New Developments in QFT: D=4 Conformal Field Theories and AdS/CFT Correspondence

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Outline

N=4 Super Yang-Mills Theory

- $\mathcal{N} = 4$ Super Yang-Mills theory is the most supersymmetric theory possible without gravity
- Field content: 1 massless gauge boson, 4 massless (Majorana) spin 1/2 fermions, 6 real (or 3 complex) massless spin 0 bosons
  All fields are in adjoint representation of the gauge group (Take $SU(N_c)$)
- The theory is exactly scale invariant, conformal field theory at quantum level, i.e. the β function identically vanishes at all orders of PT

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AdS/CFT Correspondence

- $N_c \to \infty$ (planar limit) is expected to be integrable and solvable
- Maldacena's conjecture: Planar Limit of N=4 SYM at strong coupling is dual to weakly coupled type II b supergravity in 10 dimensional $AdS_5 * S_5$ space.
- How might PT series be organized to produce simple strong coupling result?
- The amplitudes on shell possess IR singularities which should cancel in observables. What are the infrared-safe observables?

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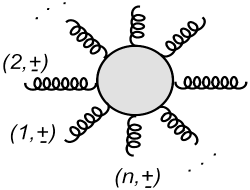

All outgoing gluons with helicity + or -
on mass shell
In the leading N_c order (planar limit)

- Colour decomposition of amplitudes in N=4 SYM theory for $N_c \to \infty$

$$\mathcal{A}_n^{(l)} = g^{n-2}\left(\frac{g^2 N_c}{16\pi^2}\right)^l \sum_{perm} Tr(T^{a_{\sigma(1)}}, ..., T^{a_{\sigma(n)}}) A_n^{(l)}(a_{\sigma(1)}, ..., a_{\sigma(n)}),$$

  where $\mathcal{A}_n$ - physical amplitude, A_n - partial amplitude, a_i - is color index of $i-th$ external "gluon"

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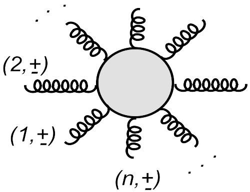

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N=4 SYM Theory: Perturbation Theory

- Bern, Dixon & Smirnov's conjecture: $M_n^{(L)}(\varepsilon) \equiv A_n^{(L)}/A_n^{(0)}$

$$\mathcal{M}_n \equiv 1 + \sum_{L=1}^{\infty} \left(\frac{g^2 N_c}{16\pi^2}\right)^L M_n^{(L)}(\varepsilon) = exp\left[\sum_{l=1}^{\infty} \left(\frac{g^2 N_c}{16\pi^2}\right)^l \left(f^{(l)}(\varepsilon) M_n^{(1)}(l\varepsilon) + C^{(l)} + E_n^{(l)}(\varepsilon)\right)\right]$$

$$f^{(l)}(\varepsilon) = f_0^{(l)}(\varepsilon) + \varepsilon f_1^{(l)}(\varepsilon) + \varepsilon^2 f_2^{(l)}(\varepsilon)$$

- $$\mathcal{M}_n(\varepsilon) = exp\left[-\frac{1}{8}\sum_{l=1}^{\infty}\left(\frac{g^2 N_c}{16\pi^2}\right)^l \left(\frac{\gamma_K^{(l)}}{(l\varepsilon)^2} + \frac{2G_0^{(l)}}{l\varepsilon}\right)\sum_{i=1}^{n}\left(\frac{\mu^2}{-s_{i,i+1}}\right)^{l\varepsilon} + \frac{1}{4}\sum_{l=1}^{\infty}\left(\frac{g^2 N_c}{16\pi^2}\right)^l \gamma_K^{(l)} F_n^{(1)}(0)\right]$$

- $$F_4^{(1)}(0) = \frac{1}{2}\log^2\left(\frac{-t}{-s}\right) + 4\zeta_2$$

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Cusp anomalous dimension

- Cusp anomalous dimension appears in RG eq. for the expectation value of a Wilson line with a cusp

  Loop expansion $$\gamma_K = \sum_{l=1}^{\infty} \left(\frac{g^2 N_c}{16\pi^2} \right)^l \gamma_K^{(l)}$$

  $$\gamma_K^{(1)} = 8, \quad \gamma_K^{(2)} = -16\zeta_2, \quad \gamma_K^{(3)} = 176\zeta_4, \ldots$$

