RENORMALIZATION GROUP FLOW IN FIELD THEORIES WITH BROKEN SYMMETRY

D. Kazakov

Bogoliubov Laboratory of Theoretical Physics

Joint Institute for Nuclear Research

RG 2002

Štrba, Slovakia, March 11-16, 2002

Outline

- ⋆ **Spontaneous Symmetry Breaking**
- ⋆ **Supersymmetry**
- ⋆ **Soft SUSY Breaking and Soft Parameters**
- ⋆ **Grassmannian Expansion**
- ⋆ **Equation - Solution - Approximate Solution**
- ⋆ **The Fixed Points**
- ⋆ **Examples**

Spontaneously Broken Theory
= Unbroken Theory in External Field

Symmetric theory $\longrightarrow$ Spontaneously Broken Symmetry

$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow$$

$$<H>=0 \qquad\qquad\qquad\qquad <H>\neq 0$$

Put the system into external field H: the couplings become dependent on external field: $g(H),\ m(H)$

Taylor expansion:

$$g(<H>) = g(0) + g'(0) <H> + \frac{<H>^2}{2} g''(0) + \ldots$$

$\star$RG Equations: $\Downarrow$

$$\boxed{\dot{g} = \beta(g) \ \longrightarrow \ \dot{g}(H) = \beta(g(H))} \quad \textbf{!!!}$$

Taylor Expansion:

$$\begin{cases} \dot{g} &= \beta(g) \\ \dot{g}' &= \frac{d\beta(g)}{dg} g' \\ \dot{g}'' &= \frac{d^2\beta(g)}{dg^2}(g')^2 + \frac{d\beta(g)}{dg} g'' \end{cases}$$

$\star$ Solution to RG Equation (one coupling for simplicity):

$$t = \int_{g_0}^{g_t} \frac{dx}{\beta(x)} \ \longrightarrow \ t = \int_{g_0(H)}^{g_t(H)} \frac{dx}{\beta(x)}$$

Taylor Expansion:

$$\boxed{g' = c_1 \beta(g), \quad g'' = c_1^2 \beta \frac{d\beta}{dg} + c_2 \beta(g), \quad g = g(t)}$$

SUPERSYMMETRY

Poincare Algebra

$[P_\mu, P_\nu] = 0,$

$[P_\mu, M_{\rho\sigma}] = i(g_{\mu\rho} P_\sigma - g_{\mu\sigma} P_\rho),$

$[M_{\mu\nu}, M_{\rho\sigma}] = i(g_{\nu\rho} M_{\mu\sigma} - g_{\nu\sigma} M_{\mu\rho} - g_{\mu\rho} M_{\nu\sigma} + g_{\mu\sigma} M_{\nu\rho}),$

Internal Algebra

$[B_r, B_s] = i C^t_{rs} B_t,$

$[B_r, P_\mu] = [B_r, M_{\mu\sigma}] = 0,$

SUSY Algebra

$[Q^i_\alpha, P_\mu] = [\bar{Q}^i_{\dot\alpha}, P_\mu] = 0,$

$[Q^i_\alpha, M_{\mu\nu}] = \frac{1}{2}(\sigma_{\mu\nu})^\beta_\alpha Q^i_\beta, \quad [\bar{Q}^i_{\dot\alpha}, M_{\mu\nu}] = -\frac{1}{2}\bar{Q}^i_{\dot\beta}(\bar\sigma_{\mu\nu})^{\dot\beta}_{\dot\alpha},$

$[Q^i_\alpha, B_r] = (b_r)^i_j Q^j_\alpha, \quad [\bar{Q}^i_{\dot\alpha}, B_r] = -\bar{Q}^j_{\dot\alpha}(b_r)^i_j,$

$\boxed{\{Q^i_\alpha, \bar{Q}^j_{\dot\beta}\} = 2\delta^{ij}(\sigma^\mu)_{\alpha\dot\beta} P_\mu},$

$\{Q^i_\alpha, Q^j_\beta\} = 2\epsilon_{\alpha\beta} Z^{ij}, \quad Z_{ij} = a^r_{ij} b_r, \quad Z^{ij} = Z^+_{ij},$

$\{\bar{Q}^i_{\dot\alpha}, \bar{Q}^j_{\dot\beta}\} = -2\epsilon_{\dot\alpha\dot\beta} Z^{ij}, \quad [Z_{ij}, anything] = 0,$

$\alpha, \dot\alpha = 1, 2 \quad i, j = 1, 2, \ldots, N.$

Supersymmetry relates particles with spin different by 1/2, boson and fermions!

$$\begin{array}{ccc} Space & \Rightarrow & Superspace \\ x_\mu & & x_\mu, \theta_\alpha, \bar\theta_{\dot\alpha} \end{array}$$

$\theta, \bar\theta$ are Grassmannian variables

$$\{\theta_\alpha, \theta_\beta\} = 0, \quad \{\bar\theta_{\dot\alpha}, \bar\theta_{\dot\beta}\} = 0, \quad \alpha, \beta = 1, 2.$$

A SUSY group element can be constructed in superspace in the same way as an ordinary translation in the usual space

$$G(x,\theta,\bar\theta) = e^{i(-x^\mu P_\mu + \theta Q + \bar\theta\bar Q)}$$

It leads to a supertranslation in superspace

$$\begin{array}{rcl} x_\mu & \longrightarrow & x_\mu + i\theta\sigma_\mu\bar\varepsilon - i\varepsilon\sigma_\mu\bar\theta, \\ \theta & \longrightarrow & \theta + \varepsilon, \\ \bar\theta & \longrightarrow & \bar\theta + \bar\varepsilon, \end{array}$$

