Conformal Deformations of N=4 Super Yang-Mills Theories

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AdS/CFT Correspondence

- Duality between $N_c \to \infty$ YM and String theory ('t Hooft, 1979)
- Duality between type IIB string theory in $AdS_5 \times S_5$ and CFT on the boundary of AdS (N=4 SYM)

  (Maldacena, Polyakov, Witten, 1998)

$$ds_{AdS_5 \times S_5} = L^2 \left(\frac{1}{u^2} \eta_{\mu\nu} dx^\mu dx^\mu + \frac{du^2}{u^2} + d\Omega_5 \right)$$

Strong version of AdS/CFT – coinciding S-Matrix

AdS/CFT Correspondence and gauge-string duality

N=4 CFT, any coupling, and gauge group rank N ⟷	Full type IIB string theory on $AdS_5 \times S_5$
Planar (N->∞) limit of N=4 CFT, $g^2N=\lambda$ - fixed 1/N - expansion ⟷	Weak coupling limit in string theory g
Strong coupling in N=4 CFT . 1/λ- expansion ⟷	Classical limit in type IIB string theory– IIB SUGRA. 1/λ- expansion

AdS/CFT Correspondence and deformed N=4 SYM

$AdS_5 \times S_5$	N=4 SYM
$AdS_5 \times S_5$	N=4 β-deformed SYM (N=1)
?	N=4 Leigh-Strassler deformed SYM (N=1)
…	…
?	**QCD** («N=0 SYM»)

Which Conformal theory? Any one?
Which Background? AdS x ?

N=4 SYM Theory

N=4 SYM in terms of N=1 superfields

$$S_{SYM}^{N=4} = \int d^8z\, Tr\left(e^{-gV}\,\bar{\Phi}_i\, e^{gV}\,\Phi^i\right) + \frac{1}{2g^2}\int d^6z\, Tr\left(W^{\alpha}W_{\alpha}\right)$$

$$+ig\int d^6z\, Tr\left(\Phi_1[\Phi_2,\Phi_3]\right) + h.c.$$

Matter fields in adjoint representation

This theory is perturbatively UV finite and
conformally invariat on quantum level: β(g)=0 in all loops

β-Deformed N=4 SYM Theory

q-deformation

$$[\Phi_1,\Phi_2]=\Phi_1\Phi_2-\Phi_2\Phi_1\Rightarrow\Phi_1\Phi_2-q\,\Phi_2\Phi_1$$

β-deformation

$$[\Phi_1,\Phi_2]\Rightarrow q\,\Phi_1\Phi_2-\frac{1}{q}\,\Phi_2\Phi_1 \qquad q=e^{i\beta}$$

! Statement: β-deformed SYM theory is conformally invariant if β is real, i.e. if q is a pure phase — Zanon et al.

The corresponding gravity background was found — Lunin, Maldacena

Leigh-Strassler deformation of N=4 SYM Theory

$$S_{LS}^{N=1} = \int d^8 z\, Tr\left(e^{-gV}\, \bar{\Phi}_i\, e^{gV}\, \Phi^i\right) + \frac{1}{2g^2}\int d^6 z\, Tr\left(W^\alpha W_\alpha\right)$$

$$+ih\int d^6 z\, Tr\left(q\Phi_1\Phi_2\Phi_3 - \frac{1}{q}\Phi_1\Phi_3\Phi_2 \right) + \frac{i\rho}{3}\sum_{i=1}^{3}\int d^6 z Tr(\Phi_i^3) + h.c.$$

- Violates global U(1) x U(1) -> Z_3
- Conformal invariance ?
- Gravity background ?

Finiteness versus Conformal Invariance

Finiteness = absence of UV divergences

Conformal invariance = vanishing of the beta-funcrion

Q:
- Is this the same or not ?
- Can one reach it simultaneously ?
- Which models satisfy these requirements ?
- Is the β-deformed N=4 SYM finite/ conf ?
- Is the LS deformed theory finite/ conf ?