- It also controls the large spin limit of anomalous dimension of leading-twist operators

  $$O_j \equiv \bar{q}(\gamma_+ \mathcal{D}^+)^j q$$

  $$\gamma_j = \frac{1}{2}\gamma_K(\alpha) \log j + \mathcal{O}(j^0), \quad j \to \infty$$

- and large x limit of the DGLAP kernel for p.d.f.

  $$P_{gg} = \frac{1}{2} \frac{\gamma_K(\alpha)}{(1-x)_+} + \ldots, \quad x \to 1, \quad \gamma(j) = -\int_0^1 dx\, x^{j-1} P(x)$$

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Strong coupling expansion/AdS

- Classical solution (Alday & Maldacena) for the scattering amplitude

$$\mathcal{M}_4(\varepsilon) = exp[-S^E_{cl}]$$

- $S^E_{cl} = \frac{1}{\varepsilon^2}\frac{\sqrt{g^2N_c}}{\pi}\left[\left(\frac{\mu^2_{IR}}{-s}\right)^{\varepsilon/2} + \left(\frac{\mu^2_{IR}}{-t}\right)^{\varepsilon/2}\right]$

$$+\frac{1}{\varepsilon}\frac{\sqrt{g^2N_c}}{2\pi}\text{(1-log 2)}\left[\left(\frac{\mu^2_{IR}}{-s}\right)^{\varepsilon/2} + \left(\frac{\mu^2_{IR}}{-t}\right)^{\varepsilon/2}\right] - \frac{\sqrt{g^2N_c}}{8\pi}\left[\log^2\left(\frac{s}{t}\right) + c\right] + \mathcal{O}(\varepsilon)$$

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Violation of BDS ansatz

- For $n = 4, 5$ the BDS ansatz goes through all checks, namely the amplitudes were calculated up to <u>three</u> loops for <u>four</u> gluons and up to <u>two</u> loops for <u>five</u> gluons.
- However, starting from $n = 6$ it fails.
    - In the strong coupling calculation in the limit $n \to \infty$ discrepancy with the BDS formula was found.
    - Starting from $n = 6$ the Regge limit factorization of the amplitude in some physical regions fails. This was also shown by explicit two-loop calculation.
- BDS formula needs to be modified by some unknown finite function, which is an open and intriguing problem.

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Virtual Correction (MHV)

- Born Term $c \equiv \cos\theta$

$$\left(\frac{d\sigma}{d\Omega}\right)_0^{--++} = \frac{\alpha^2 N_c^2}{8E^2}\frac{s^4(s^2+t^2+u^2)}{s^2t^2u^2}\left(\frac{\mu^2}{s}\right)^\varepsilon = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^\varepsilon \frac{3+c^2}{(1-c^2)^2}$$

- Virtual Correction

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$$\left. +((\frac{\mu^2}{s})^\varepsilon + (\frac{\mu^2}{-t})^\varepsilon)u^2 + ((\frac{\mu^2}{s})^\varepsilon + (\frac{\mu^2}{-u})^\varepsilon)t^2\right)$$

$$\left.\left. +\frac{16}{3}\pi^2(s^2+t^2+u^2) + 4(u^2\log^2(\frac{-s}{t}) + t^2\log^2(\frac{-s}{u}) + s^2\log^2(\frac{t}{u}))\right]\right\}$$

$$= \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\left\{\frac{\alpha N_c}{2\pi}\left[-\frac{16}{\varepsilon^2}\frac{3+c^2}{(1-c^2)^2} + \frac{4}{\varepsilon}\left(\frac{5+2c+c^2}{(1-c^2)^2}\log(\frac{1-c}{2})\right.\right.\right.$$

$$\left.\left.\left. +(c\leftrightarrow -c)\right) + \frac{16(3+c^2)\pi^2}{3(1-c^2)^2} - \frac{16}{(1-c^2)^2}\log(\frac{1-c}{2})\log(\frac{1+c}{2})\right]\right\}$$

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Virtual Correction (MHV)