⋆ Superfields ($y_\mu = x_\mu + \bar\theta\sigma_\mu\theta$)

$$\begin{array}{rcl} \Phi(y,\theta) & = & A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \\ & & \uparrow \text{scalar field} \qquad \uparrow \text{spinor field} \qquad \uparrow \text{auxiliary field} \\ & = & A(x) + i\theta\sigma^\mu\bar\theta\partial_\mu A(x) + \frac{1}{4}\theta\theta\bar\theta\bar\theta\Box A(x) \\ & + & \sqrt{2}\theta\psi(x) - \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi(x)\sigma^\mu\bar\theta + \theta\theta F(x). \end{array}$$

$$\begin{array}{rcl} V(x,\theta,\bar\theta) & = & C(x) + i\theta\chi(x) - i\bar\theta\bar\chi(x) \\ & + & \frac{i}{2}\theta\theta[M(x) + iN(x)] - \frac{i}{2}\bar\theta\bar\theta[M(x) - iN(x)] \\ & - & \theta\sigma^\mu\bar\theta v_\mu(x) + i\theta\theta\bar\theta[\lambda(x) + \frac{i}{2}\bar\sigma^\mu\partial_\mu\chi(x)] \\ & & \uparrow \text{vector field} \qquad \uparrow \text{spinor field} \qquad \updownarrow \text{auxiliary field} \\ & - & i\bar\theta\bar\theta\theta[\lambda + \frac{i}{2}\sigma^\mu\partial_\mu\bar\chi(x)] + \frac{1}{2}\theta\theta\bar\theta\bar\theta[D(x) + \frac{1}{2}\Box C(x)]. \end{array}$$

The Main Statement:

Softly Broken SUSY Theory	$\approx$	*Rigid SUSY Theory in External Spurion Superfield*

Avdeev, Kazakov & Kondrashuk NP '98
Jack & Jones, PL '97
Giudice & Rattazzi, NP '98
Yamada, PR '94

The Couplings $g \Rightarrow$ External Superfilelds $S(\theta, \bar{\theta})$

Singular Part of Effective Action:

$$S^{eff}_{Sing}(g) \Rightarrow S^{eff}_{Sing}(S, \not{D}^2 S, \not{\bar{D}}^2 S, D^2 \not{\bar{D}}^2 S)$$

Consequence:

Renormalizations of the Soft Terms Follow from those of a Rigid Theory

Renormalization Constants: $\tilde{Z}_i(g) = Z_i(S)$

Grassmannian Taylor Expansion !

Kazakov PL '98

THE RIGID N=1 SUSY GAUGE THEORY

⋆ The Lagrangian

$$\mathcal{L}_{rigid} = \int d^2\theta \, \frac{1}{4g^2} \mathrm{Tr} W^\alpha W_\alpha + \int d^2\bar{\theta} \, \frac{1}{4g^2} \mathrm{Tr} \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} + \int d^2\theta d^2\bar{\theta} \;\; \bar{\Phi}^i (e^V)_i^j \Phi_j + \int d^2\theta \;\; \mathcal{W} + \int d^2\bar{\theta} \;\; \bar{\mathcal{W}},$$

$$W_\alpha = -\frac{1}{4} \bar{D}^2 e^{-V} D_\alpha e^V, \quad \bar{W}_{\dot{\alpha}} = -\frac{1}{4} D^2 e^{-V} \bar{D}_{\dot{\alpha}} e^V,$$

$$\mathcal{W} = \frac{1}{6} y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} M^{ij} \Phi_i \Phi_j.$$

⋆ Gauge fixing

$$\mathcal{L}_{gauge-fixing} = -\frac{1}{16} \int d^2\theta d^2\bar{\theta} \mathrm{Tr} \left(\bar{f} f + f \bar{f} \right)$$

$$f = \bar{D}^2 \frac{V}{\sqrt{\xi g^2}}, \quad \bar{f} = D^2 \frac{V}{\sqrt{\xi g^2}}.$$

⋆ Ghost Term

$$\mathcal{L}_{ghost} = i \int d^2\theta \, \frac{1}{4} \mathrm{Tr} \, b \, \delta_c f - i \int d^2\bar{\theta} \, \frac{1}{4} \mathrm{Tr} \, \bar{b} \, \delta_{\bar{c}} \bar{f},$$

$$\delta_\Lambda f = \bar{D}^2 \delta_\Lambda \frac{V}{\sqrt{\xi g^2}} = i \bar{D}^2 \frac{1}{\sqrt{\xi g^2}} \mathcal{L}_{V/2} [\Lambda + \bar{\Lambda} + \coth(\mathcal{L}_{V/2})(\Lambda - \bar{\Lambda})],$$

$$
\begin{aligned}
\mathcal{L}_{ghost} \;=\; & -\int d^2\theta\; \frac{1}{4}\mathrm{Tr}\; b\bar{D}^2 \frac{1}{\sqrt{\xi g^2}}\mathcal{L}_{V/2}[c+\bar{c} \\
& \qquad\qquad + \coth(\mathcal{L}_{V/2})(c-\bar{c})] + \quad h.c. \\
= & \int d^2\theta d^2\bar{\theta}\; \mathrm{Tr}\; \left(\frac{b+\bar{b}}{\sqrt{\xi g^2}}\right)\mathcal{L}_{V/2}[c+\bar{c}+\coth(\mathcal{L}_{V/2})(c-\bar{c})] \\
= & \int d^2\theta d^2\bar{\theta}\; \mathrm{Tr}\; \left(\frac{b+\bar{b}}{\sqrt{\xi g^2}}\right)\Bigg((c-\bar{c}) + \frac{1}{2}[V,(c+\bar{c})\,] \\
& \qquad\qquad + \frac{1}{12}[V,[V,(c-\bar{c})\,]] + \ldots\Bigg)\,.
\end{aligned}
$$

where $\mathcal{L}_X Y \equiv [X, Y]$.

⋆ BRST transformations

$$
\begin{aligned}
&\delta V = \epsilon\mathcal{L}_{V/2}[c+\bar{c}+\coth(\mathcal{L}_{V/2})(c-\bar{c})], \\
&\delta\, c^a = -\frac{i}{2}\epsilon f^{abc}c^b c^c\;, \qquad \delta\,\bar{c}^a = -\frac{i}{2}\epsilon f^{abc}\bar{c}^b\bar{c}^c\;, \\
&\delta\, b^a = \frac{1}{8}\epsilon\bar{D}^2\bar{f}^a\;, \qquad \delta\,\bar{b}^a = \frac{1}{8}\epsilon D^2 f^a.
\end{aligned}
$$

SOFT SUSY BREAKING

⋆ Soft Terms

$$-\mathcal{L}_{soft-breaking} = \left[\frac{M}{2}\lambda\lambda + \frac{1}{6}A^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}B^{ij}\phi_i\phi_j + h.c.\right] + (m^2)^i_j\phi^*_i\phi^j,$$

