Field Theory in Extra Dimensions

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Questions:
- Can one construct a self-consistent QFT in D > 4 ?
- Is it possible to get rid of UV divergences ?
- Power law running: is it reliable ?

Approaches:
- Cancellation of divergences due to SUSY …
- Renormalization Group a la Wilson

Non-perturbative fixed points → a way to non-perturbative renormalizability ?

UV divergences in SUSY Gauge Theories in D > 4

One loop polarization operator

$$\Pi(p^2) = (-)^{[D/2]} \frac{1}{(p^2)^{2-D/2}} \frac{\Gamma(2-D/2)\Gamma^2(D/2)}{\Gamma(D)} \times$$

$$\left\{ -[2\frac{8(D-1)-D'}{D-2} + \frac{(D-4)(D-1)\xi(8-\xi)}{2(D-2)} + \frac{4}{D-2}]C_2(G) + 2^{[D'/2]}T(R) + \frac{4}{D-2}T(R) \right\}$$

gauge+ghost

matter

D -dim of integration

ξ - gauge fixing parameter

D' -dim of the fields

Singular part

$$: \ [-(26-D')C_2(G) + 2^{[D'/2]}T(R) + 2T(R)]$$

Cancellation of Leading Divergencies

D'=4 $\log\Lambda$	N=1	-22CA+4CA+4TR+2TR	=	-6(3CA-TR)
	N=2	-22CA+4CA+6TR+12TR	=	-12(CA-TR)
	N=4	-12CA+12CA	=	0
D'=6 Λ^2	N=1	-20CA+8CA+8TR+4TR	=	-12(CA-TR)
	N=2	-12CA+12CA	=	0
D'=10 Λ^6	N=1	-16CA+16CA	=	0

D=6 N=1 SUSY Gauge theory

Logarithmic Divergencies

Dim 6 Gauge Invariants

$$\left.\begin{aligned} I_1 &= Tr D_\rho F_{\mu\nu} D_\rho F_{\mu\nu} \\ I_2 &= Tr D_\mu F_{\mu\nu} D_\rho F_{\rho\nu} \\ I_3 &= Tr D_\rho F_{\mu\nu} D_\mu F_{\rho\nu} \\ I_4 &= Tr F_{\mu\nu} F_{\nu\rho} F_{\rho\mu} \end{aligned}\right\}$$

UV div in Feynman gauge

$$\frac{T_R - C_A}{3} Tr D_\mu F_{\mu\nu} D_\rho F_{\rho\nu}$$

Finiteness condition

$$\sum T(R) = C_2(G)$$

Gauge dependence

$$\frac{T_R - C_A(1+\xi-\xi^2/8)}{3} Tr D_\mu F_{\mu\nu} D_\rho F_{\rho\nu}$$

Eqs of Motion

$$D_\mu F_{\mu\nu} = \bar{\lambda}\gamma^\nu \lambda, \qquad \not{D}\lambda = 0$$

Finitness on shell $\ldots(D_\mu F_{\mu\nu})^2 + \ldots\bar{\lambda}\gamma^\nu D_\mu F_{\mu\nu}\lambda + \ldots(\bar{\lambda}\gamma^\nu\lambda)^2 = 0 \quad !$

Resume on Perturbative Fineteness in Higher Dimensions

- Off-shell fineteness is not valid
- On-shell fineteness in D=6 is true up to 2 loops (has been checked explicitly) Marcus &Sagnotti, Fradkin & Tseytlin
- Within the (constrained) superfield formalism one can show that allowed invariants vanish up to 2 loops, but in higher loops they exist Howe & Stelle

The situation reminds that of supergravity in D=4

Conclusion

D > 4 Quantum Field Theory
remains **perturbatively** nonrenormalizable

Wilsonian RG near the Critical Dimension

Pure gauge theory

$$L = -\frac{1}{4} Tr\, F_{\mu\nu}^2, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu]$$

Dimensions

$$[A] = \tfrac{D-2}{2}, \;\; [F] = \tfrac{D}{2}, \;\; [g] = 2 - \tfrac{D}{2}$$

D=4
critical dimension

Renormalization (ignoring all higher order operators)

$$g_B = \mu^{2-D/2} \tilde{g} Z_g(\tilde{g}) \qquad \tilde{g} \equiv g \mu^{D/2-2} \;\; \Rightarrow \;\; [\tilde{g}] = 0$$

$\mu \frac{d}{d\mu}$:

$$0 = (2 - \tfrac{D}{2})\tilde{g} + \beta(\tilde{g}) - \tilde{g}\gamma_g(\tilde{g}) \;\; \Rightarrow \;\; \mu \tfrac{d}{d\mu} \tilde{g} = \beta(\tilde{g}) = \tilde{g}(\tfrac{D}{2} - 2 + \gamma_g)$$