- Born Term $c \equiv \cos\theta$

$$\left(\frac{d\sigma}{d\Omega}\right)_0^{--++} = \frac{\alpha^2 N_c^2}{8E^2}\frac{s^4(s^2+t^2+u^2)}{s^2t^2u^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\frac{3+c^2}{(1-c^2)^2}$$

- Virtual Correction

$$\left(\frac{d\sigma}{d\Omega}\right)_{virt}^{--++} = \frac{\alpha^2 N_c^2}{8E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left\{\frac{\alpha N_c}{2\pi}\frac{s^4}{s^2t^2u^2}\left[-\frac{8}{\varepsilon^2}\left(((\frac{\mu^2}{-t})^{\varepsilon}+(\frac{\mu^2}{-u})^{\varepsilon})s^2\right.\right.\right.$$
$$\left.+((\frac{\mu^2}{s})^{\varepsilon}+(\frac{\mu^2}{-t})^{\varepsilon})u^2+((\frac{\mu^2}{s})^{\varepsilon}+(\frac{\mu^2}{-u})^{\varepsilon})t^2\right)$$
$$\left.\left.+\frac{16}{3}\pi^2(s^2+t^2+u^2)+4(u^2\log^2(\frac{-s}{t})+t^2\log^2(\frac{-s}{u})+s^2\log^2(\frac{t}{u}))\right]\right\}$$

$$= \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\left\{\frac{\alpha N_c}{2\pi}\left[-\frac{16}{\varepsilon^2}\frac{3+c^2}{(1-c^2)^2}+\frac{4}{\varepsilon}\left(\frac{5+2c+c^2}{(1-c^2)^2}\log(\frac{1-c}{2})\right.\right.\right.$$
$$\left.\left.\left.+(c\leftrightarrow -c)\right)+\frac{16(3+c^2)\pi^2}{3(1-c^2)^2}-\frac{16}{(1-c^2)^2}\log(\frac{1-c}{2})\log(\frac{1+c}{2})\right]\right\}$$

Real Emission

- MHV
$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{Born} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon^2}\frac{8(3+c^2)}{(1-c^2)^2}\right.$$
$$+\frac{1}{\varepsilon}\left[\frac{2}{(1+c)^2}\log\left(\frac{1-c}{2}\right)+\frac{2}{(1-c)^2}\log\left(\frac{1+c}{2}\right)+\frac{16\delta(2\delta-3)}{(1-c^2)^2(1-\delta)^2}\right.$$
$$\left.\left.+\frac{12(3+c^2)(\log(1-\delta)-\log(\delta))}{(1-c^2)^2}\right]+\text{Finite part}\right\}$$
- Anti MHV
$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{Born} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon^2}\frac{8(3+c^2)}{(1-c^2)^2}\right.$$
$$+\frac{2}{\varepsilon}\left[\frac{64}{3(1-c^2)^3}+\delta\frac{(6\delta^2-3\delta+30)c^2+(10\delta^2-57\delta+66)}{3(1-c^2)^2}-\frac{6(c^2+3)\log\delta}{(1-c^2)^2}+\right.$$
$$\left(\frac{3c^2-24c+85}{(1-c)(1+c)^3}\log\frac{1-c}{2}-\frac{4(c^2-6c+21)}{(1-c)(1+c)^3}\log\frac{1+\delta-(1-\delta)c}{2}\right.$$
$$\left.\left.+\frac{16\delta(5-c)}{(1-c^2)^2(1+\delta-(1-\delta)c)}-\frac{32(1-c)}{3(1+c)^3(1+\delta-(1-\delta)c)^3}+(c\leftrightarrow -c)\right)\right]$$
$$\left.+\text{Finite part}\right\}$$

Real Emission

- MHV

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{Born} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon^2}\frac{8(3+c^2)}{(1-c^2)^2} + \frac{1}{\varepsilon}\left[\frac{2}{(1+c)^2}\log(\frac{1-c}{2}) + \frac{2}{(1-c)^2}\log(\frac{1+c}{2}) + \frac{16\delta(2\delta-3)}{(1-c^2)^2(1-\delta)^2} + \frac{12(3+c^2)(\log(1-\delta)-\log(\delta))}{(1-c^2)^2}\right] + \text{Finite part}\right\}$$