A: Yes! to all of these questions

How to Reach Finiteness/Conf Inv

1. In terms of N=1 superfields due to non-renormalization theorems all chiral vertices are finite
2. In background gauge any gauge vertex is renormalized like the gauge propagator
3. Due to Grisaru-Girardelli theorem or Novikov-Shifman-Vainshtein-Zakharov beta-function the gauge propagator is finite if this is true for the chiral ones

$$\beta_g = g^2 \frac{\sum T(R) - 3C_2 - \sum \gamma(R)}{1 - 3g^2 C_2}$$ NSVZ

4. To reach the vanishing of anomalous dimensions γ(R) on can adjust the Yukawa coupling order by order in PT

$$h^2 = c_0 g^2 + c_1 g^4 + c_2 g^6 + \ldots$$ EKT, Jones

Application to Deformed N=4 SYM Theories

Notation

$$h \cdot q = h_1 \quad \frac{h}{q} = h_2 \quad \rho = h_3$$

One loop cancellation condition

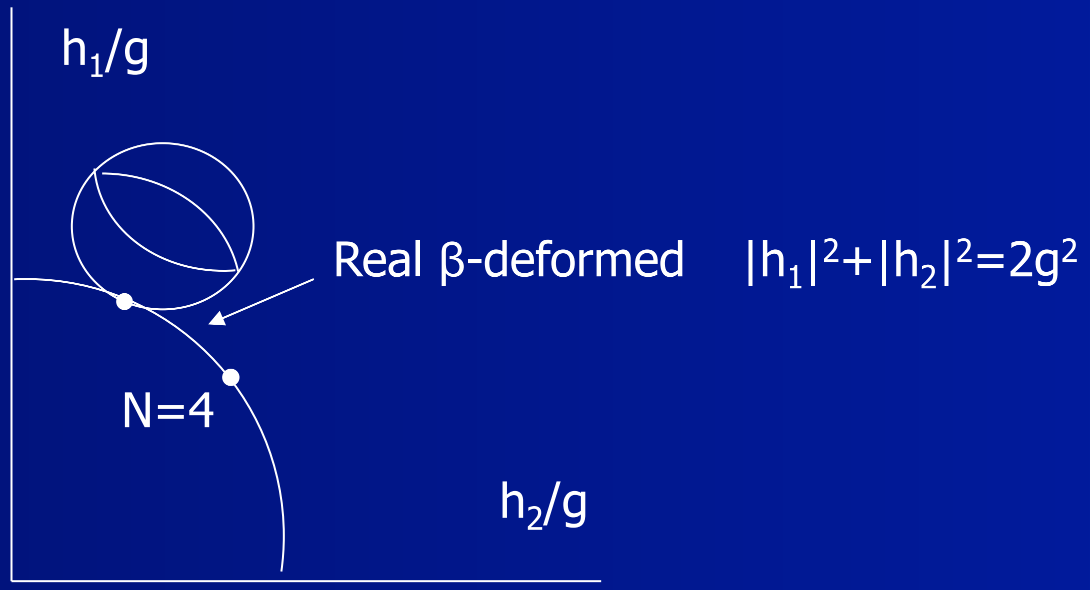
$$f_{ik} h_i \bar{h}_k = |h_1|^2 + |h_2|^2 + |h_3|^2 - \frac{2}{N^2} |h_1 - h_2|^2 - \frac{4}{N^2} |h_3|^2 = 2g^2$$

β- deformed case: $h_3=0$

Planar limit: N->∞

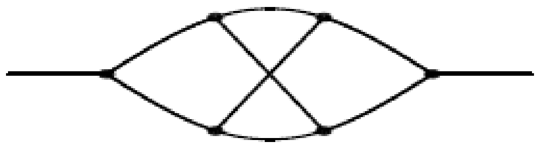
N=4 case: $h_1=h_2=g$, $h_3=0$

Illustration (One loop)



For real β this is valid in any loop order

Higher Loops



Three-loop conformal condition in the non-planar limit

$$\sim\left[(|h_1|^2+|h_2|^2+|h_3|^2-\frac{2}{N^2}|h_1-h_2|^2-\frac{4}{N^2}|h_3|^2)-2g^2\right]P_{31}(g,h_i,N)+$$

$$+|h_1-h_2|^2(N^2|h_1^2+h_2^2+h_1h_2|^2-9N^2|h_1|^2|h_2|^2+5|h_1-h_2|^4)-$$
$$-18|h_3|^2((N^2-5)|h_1^2+h_2^2|^2-(N^2-10)(\bar{h}_1\bar{h}_2(h_1^2+h_2^2)+c.c)-20|h_1|^2|h_2|^2)+$$
$$+(\bar{h}_3(h_1-h_2)((N^2+20)(h_1^2+h_2^2)+10(N^2-4)h_1h_2)+c.c)-8(N^2-10)(|h_3|^2)^3$$

