Renormalizable 1/N_f-Expansion for Gauge Theories in Extra Dimensions

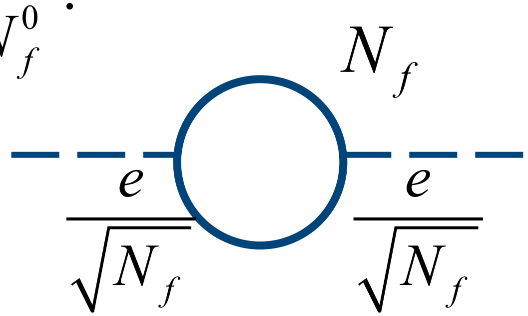
D.I.Kazakov and G.S.Vartanov

based on hep-th/0607177
hep-th/0702004

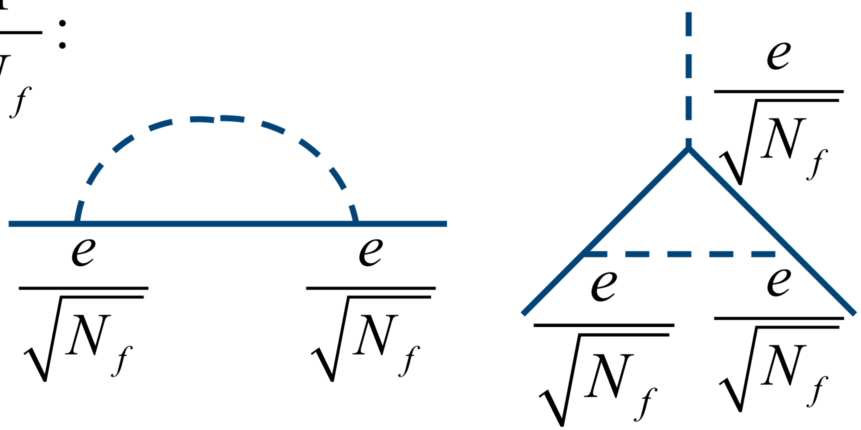
QED with N_f fermions

$$L = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2\alpha}(\partial_\mu A_\mu)^2 + i\bar{\psi}_i \not\partial \psi_i - m\bar{\psi}_i\psi_i + \frac{e}{\sqrt{N_f}}\bar{\psi}_i \not A \psi_i$$

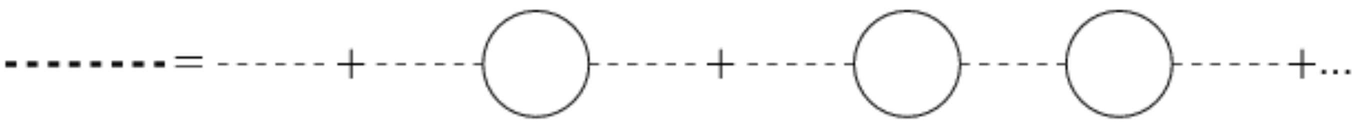
$\frac{1}{N_f^0}$:



$\frac{1}{N_f}$:



Resummation of the gauge propagator



Summing only the zeroth order diagrams to the propagator of the gauge field gives

$$D_{\mu\nu}(p^2) = -\frac{i}{p^2}\frac{(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2})}{1 + e^2 f(D)(-p^2)^{D/2-2}},$$

$$f(D) = \frac{\Gamma^2(D/2)\Gamma(2-D/2)}{2^{[D/2]-1}\Gamma(D)\pi^{D/2}}.$$

No divergences for odd D in dim reg.

Degree of divergence

Не удается отобразить рисунок. Возможно, рисунок поврежден или недостаточно памяти для его открытия. Перезагрузите компьютер, а затем снова откройте файл. Если вместо рисунка все еще отображается красный крестик, попробуйте удалить рисунок и вставить его заново.