⋆ Spurion Fields $\quad \eta = \theta^2$ and $\bar{\eta} = \bar{\theta}^2$

Girardello & Grisaru
NP '82

$$\mathcal{L}_{soft} = \int d^2\theta\, \frac{1}{4g^2}(1-2M\eta)\mathrm{Tr}W^\alpha W_\alpha + \int d^2\bar{\theta}\, \frac{1}{4g^2}(1-2\bar{M}\bar{\eta})\mathrm{Tr}\bar{W}^{\dot{\alpha}}\bar{W}_{\dot{\alpha}}$$
$$+ \int d^2\theta d^2\bar{\theta}\;\; \bar{\Phi}^i(\delta^k_i - (m^2)^k_i\eta\bar{\eta})(e^V)^j_k\Phi_j$$
$$+ \int d^2\theta \left[\frac{1}{6}(y^{ijk} - A^{ijk}\eta)\Phi_i\Phi_j\Phi_k + \frac{1}{2}(M^{ij} - B^{ij}\eta)\Phi_i\Phi_j\right] + h.c.$$

⋆ Replacement

$$\frac{1}{g^2} \;\to\; \frac{1}{\tilde{g}^2} = \frac{1 - M\theta^2 - \bar{M}\bar{\theta}^2}{g^2},$$

$$y^{ijk} \;\to\; \tilde{y}^{ijk} = y^{ijk} - A^{ijk}\eta,$$

$$M^{ij} \;\to\; \tilde{M}^{ij} = M^{ij} - B^{ij}\eta,$$

⋆ NEW!

$$\frac{1}{g^2} \to \frac{1}{\tilde{g}^2} = \frac{1 - M\theta^2 - \bar{M}\bar{\theta}^2 - \Delta\theta^2\bar{\theta}^2}{g^2},$$

$$\tilde{g}^2 = g^2 \left(1 + M\theta^2 + \bar{M}\bar{\theta}^2 + 2M\bar{M}\theta^2\bar{\theta}^2 + \Delta\theta^2\bar{\theta}^2\right).$$

⋆ Gauge Parameter

$$\tilde{\xi} = \xi \left(1 + x\theta^2 + \bar{x}\bar{\theta}^2 + (x\bar{x} + z)\theta^2\bar{\theta}^2\right),$$

⋆ Modified Gauge Fixing

$$f \to \bar{D}^2 \frac{V}{\sqrt{\tilde{\xi}\tilde{g}^2}}, \quad \bar{f} \to D^2 \frac{V}{\sqrt{\tilde{\xi}\tilde{g}^2}}.$$

$$\mathcal{L}_{gauge-fixing} = -\frac{1}{8}\int d^2\theta d^2\bar{\theta}\ \mathrm{Tr}\left(\bar{D}^2\frac{V}{\sqrt{\tilde{\xi}\tilde{g}^2}} D^2\frac{V}{\sqrt{\tilde{\xi}\tilde{g}^2}}\right),$$

⋆ Modified Ghost Term

$$\mathcal{L}_{ghost} = \int d^2\theta d^2\bar{\theta}\ \mathrm{Tr}\frac{1}{\sqrt{\tilde{\xi}\tilde{g}^2}}\left(b + \bar{b}\right)\mathcal{L}_{V/2}[c + \bar{c} + \coth(\mathcal{L}_{V/2})(c - \bar{c})].$$

$$\mathcal{L}^{(2)}_{ghost} = \int d^2\theta d^2\bar{\theta}\ \mathrm{Tr}\frac{1}{\sqrt{\xi g^2}}\left(1 - \frac{1}{2}\Delta\xi\theta^2\bar{\theta}^2\right)\left(b + \bar{b}\right)\left(c - \bar{c}\right) - \frac{1}{2}\int d^2\theta\ \mathrm{Tr}\frac{1}{\sqrt{\xi g^2}} M\xi bc + \frac{1}{2}\int d^2\bar{\theta}\ \mathrm{Tr}\frac{1}{\sqrt{\xi g^2}}\bar{M}\xi\bar{b}\bar{c},$$

$$
\begin{aligned}
V(x,\theta,\bar\theta) \;=\; & \mathbb{C}(x) + i\theta\chi(x) - i\bar\theta\bar\chi(x) + \frac{i}{2}\theta\theta N(x) - \frac{i}{2}\bar\theta\bar\theta\bar N(x) \\
& -\theta\sigma^{\mu}\bar\theta v_{\mu}(x) + i\theta\theta\bar\theta[\bar\lambda(x) + \frac{i}{2}\bar\sigma^{\mu}\partial_{\mu}\chi(x)] \\
& -i\bar\theta\bar\theta\theta[\lambda + \frac{i}{2}\sigma^{\mu}\partial_{\mu}\bar\chi(x)] + \frac{1}{2}\theta\theta\bar\theta\bar\theta[D(x) - \frac{1}{2}\Box\mathbb{C}(x)].
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{g-f} \;=\; & \frac{1}{2\xi g^2}\Big[-(D - \Box\mathbb{C} - \Delta\xi\mathbb{C} + \frac{i}{2}M\xi\bar N - \frac{i}{2}\bar M\xi N)^2 \\
& - (\partial^{\mu}v_{\mu})^2 + (\bar N - i\bar M\xi\mathbb{C})\Box(N + iM\xi\mathbb{C}) \\
& - i(\lambda + \frac{1}{2}\bar M\xi\chi)\sigma^{\mu}\partial_{\mu}(\bar\lambda + \frac{1}{2}M\xi\bar\chi) \\
& - (\lambda + \frac{1}{2}\bar M\xi\chi)\Box\chi - (\bar\lambda + \frac{1}{2}M\xi\bar\chi)\Box\bar\chi - i\Box\chi\sigma^{\mu}\partial_{\mu}\bar\chi\Big].
\end{aligned}
$$