Conformal and finiteness condition

$$\hat{G}_{31}=-\frac{N^2-4}{2^6N^6}\frac{3\varsigma_3}{(4\pi)^6}\cdot$$

$$f_{ik}h_i\bar{h}_k=2g^2+\frac{\hat{G}_{31}}{6(4\pi)^6}g^2\varepsilon^2+\frac{\hat{G}_{31}}{3(4\pi)^4}g^4\varepsilon-\frac{3\hat{G}_{31}}{(4\pi)^2}g^6+\dots$$

Comment on Dimensional Reduction

To reach cancellation of pole terms and anomalous di,mensions simultaneously

$$Z_2 = 1 + \frac{c_{11}}{\varepsilon} + \left(\frac{c_{22}}{\varepsilon^2} + \frac{c_{21}}{\varepsilon}\right) + \left(\frac{c_{33}}{\varepsilon^3} + \frac{c_{32}}{\varepsilon^2} + \frac{c_{31}}{\varepsilon}\right) + \ldots$$

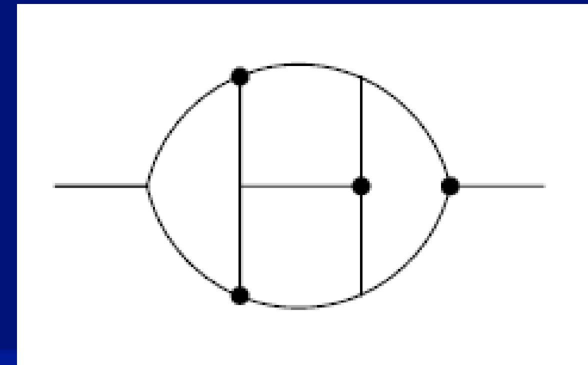
$$\gamma = c_{11} + 2c_{21} + 3c_{31} + \ldots$$

One needs double perturbative expansion

$$\begin{aligned} h^2 &= c_0(\varepsilon)g^2 + c_1(\varepsilon)g^4 + c_2(\varepsilon)g^6 + c_3(\varepsilon)g^8 + \ldots \\ &= c_{00}g^2 + c_{10}g^4 + c_{20}g^6 + c_{30}g^8 + \ldots \\ &\quad + c_{01}\varepsilon g^2 + c_{11}\varepsilon g^4 + c_{21}\varepsilon g^6 + \ldots \\ &\quad + c_{02}\varepsilon^2 g^2 + c_{12}\varepsilon^2 g^4 + \ldots \\ &\quad + c_{03}\varepsilon^3 g^2 + \ldots \end{aligned}$$

Higher Loops

Four-loop conformal condition in the planar limit



$$\sim (|h_1|^2+|h_2|^2+|h_3|^2)^4-(2g^2)^4+$$

$$+(|h_1|^2-|h_2|^2)^4+(|h_3|^2)^4-4|h_3|^2(|h_1|^2+|h_2|^2)^3+6(|h_3|^2)^2(|h_1|^2+|h_2|^2)^2-$$
$$-4(|h_3|^2)^3(|h_1|^2+|h_2|^2)+24|h_1|^2|h_2|^2|h_3|^2(|h_1|^2+|h_2|^2)+8h_3^3(|h_2|^2\bar{h}_1^3-|h_1|^2\bar{h}_2^3)+$$
$$+8\bar{h}_3^3(|h_2|^2h_1^3-|h_1|^2h_2^3)-8|h_3|^2(h_2^3\bar{h}_1^3+h_1^3\bar{h}_2^3)$$

$$\hat{G}_{41}=\frac{5\varsigma_5N^4\cdot}{2g^8(4\pi)^8}$$

Conformal and finiteness condition

$$|h_1|^2+|h_2|^2+|h_3|^2=g^2\Big[2+\frac{5}{18}\varsigma_5\hat{G}_{41}\varepsilon^3+\frac{5}{3}\varsigma_5\hat{G}_{41}\Big(\frac{g^2N}{16\pi^2}\Big)\varepsilon^2+$$
$$+5\varsigma_5\hat{G}_{41}\Big(\frac{g^2N}{16\pi^2}\Big)^2\varepsilon+10\varsigma_5\hat{G}_{41}\Big(\frac{g^2N}{16\pi^2}\Big)^3+\dots\Big]$$