\# of loops   \# of ψ fields   \# of A fields

- ψ propagator: $\omega(G)=L*D-(2L-1)*1-L*(D-2)=1$

-> logarithmic divergence! in any D

- ψψA vertex: $\omega(G)= L*D-2L*1-L*(D-2)=0$

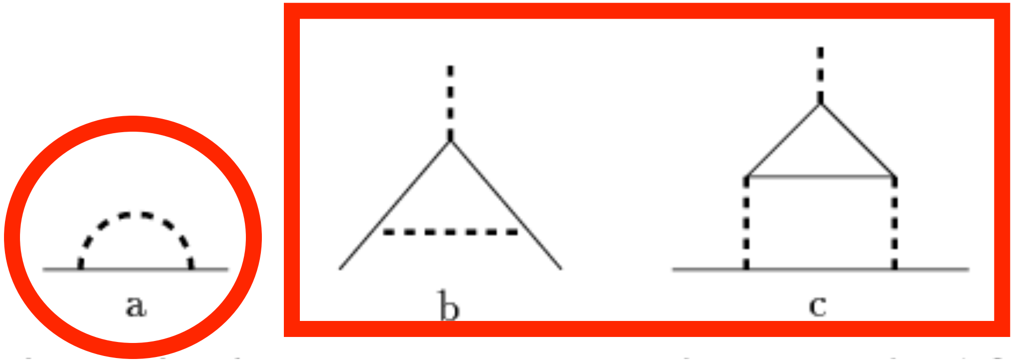
-> logarithmic divergence! in any D

- A propagator: $\omega(G)= L*D-2L*1-(L-1)*(D-2)=D-2$

-> no genuine divs, only divergent subgraphs

No other divergences!!!

Leading order of 1/N expansion



$$Diag.a \Rightarrow \frac{1}{N\varepsilon}A, \quad Diag.b \Rightarrow \frac{1}{N\varepsilon}B, \quad Diag.c \Rightarrow \frac{1}{N\varepsilon}C,$$

$$A = \frac{-\Gamma(D)(D-1)(2-D/2)}{2^{D/2}\Gamma(2-D/2)\Gamma(D/2+1)\Gamma^2(D/2)}, B = -A, C = 0$$

The consequence of the Ward identities

The consequence of the Fahri theorem

Problem I – no coupling constant

$$Z_2^{-1} = 1 - \frac{1}{\varepsilon}\frac{A}{N},$$

$$Z_1 = 1 - \frac{1}{\varepsilon}\frac{B+C}{N},$$

$$Z_3^{-1} = 1$$

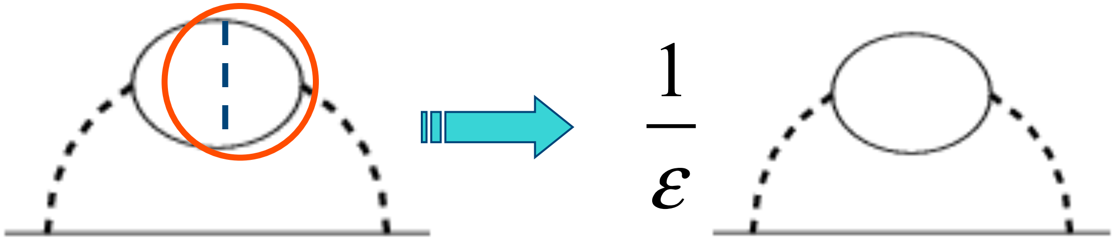
The coupling disappeared (like the mass term)

$$\Rightarrow \quad \frac{1}{N_B} = \frac{1}{N}\left(\mu^2\right)^{\varepsilon} Z_1 Z_2^{-1} Z_3^{-1/2}$$

What to renormalize?

Problem II – "forbidden" diagrams

"Forbidden" diagram appears as a counter term



And leads to higher order pole term not taken into account in "pole eqs"

New dimensionless coupling

Rescaling

$$A \to A/e,\quad \frac{e}{\sqrt{N_f}}\bar{\psi}_i \not{A}\psi_i \to \frac{\sqrt{h}}{\sqrt{N_f}}\bar{\psi}_i \not{A}\psi_i$$

Summation in the propagator

$$\frac{1}{1/e^2-\mathrm{d}}+\frac{1}{1/e^2-\mathrm{d}}h\mathrm{d}\frac{1}{1/e^2-\mathrm{d}}+\frac{1}{1/e^2-\mathrm{d}}h\mathrm{d}\frac{1}{1/e^2-\mathrm{d}}h\mathrm{d}\frac{1}{1/e^2-\mathrm{d}}+\mathrm{K}$$

$$=\frac{1}{1/e^2-\mathrm{d}}\left(\frac{1}{1-h\mathrm{d}/(1/e^2-\mathrm{d})}\right)=\frac{1}{1/e^2-(1+h)\mathrm{d}}$$