⋆ The Meaning of Masses

M – gaugino field λ mass
– auxiliary field χ mass
– spinor ghost mass

Δ – soft scalar ghost mass
– auxiliary fields $\mathbb{C}$ and N mass

RENORMALIZATION OF SOFT VERSUS RIGID THEORY

The Statement *Let a rigid theory be renormalized via introduction of the renormalization constants* Z_i,*defined within some minimal subtraction massless scheme. Then, a softly broken theory is renormalized via introduction of the renormalization superfields* $\tilde{Z}_i$ *which are related to* Z_i *by the coupling constants redefinition*

$$\tilde{Z}_i(g^2, y, \bar{y}) = Z_i(\tilde{g}^2, \tilde{y}, \tilde{\bar{y}}),$$

where the redefined couplings are

$$\begin{aligned}
\tilde{g}_i^2 &= g_i^2(1 + M_i\eta + \bar{M}_i\bar{\eta} + (2M_i\bar{M}_i + \Delta_i)\eta\bar{\eta}) \\
\tilde{y}^{ijk} &= y^{ijk} - A^{ijk}\eta + \tfrac{1}{2}(y^{njk}(m^2)^i_n + y^{ink}(m^2)^j_n + y^{ijn}(m^2)^k_n)\eta\bar{\eta} \\
\tilde{\bar{y}}_{ijk} &= \bar{y}_{ijk} - \bar{A}_{ijk}\bar{\eta} + \tfrac{1}{2}(y_{njk}(m^2)_i^n + y_{ink}(m^2)_j^n + y_{ijn}(m^2)_k^n)\eta\bar{\eta}
\end{aligned}$$

Summary on Soft Term Renormalizations

The Rigid Terms	The Soft Terms
$\beta_{\alpha_i} = \alpha_i \gamma_{\alpha_i}$	$\beta_{m_{Ai}} = D_1 \gamma_{\alpha i}$
$\beta_M^{ij} = \frac{1}{2}(M^{il}\gamma_l^j + M^{lj}\gamma_l^i)$	$\beta_B^{ij} = \frac{1}{2}(B^{il}\gamma_l^j + B^{lj}\gamma_l^i)$ $-(M^{il}D_1\gamma_l^j + M^{lj}D_1\gamma_l^i)$
$\beta_y^{ijk} = \frac{1}{2}(y^{ijl}\gamma_l^k + \text{perm's})$ $\Uparrow$ chiral anomalous dim.	$\beta_A^{ijk} = \frac{1}{2}(A^{ijl}\gamma_l^k + \text{perm's})$ $-(y^{ijl}D_1\gamma_l^k + \text{perm's})$ $(\beta_{m^2})_j^i = D_2 \gamma_j^i$

$$D_1 = m_{A_i}\alpha_i \frac{\partial}{\partial \alpha_i} - A^{ijk}\frac{\partial}{\partial y^{ijk}}$$

$$\bar{D}_1 = m_{A_i}\alpha_i \frac{\partial}{\partial \alpha_i} - A_{ijk}\frac{\partial}{\partial y_{ijk}}$$

$$D_2 = \bar{D}_1 D_1 + \Sigma_{\alpha_i}\alpha_i \frac{\partial}{\partial \alpha_i} + \frac{1}{2}(m^2)^a_n \left(y^{nbc}\frac{\partial}{\partial y^{abc}} + y^{bnc}\frac{\partial}{\partial y^{bac}} + y^{bcn}\frac{\partial}{\partial y^{bca}} + y_{abc}\frac{\partial}{\partial y_{nbc}} + y_{bac}\frac{\partial}{\partial y_{bnc}} + y_{bca}\frac{\partial}{\partial y_{bcn}} \right)$$

$$\Sigma_{\alpha_i} = M_i \bar{M}_i + \Delta_i$$

Jack, Jones & Pickering
PL '98 $(\Delta \equiv X)$

Example: Pure gauge theory

Notation: $\alpha \equiv g_i^2/16\pi^2$.

$$\alpha^{Bare} = Z_\alpha \alpha \quad \Rightarrow \quad \tilde{\alpha}^{Bare} = \tilde{Z}_\alpha \tilde{\alpha}$$

Expand over η:

$$\alpha^{Bare}(1 + m_A^{Bare}\eta) = \alpha(1 + m_A\eta) Z_\alpha(\tilde{\alpha})|_{\bar{\eta}=0}.$$

$$\begin{aligned} \alpha^{Bare} &= \alpha Z_\alpha(\alpha), \\ m_A^{Bare}\alpha^{Bare} &= m_A \alpha Z_\alpha(\alpha) + \alpha D_1 Z_\alpha \end{aligned}$$

$$D_1 = m_A \alpha \frac{\partial}{\partial \alpha}$$

Differential operator extracting linear term over η

Result:

$$m_A^{Bare} = m_A + D_1 \ln Z_\alpha$$

RG Functions

$$\beta_\alpha = \alpha \gamma_\alpha, \qquad \beta_{m_A} = D_1 \gamma_\alpha$$

Consequence:

$$\frac{m_A \alpha}{\beta_\alpha} \quad \textit{is RG invariant}$$

J.Hisano & M.Shifman
Phys.Rev.D56 (1997)5745

ILLUSTRATION

⋆ One-loop Renormalization functions

$$\gamma_\alpha^{(1)} = \alpha Q, \quad Q = T(R) - 3C(G),$$
$$\gamma_j^{i\ (1)} = \frac{1}{2} y^{ikl} y_{jkl} - 2\alpha C(R)^i_j,$$

⋆ Casimir Operators

$$T(R)\delta_{AB} = Tr(R_A R_B), \quad C(G)\delta_{AB} = f_{ACD} f_{BCD},$$
$$C(R)^i_j = (R_A R_A)^i_j$$

⋆ Soft RG Functions

$$\beta_{m_A}^{(1)} = \alpha m_A Q,$$
$$\beta_B^{ij\ (1)} = \frac{1}{2} B^{il} (\frac{1}{2} y^{jkm} y_{lkm} - 2\alpha C(R)^j_l)$$
$$+ M^{il} (\frac{1}{2} A^{jkm} y_{lkm} + 2\alpha m_A C(R)^j_l) + (i \leftrightarrow j),$$
$$\beta_A^{ijk\ (1)} = \frac{1}{2} A^{ijl} (\frac{1}{2} y^{kmn} y_{lmn} - 2\alpha C(R)^k_l)$$
$$+ y^{ijl} (\frac{1}{2} A^{kmn} y_{lmn} + 2\alpha m_A C(R)^k_l)$$
$$+ (i \leftrightarrow j) + (i \leftrightarrow k),$$
$$[\beta_{m^2}]^{i\ (1)}_j = \frac{1}{2} A^{ikl} A_{jkl} - 4\alpha m_A^2 C(R)^i_j$$
$$+ \frac{1}{4} y^{nkl} (m^2)^i_n y_{jkl} + \frac{1}{4} y^{ikl} (m^2)^n_j y_{nkl} + \frac{4}{4} y^{isl} (m^2)^k_s y_{jkl}.$$