Background gauge

$$\gamma_g = \tfrac{1}{2} \gamma_A \longleftarrow$$

The gauge field
anomalous dimension

$$\beta(\tilde{g}) = \tilde{g}(\tfrac{D}{2} - 2 + \tfrac{1}{2}\gamma_A(\tilde{g}))$$

Solution to the RG Equation

Perturbative solution

1 loop $\gamma_A = b\tilde{g} \longrightarrow \mu \frac{d}{d\mu}\tilde{g} = \tilde{g}\left(\frac{D}{2} - 2 + b\tilde{g}\right)$

$\frac{D}{2} - 2 = \varepsilon$

$$\frac{1}{\tilde{g}} = \frac{1}{\tilde{g}_0}\left(\frac{\mu}{Q}\right)^{\varepsilon} + \frac{b}{\varepsilon}\left[\left(\frac{\mu}{Q}\right)^{\varepsilon} - 1\right]$$

Power law running

$\varepsilon \to 0$

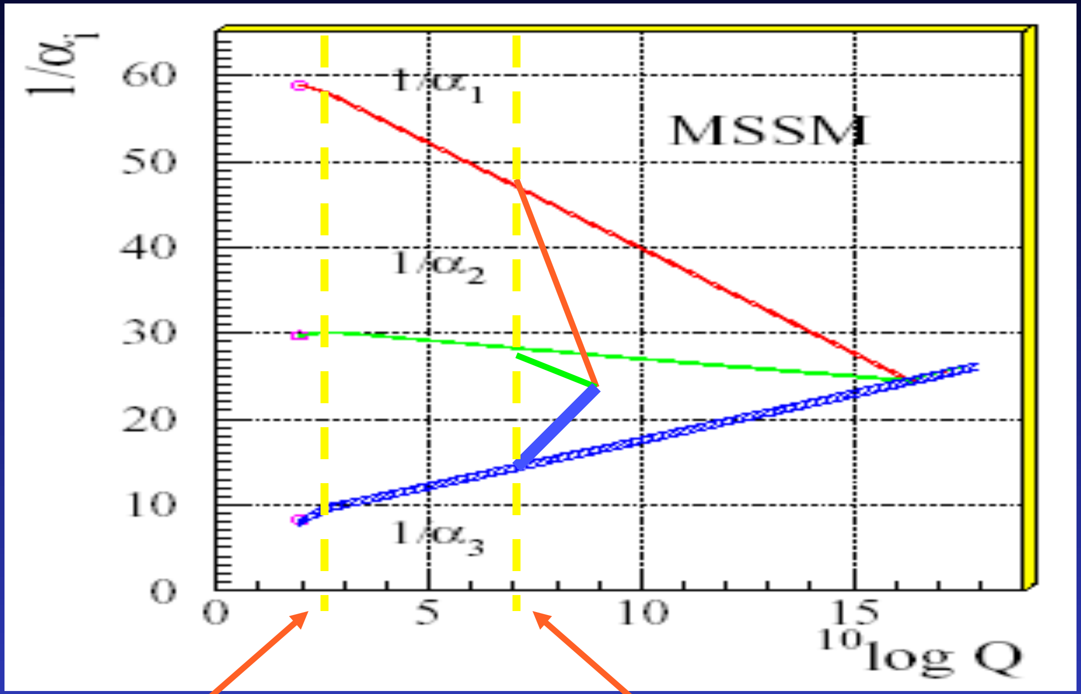
$$\frac{1}{\tilde{g}} = \frac{1}{\tilde{g}_0} + b\log\frac{\mu}{Q}$$

Log running

When applied to GUTs:

1) the unification point does not depend on ε
2) can provide unification at any scale ?!

Power law Running Couplings



Susy Threshold Extra Dim Threshold

Dienes, Dudas, Ghergetta

However,

- This picture ignores all higher dimensional operators which are **relevant** in the UV
- The correct meaning: IR running ONLY!!!!