QED – effective lagrangian

$$L_{eff} = -\frac{1}{4} F_{\mu\nu} \left(\frac{1}{e^2} + f(D)(\partial^2)^{D/2-2}(1+h) \right) F_{\mu\nu} - \frac{1}{2\alpha e^2}(\partial_\mu A_\mu)^2$$

$$+ i\bar{\psi}_i \not{\partial} \psi_i - m\bar{\psi}_i \psi_i + \frac{\sqrt{h}}{\sqrt{N_f}} \bar{\psi}_i \not{A} \psi_i$$

- New dimensionless coupling *h* enters the gauge transformation and plays the role of a charge (logarithmically renormalized)

- Old dimensionful coupling *e* plays the role of a mass and is also logarithmically renormalized

Renormalization and RG

$$h_B = (\mu^2)^\varepsilon h Z_1 Z_2^{-1} Z_3^{-1/2}$$

$$\psi_B = Z_2^{1/2}\psi, \quad A_B = Z_3^{1/2} A$$

$$Z_1 = Z_2, \; Z_3 = 1$$

$$h = const$$

Non-trivial RG

$$Z_i = 1 + \sum_{n=1}^{\infty} \frac{c_n^i(h, 1/N)}{\varepsilon^n}$$

First order in 1/N

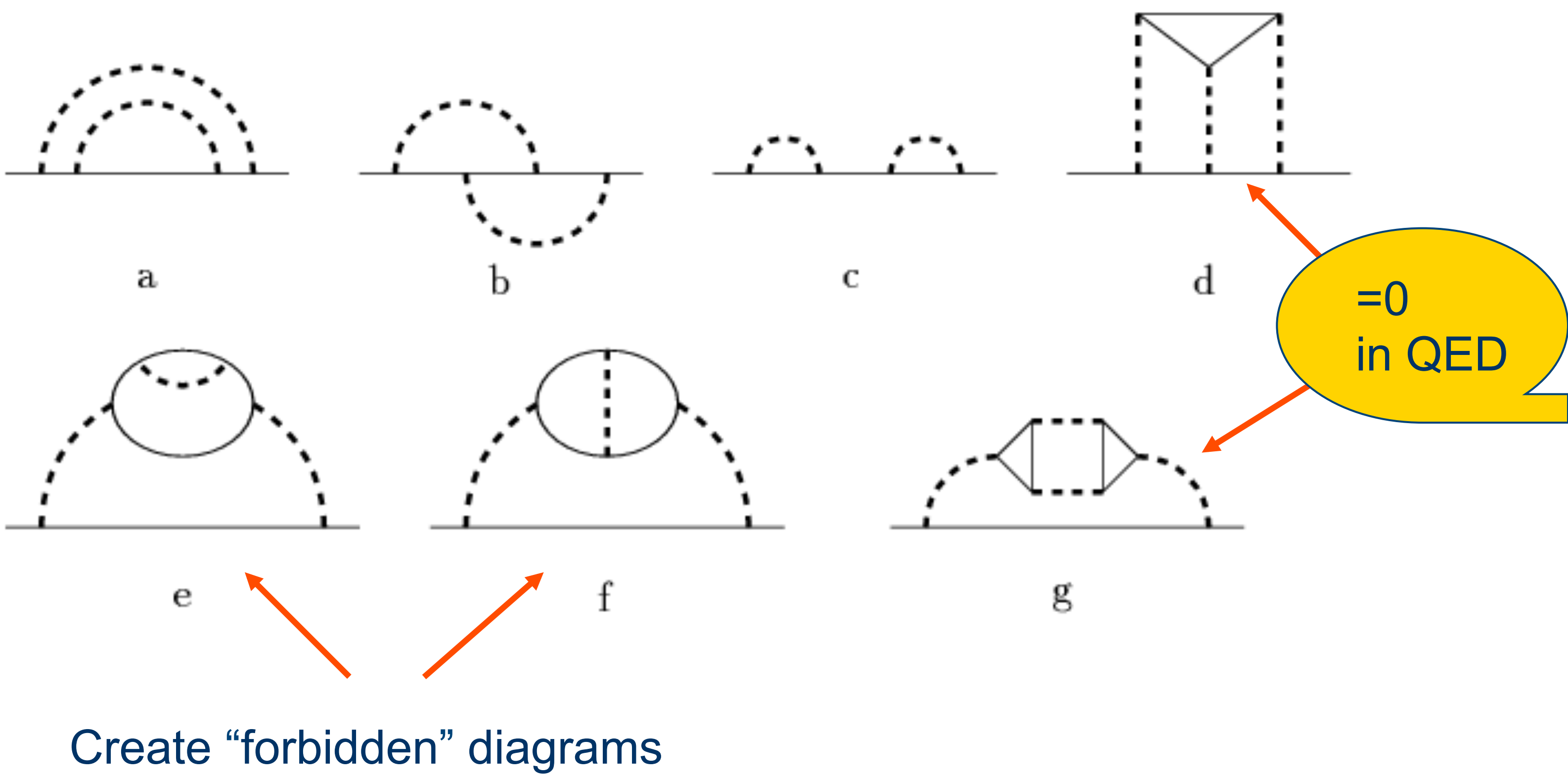
$$Z_1 = 1 - \frac{A}{N_f \varepsilon} \frac{h}{1+h}$$