Three-loop gaugino mass renormalization

The RG β functions for the gauge couplings:

$$\beta_{\alpha_i} = b_i\alpha_i^2 + \alpha_i^2\left(\sum_j b_{ij}\alpha_j - \sum_f a_{if}Y_f\right) + \alpha_i^2\left[\sum_{jk} b_{ijk}\alpha_j\alpha_k - \sum_{jf} a_{ijf}\alpha_j Y_f + \sum_{fg} a_{ifg}Y_f Y_g\right] + \ldots,$$

Y_f means Y_t, Y_b, Y_τ and the coefficients $b_i, b_{ij}, a_{if}, b_{ijk}$, a_{ijf}, a_{ifg} are given in Ferreira, Jack & Jones, PL B387(1996)80

The RG β functions for the gaugino masses:

$$\begin{aligned}\beta_{m_{A_i}} &= b_i\alpha_i m_{A_i} \\ &+ \alpha_i\left(\sum_j b_{ij}\alpha_j(m_{A_i}+m_{A_j}) - \sum_f a_{if}Y_f(m_{A_i}-A_f)\right) \\ &+ \alpha_i\left[\sum_{jk} b_{ijk}\alpha_j\alpha_k(m_{A_i}+m_{A_j}+m_{A_k}) - \sum_{jf} a_{ijf}\alpha_j Y_f(m_{A_i}+m_{A_j}-A_f) + \sum_{fg} a_{ifg}Y_f Y_g(m_{A_i}-A_f-A_g)\right] + \ldots.\end{aligned}$$

⋆ Illustration (3 loops)

Jack, Jones & North
PL '96

$$
\begin{aligned}
\gamma_\alpha &= \alpha Q + 2\alpha^2 QC(G) - \frac{2}{r}\alpha \gamma_j^{i\,(1)} C(R)_i^j - \alpha^3 Q^2 C(G) \\
&+ 4\alpha^3 QC^2(G) - \frac{6}{r}\alpha^3 QC(R)_j^i C(R)_i^j - \frac{4}{r}\alpha^2 C(G)\gamma_j^{i\,(1)} C(R)_i^j \\
&+ \frac{3}{r}\alpha y^{ikm} y_{jkn} \gamma_m^{n\,(1)} C(R)_i^j \\
&+ \frac{1}{r}\alpha \gamma_j^{i\,(1)} \gamma_p^{j\,(1)} C(R)_i^p + \frac{6}{r}\alpha^2 \gamma_j^{i\,(1)} C(R)_p^j C(R)_i^p, \\
\gamma_j^i &= \frac{1}{2} y^{ikl} y_{jkl} - 2\alpha C(R)_j^i \\
&- (y^{imp} y_{jmn} + 2\alpha C(R)_j^p \delta_n^i)(\frac{1}{2} y^{nkl} y_{pkl} - 2\alpha C(R)_p^n) \\
&\quad +2\alpha^2 QC(R)_j^i \ ,
\end{aligned}
$$

⋆ Solution for Σ_α:

Jack, Jones & Pickering
PL '98 $(\Delta \equiv X)$

$$
\begin{aligned}
{\Sigma_\alpha}^{(1)} &= M^2, \\
{\Sigma_\alpha}^{(2)} &= \Delta_\alpha^{(2)} = -2\alpha[\frac{1}{r}(m^2)^i_j C(R)_i^j - M^2 C(G)], \\
{\Sigma_\alpha}^{(3)} &= \Delta_\alpha^{(3)} = \frac{\alpha}{2r}[\frac{1}{2}(m^2)^i_n y^{nkl} y_{jkl} + \frac{1}{2}(m^2)^n_j y^{ikl} y_{nkl} \\
&+ 2(m^2)^m_n y^{ikn} y_{jkm} + A^{ikl} A_{jkl} - 8\alpha M^2 C(R)_j^i] C(R)_i^j \\
&- 2\alpha^2 QC(G) M^2 - 4\alpha^2 C(G)[\frac{1}{r}(m^2)^i_j C(R)_i^j - M^2 C(G)].
\end{aligned}
$$

⋆ All loop Solution in NSVZ Scheme

Novikov, Shifman,
Vainshtein & Zakharov
NP '83

$$\gamma_\alpha^{NSVZ} = \alpha \frac{Q - 2r^{-1}\mathrm{Tr}[\gamma C(R)]}{1 - 2C(G)\alpha}$$

Solition:

$$\Delta_\alpha^{NSVZ} = -2\alpha \frac{r^{-1}\mathrm{Tr}[m^2 C(R)] - M^2 C(G)}{1 - 2C(G)\alpha}$$

Jack, Jones & Pickering
PL '98
$\Delta \equiv X$

ILLUSTRATION: THE MSSM

⋆ Notation:

$$\alpha_i \equiv \frac{g_i^2}{16\pi^2}, \quad i = 1,2,3; \quad Y_k \equiv \frac{y_k^2}{16\pi^2}, \quad k = t, b, \tau.$$

⋆ Substitution:

$$\begin{aligned}\tilde{\alpha}_i &= \alpha_i \left(1 + M_i\eta + \bar{M}_i\bar{\eta} + (M_i\bar{M}_i + \Sigma_{\alpha_i})\eta\bar{\eta}\right)\\ \tilde{Y}_k &= Y_k \left(1 - A_k\eta - \bar{A}_k\bar{\eta} + (A_k\bar{A}_k + \Sigma_k)\eta\bar{\eta}\right)\end{aligned}$$

$$\Sigma_t = \tilde{m}_Q^2 + \tilde{m}_U^2 + m_{H_2}^2, \ \Sigma_b = \tilde{m}_Q^2 + \tilde{m}_D^2 + m_{H_1}^2, \ \Sigma_\tau = \tilde{m}_L^2 + \tilde{m}_E^2 + m_{H_1}^2$$