Solution to the RG Equation

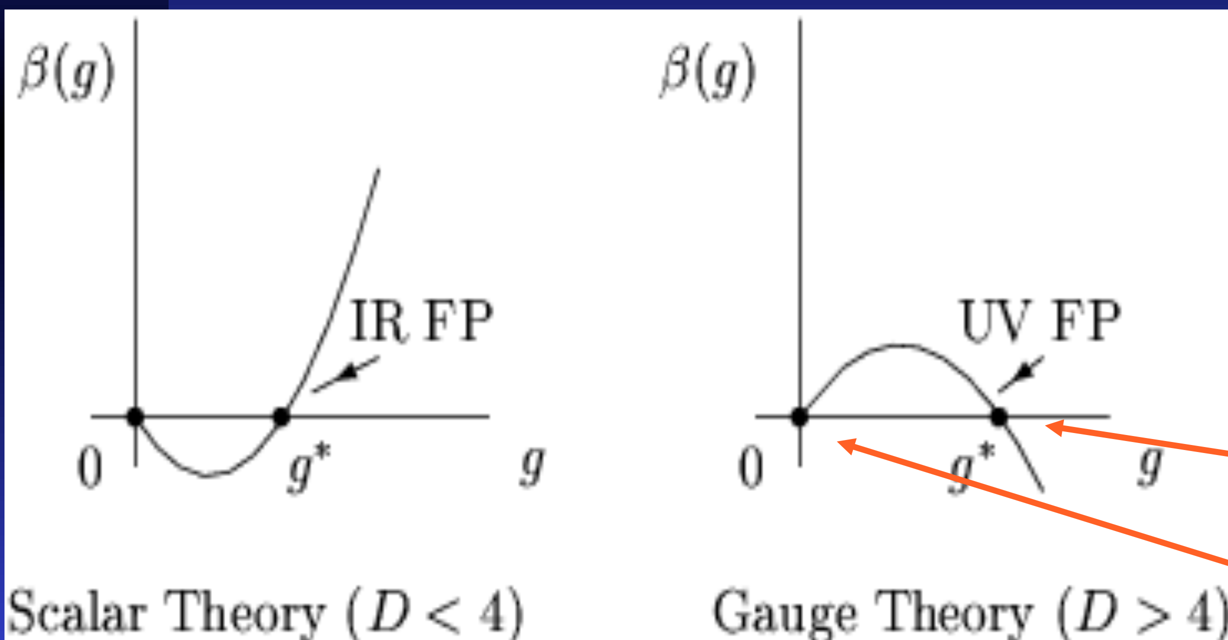
Nonperturbative solution

$$\beta(\tilde{g}) = \tilde{g}\left(\frac{D}{2} - 2 + \frac{1}{2}\gamma_A(\tilde{g})\right) = 0$$

The fixed points:

1) $\tilde{g} = 0 \rightarrow g = 0, \ \gamma_A = 0$

2) $\tilde{g} = g^*, \ \gamma_A = 4 - D$



- g^* is unknown
- $\gamma_A(g^*)$ is known exactly !

Non-Gaussian fixed point

Gaussian fixed point

IR freedom

Asymptotic freedom

Dimensional Counting

$$g = g^* \qquad [A] = \frac{D-2}{2} + \frac{1}{2}\gamma_A = \frac{D-2}{2} + \frac{4-D}{2} = 1$$

$$g\partial A[A,A] \quad \rightarrow \quad D = [g] + 1 + 3[A] + \gamma_V$$

background gauge $\quad \gamma_V = -\gamma_A$

$$[g] = D - 4 - \gamma_V = D - 4 + \gamma_A = 0$$

At the fixed point the coupling is dimensionless in any D !

- This statement does not depend on the gauge

The theory at the fixed point is perturbatively nonrenormalizable, but nonperturbatively renormalizable!

Modified Perturbation Theory

$g = 0$	$g = g^*$
$\bar{A}A : \frac{1}{(x^2)^{\frac{D-2}{2}}}$	$\bar{A}A : \frac{1}{(x^2)^1}$
$\int \frac{d^D x \, e^{ipx}}{(x^2)^{\frac{D-2}{2}}} : \frac{1}{p^2}$	$\int \frac{d^D x \, e^{ipx}}{(x^2)} : \frac{1}{(p^2)^{\frac{D-2}{2}}}$

New Feynman Rules

→ $D = 6$

$1/p^4$

$$\frac{D+2-2+2\gamma_V}{4-2\gamma_A} \Rightarrow D-4,$$

$$\frac{2D+4-2+4\gamma_V}{10-5\gamma_A} \Rightarrow 2D-8+\gamma_A = D-4,$$

. . . .

$$\Rightarrow \quad D-4.$$

An Effective Theory at the Fixed Point

D=6

$$L_{eff} : Tr(D_\mu F_{\mu\nu})^2$$

Properties

- No scale, the coupling is dimesionless
- Scale (conformal) invariance
- The exact anomalous dimensions are included
- Vanish on shell ($D^{\mu}F^{\mu\nu} = 0$)

Questions

- Can one guarantee that this fixed point exist?
- Is it perturbatively reachable?
- Can one calculate anything at the FP?