- Anti MHV

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{Born} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon^2}\frac{8(3+c^2)}{(1-c^2)^2} + \frac{2}{\varepsilon}\left[\frac{64}{3(1-c^2)^3} + \delta\frac{(6\delta^2-3\delta+30)c^2+(10\delta^2-57\delta+66)}{3(1-c^2)^2} - \frac{6(c^2+3)\log\delta}{(1-c^2)^2} + \left(\frac{3c^2-24c+85}{(1-c)(1+c)^3}\log\frac{1-c}{2} - \frac{4(c^2-6c+21)}{(1-c)(1+c)^3}\log\frac{1+\delta-(1-\delta)c}{2} + \frac{16\delta(5-c)}{(1-c^2)^2(1+\delta-(1-\delta)c)} - \frac{32(1-c)}{3(1+c)^3(1+\delta-(1-\delta)c)^3} + (c\leftrightarrow -c)\right)\right] + \text{Finite part}\right\}$$

Real Emission

- MHV

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{Born} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon^2}\frac{8(3+c^2)}{(1-c^2)^2} + \frac{1}{\varepsilon}\left[\frac{2}{(1+c)^2}\log(\frac{1-c}{2}) + \frac{2}{(1-c)^2}\log(\frac{1+c}{2}) + \frac{16\delta(2\delta-3)}{(1-c^2)^2(1-\delta)^2} + \frac{12(3+c^2)(\log(1-\delta)-\log(\delta))}{(1-c^2)^2}\right] + \text{Finite part}\right\}$$

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$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{Born} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon^2}\frac{8(3+c^2)}{(1-c^2)^2}\right.$$

$$+\frac{2}{\varepsilon}\left[\frac{64}{3(1-c^2)^3} + \delta\frac{(6\delta^2-3\delta+30)c^2+(10\delta^2-57\delta+66)}{3(1-c^2)^2} - \frac{6(c^2+3)\log\delta}{(1-c^2)^2} + \left(\frac{3c^2-24c+85}{(1-c)(1+c)^3}\log\frac{1-c}{2} - \frac{4(c^2-6c+21)}{(1-c)(1+c)^3}\log\frac{1+\delta-(1-\delta)c}{2} + \frac{16\delta(5-c)}{(1-c^2)^2(1+\delta-(1-\delta)c)} - \frac{32(1-c)}{3(1+c)^3(1+\delta-(1-\delta)c)^3} + (c\leftrightarrow -c)\right)\right]$$

$$\left. + \text{Finite part}\right\}$$

Real Emission (Matter)

- Fermions

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q)}_{Real} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{4}{\varepsilon}\left[\frac{32(79-25c^2)}{3(1-c^2)^2)} + \frac{64(3-c)^2}{(1-c)(1+c)^3}\right)\log(\frac{1-c}{2}) + \frac{64(3+c)^2}{(1-c)^3(1+c)}\log(\frac{1+c}{2})\right] + \text{Finite part}\right\}$$

- Sfermions

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\tilde{\bar{q}}\tilde{q})}_{Real} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon}\left[-\frac{128(10+7c^2)}{(1-c^2)^2} - \frac{192(5-c)}{(1+c)^3}\log(\frac{1-c}{2}) - \frac{192(5+c)}{(1-c)^3}\log(\frac{1+c}{2})\right] + \text{Finite part}\right\}$$

Real Emission (Matter)

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$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q)}_{Real} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{4}{\varepsilon}\left[\frac{32(79-25c^2)}{3(1-c^2)^2)}\right.\right.$$
$$\left.\left.+\frac{64(3-c)^2}{(1-c)(1+c)^3})\log(\frac{1-c}{2}) + \frac{64(3+c)^2}{(1-c)^3(1+c)}\log(\frac{1+c}{2})\right] + \text{Finite part}\right\}$$

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$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\tilde{\bar{q}}\tilde{q})}_{Real} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon}\left[-\frac{128(10+7c^2)}{(1-c^2)^2}\right.\right.$$
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$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\tilde{\bar{q}}\tilde{q})}_{Real} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon}\left[-\frac{128(10+7c^2)}{(1-c^2)^2} - \frac{192(5-c)}{(1+c)^3}\log(\frac{1-c}{2}) - \frac{192(5+c)}{(1-c)^3}\log(\frac{1+c}{2})\right] + \text{Finite part}\right\}$$