⋆ RGE:

$$\dot{a}_i = a_i\gamma_i(a), \qquad a_i = \{\alpha_i, Y_k\},$$

⋆ Soft RGE: Kazakov PL '98

$$\dot{\tilde{a}}_i = \tilde{a}_i\gamma_i(\tilde{a}).$$

$$\tilde{a}_i = a_i(1 + m_i\eta + \bar{m}_i\bar{\eta} + S_i\eta\bar{\eta}),$$

$$m_i = \{M_i, -A_k\}, \quad S_i = \{M_i\bar{M}_i + \Sigma_{\alpha_i}, A_k\bar{A}_k + \Sigma_k\}$$

$$\begin{aligned}\dot{m}_i &= \gamma_i(\tilde{a})|_F = \sum_j a_j \frac{\partial\gamma_i}{\partial a_j} m_j.\\ \dot{\Sigma}_i &= \gamma_i(\tilde{a})|_D = \sum_j a_j \frac{\partial\gamma_i}{\partial a_j}(m_j m_j + \Sigma_j) + \sum_{j,k} a_j a_k \frac{\partial^2\gamma_i}{\partial a_j \partial a_k} m_j m_k\end{aligned}$$

⋆ Solutions for Σ_α:

$$\begin{aligned}
\Sigma_{\alpha_1} &= M_1^2 - \alpha_1\sigma_1 - \frac{199}{25}\alpha_1^2 M_1^2 - \frac{27}{5}\alpha_1\alpha_2 M_2^2 - \frac{88}{5}\alpha_1\alpha_3 M_3^2 \\
&+ \frac{13}{5}\alpha_1 Y_t(\Sigma_t + A_t^2) + \frac{7}{5}\alpha_1 Y_b(\Sigma_b + A_b^2) + \frac{9}{5}\alpha_1 Y_\tau(\Sigma_\tau + A_\tau^2) \\
\Sigma_{\alpha_2} &= M_2^2 - \alpha_2(\sigma_2 - 4M_2^2) - \alpha_2^2(4\sigma_2 + 9M_2^2) - \frac{9}{5}\alpha_2\alpha_1 M_1^2 \\
&-24\alpha_2\alpha_3 M_3^2 + 3\alpha_2 Y_t(\Sigma_t + A_t^2) + 3\alpha_2 Y_b(\Sigma_b + A_b^2) \\
&+\alpha_2 Y_\tau(\Sigma_\tau + A_\tau^2) \\
\Sigma_{\alpha_3} &= M_3^2 - \alpha_3(\sigma_3 - 6M_3^2) - \alpha_3^2(6\sigma_3 - 22M_3^2) - \frac{11}{5}\alpha_3\alpha_1 M_1^2 \\
&-9\alpha_3\alpha_2 M_2^2 + 2\alpha_3 Y_t(\Sigma_t + A_t^2) + 2\alpha_3 Y_b(\Sigma_b + A_b^2)
\end{aligned}$$

$$\begin{aligned}
\sigma_1 &= \frac{1}{5}\left[3(m_{H_1}^2 + m_{H_2}^2) + 3(\tilde{m}_Q^2 + 3\tilde{m}_L^2 + 8\tilde{m}_U^2 + 2\tilde{m}_D^2 + 6\tilde{m}_E^2)\right] \\
\sigma_2 &= m_{H_1}^2 + m_{H_2}^2 + 3(3\tilde{m}_Q^2 + \tilde{m}_L^2) \\
\sigma_3 &= 3(2\tilde{m}_Q^2 + \tilde{m}_U^2 + \tilde{m}_D^2).
\end{aligned}$$

Martin & Vaughn
PR '94

ALTERNATIVE FORMULATION

⋆ RG Equation

$$\frac{d}{dt}\alpha = \beta(\alpha) \;\Rightarrow\; \frac{d}{dt}\alpha(1+M_A\theta^2) = \beta(\alpha(1+M_A\theta^2))$$

Expand over θ^2:

$$\begin{array}{l} \dot{\alpha} = \beta(\alpha) \\ \dot{\alpha}M_A + \alpha\dot{M}_A = \frac{d\beta}{d\alpha}\alpha M_A \end{array} \;\Rightarrow\; \boxed{\dot{M}_A = M_A\alpha\frac{d}{d\alpha}\left(\frac{\beta(\alpha)}{\alpha}\right)}$$

⋆ Solution to RG Equation

$$\int^{\alpha} \frac{d\alpha'}{\beta(\alpha')} = \ln\frac{Q^2}{\Lambda^2}$$

$$\int^{\alpha(1+M_A\theta^2)} \frac{d\alpha'}{\beta(\alpha')} = \ln\frac{Q^2}{\Lambda^2(1+c\theta^2)}$$

Expand over θ^2:

$$\frac{\alpha M_A}{\beta(\alpha)} = -c \;\Rightarrow\; \boxed{M_A = const\,\frac{\beta(\alpha)}{\alpha}}$$

EXAMPLES

1. MSSM in low $\tan\beta$ regime

RQ Eqs.
$$\begin{cases} \dot{\alpha}_i = -b_i\alpha_i^2, \quad i=1,2,3 \\ \dot{Y}_t = Y_t(\frac{16}{3}\alpha_3 + 3\alpha_2 + \frac{13}{15}\alpha_1 - 6Y_t) \end{cases}$$

Solution
$$\begin{cases} \alpha_i(t) = \frac{\alpha_0}{1+b_i\alpha_0 t}, \quad t = \ln\frac{M_X^2}{Q^2} \\ Y_t(t) = \frac{Y_0 E(t)}{1+6Y_0F(t)} \end{cases}$$

$$E(t) = \prod_i (1+b_i\alpha_0 t)^{c_i/b_i}, \quad F(t) = \int_0^t E(t')dt'$$

To get solutions for the soft terms one has to make substitution: $\alpha \to \tilde{\alpha}, Y \to \tilde{Y}$ and expand over $\eta, \bar{\eta}$

$$\alpha_i M_{A_i} = \frac{\alpha_0 M_0}{1+b_i\alpha_0 t} - \frac{\alpha_0 b_i \alpha_0 M_0 t}{(1+b_i\alpha_0 t)^2} = \frac{\alpha_0}{1+b_i\alpha_0 t}\cdot\frac{M_0}{1+b_i\alpha_0 t}$$

$$M_{A_i} = \frac{M_0}{1+b_i\alpha_0 t}$$

$$A_t(t) = \frac{A_0}{1+6Y_0F} - M_0[\frac{t}{E}\frac{dE}{dt} - \frac{6Y_0}{1+6Y_0F}(tE-F)]$$