Unitary Equivalent Points in moduli space of the LS theory

$$W_\beta = ih\int d^6z\, Tr\left(q\Phi_1\Phi_2\Phi_3 - \frac{1}{q}\Phi_1\Phi_3\Phi_2 \right)$$

$$\begin{pmatrix}\Phi_1\\ \Phi_2\\ \Phi_3\end{pmatrix} = U \begin{pmatrix}\Psi_1\\ \Psi_2\\ \Psi_3\end{pmatrix}$$

$$\tilde{W}_\beta = i\int d^6z\, Tr\left(\tilde{h}_1\Phi_1\Phi_2\Phi_3 - \tilde{h}_2\Phi_1\Phi_3\Phi_2 \right) + i\frac{\tilde{h}_3}{3}\int d^6z \sum_{i=1}^{3}\Phi_i^3$$

$$UU^+ = 1$$

$$\begin{cases} \tilde{h}_1 = -a + ib \\ \tilde{h}_2 = a + ib \\ \tilde{h}_3 = 2a \end{cases} \qquad \begin{cases} a = \pm\dfrac{h}{2\sqrt{3}}\left(q - \dfrac{1}{q}\right) \\ b = \pm\dfrac{h}{2}\left(q + \dfrac{1}{q}\right) \\ b^2 = 1 + 3a^2 \end{cases}$$

Particularly $|q| = 1 \quad |h|^2 = g^2$

is exactly superconformal in the planar limit

Looking for new solutions in the planar limit

$$W_\beta = ih\int d^6z\, Tr\left(q\Phi_1\Phi_2\Phi_3 - \frac{1}{q}\Phi_1\Phi_3\Phi_2 \right)$$

$$\Uparrow \quad q = e^{\frac{i\pi m}{3}}$$

Is exactly superconformal in the planar limit!!!

Zanon et al. hep-th/0507282

$$\begin{cases} \dfrac{h_1}{g} = e^{i\pi\alpha} \\ \dfrac{h_2}{g} = 0 \\ \dfrac{h_3}{g} = 1 \end{cases}$$

$$\alpha \neq \frac{2m}{3}$$

$$\begin{cases} \dfrac{h_1}{g} = 0 \\ \dfrac{h_2}{g} = -e^{i\pi\alpha} \\ \dfrac{h_3}{g} = 1 \end{cases}$$

$$W_{LS^*} = ih\int d^6z\left[qTr(\Phi_1\Phi_2\Phi_3) + \frac{1}{q}\sum_{i=1}^{3}\frac{\Phi_i^3}{3} \right]$$