Splitting of Massless states

- There are NO isolated massless asymptotic states!
- Massless particle can split into two (or more) indistinguishable collinear particles.
- One has to consider coherent states of parallel massless particles.

$$d\sigma^{spl,init}_{2\to2} = \frac{\alpha}{2\pi}\frac{1}{\epsilon}\left(\frac{\mu^2}{Q_f^2}\right)^\epsilon \int_0^1 dz P_{gg}(z) \sum_{i,j=1,2;\; i\neq j} d\sigma_{2\to2}(zp_i,p_j,p_3,p_4) S_2^{spl,init}(z)$$

$$d\sigma^{spl,fin}_{2\to2} = \frac{\alpha}{2\pi}\frac{1}{\epsilon}\left(\frac{\mu^2}{Q_f^2}\right)^\epsilon d\sigma_{2\to2}(p_1,p_2,p_3,p_4)\int_0^1 dz \sum_{l=g,q,\Lambda} P_{gl}(z) S_2^{spl,fin}(z)$$

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Initial and final state splitting (MHV)

- Initial

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{InSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}$$

$$\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon}\left[-\frac{4(c^2+3)}{(1-c^2)^2}\left(\log\frac{1-c}{2}+\log\frac{1+c}{2}\right)-\frac{8(c^2+3)}{(1-c^2)^2}\log\frac{1-\delta}{\delta}\right.\right.$$
$$\left.\left.-\frac{16\delta(2\delta-3)}{(1-\delta^2)(1-c^2)^2}\right]+\text{Finite part}\right\}$$

- Final

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{FnSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}$$

$$\frac{\alpha N_c}{2\pi}\left\{-\frac{1}{\varepsilon}\frac{4(c^2+3)}{(1-c^2)^2}\log\frac{1-\delta}{\delta}+\text{Finite part}\right\}$$

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$$\left.\left.-\frac{16\delta(2\delta-3)}{(1-\delta^2)(1-c^2)^2}\right]+\text{Finite part}\right\}$$

- Final

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{FnSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}$$

$$\frac{\alpha N_c}{2\pi}\left\{-\frac{1}{\varepsilon}\frac{4(c^2+3)}{(1-c^2)^2}\log\frac{1-\delta}{\delta}+\text{Finite part}\right\}$$

Infrared-free sets

- $$A^{MHV} = \frac{1}{2}\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++)}_{Virt} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{Real} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{InSplit} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{FnSplit}$$
- $$B^{AntiMHV} = \frac{1}{2}\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++)}_{Virt} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{Real} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{InSplit} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{FnSplit}$$
- $$C^{Matter} = \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q,\tilde{\bar{q}}\tilde{q})}_{Real} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q,\tilde{\bar{q}}\tilde{q})}_{InSplit} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q,\tilde{\bar{q}}\tilde{q})}_{FnSplit}$$

Infrared-free sets

- $$A^{MHV} = \frac{1}{2}\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++)}_{Virt} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{Real} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{InSplit} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{FnSplit}$$

- $$B^{AntiMHV} = \frac{1}{2}\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++)}_{Virt} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{Real} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{InSplit} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{FnSplit}$$

- $$C^{Matter} = \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q,\bar{\tilde{q}}\tilde{q})}_{Real} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q,\bar{\tilde{q}}\tilde{q})}_{InSplit} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q,\bar{\tilde{q}}\tilde{q})}_{FnSplit}$$

Infrared-free sets

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- $$B^{AntiMHV} = \frac{1}{2}\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++)}_{Virt} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{Real} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{InSplit} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{FnSplit}$$
- $$C^{Matter} = \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q,\tilde{\bar{q}}\tilde{q})}_{Real} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q,\tilde{\bar{q}}\tilde{q})}_{InSplit} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q,\tilde{\bar{q}}\tilde{q})}_{FnSplit}$$

Infrared-free observables

- Registration of two fastest gluons of positive chirality

$$A^{MHV}\Big|_{\delta=1/3} + B^{AntiMHV}\Big|_{\delta=1}$$

- Registration of one fastest gluon of positive chirality

$$A^{MHV}\Big|_{\delta=1/3} + B^{AntiMHV}\Big|_{\delta=1/3} + C^{Matter}\Big|_{\delta=1}$$