Pole eqs

$$h \frac{dc_n^i}{dh} = \gamma_i c_{n-1}^i + \beta \frac{dc_{n-1}^i}{dh},$$

Can be checked explicitly

Check of pole equations (the second order contribution to the gauge propagator)



Check of the [pole equations (the second order contribution to the gauge propagator)

$$c_1(h,N) = -\frac{1}{N}\frac{Ah}{1+h}, \quad \gamma = \frac{A}{N}\frac{h}{(1+h)^2}$$

$$c_2(h,N) = \frac{1}{N^2}\left(\frac{3}{2}\frac{A^2h^2}{(1+h)^2} + \frac{ABh^2}{(1+h)^2} + \frac{2}{3}\frac{A^2h^3}{(1+h)^3} + \frac{2}{3}\frac{ABh^3}{(1+h)^3} + \frac{4}{3}\frac{ACh^3}{(1+h)^3} + \frac{ACh^4}{(1+h)^4} \right)$$

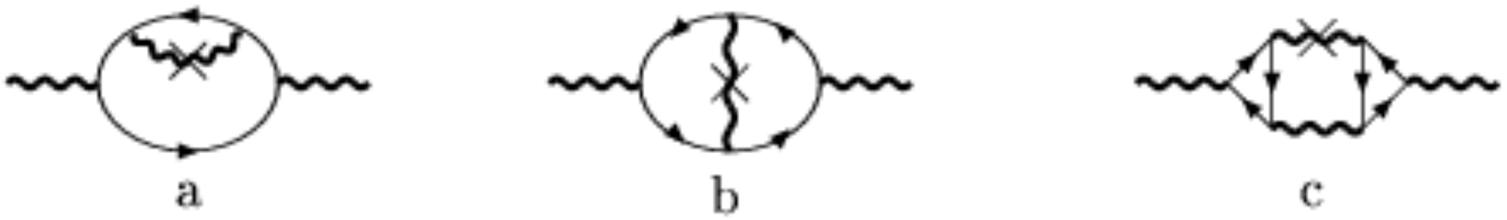
coefficients of the anomalous dimensions for the A field in 1/N expansion.

$$h\frac{dc_2}{dh} = \gamma c_1 + \beta \frac{dc_1}{dh},$$

It works!

$$h\frac{dc_2}{dh} = \frac{1}{N^2}\frac{Ah}{(1+h)^2}\frac{Ah}{1+h} + \frac{1}{N^2}\left(\frac{2(A+B)h^2}{(1+h)^2} + \frac{4Ch^3}{(1+h)^3} \right)\frac{A}{(1+h)^2}$$

Running of the coupling *e* in the leading order



$$Diag.a \Rightarrow \frac{h^2}{N_f(1+h)^2\varepsilon}F, Diag.b \Rightarrow \frac{h^2}{N_f(1+h)^2\varepsilon}E, Diag.c \Rightarrow 0,$$

$$F = \frac{\Gamma(D+1)(D/2-1)(D-1)^2(2-D/2)}{2^{D/2+1}\Gamma(2-D/2)\Gamma(D/2+2)\Gamma^2(D/2)}, E = -\frac{D^2+D/2-9}{D/2(D/2-1)(D-1)}F$$