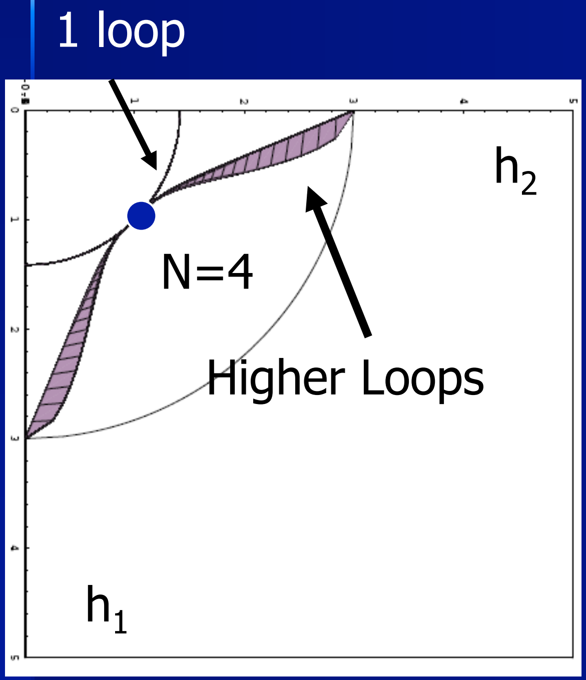
$$\Downarrow \quad q = \pm i$$

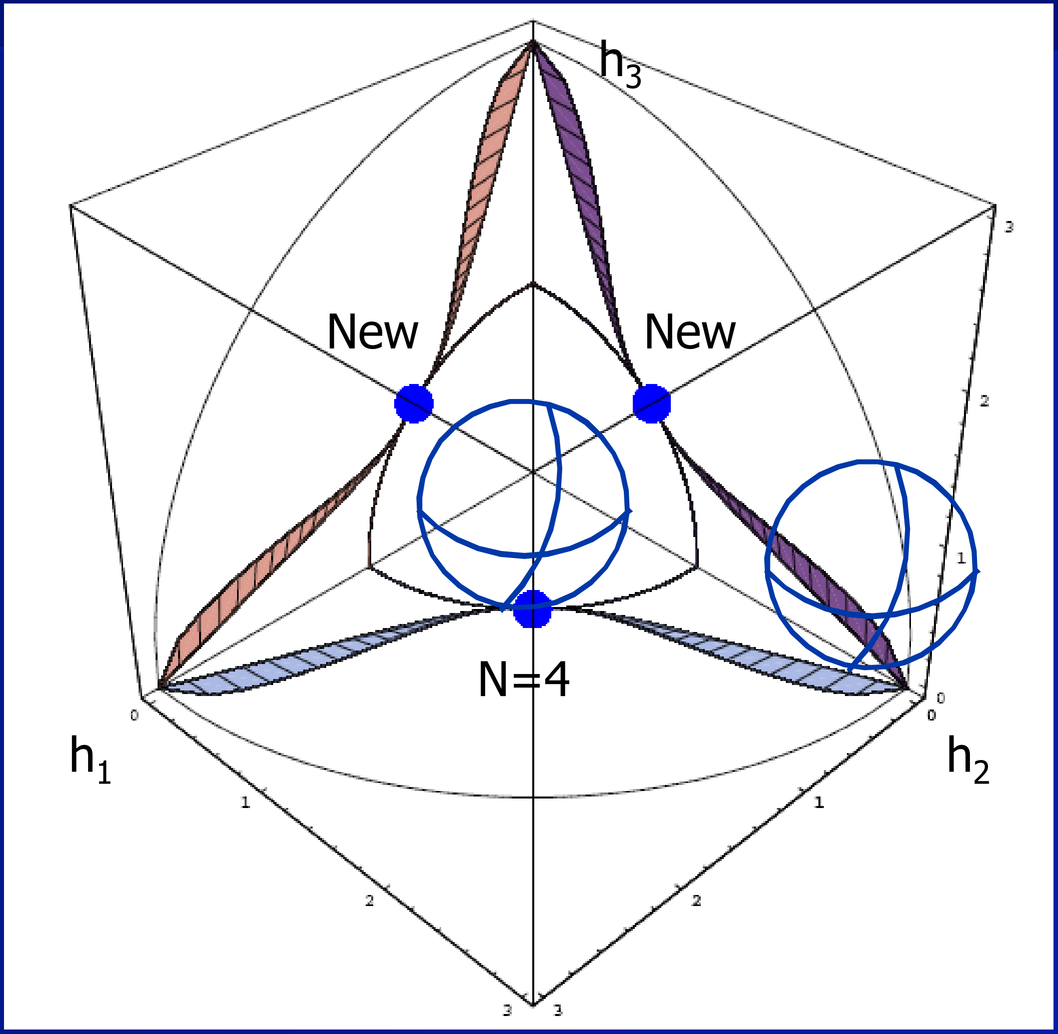
$$\begin{cases} \dfrac{h_1}{g} = \pm\dfrac{1}{2} - i \pm \dfrac{1}{2\sqrt{3}} \\ \dfrac{h_2}{g} = \pm\dfrac{1}{2} + i \pm \dfrac{1}{2\sqrt{3}} \\ \dfrac{h_3}{g} = - \pm \dfrac{2i}{\sqrt{3}} \end{cases}$$

Is exactly superconformal in the planar limit???