- Anti MHV cross-section

$$B^{AntiMHV}\Big|_{\delta=1} + C^{Matter}\Big|_{\delta=1} \quad \Rightarrow \text{Finite Part}$$

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$$B^{AntiMHV}\Big|_{\delta=1} + C^{Matter}\Big|_{\delta=1} \quad \Rightarrow \text{Finite Part}$$

The simplest IR finite answer so far ($Q_f = E$): N=4 SYM Anti MHV

L.Bork, D.K., G.Vartanov, A.Zhiboedov

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{AntiMHV} = \frac{\alpha^2 N_c^2}{E^2}\left\{\frac{3+c^2}{(1-c^2)^2}\right.$$
$$-\frac{\alpha N_c}{2\pi}\left[2\frac{(c^4+2c^3+4c^2+6c+19)\log^2(\frac{1-c}{2})}{(1-c)^2(1+c)^4} + 2\frac{(c^4-2c^3+4c^2-6c+19)\log^2(\frac{1+c}{2})}{(1-c)^4(1+c)^2}\right.$$
$$-8\frac{(c^2+1)\log(\frac{1+c}{2})\log(\frac{1-c}{2})}{(1-c^2)^2} - \frac{6\pi^2(c^2-1)+5(61c^2+99)}{9(1-c^2)^2}$$
$$\left.\left.+2\frac{(11c^3+31c^2-47c+133)\log(\frac{1+c}{2})}{3(1-c)^3(1+c)^2} - 2\frac{(11c^3-31c^2-47c-133)\log(\frac{1-c}{2})}{3(1+c)^3(1-c)^2}\right]\right\}$$

Summary For Part I

- In observable cross-sections the IR divergences do cancel in accordance with Kinoshita-Lee-Nauenberg theorem

$$d\sigma_{obs}^{incl} = \sum_{n=2}^{\infty}\int_0^1 dz_1 q_1(z_1,\frac{Q_f^2}{\mu^2})\int_0^1 dz_2 q_2(z_2,\frac{Q_f^2}{\mu^2})\prod_{i=1}^{n}\int_0^1 dx_i q_i(x_i,\frac{Q_f^2}{\mu^2}) \times$$
$$\times\, d\sigma^{2\to n}(z_1p_1,z_2p_2,...)S_n(\{z\},\{x\}) = g^4\sum_{L=0}^{\infty}\left(\frac{g^2}{16\pi^2}\right)^L d\sigma_L^{Finite}(s,t,u,Q_f^2)$$

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- In some cases the cancellation of complicated functions occurs, though not always;
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- What are the IR safe observables in the strong coupling limit?
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UV Divergences in Gravity

Power counting in gravity and supergravity in D spacetime dimensions

$$\omega = (D-2)L + 2 \qquad D = 4 \to \omega = 2L + 2$$

Restrict analysis to ON-SHELL amplitudes only!

- Pure Gravity in D=4: Finite in 1 loop
  $R_{ij}^2,\ R^2, \to 0,\ R_{ijkl}^2 \to\ Gauss - Bonet\ term \to 0$
- Gravity with matter in D=4: Divergence in 1 loop
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- Local supersymmetry implies that the pure curvature part of such a D=4, 3-loop divergent structure must be built from the square of the Bel-Robinson tensor

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- This is directly related to the α'^3 corrections in the superstring effective action, except that in the string context such contributions occur with finite coefficients.
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Non-Renormalization Theorems

- Powerful SUSY "non-renormalization theorems" exclude infinite renormalization within D=4, N=1 supersymmetry of chiral invariants, given in N=1 superspace by integrals over half the superspace

$$\int d^2\theta W(\Phi(x,\theta,\bar{\theta})), \quad \bar{D}\Phi = 0$$

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- So, in D=4, N=1 supersymmetry, full superspace integrals like (or "D terms") are allowed, but chiral integrals like (or "F terms") are not.