The running of the coupling e in QED

$$\frac{1}{e^2} = \frac{1}{e_0^2}\left(\frac{p^2}{p_0^2}\right)^{\gamma},$$

$$\gamma = \frac{\Gamma(D)(D-1)(D/2-2)(D-3)(D+2)(D-6)}{2^{D/2+1}\Gamma(D/2+2)\Gamma^2(D/2)\Gamma(2-D/2)N_f}\,\frac{h^2}{(1+h)^3}$$

The power like behaviour of the initial coupling constant (a'la Kaluza-Klein)

QCD *with N_f fermions*

$$L = -\frac{1}{4}(F^a_{\mu\nu})^2 - \frac{1}{2\alpha}(\partial_\mu A^a_\mu)^2 + i\bar{\psi}_i \not{\partial} \psi_i$$

$$-m\bar{\psi}_i\psi_i + \frac{e}{\sqrt{N_f}}\bar{\psi}_i \not{A}^a \psi_i + \partial_\mu \bar{c^a} D_\mu c_a ,$$

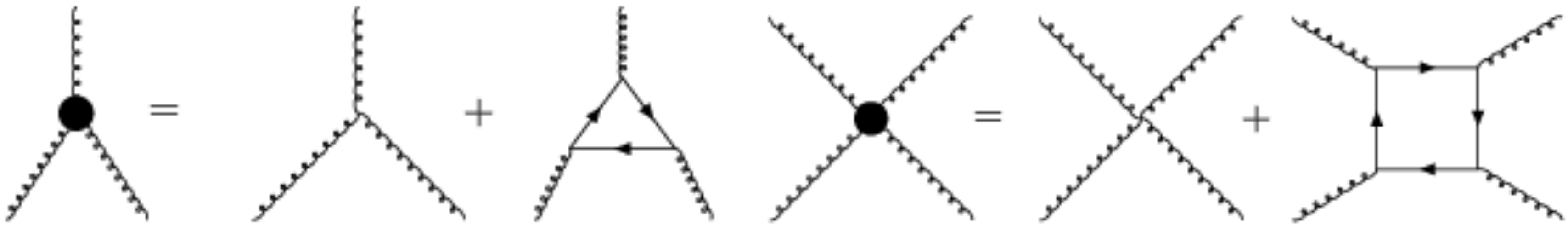
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \frac{g}{\sqrt{N_f}} f^{abc} A^b_\mu A^c_\nu , D_\mu = \partial_\mu + \frac{g}{\sqrt{N_f}} [A_\mu , \_]$$

QCD – the gauge propagator in 1/N^0

$$G^{ab}_{\mu\nu}(p^2) = -\frac{i\delta^{ab}}{p^2}\frac{(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2})}{1 + g^2 f(D)(-p^2)^{D/2-2}},$$

$$f(D) = \frac{\Gamma^2(D/2)\Gamma(2-D/2)}{2^{[D/2]-1}\Gamma(D)\pi^{D/2}}T(R).$$

QCD – effective lagrangian



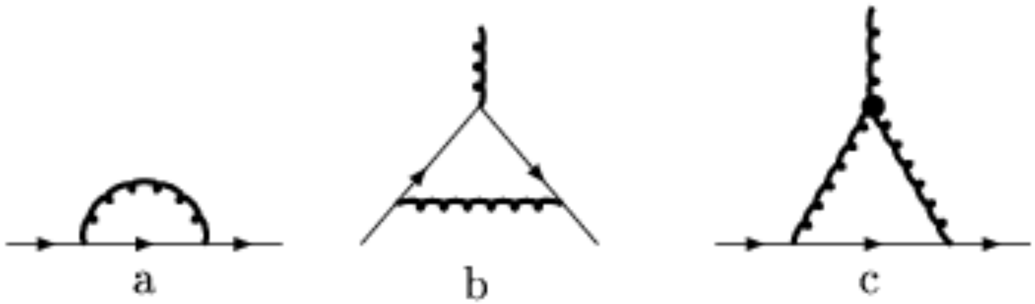
$$\mathcal{L}_{eff} = -\frac{1}{4g^2}(F^a_{\mu\nu})^2 - ($$

