}{\epsilon} \left[\frac{(79 - 25c^2)}{3(1 - c^2)^2} + \frac{2(3 - c)^2}{(1 - c)(1 + c)^3} \right] \log\left(\frac{1 - c}{2}\right) + \frac{2(3 + c)^2}{(1 - c)^3(1 + c)} \log\left(\frac{1 + c}{2}\right) \right\} + \text{Finite part}$$

- Sfermions

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{InSplit}^{(--+\bar{q}q)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s}\right)^\epsilon \left(\frac{\mu^2}{Q_f^2}\right)^\epsilon \frac{\alpha N_c}{2\pi} \left\{ \frac{8}{\epsilon} \left[-\frac{2(10 + 7c^2)}{(1 - c^2)^2} - \frac{3(5 - c)}{(1 + c)^3} \log\left(\frac{1 - c}{2}\right) - \frac{3(5 + c)}{(1 - c)^3} \log\left(\frac{1 + c}{2}\right) \right] + \text{Finite part} \right\}$$

Initial state splitting (Matter) ($\delta = 1$)

- Fermions

$$\left(\frac{d\sigma}{d\Omega_{14}} \right)_{InSplit}^{(--+\bar{q}q)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s} \right)^\epsilon \left(\frac{\mu^2}{Q_f^2} \right)^\epsilon \frac{\alpha N_c}{2\pi} \left\{ \frac{16}{\epsilon} \left[\frac{(79 - 25c^2)}{3(1 - c^2)^2} + \frac{2(3 - c)^2}{(1 - c)(1 + c)^3} \log\left(\frac{1 - c}{2}\right) + \frac{2(3 + c)^2}{(1 - c)^3(1 + c)} \log\left(\frac{1 + c}{2}\right) \right] + \text{Finite part} \right\}$$

- Sfermions

$$\left(\frac{d\sigma}{d\Omega_{14}} \right)_{InSplit}^{(--+\bar{q}q)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s} \right)^\epsilon \left(\frac{\mu^2}{Q_f^2} \right)^\epsilon \frac{\alpha N_c}{2\pi} \left\{ \frac{8}{\epsilon} \left[-\frac{2(10 + 7c^2)}{(1 - c^2)^2} - \frac{3(5 - c)}{(1 + c)^3} \log\left(\frac{1 - c}{2}\right) - \frac{3(5 + c)}{(1 - c)^3} \log\left(\frac{1 + c}{2}\right) \right] + \text{Finite part} \right\}$$

Initial state splitting (Matter) ($\delta = 1$)

- Fermions

$$\left(\frac{d\sigma}{d\Omega_{14}} \right)_{InSplit}^{(---\bar{q}q)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s} \right)^\epsilon \left(\frac{\mu^2}{Q_f^2} \right)^\epsilon \frac{\alpha N_c}{2\pi} \left\{ \frac{16}{\epsilon} \left[\frac{(79 - 25c^2)}{3(1 - c^2)^2} \right. \right. \\ \left. \left. + \frac{2(3 - c)^2}{(1 - c)(1 + c)^3} \right) \log\left(\frac{1 - c}{2}\right) + \frac{2(3 + c)^2}{(1 - c)^3(1 + c)} \log\left(\frac{1 + c}{2}\right) \right] + \text{Finite part} \right\}$$

- Sfermions

$$\left(\frac{d\sigma}{d\Omega_{14}} \right)_{InSplit}^{(---\bar{q}q)} = \frac{\alpha^2 N_c^2}{E^2} \left(\frac{\mu^2}{s} \right)^\epsilon \left(\frac{\mu^2}{Q_f^2} \right)^\epsilon \frac{\alpha N_c}{2\pi} \left\{ \frac{8}{\epsilon} \left[-\frac{2(10 + 7c^2)}{(1 - c^2)^2} \right. \right. \\ \left. \left. - \frac{3(5 - c)}{(1 + c)^3} \log\left(\frac{1 - c}{2}\right) - \frac{3(5 + c)}{(1 - c)^3} \log\left(\frac{1 + c}{2}\right) \right] + \text{Finite part} \right\}$$

Infrared-free sets (for any arbitrary δ)

- $$A^{MHV} = \frac{1}{2} \left(\frac{d\sigma}{d\Omega_{14}} \right)_{Virt}^{(----)} + \left(\frac{d\sigma}{d\Omega_{14}} \right)_{Real}^{(----)} + \left(\frac{d\sigma}{d\Omega_{14}} \right)_{InSplit}^{(----)} + \left(\frac{d\sigma}{d\Omega_{14}} \right)_{FnSplit}^{(----)}$$