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The First Divergent Loop Orders

- The strength of a given supersymmetric non-renormalization theorem depends on the extent of linearly realizable, or "off-shell" supersymmetry.
- Assuming that 1/2 supersymmetry is linearly realizable and requiring gauge and supersymmetry invariances, predictions were derived for the first divergent loop orders in maximal ($N = 4 <-> 16$ supercharges) SYM and ($N = 8 <-> 32$ sc.) SUGRA

P.Howe,K.Stelle,P.Townsend

Max. SYM first divergences, assuming half SUSY off-shell (8 supercharges)

Dimension D	10	8	7	6	5	4
Loop order L	1	1	2	3	4	∞
Gen. form	$\partial^2 F^4$	F^4	$\partial^2 F^4$	$\partial^2 F^4$	F^4	finite

Max. SUGRA first divergences, assuming half SUSY off-shell (16 supercharges)

Dimension D	11	10	8	7	6	5	4
Loop order L	2	2	1	2	3	2	3
Gen. form	$\partial^{12} R^4$	$\partial^{10} R^4$	R^4	$\partial^6 R^4$	$\partial^6 R^4$	R^4	R^4

Unitarity-based calculations

Bern, Carrasco, Dixon, Dunbar, Johansson,

Kosower, Perelstein, Roiban, Rozowsky et al.

- Within the last decade, there have been significant advances in the computation of loop corrections in quantum field theory.
- These developments include the organization of amplitudes into a new kind of perturbation theory starting with maximal helicity violating amplitudes (MHV), then next-to-MHV (NMHV), etc.
- They also incorporate a specific use of dimensional regularization together with a clever use of unitarity cutting rules.

Cut Construction Procedure

- Normally, one thinks of unitarity relations such as the optical theorem as giving information only about the imaginary parts of amplitudes. However, if one keeps all orders in an expansion in $\varepsilon = 4 - D$, then loop integrals like require integrands to have an additional momentum dependence

$$f(s) \to f(s)s^{-\varepsilon/2}$$

- Then, since

$$s^{-\varepsilon/2} = 1 - \varepsilon/2 \log(s)$$

and

$$\log(s) = \log(|s|) + i\pi\Theta(s),$$

one can learn about the real parts of an amplitude by retaining imaginary terms at order ε. This gives rise to a procedure for the *cut construction* of higher-loop diagrams.

Kawai-Lewellen-Tai relation

- For maximal supergravity, another relation allowing amplitudes to be evaluated is the Kawai-Lewellen-Tai (KLT)relation between open- and closed-string amplitudes. This gives rise to tree-level relations between field-theoretic max. SUGRA and max. SYM field-theory amplitudes, e.g.

$$M_4^{tree}(1,2,3,4) = -is_{12}\, A_4^{tree}(1,2,3,4)\, A_4^{tree}(1,2,4,3)$$

- Combining this with unitarity-based calculations, in which all amplitudes are ultimately reduced to integrals over products of tree amplitudes, one has a way to obtain higher-loop supergravity amplitudes from SYM amplitudes.

On-shell Simplification

- Another remarkable aspect of the unitarity-based methods is the simplification of vertices. The off-shell 3- graviton vertex has the form (with about 100 terms)

$$
\begin{aligned}
&G_{3,\mu\alpha,\nu\beta,\sigma\gamma}(k_1,k_2,k_3) = \kappa Sym[\\
&-\frac{1}{2}P_3(k_1\cdot k_2\eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\sigma\gamma}) - \frac{1}{2}P_6(k_{1\nu}k_{1\beta}\eta_{\mu\alpha}\eta_{\sigma\gamma}) + \frac{1}{2}P_3(k_1\cdot k_2\eta_{\mu\nu}\eta_{\alpha\beta}\eta_{\sigma\gamma})\\
&+P_6(k_1\cdot k_2\eta_{\mu\alpha}\eta_{\nu\sigma}\eta_{\beta\gamma}) + 2P_3(k_{1\nu}k_{2\gamma}\eta_{\mu\alpha}\eta_{\beta\sigma}) - P_3(k_{1\beta}k_{2\mu}\eta_{\alpha\nu}\eta_{\sigma\gamma})\\
&+P_3(k_{1\sigma}k_{2\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + P_6(k_{1\sigma}k_{1\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + 2P_6(k_{1\nu}k_{2\gamma}\eta_{\beta\mu}\eta_{\alpha\sigma})\\
&+2P_3(k_{1\nu}k_{2\mu}\eta_{\beta\sigma}\eta_{\gamma\alpha}) - 2P_3(k_1\cdot k_2\eta_{\alpha\nu}\eta_{\beta\sigma}\eta_{\gamma\mu})]
\end{aligned}
$$