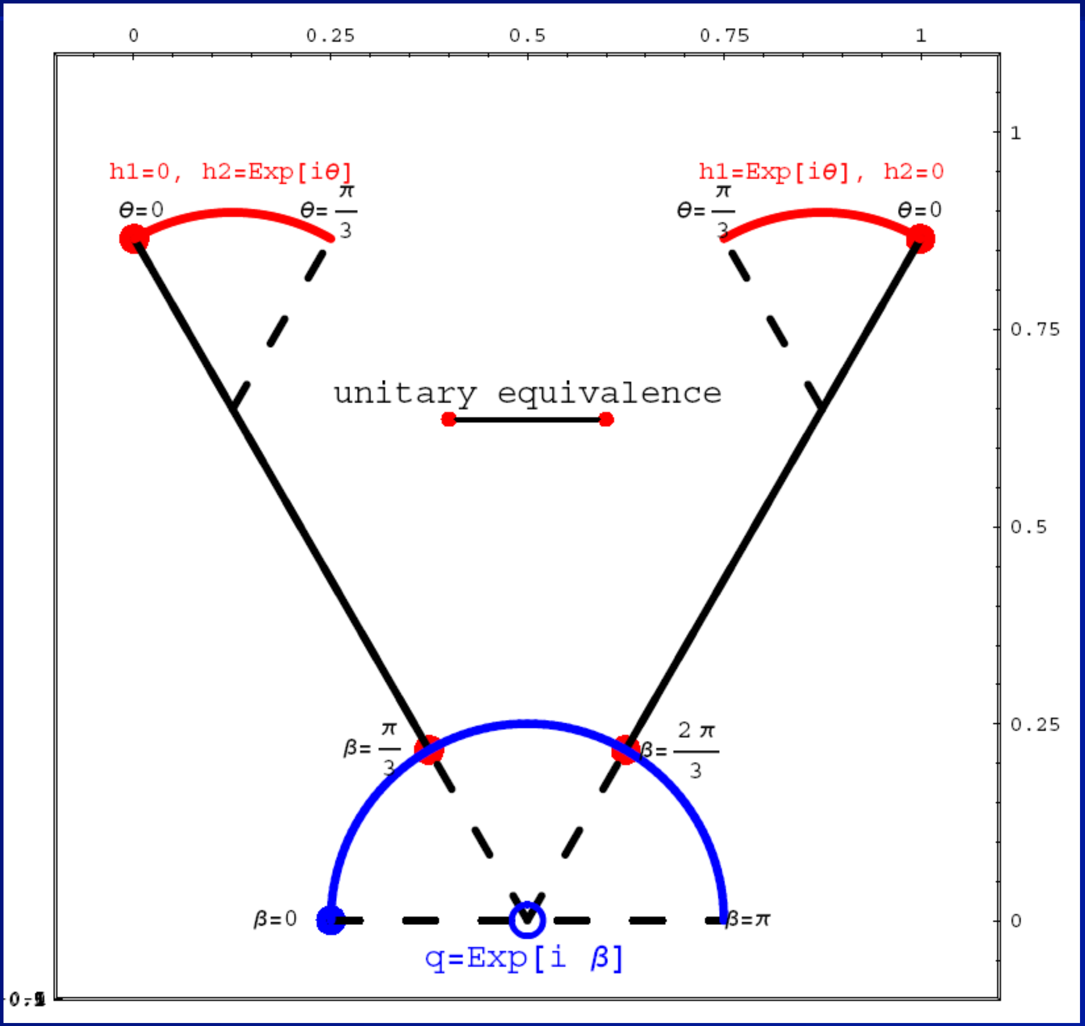
$$|\,q\,| = 1 \qquad |h|^2 = g^2$$

Illustration (Higher Loops)





Unitary Equivalence



Conclusion

• We have found conditions of conformal invariance and finiteness of the full Leigh-Strassler deformation of N=4 SYM

- up to four loops in the planar limit
- up to three loops in the non-planar limit

• We have found a family of solutions which might be exactly conformal in the planar limit