- $$B^{AntiMHV} = \frac{1}{2} \left(\frac{d\sigma}{d\Omega_{14}} \right)_{Virt}^{(----)} + \left(\frac{d\sigma}{d\Omega_{14}} \right)_{Real}^{(----)} + \left(\frac{d\sigma}{d\Omega_{14}} \right)_{InSplit}^{(----)} + \left(\frac{d\sigma}{d\Omega_{14}} \right)_{FnSplit}^{(----)}$$

- $$C^{Matter} = \left(\frac{d\sigma}{d\Omega_{14}} \right)_{Real}^{(---+\bar{q}q,\bar{q}\bar{q})} + \left(\frac{d\sigma}{d\Omega_{14}} \right)_{InSplit}^{(---+\bar{q}q,\bar{q}\bar{q})} + \left(\frac{d\sigma}{d\Omega_{14}} \right)_{FnSplit}^{(---+\bar{q}q,\bar{q}\bar{q})}$$

Infrared-free sets (for any arbitrary δ)

- $$A^{MHV} = \frac{1}{2} \left(\frac{d\sigma}{d\Omega_{14}} \right)_{Virt}^{(----)} + \left(\frac{d\sigma}{d\Omega_{14}} \right)_{Real}^{(----)} + \left(\frac{d\sigma}{d\Omega_{14}} \right)_{InSplit}^{(----)} + \left(\frac{d\sigma}{d\Omega_{14}} \right)_{FnSplit}^{(----)}$$

- $$B^{AntiMHV} = \frac{1}{2} \left(\frac{d\sigma}{d\Omega_{14}} \right)_{Virt}^{(----)} + \left(\frac{d\sigma}{d\Omega_{14}} \right)_{Real}^{(----)} + \left(\frac{d\sigma}{d\Omega_{14}} \right)_{InSplit}^{(----)} + \left(\frac{d\sigma}{d\Omega_{14}} \right)_{FnSplit}^{(----)}$$

- $$C^{Matter} = \left(\frac{d\sigma}{d\Omega_{14}} \right)_{Real}^{(---+\bar{q}q,\bar{q}\bar{q})} + \left(\frac{d\sigma}{d\Omega_{14}} \right)_{InSplit}^{(---+\bar{q}q,\bar{q}\bar{q})} + \left(\frac{d\sigma}{d\Omega_{14}} \right)_{FnSplit}^{(---+\bar{q}q,\bar{q}\bar{q})}$$

Infrared-free sets (for any arbitrary δ)



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Infrared-free observables

- Registration of two fastest gluons of positive chirality

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The simplest IR finite answer so far ($Q_f = E$): **N=4 SYM Anti MHV**

$$\left(\frac{d\sigma}{d\Omega_{14}} \right)_{AntiMHV} = \frac{\alpha^2 N_c^2}{E^2} \left\{ \frac{3 + c^2}{(1 - c^2)^2} - \frac{\alpha N_c}{2\pi} \left[2 \frac{(c^4 + 2c^3 + 4c^2 + 6c + 19) \log^2\left(\frac{1-c}{2}\right)}{(1-c)^2(1+c)^4} + 2 \frac{(c^4 - 2c^3 + 4c^2 - 6c + 19) \log^2\left(\frac{1+c}{2}\right)}{(1-c)^4(1+c)^2} - 8 \frac{(c^2 + 1) \log\left(\frac{1+c}{2}\right) \log\left(\frac{1-c}{2}\right)}{(1 - c^2)^2} + \frac{6\pi^2(3c^2 + 13) - 5(61c^2 + 99)}{9(1 - c^2)^2} + 2 \frac{(11c^3 + 31c^2 - 47c + 133) \log\left(\frac{1+c}{2}\right)}{3(1-c)^3(1+c)^2} - 2 \frac{(11c^3 - 31c^2 - 47c - 133) \log\left(\frac{1-c}{2}\right)}{3(1+c)^3(1-c)^2} \right] \right\}$$

Summary

- In observable cross-sections the IR divergences do cancel in accordance with Kinoshita-Lee-Nauenberg theorem

$$d\sigma_{obs}^{incl} = \sum_{n=2}^{\infty} \int_0^1 dz_1 q_1(z_1, \frac{Q_f^2}{\mu^2}) \int_0^1 dz_2 q_2(z_2, \frac{Q_f^2}{\mu^2}) \prod_{i=1}^n \int_0^1 dx_i q_i(x_i, \frac{Q_f^2}{\mu^2}) \times$$

$$\times d\sigma^{2 \rightarrow n}(z_1 p_1, z_2 p_2, \dots) S_n(\{z\}, \{x\}) = g^4 \sum_{L=0}^{\infty} \left(\frac{g^2}{16\pi^2} \right)^L d\sigma_L^{Finite}(s, t, u, Q_f^2)$$

- The simple structure of the MHV amplitude DOES NOT reveal at the level of IR finite cross-sections;
- In some cases the cancellation of complicated functions occurs, though not always;
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