- Putting this vertex on-shell for tree diagrams simplifies it to

$$i\kappa(\eta_{\mu\nu}(k_1-k_2)_\rho + cyclic) \times (\eta_{\alpha\beta}(k_1-k_2)_\gamma + cyclic)$$

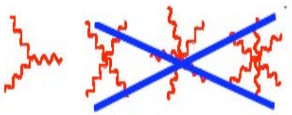
which is just the square of the colour-stripped version of the SYM amplitude,

$$-gf^{abc}(\eta_{\mu\nu}(k_1-k_2)_\rho + cyclic)$$

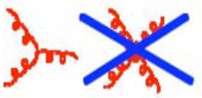
in agreement with KLT.

Simplifies Feynman Rules

- More remarkable still is the fact that on-shell tree amplitudes can be built exclusively using 3-point vertices: contributions from the infinite numbers of higher-point vertices all cancel out:



- This is also a reflection of a simplification in SYM, where the 4-point vertex does not contribute to on-shell tree amplitude calculations:



- Using these on-shell simplifications inside general tree diagrams requires also a technique of making complex shifts of external momenta.

New expecations for UV divergences

- In this way, a different set of anticipated first loop orders for ultraviolet divergences arose using the unitarity-based approach:

Max. SYM first divergences, early unitarity-based predictions

Dimension D	10	8	7	6	5	4
Loop order L	1	1	2	3	6	∞
Gen. form	$\partial^2 F^4$	F^4	$\partial^2 F^4$	$\partial^2 F^4$	$\partial^2 F^4$	finite

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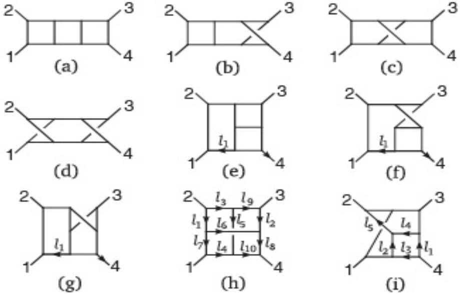
Dimension D	11	10	8	7	6	5	4
Loop order L	2	2	1	2	3	4	5
Gen. form	$\partial^{12} R^4$	$\partial^{10} R^4$	R^4	$\partial^6 R^4$	$\partial^6 R^4$	$\partial^6 R^4$	$\partial^4 R^4$

- These anticipations were based on iterated 2-particle cuts, however. Full calculations can reveal different behavior, however.

Explicit Calculations

- An important development is the subsequent completion of the 3-loop calculation: Bern, Carrasco, Dixon, Johansson, Kosower, Roiban.

Normal Feynman diagram calculation of these would involve about 10^{20} terms

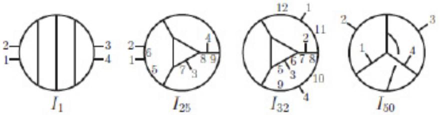


The result is finite at L=3 in D=4.

Explicit Calculations II

- Meanwhile, the 4-loop calculation has now been done (May 2009).

Bern, Carrasco, Dixon, Johansson & Roiban



+ 46 more topologies

The result

$$M_4^{4-loop} = \left(\frac{\kappa}{2}\right)^{10} s\,t\,u\,M_4^{tree} \sum_{S_4} \sum_{i=1}^{50} c_i I_i$$

is ultraviolet finite in D=4 (as expected) and in D=5 (unexpected).

IR Divergences in QG

L.Bork, D.K., G.Vartanov, A.Zhiboedov

- For the MHV amplitudes

$$A^{MHV}_{Gravity} = \frac{1}{2}\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++)}_{Virt} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{Real} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{InSplit} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{FnSplit}$$

All the IR and Collinear divergences CANCEL like in SYM theory!

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- There is new hope for the finiteness of maximally supersymmetric Quantum gravity

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Conclusion

- It may well be that maximally supersymmetric N=4 SYM theory and N=8 supergravity are finite and integrable.
- One may hope to obtain exact solution of the models in all orders of PT and beyond
- Any possible solution seems to be related by duality to a proper string model in AdS space.