$$)\ (1+h)$$

$$- \frac{1}{2\alpha g^2}(\partial_\mu A^a_\mu)^2 + i\bar{\psi}_i\hat{\partial}\psi_i - m\bar{\psi}_i\psi_i + \frac{h}{\sqrt{N_f}}\bar{\psi}_i\hat{A}^a T^a\psi_i + \partial_\mu \bar{c}^a D_\mu c_a$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \frac{\sqrt{h}}{\sqrt{N_f}} f^{abc} A^b_\mu A^c_\nu, D_\mu c_a = \partial_\mu + \frac{\sqrt{h}}{\sqrt{N_f}} f^{abc} A^b_\mu c_c$$

QCD – fermionic sector



$$Diag.a \Rightarrow \frac{h}{N_f(1+h)\varepsilon}A, Diag.b \Rightarrow \frac{h}{N_f(1+h)\varepsilon}B, Diag.c \Rightarrow \frac{h}{N_f(1+h)\varepsilon}C,$$

$$A = -\frac{\Gamma(D)(D-1)(2-D/2)C_F}{2^{D/2}\Gamma(2-D/2)\Gamma(D/2+2)\Gamma^2(D/2)T}, B = -\frac{C_F - C_A/2}{C_F}A,$$

$$C = -\frac{(1-D/2)C_A}{2(2-D/2)C_F}A.$$

QCD – ghost sector

$$Diag.a \Rightarrow \frac{h}{N_f(1+h)\varepsilon} A', Diag.b \Rightarrow \frac{h}{N_f(1+h)\varepsilon} B', Diag.c \Rightarrow \frac{h}{N_f(1+h)\varepsilon} C',$$

$$A' = -\frac{\Gamma(D)(D-1)C_A}{2^{D/2}\Gamma(2-D/2)\Gamma(D/2+2)\Gamma^2(D/2)}, B' = 0, C' = 0.$$

QCD – gauge invariance

$$Z_2^{-1} = 1 - \frac{A}{N\varepsilon}\frac{h}{1+h}, \; Z_1 = 1 - \frac{B+C}{N\varepsilon}\frac{h}{1+h}, \; Z_3^{-1} = 1,$$

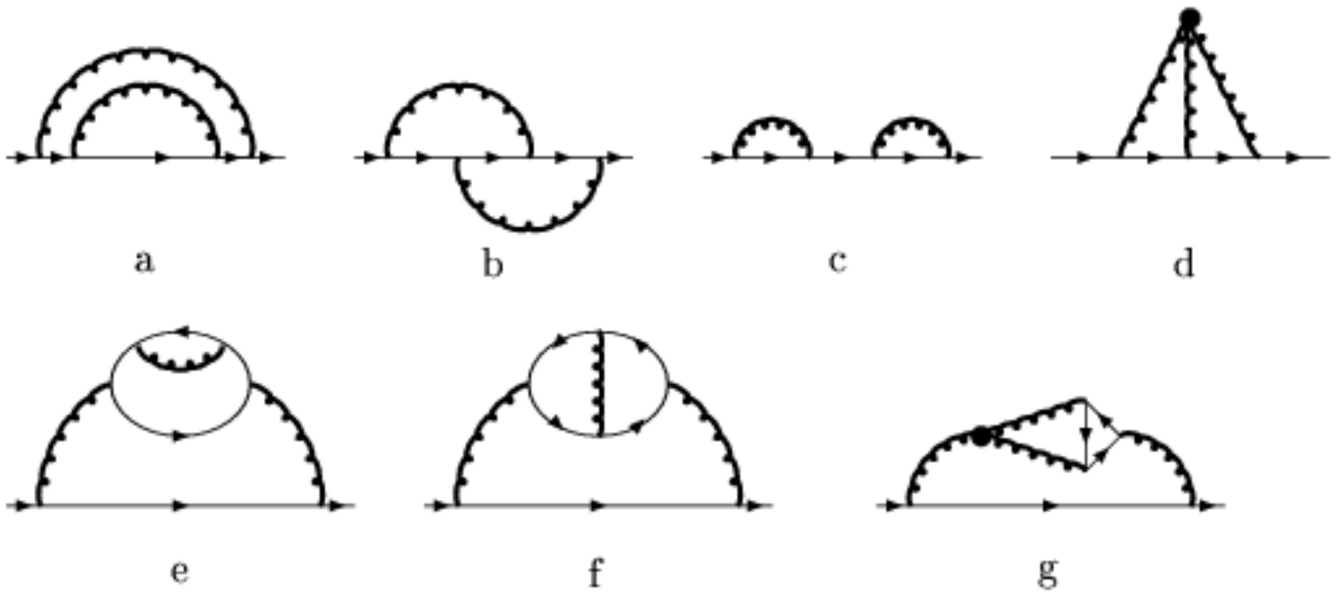
$$\tilde{Z}_2^{-1} = 1 - \frac{A'}{N\varepsilon}\frac{h}{1+h}, \; \tilde{Z}_1 = 1, \; Z_3^{-1} = 1,$$