Expand up to $\eta\bar{\eta}$:

$$\Sigma_t(t) = \frac{\Sigma_0 - A_0^2}{1+6Y_0F} + \frac{[A_0+6M_0Y_0(tE-F)]^2}{(1+6Y_0F)^2} + M_0^2[\frac{d}{dt}(\frac{t^2}{E}\frac{dE}{dt}) - \frac{6Y_0}{1+6Y_0F}t^2\frac{dE}{dt}]$$

Infrared Quasi Fixed Points

$$Y_0 \to \infty \quad \Rightarrow Y_t(t) \to \boxed{Y_t^{FP} = \frac{E}{6F}}$$

Expand the FP over $\eta, \bar{\eta}$

$$\begin{aligned} A_t^{FP} &= -M_0 \left(\frac{t}{E}\frac{dE}{dt} - \frac{tE - F}{F} \right) \\ \Sigma_t^{FP} &= M_0^2 \left[(\frac{tE - F}{F})^2 + \frac{d}{dt}(\frac{t^2}{E}\frac{dE}{dt}) - \frac{t^2}{F}\frac{dE}{dt} \right] \end{aligned}$$

2. MSSM in high $\tan\beta$ regime

$$\text{RQ Eqs.} \begin{cases} \dot{\alpha}_i = -b_i \alpha_i^2, \quad i = 1, 2, 3 \\ \dot{Y}_t = Y_t(\frac{16}{3}\alpha_3 + 3\alpha_2 + \frac{13}{15}\alpha_1 - 6Y_t - Y_b) \\ \dot{Y}_b = Y_t(\frac{16}{3}\alpha_3 + 3\alpha_2 + \frac{7}{15}\alpha_1 - Y_t - 6Y_b - Y_\tau) \\ \dot{Y}_\tau = Y_\tau(3\alpha_2 + \frac{9}{5}\alpha_1 - 3Y_b - 4Y_\tau) \end{cases}$$

No explicit solution $\Rightarrow \begin{cases} \text{Approximate Solution} \\ \text{Iterative Solution} \end{cases}$

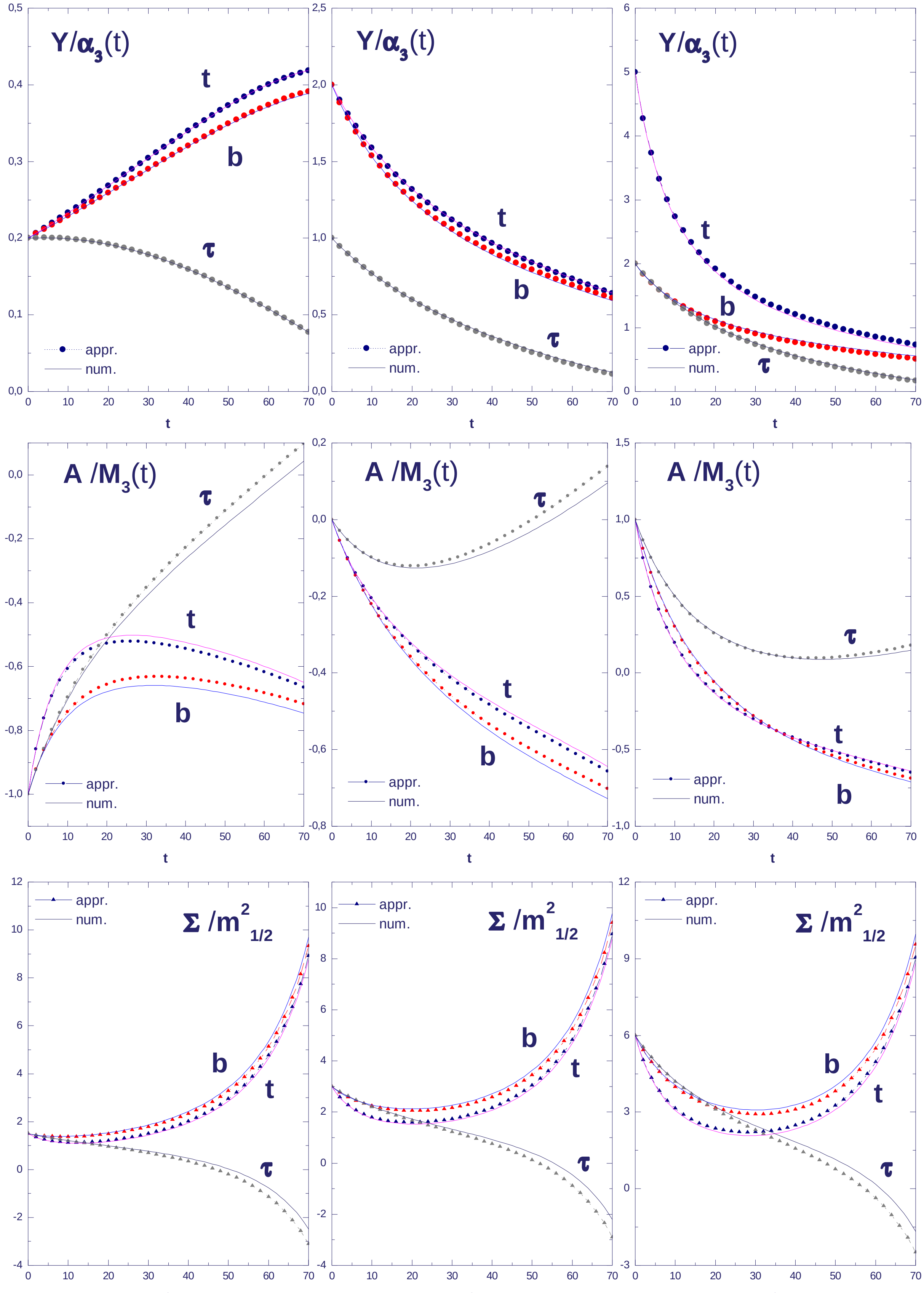
In both the cases Grassmannian Expansion leads to the corresponding solutions for the soft terms