Fixed Points in SUSY Gauge Theories

Critical dimension D=4 superfield formalism N=1 SUSY

$$L = \int d^4\theta\, \overline{\Phi}_i \Phi_i + \int d^2\theta\, \mathrm{W} + h.c. \quad \mathrm{W} = y\Phi_1\Phi_2\Phi_3$$

- Dimensions

$$[L] = D,\ [d\theta] = 1/2,\ [\mathrm{W}] = \mathrm{D}\text{-}1,$$

$$[\Phi] = \tfrac{\mathrm{D}-2}{2},\ [y] = D - 1 - 3\tfrac{\mathrm{D}-2}{2} = 2 - D/2$$

- RG Eqs $\quad \tilde{y} \equiv y\mu^{D/2-2} \quad [\tilde{y}] = 0$

$$\mu \tfrac{d}{d\mu}\tilde{y} \equiv \frac{\tilde{y}}{2}(\gamma_1 + \gamma_2 + \gamma_3 + D - 4)$$

chiral field anomalous dimensions

Fixed Points

1) $\tilde{y} \equiv 0 \rightarrow y = 0,\ \gamma_i = 0$

2) $\tilde{y} \equiv y^*,\ \gamma_i = (4 - D)/3$

Fixed Point Properties

- Dimension $[y^*] = D - 1 - 3\frac{D-2}{2} - \frac{\gamma_1 + \gamma_2 + \gamma_3}{2} = 0$ in any D

- Sign $\gamma_i < 0 \;\; for \;\; D > 4$ Possible only in gauge theory

- Conformal (superconformal) fixed point ? $g = g^*, \;\; y = y^*$

Subtlety: N=1 SUSY gauge theory is possible only for $D \leq 10$

Nahm's classification of superfconformal algebras: $D \leq 6$

Alternative: either th FP is not valid for D > 6
or superconformal algebra is not realized ?

Perturbative check: Exact β function

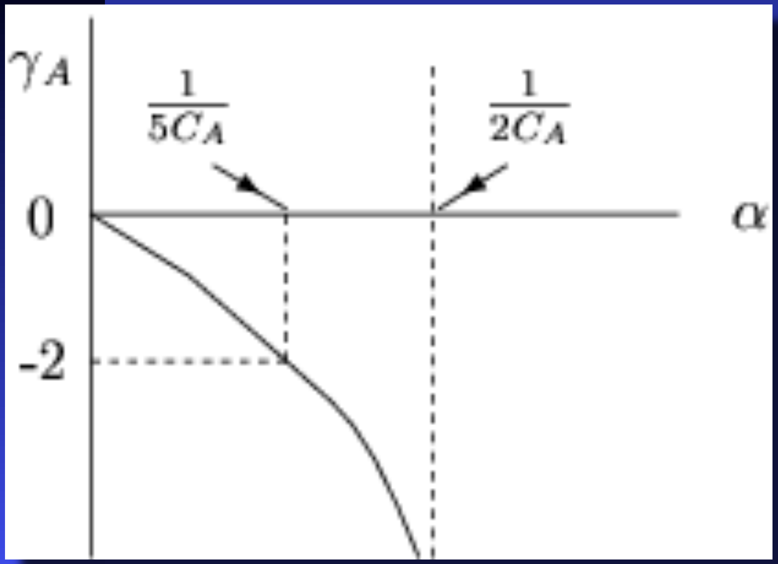
Exact all loop β function

Novikov, Shifman, Vainshtein, Zakharov

$$\gamma_A = 2\alpha \frac{T_R - 3C_A - \frac{2}{r}\sum C_R \gamma_R}{1 - 2C_A \alpha} \qquad \alpha \equiv \frac{g^2}{16\pi^2}$$

Fixed point condition

$$\gamma_A = 2\alpha \frac{-3C_A}{1 - 2C_A \alpha} = 4 - D$$



Solution for the coupling

$$\alpha^* = \frac{D-4}{D-1}\frac{1}{2C_A}$$

D=6 $\quad \alpha^* = \frac{1}{5C_A}$

Conclusions

- Perturbative finiteness (renormalizability) above D=4 seems not to be valid
- Within the Wilson RG approach the nontrivial FP may lead to nonperturbative renormalizability
- Effective renormalizable theories with unusual properties may exist
- At the FP the couplings are dimensionless and the theory possesses the conformal invariance
- At the FP the anomalous dimensions of the fields are known exactly and one may hope to obtain the correlators of the gauge invariant operators