- A+B+C = A'+B'+C' ! - the gauge invariance!

$$\beta(h,N) = -\frac{A'}{N}\frac{h^2}{(1+h)^2} = \frac{\Gamma(D)(D-1)C_A/N_f}{2^{D/2}\Gamma(2-D/2)\Gamma(D/2+2)\Gamma^2(D/2)}\frac{h^2}{(1+h)^2}$$

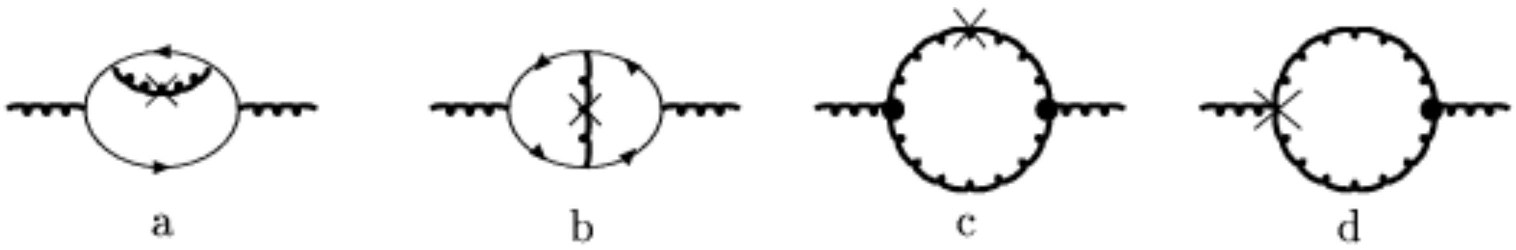
$$\beta(h) < 0, \text{ for } D = 5,9,..; \; \beta(h) > 0, \text{ for } D = 7,11,..;$$

QCD – second order of 1/N expansion



Explicit check that pole equations work!

QCD – running of the initial coupling g



$$Diag.a \Rightarrow \frac{h^2}{N_f(1+h)^2\varepsilon}F, Diag.b \Rightarrow \frac{h^2}{N_f(1+h)^2\varepsilon}E,$$

$$Diag.c \Rightarrow \frac{h^2}{N_f(1+h)^2\varepsilon}G, Diag.d \Rightarrow \frac{h^2}{N_f(1+h)^2\varepsilon}H,$$

Logarithmic renormalization (running)

Singularities of the gauge propagator

$$D_{\mu\nu}(p^2) = -\frac{i}{p^2}\frac{(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2})}{1+e^2 f(D)(-p^2)^{D/2-2}},$$

$$f(D) = \frac{\Gamma^2(D/2)\Gamma(2-D/2)}{2^{[D/2]-1}\Gamma(D)\pi^{D/2}}.$$

$$f(D) < 0,\ \text{for } D = 5,9,..;\ f(D) > 0,\ \text{for } D = 7,11,..;$$

Landau pole

Safe but may have complex poles

Inconsistent with Kallen-Lehmann representation

Conclusion ($1/N_f$)

\+ renormalizable (!!) expansions in D>4,
conformal theory (up to logs),
decreasing x-sections

\- Problem with analyticity (Landau pole, complex poles, negative spectral density in K-L rep)
No role of non-Abelian interaction (N_c/N_f)