⋆ Approximate Solution

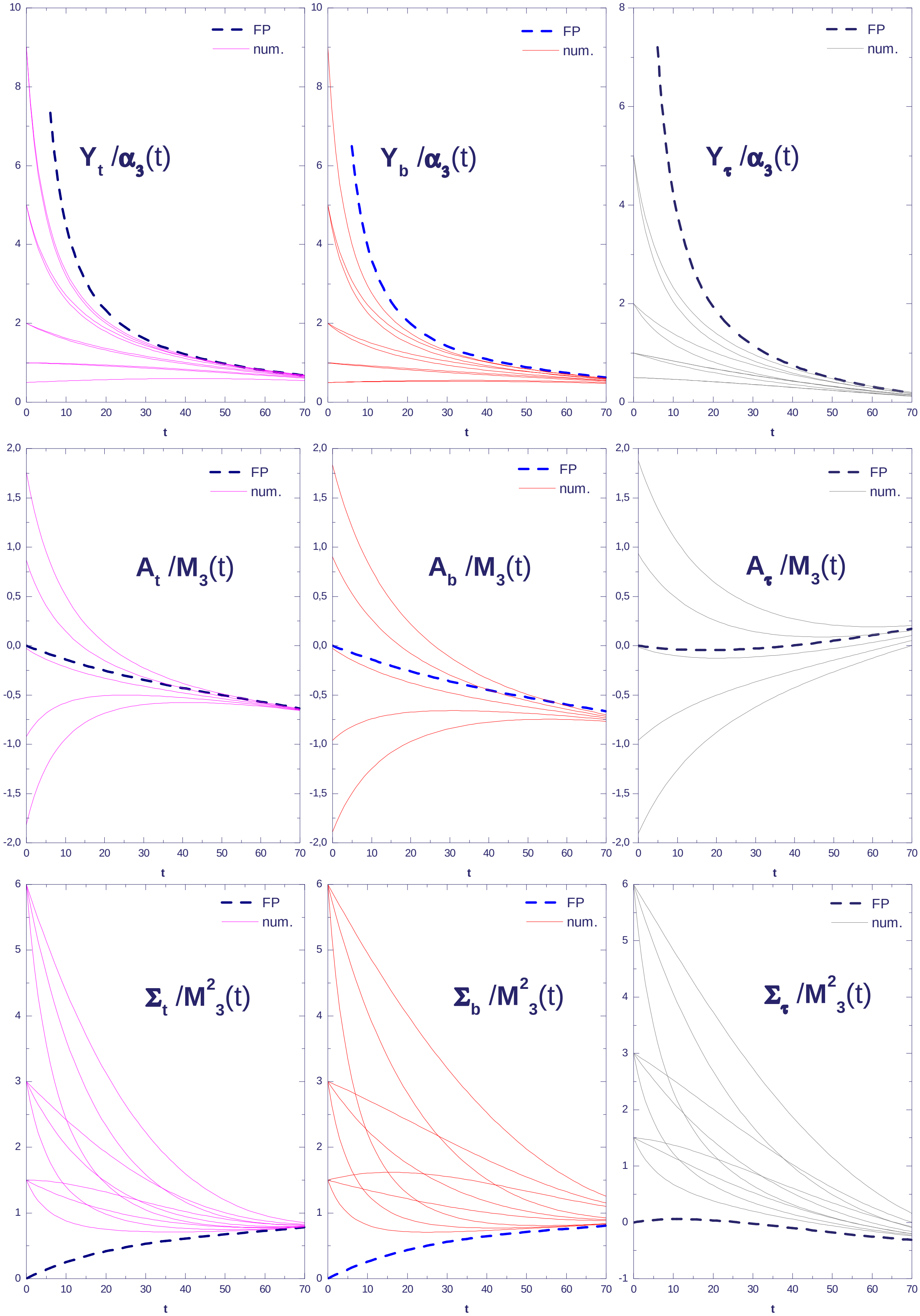
$$Y_t^{app}(t) = \frac{Y_{t0}E_t(t)}{[1 + \frac{7}{2}(Y_{t0}F_t(t) + Y_{b0}F_b(t))]^{2/7}[1 + 7Y_{t0}F_t(t)]^{5/7}}$$

...

⋆ Comparison to Numerical Solution



⋆ The Fixed Points



3. Totally all loop finite N=1 SUSY gauge theories

Conditions:

$$\begin{cases} \sum_R T(R) = 3C_2(G) \\ Y_i = Y_i(\alpha),\ \ Y_i(\alpha) = c_1^i \alpha + c_2^i \alpha^2 + \ldots \end{cases}$$

c_n^i are calculated algebraically in n-th order of PT.

Softly Broken Finite Theory

- Replace $\alpha \to \tilde{\alpha},\ Y \to \tilde{Y}$
- Expand over $\eta, \bar{\eta}$

$$\tilde{Y}_i = Y_i(\tilde{\alpha}) \ \Rightarrow \begin{cases} A_i = -M \frac{d\ln Y_i}{d\ln\alpha} \\ \\ \Sigma_i = M^2 \frac{d}{d\alpha} \alpha \frac{d\ln Y_i}{d\ln\alpha} \end{cases}$$

This expressions lead to a totally finite softly broken SUSY field theory!

Alternative formulation for the bare couplings

In Dimensional regularization

$$Y_i^{Bare} = \alpha_{Bare} \cdot f_i(\varepsilon), \quad f_i(\varepsilon) = c_i^{(1)} + c_i^{(2)}\varepsilon + c_i^{(3)}\varepsilon^2 + \ldots$$

Soft SUSY Breaking

$$\tilde{Y}_i^{Bare} = \tilde{\alpha}_{Bare} \cdot f_i(\varepsilon) \ \Rightarrow \ \boxed{\begin{array}{l} A_i^{Bare} = -M_{Bare} \\ \Sigma_i^{Bare} = M_{Bare}^2 \end{array}}$$

Universality of the soft terms!

CONCLUSION

One has to write down a spontaneously broken theory in terms of a rigid one in external field and to be able to perform renormalization for the arbitrary filed. Then:

- All the renormalizations are defined in a rigid theory. No independent renormalizations in softly broken theory.
- RG flow in Softly Broken Theories follows those in rigid theories.
- This statement is true for RG equations, solutions to these equations, particular (fixed point) solutions, approximate solutions, etc.